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**1.2. Corollary.** *Let  $\pi: X \rightarrow X'$  be a proper open surjective morphism. Suppose  $X$  is Kähler and  $X'$  normal. Then  $X'$  is Kähler.*

Many other consequences may be formulated. For example

**1.3. Corollary.** *Let  $\pi: X \rightarrow X'$  be a flat projective morphism. Suppose  $X$  is Kähler and  $X'$  reduced. Then  $X'$  is Kähler.*

*Proof.* The fibers of a projective morphism have smoothly embeddable neighborhoods by construction of  $\mathbb{P}(\mathcal{F})$  for a coherent sheaf  $\mathcal{F}$ .

**1.4. Remark.** Conditions (i) and (ii) of Theorem 3 are actually unnecessary. See note 3.6 of Chap. I.

## 2. The Space of Cycles of a Kähler Space

We use the notations of Chap. I, 3.

**2.1. Theorem 4.** *Let  $X$  be a Kähler space and  $m \geq 0$  an integer. Then the Barlet space  $\mathbf{B}_m(X)$  of  $m$ -cycles of  $X$  is weakly Kähler. Moreover, the open subset  $\mathbf{B}_m(X)^{(0)}$  of  $\mathbf{B}_m(X)$  is Kähler.*

*Proof.* By an argument similar to the above, set

$$W_\alpha := \{c \in \mathbf{B}_m(X) \mid |c| \subset U_\alpha\}$$

$$W_{\alpha\beta}^j := \{c \in \mathbf{B}_m(X) \mid |c| \subset U_{\alpha\beta}^j\}$$

$$\Phi_\alpha := F_{\chi_\alpha}, \quad G_{\alpha\beta}^j := F_{\tau_{\alpha\beta}^j},$$

Then  $\Phi_\alpha \in SP^0(W_\alpha)$ ,  $G_{\alpha\beta}^j$  is weakly holomorphic on  $W_{\alpha\beta}^j$  and holomorphic on  $W_{\alpha\beta}^j \cap \mathbf{B}_m(X)^{(0)}$ ,  $(\Phi_\alpha - \Phi_\beta)|_{W_{\alpha\beta}^j} = G_{\alpha\beta}^j + \bar{G}_{\alpha\beta}^j$  and the result follows.

**2.2. Corollary.** *Let  $X$  be a Kähler space. Then the weak normalization of  $\mathbf{B}_m(X)$  is Kähler.*

*Proof.* By a well-known result [5, 12, 18] every connected component of  $\mathbf{B}_m(X)$  is compact and, by Theorem 4 above, weakly Kähler. The result follows from Proposition 4.2.4 of Chap. II.

## 3. Fujiki's Class $\mathcal{C}$

**3.1. Definition** (Fujiki [12]). A reduced compact complex space  $X$  is said to belong to class  $\mathcal{C}$  if it is a holomorphic image of a compact Kähler space.

By Hironaka's resolution of singularities it is sufficient to take holomorphic images of compact Kähler manifolds.

Let us define for the moment the class  $\mathcal{C}^*$  of reduced compact spaces bimeromorphically equivalent to compact Kähler manifolds, i.e. admitting compact Kähler modifications.

It is then true that  $\mathcal{C}$  is stable under holomorphic images and subspaces; but it seems difficult to prove, for example, that a reduced subspace of a space in  $\mathcal{C}^*$  is in  $\mathcal{C}^*$ . Of course,  $\mathcal{C}^* \subset \mathcal{C}$ .

On the other hand, several important results are valid for compact manifolds in  $\mathcal{C}^*$ . For example:

(i) If  $X$  is a manifold in  $\mathcal{C}^*$  and  $H^0(X, \Omega_X^2) = 0$  then  $X$  is Moisëzon [14].

(ii) If  $X$  is a manifold in  $\mathcal{C}^*$ ,  $n = \dim X$  and  $\pi: X \rightarrow S$  a surjective morphism of  $X$  on a complex space  $S$ , then  $R^q \pi_* (\Omega_X^n) = 0$  for all  $q > \dim X - \dim S$  (Takegoshi [22]). It seems difficult to prove such results with the hypothesis  $X \in \mathcal{C}$ . But we have

### 3.2. Theorem 5. $\mathcal{C} = \mathcal{C}^*$ .

*Proof.* Let  $X$  be a compact complex space in  $\mathcal{C}$ . By definition there is a compact Kähler space  $X_1$  and a surjective morphism  $\varrho: X_1 \rightarrow X$ . By Hironaka's flattening theorem [16], there is a commutative diagram

$$\begin{array}{ccc} X_1 & \xleftarrow{\sigma_1} & Y_1 \\ \varrho \downarrow & & \downarrow \pi \\ X & \xleftarrow{\sigma} & Y, \end{array}$$

where  $\sigma, \sigma_1$  are projective modifications and  $\pi$  is flat. Since  $\sigma_1$  is a Kähler morphism and  $X_1$  a compact Kähler space,  $Y_1$  is Kähler. Moreover  $Y$  can be chosen to be normal, since flatness is preserved by base-change. If we apply Corollary 1.2 to  $\pi: Y_1 \rightarrow Y$ , then we deduce that  $Y$  is Kähler and  $X \in \mathcal{C}^*$  as required.

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