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(ii) Y admits in X a fundamental system of neighborhoods V such that $H^k(V, \mathbb{R})=0$ for $k>2m$ [23, Lemma 3.5].

3.5.2. Definition. An open $U \subset X$ is said to be m -admissible if

(i) U is m -complete.

(ii) There is an open V such that $U \subset V \subset X$ and $H^k(V, \mathbb{R})=0$ for all $k>2m$.

3.5.3. Remark. If X is a Kähler manifold with a Kähler form ω and $U \subset X$ is 0-admissible, then one easily sees that $\omega|_U = i\partial\bar{\partial}\varphi$ for some $\varphi \in SP^\infty(U)$. This is the most trivial particular case of our Theorem 2.

3.5.3. Proposition. (i) If $U \subset X$ is m -admissible and $k>2m$, then the canonical morphism $H^k(X, \mathbb{R}) \rightarrow H^k(U, \mathbb{R})$ is zero.

(ii) Any compact m -dimensional complex-analytic subset of X admits a fundamental system of m -admissible neighborhoods.

Proof. (i) Is obvious by the definitions and (ii) is a restatement of 3.5.1.

3.5.4. Proposition. Let $\mathbf{B}_m(X)^{(o)}$ be the open set of $\mathbf{B}_m(X)$ consisting of cycles whose support admits in X a smoothly embeddable neighborhood. Let $\xi \in H^m(X, \Omega_X^m)$. Then F_ξ is holomorphic on $\mathbf{B}_m(X)^{(o)}$.

Sketch of Proof. For $c \in \mathbf{B}_m(X)^{(o)}$, $|c|$ admits a smoothly embeddable neighborhood V therefore by 3.5.1 a neighborhood U with an embedding $\sigma: U \rightarrow U_1$ in a smooth m -complete U_1 .

If \mathcal{N} is the coherent sheaf on U_1 defined by the exact sequence

$$0 \rightarrow \mathcal{N} \rightarrow \Omega_{U_1}^m \rightarrow \sigma_* \Omega_U^m \rightarrow 0,$$

then $H^{m+1}(U_1, \mathcal{N})=0$ and hence $\xi|_U$ is induced by some $\xi_1 \in H^m(U_1, \Omega_{U_1}^m)$. By 3.4.1(v), F_{ξ_1} is holomorphic on $\mathbf{B}_m(U_1)$ so F_ξ is holomorphic near c .

3.5.5. Corollary. If $\pi: X \rightarrow X'$ is geometrically flat with m -dimensional fibers and U' is the set of $x' \in X'$ such that $\pi^{-1}(x')$ admits in X smoothly embeddable neighborhoods then for any $\xi \in H^m(X, \Omega_X^m)$, $\pi_* \xi|_{U'}$ is holomorphic.

3.6. Note Added in Proof. After having submitted the manuscript, the author together with D. Barlet solved problem D of the Introduction. Proposition 3.5.4 and Corollary 3.5.5 above are now true with $\mathbf{B}_m(X)$ instead of $\mathbf{B}_m(X)^{(o)}$. The notion of a weakly Kähler space loses its importance and Theorems 3 and 4 below (Ch. IV) become

Theorem 3'. If $\pi: X \rightarrow X'$ is geometrically flat with X Kähler and X' reduced, then X' is Kähler.

Theorem 4'. If X is Kähler then $\mathbf{B}_m(X)$ is Kähler.

II. Theorem 1 and its First Consequences

1. Kähler Spaces and Kähler Metrics

Let X be a complex space.