(ii) *Y admits in X a fundamental system of neighborhoods V such that* $H^k(V,\mathbb{R})=0$ *for* $k>2m$ [23, *Lemma 3.5*].

3.5.2. Definition. An open $U\subset X$ is said to be *m-admissible* if

(i) U is m-complete.

(ii) There is an open V such that $U\subset V\subset X$ and $H^k(V,\mathbb{R})=0$ for all $k>2m$.

3.5.3. Remark. If X is a Kähler manifold with a Kähler form ω and $U\subset X$ is 0-admissible, then one easily sees that $\omega|_U=i\partial\bar{\partial}\varphi$ for some $\varphi\in SP^\infty(U)$. This is the most trivial particular case of our Theorem 2.

3.5.3. Proposition. (i) *If* $U\subset X$ *is m-admissible and* $k>2m$, *then the canonical morphism* $H^k(X,\mathbb{R})\to H^k(U,\mathbb{R})$ *is zero.*

(ii) *Any compact m-dimensional complex-analytic subset of X admits a fundamental system of m-admissible neighborhoods.*

Proof. (i) Is obvious by the definitions and (ii) is a restatement of 3.5.1.

3.5.4. Proposition. *Let* $\mathbf{B}_m(X)^{(o)}$ *be the open set of* $\mathbf{B}_m(X)$ *consisting of cycles whose support admits in X a smoothly embeddable neighborhood. Let* $\xi\in H^m(X,\Omega_X^m)$. *Then* F_ξ *is holomorphic on* $\mathbf{B}_m(X)^{(o)}$.

Sketch of Proof. For $c\in\mathbf{B}_m(X)^{(o)}$, $|c|$ admits a smoothly embeddable neighborhood V therefore by 3.5.1 a neighborhood U with an embedding $\sigma: U\to U_1$ in a smooth m-complete U_1.

If $\mathcal{N}$ is the coherent sheaf on U_1 defined by the exact sequence

$$0\to\mathcal{N}\to\Omega_{U_1}^m\to\sigma_*\Omega_U^m\to 0,$$

then $H^{m+1}(U_1,\mathcal{N})=0$ and hence $\xi|_U$ is induced by some $\xi_1\in H^m(U_1,\Omega_{U_1}^m)$. By 3.4.1(v), F_{ξ_1} is holomorphic on $\mathbf{B}_m(U_1)$ so F_ξ is holomorphic near c.

3.5.5. Corollary. *If* $\pi: X\to X'$ *is geometrically flat with m-dimensional fibers and* U' *is the set of* $x'\in X'$ *such that* $\pi^{-1}(x')$ *admits in X smoothly embeddable neighborhoods then for any* $\xi\in H^m(X,\Omega_X^m)$, $\pi_*\xi|_{U'}$ *is holomorphic.*

3.6. Note Added in Proof. After having submitted the manuscript, the author together with D. Barlet solved problem D of the Introduction. Proposition 3.5.4 and Corollary 3.5.5 above are now true with $\mathbf{B}_m(X)$ instead of $\mathbf{B}_m(X)^{(0)}$. The notion of a weakly Kähler space loses its importance and Theorems 3 and 4 below (Ch. IV) become

Theorem 3'. *If* $\pi: X\to X'$ *is geometrically flat with X Kähler and X' reduced, then X' is Kähler.*

Theorem 4'. *If X is Kähler then* $\mathbf{B}_m(X)$ *is Kähler.*

II. Theorem 1 and its First Consequences

1. Kähler Spaces and Kähler Metrics

Let X be a complex space.