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1.2. Corollary. *Let $\pi: X \to X'$ be a proper open surjective morphism. Suppose X is Kähler and X' normal. Then X' is Kähler.*

Many other consequences may be formulated. For example

1.3. Corollary. *Let $\pi: X \to X'$ be a flat projective morphism. Suppose X is Kähler and X' reduced. Then X' is Kähler.*

Proof. The fibers of a projective morphism have smoothly embeddable neighborhoods by construction of $\mathbb{P}(\mathscr{F})$ for a coherent sheaf $\mathscr{F}$.

1.4. Remark. Conditions (i) and (ii) of Theorem 3 are actually unnecessary. See note 3.6 of Chap. I.

2. The Space of Cycles of a Kähler Space

We use the notations of Chap. I, 3.

2.1. Theorem 4. *Let X be a Kähler space and $m \geqq 0$ an integer. Then the Barlet space $\mathbf{B}_m(X)$ of m-cycles of X is weakly Kähler. Moreover, the open subset $\mathbf{B}_m(X)^{(0)}$ of $\mathbf{B}_m(X)$ is Kähler.*

Proof. By an argument similar to the above, set

$$W_\alpha := \{c \in \mathbf{B}_m(X) \mid |c| \subset U_\alpha\}$$

$$W_{\alpha\beta}^j := \{c \in \mathbf{B}_m(X) \mid |c| \subset U_{\alpha\beta}^j\}$$

$$\Phi_\alpha := F_{\chi_\alpha}, \qquad G_{\alpha\beta}^j := F_{\tau_{\alpha\beta}^j},$$

Then $\Phi_\alpha \in SP^0(W_\alpha)$, $G_{\alpha\beta}^j$ is weakly holomorphic on $W_{\alpha\beta}^j$ and holomorphic on $W_{\alpha\beta}^j \cap \mathbf{B}_m(X)^{(0)}$, $(\Phi_\alpha - \Phi_\beta)|_{W_{\alpha\beta}^j} = G_{\alpha\beta}^j + \bar{G}_{\alpha\beta}^j$ and the result follows.

2.2. Corollary. *Let X be a Kähler space. Then the weak normalization of $\mathbf{B}_m(X)$ is Kähler.*

Proof. By a well-known result [5, 12, 18] every connected component of $\mathbf{B}_m(X)$ is compact and, by Theorem 4 above, weakly Kähler. The result follows from Proposition 4.2.4 of Chap. II.

3. Fujiki's Class $\mathscr{C}$

3.1. Definition (Fujiki [12]). A reduced compact complex space X is said to belong to class $\mathscr{C}$ if it is a holomorphic image of a compact Kähler space.

By Hironaka's resolution of singularities it is sufficient to take holomorphic images of compact Kähler manifolds.

Let us define for the moment the class $\mathscr{C}^*$ of reduced compact spaces bimeromorphically equivalent to compact Kähler manifolds, i.e. admitting compact Kähler modifications.

It is then true that $\mathscr{C}$ is stable under holomorphic images and subspaces; but it seems difficult to prove, for example, that a reduced subspace of a space in $\mathscr{C}^*$ is in $\mathscr{C}^*$. Of course, $\mathscr{C}^* \subset \mathscr{C}$.