Proof. (i) Is a consequence of Lemma 4.1.3 above.

(ii) Since n is finite, it is Kähler morphism by 1.3.1 (v). We apply 4.2.2 and 4.2.1 to conclude.

4.2.4. Corollary. *If X is compact, then $\hat{X}$ is Kähler iff X is weakly Kähler.*

III. Theorem 2

1. Čech Spaces and Čech Open Sets

1.1. Definitions. A (topological or complex-analytic) *Čech space* will be by definition a pair

$$\underline{X}=(X,\mathscr{X}),$$

where X is a (topological or complex) space and $\mathscr{X}$ an open covering of X. We call X the *space underlying to* $\underline{X}$ and always denote both by the same letter. We will deal only with complex-analytic Čech spaces. If $\mathscr{X}=(X_\lambda)_{\lambda\in\Lambda}$, the X_λ will be called the *elementary* open sets of $\underline{X}$.

Suppose $\underline{X}=(X,(X_\lambda)_{\lambda\in\Lambda})$ and $\underline{Y}=(Y,(Y_\mu)_{\mu\in M})$ are two Čech spaces. A morphism

$$F:\underline{X}\to\underline{Y}$$

will be a pair $F=(f,\mu)$ where $f:X\to Y$ is a morphism in the ordinary sense and $\mu:\Lambda\to M$ a map such that

$$X_\lambda\subset f^{-1}(Y_{\mu(\lambda)}) \tag{1.1.1}$$

for all $\lambda\in\Lambda$. We call f the *morphism underlying to F*. We will say that F is an *open inclusion* if f is one.

A *Čech open set* $\underline{U}\leqslant\underline{X}$ will be a Čech space chose underlying space is an open subset of X together with an open inclusion

$$j:\underline{U}\to\underline{X}.$$

Or course, j is not uniquely determined by $\underline{U}$.

If $\underline{U}_1=(U_1,(U_{1,\alpha})_{\alpha\in A_1})$ and $\underline{U}_2=(U_2,(U_{2,\beta})_{\beta\in A_2})$ are two Čech open sets of $\underline{X}$, define

$$\underline{U}_1\cap\underline{U}_2:=(U_1\cap U_2,(U_{1,\alpha}\cap U_{2,\beta})_{(\alpha,\beta)\in A_1\times A_2}). \tag{1.1.2}$$

Notice that there are two open inclusions

$$j_1,j_2:\underline{U}_1\cap\underline{U}_2\to\underline{X}$$

each factoring through $\underline{U}_1$ and $\underline{U}_2$, respectively.

If $\underline{X}=(X,\mathscr{X})$ is a Čech space and $\mathscr{F}$ a sheaf of abelian groups on X, write

$$C^q(\underline{X},\mathscr{F}),\ Z^q(\underline{X},\mathscr{F}),\ H^q(\underline{X},\mathscr{F})$$

for the groups of Čech contains, cocycles and cohomology classes of degree q of the covering $\mathscr{X}$ with coefficients in $\mathscr{F}$. Denote by

$$\varepsilon:H^0(X,\mathscr{F})\to C^0(\underline{X},\mathscr{F}) \tag{1.1.3}$$