

Werk

Titel: Mathematische Annalen

Verlag: Springer

Jahr: 1989

Kollektion: Mathematica

Werk Id: PPN235181684_0283

PURL: http://resolver.sub.uni-goettingen.de/purl?PID=PPN235181684_0283 | LOG_0028

Terms and Conditions

The Goettingen State and University Library provides access to digitized documents strictly for noncommercial educational, research and private purposes and makes no warranty with regard to their use for other purposes. Some of our collections are protected by copyright. Publication and/or broadcast in any form (including electronic) requires prior written permission from the Goettingen State- and University Library.

Each copy of any part of this document must contain these Terms and Conditions. With the usage of the library's online system to access or download a digitized document you accept the Terms and Conditions.

Reproductions of material on the web site may not be made for or donated to other repositories, nor may be further reproduced without written permission from the Goettingen State- and University Library.

For reproduction requests and permissions, please contact us. If citing materials, please give proper attribution of the source.

Contact

Niedersächsische Staats- und Universitätsbibliothek Göttingen
Georg-August-Universität Göttingen
Platz der Göttinger Sieben 1
37073 Göttingen
Germany
Email: gdz@sub.uni-goettingen.de

1.2. Corollary. *Let $\pi: X \rightarrow X'$ be a proper open surjective morphism. Suppose X is Kähler and X' normal. Then X' is Kähler.*

Many other consequences may be formulated. For example

1.3. Corollary. *Let $\pi: X \rightarrow X'$ be a flat projective morphism. Suppose X is Kähler and X' reduced. Then X' is Kähler.*

Proof. The fibers of a projective morphism have smoothly embeddable neighborhoods by construction of $\mathbb{P}(\mathcal{F})$ for a coherent sheaf \mathcal{F} .

1.4. Remark. Conditions (i) and (ii) of Theorem 3 are actually unnecessary. See note 3.6 of Chap. I.

2. The Space of Cycles of a Kähler Space

We use the notations of Chap. I, 3.

2.1. Theorem 4. *Let X be a Kähler space and $m \geq 0$ an integer. Then the Barlet space $\mathbf{B}_m(X)$ of m -cycles of X is weakly Kähler. Moreover, the open subset $\mathbf{B}_m(X)^{(0)}$ of $\mathbf{B}_m(X)$ is Kähler.*

Proof. By an argument similar to the above, set

$$W_\alpha := \{c \in \mathbf{B}_m(X) \mid |c| \subset U_\alpha\}$$

$$W_{\alpha\beta}^j := \{c \in \mathbf{B}_m(X) \mid |c| \subset U_{\alpha\beta}^j\}$$

$$\Phi_\alpha := F_{\chi_\alpha}, \quad G_{\alpha\beta}^j := F_{\tau_{\alpha\beta}^j},$$

Then $\Phi_\alpha \in SP^0(W_\alpha)$, $G_{\alpha\beta}^j$ is weakly holomorphic on $W_{\alpha\beta}^j$ and holomorphic on $W_{\alpha\beta}^j \cap \mathbf{B}_m(X)^{(0)}$, $(\Phi_\alpha - \Phi_\beta)|_{W_{\alpha\beta}^j} = G_{\alpha\beta}^j + \bar{G}_{\alpha\beta}^j$ and the result follows.

2.2. Corollary. *Let X be a Kähler space. Then the weak normalization of $\mathbf{B}_m(X)$ is Kähler.*

Proof. By a well-known result [5, 12, 18] every connected component of $\mathbf{B}_m(X)$ is compact and, by Theorem 4 above, weakly Kähler. The result follows from Proposition 4.2.4 of Chap. II.

3. Fujiki's Class \mathcal{C}

3.1. Definition (Fujiki [12]). A reduced compact complex space X is said to belong to class \mathcal{C} if it is a holomorphic image of a compact Kähler space.

By Hironaka's resolution of singularities it is sufficient to take holomorphic images of compact Kähler manifolds.

Let us define for the moment the class \mathcal{C}^* of reduced compact spaces bimeromorphically equivalent to compact Kähler manifolds, i.e. admitting compact Kähler modifications.

It is then true that \mathcal{C} is stable under holomorphic images and subspaces; but it seems difficult to prove, for example, that a reduced subspace of a space in \mathcal{C}^* is in \mathcal{C}^* . Of course, $\mathcal{C}^* \subset \mathcal{C}$.