On the other hand, several important results are valid for compact manifolds in $\mathscr{C}^*$. For example:

(i) If X is a manifold in $\mathscr{C}^*$ and $H^0(X, \Omega_X^2)=0$ then X is Moišezon [14].

(ii) If X is a manifold in $\mathscr{C}^*$, $n=\dim X$ and $\pi: X \to S$ a surjective morphism of X on a complex space S, then $R^q\pi_*(\Omega_X^n)=0$ for all $q > \dim X - \dim S$ (Takegoshi [22]). It seems difficult to prove such results with the hypothesis $X \in \mathscr{C}$. But we have

3.2. Theorem 5. $\mathscr{C}=\mathscr{C}^*$.

Proof. Let X be a compact complex space in $\mathscr{C}$. By definition there is a compact Kähler space X_1 and a surjective morphism $\varrho: X_1 \to X$. By Hironaka's flattening theorem [16], there is a commutative diagram

$$\begin{array}{ccc} X_1 & \xleftarrow{\sigma_1} & Y_1 \\ \varrho\big\downarrow & & \big\downarrow\pi \\ X & \xleftarrow{\sigma} & Y, \end{array}$$

where σ, σ_1 are projective modifications and π is flat. Since σ_1 is a Kähler morphism and X_1 a compact Kähler space, Y_1 is Kähler. Moreover Y can be chosen to be normal, since flatness is preserved by base-change. If we apply Corollary 1.2 to $\pi: Y_1 \to Y$, then we deduce that Y is Kähler and $X \in \mathscr{C}^*$ as required.

References

1. Andreotti, A., Grauert, H.: Théorèmes de finitude pour la cohomologie des espaces complexes. Bull. Soc. Math. France **90**, 193–259 (1962)
2. Andreotti, A., Norguet, F.: La convexité holomorphe dans l'espace analytique des cycles d'une variété algébrique. Ann. Sc. Norm. Super. Pisa Cl. Sci., IV. Ser. **21**, 31–82 (1967)
3. Barlet, D.: Espace analytique réduit des cycles analytiques complexes compacts d'un espace analytique complexe de dimension finie – Séminaire F. Norguet. (Lecture Notes Mathematics, Vol. 482, pp. 1–158.) Berlin Heidelberg New York: Springer 1975
4. Barlet, D.: Familles analytiques de cycles et classes fondamentales relatives – Séminaire F. Norguet. (Lecture Notes Mathematics, Vol. 807, pp. 1–24.) Berlin Heidelberg, New York: Springer 1977–79
5. Barlet, D.: Convexité de l'espace des cycles. Bull. Soc. Math. France **106**, 373–397 (1978)
6. Barlet, D.: Convexité au voisinage d'un cycle – Séminaire F. Norguet. (Lecture Notes Mathematics, Vol. 807, pp. 102–121.) Berlin Heidelberg New York: Springer 1977–79
7. Bigolin, B.: Osservazioni sulla coomologia del $\partial\bar{\partial}$. Ann. Sc. Norm. Super. Pisa, Cl. Sci., IV. Ser. **24**, 571–583 (1970)
8. Bingener, J.: Deformations of Kähler spaces. I. Math. Z. **182**, 505–535 (1983); II. Arch. Math. **41**, 517–530 (1983)
9. Douady, A.: Le problème des modules pour les sous-espaces analytiques complexes d'un espace analytique donné. Ann. Inst. Fourier, Grenoble **16**, 1–95 (1966)
10. Fornaess, J.E., Narasimhan, R.: The Levi problem on complex spaces with singularities. Math. Ann. **248**, 47–72 (1980)
11. Frisch, J.: Points de platitude d'un morphisme d'espaces analytiques complexes. Invent. Math. **4**, 118–138 (1967)
12. Fujiki, A.: Closedness of the Douady space of compact Kähler spaces. Pub. Res. Inst. Math. Sci. Kyoto **14**, 1–52 (1978)
13. Fujiki, A.: Kählerian normal complex surfaces. Tohôku. Math. J. **35**, 101–117 (1983)

14. Fujiki, A.: On a complex manifold in $\mathscr{C}$ without holomorphic 2-forms. Publ. Res. Inst. Math. Sci. Kyoto **19**, 193–202 (1983)
15. Grauert, H.: Über Modifikationen und exzeptionelle analytische Mengen. Math. Ann. **146**, 331–368 (1962)
16. Hironaka, H.: Flattening theorem in complex-analytic geometry. Am. J. Math. **97**, 503–547 (1975)
17. Hironaka, H.: Fundamental problems on Douady spaces. Report at the Symposium at Kinosaki (1977) pp. 253–261
18. Lieberman, D.: Closedness of the Chow scheme – Séminaire F. Norguet (Lecture Notes Mathematics, Vol. 670, pp. 140–185.) Berlin Heidelberg New York: Springer 1976
19. Malgrange, B.: Ideals of differentiable functions. Oxford University Press, 1966
20. Moišezon, B.: Singular Kählerian spaces – Proceedings of the international conference on manifolds, Tokyo (1973) pp. 343–351
21. Richberg, R.: Stetige streng pseudokonvexe Funktionen. Math. Ann. **175**, 257–286 (1968)
22. Takegoshi, K.: Relative vanishing theorems in analytic spaces. Duke Math. J. **52**, 273–279 (1985)
23. Varouchas, J.: Stabilité de la classe des variétés kählériennes par certains morphismes propres. Invent. Math. **77**, 117–127 (1984)
24. Varouchas, J.: Sur l'image d'une variété kählérienne compacte – Fonctions de plusieurs variables complexes *V*. (Lecture Notes Mathematics, Vol. 1188, pp. 245–259.) Berlin Heidelberg New York: Springer 1985
25. Varouchas, J., Barlet, D.: Fonctions holomorphes sur l'espace des cycles (to appear)

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