Werk

Titel: Mathematische Annalen

Verlag: Springer

Jahr: 1989

Kollektion: Mathematica

Werk Id: PPN235181684_0283

PURL: http://resolver.sub.uni-goettingen.de/purl?PID=PPN235181684_0283|LOG_0029

1.2. Corollary. *Let $\pi: X \to X'$ be a proper open surjective morphism. Suppose X is Kähler and X' normal. Then X' is Kähler.*

Many other consequences may be formulated. For example

1.3. Corollary. *Let $\pi: X \to X'$ be a flat projective morphism. Suppose X is Kähler and X' reduced. Then X' is Kähler.*

Proof. The fibers of a projective morphism have smoothly embeddable neighborhoods by construction of $\mathbb{P}(\mathscr{F})$ for a coherent sheaf $\mathscr{F}$.

1.4. Remark. Conditions (i) and (ii) of Theorem 3 are actually unnecessary. See note 3.6 of Chap. I.

2. *The Space of Cycles of a Kähler Space*

We use the notations of Chap. I, 3.

2.1. Theorem 4. *Let X be a Kähler space and $m \geqq 0$ an integer. Then the Barlet space $\mathbf{B}_m(X)$ of m-cycles of X is weakly Kähler. Moreover, the open subset $\mathbf{B}_m(X)^{(0)}$ of $\mathbf{B}_m(X)$ is Kähler.*

Proof. By an argument similar to the above, set

$$W_\alpha := \{c \in \mathbf{B}_m(X) \mid |c| \subset U_\alpha\}$$

$$W^j_{\alpha\beta} := \{c \in \mathbf{B}_m(X) \mid |c| \subset U^j_{\alpha\beta}\}$$

$$\Phi_\alpha := F_{\chi_\alpha}, \qquad G^j_{\alpha\beta} := F_{\tau^j_{\alpha\beta}},$$

Then $\Phi_\alpha \in SP^0(W_\alpha)$, $G^j_{\alpha\beta}$ is weakly holomorphic on $W^j_{\alpha\beta}$ and holomorphic on $W^j_{\alpha\beta} \cap \mathbf{B}_m(X)^{(0)}$, $(\Phi_\alpha - \Phi_\beta)|_{W^j_{\alpha\beta}} = G^j_{\alpha\beta} + \bar{G}^j_{\alpha\beta}$ and the result follows.

2.2. Corollary. *Let X be a Kähler space. Then the weak normalization of $\mathbf{B}_m(X)$ is Kähler.*

Proof. By a well-known result [5, 12, 18] every connected component of $\mathbf{B}_m(X)$ is compact and, by Theorem 4 above, weakly Kähler. The result follows from Proposition 4.2.4 of Chap. II.

3. *Fujiki's Class $\mathscr{C}$*

3.1. Definition (Fujiki [12]). A reduced compact complex space X is said to belong to class $\mathscr{C}$ if it is a holomorphic image of a compact Kähler space.

By Hironaka's resolution of singularities it is sufficient to take holomorphic images of compact Kähler manifolds.

Let us define for the moment the class $\mathscr{C}^*$ of reduced compact spaces bimeromorphically equivalent to compact Kähler manifolds, i.e. admitting compact Kähler modifications.

It is then true that $\mathscr{C}$ is stable under holomorphic images and subspaces; but it seems difficult to prove, for example, that a reduced subspace of a space in $\mathscr{C}^*$ is in $\mathscr{C}^*$. Of course, $\mathscr{C}^* \subset \mathscr{C}$.

On the other hand, several important results are valid for compact manifolds in $\mathscr{C}^*$. For example:

(i) If X is a manifold in $\mathscr{C}^*$ and $H^0(X, \Omega_X^2)=0$ then X is Moišezon [14].

(ii) If X is a manifold in $\mathscr{C}^*$, $n=\dim X$ and $\pi: X \to S$ a surjective morphism of X on a complex space S, then $R^q\pi_*(\Omega_X^n)=0$ for all $q>\dim X-\dim S$ (Takegoshi [22]). It seems difficult to prove such results with the hypothesis $X \in \mathscr{C}$. But we have

3.2. Theorem 5. $\mathscr{C}=\mathscr{C}^*$.

Proof. Let X be a compact complex space in $\mathscr{C}$. By definition there is a compact Kähler space X_1 and a surjective morphism $\varrho: X_1 \to X$. By Hironaka's flattening theorem [16], there is a commutative diagram

$$\begin{array}{ccc} X_1 & \xleftarrow{\sigma_1} & Y_1 \\ {\scriptstyle\varrho}\downarrow & & \downarrow{\scriptstyle\pi} \\ X & \xleftarrow{\sigma} & Y, \end{array}$$

where σ, σ_1 are projective modifications and π is flat. Since σ_1 is a Kähler morphism and X_1 a compact Kähler space, Y_1 is Kähler. Moreover Y can be chosen to be normal, since flatness is preserved by base-change. If we apply Corollary 1.2 to $\pi: Y_1 \to Y$, then we deduce that Y is Kähler and $X \in \mathscr{C}^*$ as required.

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