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Intersection homology $\mathscr{D}$-module on local complete intersections with isolated singularities

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Introduction

Let M be a complex manifold and X a subvariety of codimension d in M. In [BK] Brylinski and Kashiwara proved the existence of a holonomic $\mathscr{D}$-module $\mathscr{L}(X, M)$ with regular singularities such that its deRham complex satisfies $DR(\mathscr{L}(X, M)) = IC_X^{\cdot}[-d]$. Here $IC_X^{\cdot}$ denotes the Goresky-MacPherson intersection homology complex for the middle perversity [GM]. Although this proves the existence of $\mathscr{L}(X, M)$ no explicit construction for it is known. In this paper we accomplish this for varieties X that are local complete intersections with isolated singularities. One potential application of the results in this paper is to explicitly study the conjectured pure Hodge structure of Brylinski on intersection homology [Br 1]. Another application to the fundamental class of a variety is given here.

In the first section we first recall some facts about $\mathscr{D}$-modules and preverse sheaves and then we establish two basic propositions.

In the second section we construct a local Alexander-deRham duality which implies our main result. For simplicity we state it here for hypersurfaces. We recall that $\mathscr{O}_M[*X]$ denotes the sheaf of meromorphic functions on M with poles along X.

Theorem. *Let X be a hypersurface with an isolated singularity at x in M^n. Then $F \in \mathscr{L}(X, M) \subset \mathscr{O}_M[*X]/\mathscr{O}_M$ if and only if*

$$\int_\gamma F\omega = 0$$

for all $\gamma \in H_n(B - B \cap X)$ and all $\omega \in \Omega^n_{M,x}$, where B is a small ball centered at x in M.

As a corollary we prove that the fundamental class $c_{X/M} \in H^d(M, \Omega^d_M)$ of X [AE] generates the $\mathscr{D}$-module $\mathscr{L}(X, M)$.

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1. Preliminaries

We start by recalling some basic facts about the theory of $\mathscr{D}$-modules and intersection homology. Let M be complex manifold of dimension n. We denote by $\mathscr{D}$ the sheaf of linear differential operators on M with holomorphic coefficients. If $\mathscr{M}$ is a (left) $\mathscr{D}$-module, i.e., a sheaf with a (left) action by $\mathscr{D}$ on it, we define the deRham complex

$$DR(\mathscr{M})=\Omega_M^{\cdot}\otimes_{\mathscr{O}_M}\mathscr{M}$$

where the differential d is given in local coordinates $x_1, \ldots, x_n$ by $d(\omega\otimes m)=d\omega\otimes m+\sum dx_i\wedge\omega\otimes\frac{\partial m}{\partial x_i}$.

If X is any subvariety of M we denote by $IC_X^{\cdot}$ the (middle) intersection homology complex of X with $\mathbb{C}$-coefficients [GM]. We will index the complex $IC_X^{\cdot}$ in such a manner that $H^0(IC_X^{\cdot})=\mathbb{C}_X$. By the Riemann-Hilbert correspondence ([K 1], [Be], [M]) there exists a holonomic $\mathscr{D}$-module $\mathscr{L}(X,M)$ with regular singularities such that

$$DR(\mathscr{L}(X,M))=IC_X^{\cdot}[-\operatorname{codim} X].$$

Let $\mathscr{I}$ be the sheaf of ideals of X in M. If $\mathscr{M}$ is any $\mathscr{D}$-module we also have the following two $\mathscr{D}$-modules associated to $\mathscr{M}$

$$\Gamma_{[X]}(\mathscr{M})=\varinjlim_k \mathscr{H}om_{\mathscr{O}_M}(\mathscr{O}_M/\mathscr{I}^k, \mathscr{M}),$$

$$\Gamma_{[M|X]}(\mathscr{M})=\varinjlim_k \mathscr{H}om_{\mathscr{O}_M}(\mathscr{I}^k, \mathscr{M}).$$

According to Kashiwara and Kawai [K 2], [KK] if $\mathscr{M}$ is holonomic with regular singularities so are the modules $\mathscr{H}^k_{[X]}(\mathscr{M})=R^k\Gamma_{[X]}(\mathscr{M})$ and $\mathscr{H}^k_{[M|X]}(\mathscr{M})=R^k\Gamma_{[M|X]}(\mathscr{M})$.

We also have the following triangle (with no hypothesis on $\mathscr{M}$)

$$\begin{array}{ccc} & \nearrow & R\Gamma_{[X]}(\mathscr{M}) \\ \mathscr{M} & & \downarrow {\scriptstyle (1)} \\ & \nwarrow & R\Gamma_{[M|X]}(\mathscr{M}). \end{array}$$

If we denote by i: $X\to M$ and j: $M-X\to M$ the natural inclusions then we have by the Riemann-Hilbert correspondence [M], [Be]:

$$DR(R\Gamma_{[X]}(\mathscr{M}))=i^!DR(\mathscr{M})[\operatorname{codim} X], \tag{1.1}$$

$$DR(R\Gamma_{[M|X]}(\mathscr{M}))=Rj_*j^*(DR(\mathscr{M})). \tag{1.2}$$

As a special case of (1.2) we have using the notation of [BBD]

$$DR(\mathscr{H}^0_{[M|X]}(\mathscr{M}))={}^pj_*j^*DR(\mathscr{M}). \tag{1.3}$$

As an immediate corollary of (1.1) we get

Proposition 1. *If $X \subset M$ is a subvariety of codimension d then*

$$DR(R\Gamma_{[X]}(\mathcal{O}_M)) = \mathbb{D}^{\boldsymbol{\cdot}}_X[-2n+d].$$

We also have using the notation of [BBD].

Proposition 2. *Let $X \subset M$ be a normal subvariety of codimension d with singular set Z. Then*

$$DR(\mathscr{H}^d_{[X]}(\mathcal{O}_M)) = {}^p j_* \mathbb{C}_{X-Z}[-d]$$

where $j\colon X-Z \to X$ is the natural inclusion.

Proof. Apply the triangle $R\Gamma_{[Z]} \to R\Gamma_{[M]} \to R\Gamma_{[M|Z]}$ to $R\Gamma_{[X]}(\mathcal{O}_M)$ to get

$$\begin{array}{ccc} & \nearrow & R\Gamma_{[M|Z]}R\Gamma_{[X]}(\mathcal{O}_M) \\ R\Gamma_{[X]}(\mathcal{O}_M) & & \downarrow (1) \\ & \nwarrow & R\Gamma_{[Z]}(\mathcal{O}_M). \end{array}$$

Because Z is of codimension $\geqq 2$ in X we have $\mathscr{H}^k_{[Z]}(\mathcal{O}_M) \simeq 0$ for $k \leqq d+1$. Therefore

$$\mathscr{H}^d_{[X]}(\mathcal{O}_M) \simeq \mathscr{H}^d(R\Gamma_{[M|Z]}R\Gamma_{[X]}(\mathcal{O}_M)).$$

Because $\mathscr{H}^k_{[X]}(\mathcal{O}_M) \simeq 0$ for $k<d$ and $\mathscr{H}^k_{[M|X]}(\mathscr{M}) \simeq 0$ for $k<0$ for any $\mathscr{M}$ an easy argument using the Grothendieck spectral sequence shows that

$$\mathscr{H}^d_{[X]}(\mathcal{O}_M) \simeq \mathscr{H}^0_{[M|Z]}(\mathscr{H}^d_{[X]}(\mathcal{O}_M)). \tag{1.4}$$

Using the following notation

$$\begin{array}{ccc} M-Z & \xrightarrow{\;\tilde{j}\;} & M \\ \uparrow \tilde{i} & & \uparrow i \\ X-Z & \xrightarrow[\;j\;]{} & X \end{array}$$

we get by applying DR on both sides of (1.4) and using (1.3) that

$$\begin{aligned} DR(\mathscr{H}^d_{[X]}(\mathcal{O}_M)) &\simeq \mathrm{DR}(\mathscr{H}^0_{[M|Z]}(\mathscr{H}^d_{[X]}(\mathcal{O}_M))) \simeq {}^p\tilde{j}_*\tilde{j}^* DR(\mathscr{H}^d_{[X]}(\mathcal{O}_M)) \\ &\simeq {}^p\tilde{j}_* DR(\mathscr{H}^d_{[X-Z]}(\mathcal{O}_{M-Z})) \simeq {}^p\tilde{j}_*\tilde{i}_*\mathbb{C}_{X-Z}[-d] = i_*{}^p j_*\mathbb{C}_{X-Z}[-d]. \end{aligned}$$

Remark. The normality hypothesis in the theorem could be dropped if we replaced ${}^p j_*$ with the appropriate functor. All the results in this paper would remain true.

For simplicity we will assume from now on that all of our varieties are normal and irreducible. We conclude this section by two examples which compute $\mathscr{H}^k_{[X]}(\mathcal{O}_M)$ explicitly. They will play a crucial role in Sect. 2.

Example 1. Let $X \subset M$ be a hypersurface. Then we have the following exact sequence

$$0 \to \mathcal{O}_M \to \mathcal{O}_M[*X] \to \mathscr{H}^1_{[X]}(\mathcal{O}_M) \to 0.$$

Therefore

$$\mathscr{H}^1_{[X]}(\mathcal{O}_M) \simeq \mathcal{O}_M[*X]/\mathcal{O}_M$$

and

$$\mathscr{H}^k_{[X]}(\mathcal{O}_M) \simeq \mathscr{H}^{k-1}_{[M|X]}(\mathcal{O}_M) \simeq 0 \quad \text{for } k>1$$

because $M-X$ is locally Stein along X.

Example 2. Let X be a local complete intersection given locally as $X = X_1 \cap \ldots \cap X_d$, where the X_i are hypersurfaces. Also assume that $d \geqq 2$. Then

$$\mathscr{H}^d_{[X]}(\mathcal{O}_M) \simeq \frac{\mathcal{O}_M[*X_1 \cup \ldots \cup X_d]}{\sum_{i=1}^{d} \mathcal{O}_M[*X_1 \cup \ldots \cup \hat{X}_i \cup \ldots \cup X_d]},$$

$$\mathscr{H}^k_{[X]}(\mathcal{O}_M) \simeq 0 \quad \text{for } k \neq d.$$

This can be seen as follows. First observe that by a long exact sequence argument $\mathscr{H}^k_{[X]}(\mathcal{O}_M) \simeq \mathscr{H}^{k-1}_{[M|X]}(\mathcal{O}_M)$ for all k. An easy computation using the cover $U_i = M - X_i$ which is Stein locally along X yields the result.

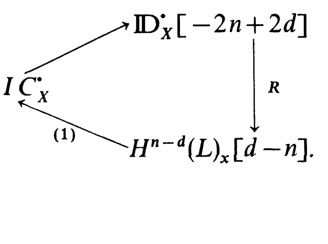
Remark. For a local complete intersection X of codimension d $\mathbb{D}^{\cdot}_X[-2n+d] = DR(\mathscr{H}^d_{[X]}(\mathcal{O}_M))$ and is therefore perverse. Notice also that this implies the theorem of Hamm that $H_k(X, X-x) \simeq 0$ for $k < \dim_{\mathbb{C}} X$ for a l.c.i. X.

Because $IC^{\cdot}_X$ and $\mathscr{L}(X,M)$ are simple [BBD] and there is a natural map $IC^{\cdot}_X \to {}^p j_* \mathbb{C}_{X-Z}$ we get that $\mathscr{L}(X,M) \subset \mathscr{H}^d_{[X]}(\mathcal{O}_M)$. If X is a l.c.i. then we have an explicit description of $\mathscr{H}^d_{[X]}(\mathcal{O}_M)$ as shown in example 2. In the next section we will describe $\mathscr{L}(X,M) \subset \mathscr{H}^d_{[X]}(\mathcal{O}_M)$ by vanishing of local residues if X has isolated singularities.

2. Construction of duality by residues

To motivate the later constructions we will start with some topological considerations. As before we are considering a complex manifold M with a normal, irreducible subvariety $X \subset M$ of codimension d with an isolated singularity at x. We also assume that X is a local complete intersection given by $X = \bigcap_{i=1}^{d} X_i$ in a neighborhood of x. Let B be a small ball in M centered at x with boundary S and let $L = S \cap X$. Because X is Whitney stratified we can choose B sufficiently small so that S is transverse to X and topologically the pair $(B, B \cap X)$ is a cone over $(S, S \cap X)$. We will denote by i: $\{x\} \to X$ and j: $X - \{x\} \to X$ the natural inclusions.

By definition we have $IC^{\cdot}_X \simeq \tau_{\leqq d-1} Rj_* \mathbb{C}_{X-\{x\}} \simeq \tau_{\leqq d-1} \mathbb{D}^{\cdot}_X$. In the last section we saw that $\mathbb{D}^{\cdot}_X$ is perverse and therefore in this case we have the following triangle



Using the abelian category structure on perverse sheaves we see that

$$I C_X^{\cdot} = \ker(\mathbb{D}_X^{\cdot}[2d-2n] \xrightarrow{R} H^{n-d}(L)_x[d-n]).$$

But

$$\begin{aligned}&\mathrm{Hom}[\mathbb{D}_X^{\cdot}[2d-2n], H^{n-d}(L)_x[d-n]) \\ &\quad \simeq \mathrm{Hom}(i^*\mathbb{D}_X^{\cdot}[2d-2n], H^{n-d}(L)[d-n]) \simeq \mathrm{Hom}(H^{n-d}(L), H^{n-d}(L)),\end{aligned}$$

and under the above identifications the morphism R corresponds to the identity.

Just as above we get that

$$\mathrm{Hom}(\mathbb{D}_X^{\cdot}[2d-2n], i_*\mathbb{C}[d-n]) \simeq \mathrm{Hom}(H^{n-d}(L), \mathbb{C}) \simeq H^{n-d-1}(L).$$

The identification

$$P: H^{n-d-1}(L) \xrightarrow{\simeq} \mathrm{Hom}(H^{n-d}(L), \mathbb{C})$$

is of course given by $P(\alpha)(\beta) = \alpha \cup \beta$.

Now it is easy to see that $I C_X^{\cdot}$ is the common kernel of all the homomorphisms parametrized by $H^{n-d-1}(L)$.

We want to proceed in the analogous manner for $\mathscr{D}$-modules and construct an explicit Alexander-deRham pairing

$$H^n(\Omega_M^{\cdot} \otimes_{\mathcal{O}_M} \mathscr{H}_{[X]}^d(\mathcal{O}_M)) \otimes H^{n-d-1}(L) \to \mathbb{C}.$$

Let X be an arbitrary codimension d subvariety of M and x a point on X. Then using the notation at the beginning of this section we have

$$\begin{aligned}H_{n+d-1}(B - B \cap X) &\simeq H_{n+d-1}(S - S \cap X) \xleftarrow{\simeq} H_{n+d}(S, S - S \cap X) \\ &\simeq H^{n-d-1}(S \cap X).\end{aligned}$$

So we get an isomorphism

$$\tilde{\tau}: H^{n-d-1}(L) \to H_{n+d-1}(B - B \cap X).$$

Assume now that X is a local complete intersection given by $X = \bigcap_{i=1}^{d} X_i$. Because $\Omega_M^n \otimes_{\mathcal{O}_M} \mathscr{H}_{[X]}^d(\mathcal{O}_M)$ is in this case given by meromorphic n-forms with poles along $\bigcup_{i=1}^{d} X_i$ and the duality should be given by an integral we want to construct a map $H_{n+d-1}(B - B \cap X) \to H_n(B - B \cap (X_1 \cup \ldots \cup X_d))$.

Lemma. *If X is a local complete intersection given locally in a small ball B by $\bigcap_{i=1}^{d} X_i = X$ then*

$$\begin{aligned}H_{n+d-1}(B - B \cap X) &\simeq \mathrm{Ker}(H_n(B - B \cap (X_1 \cup \ldots \cup X_d)) \\ &\qquad \to \bigoplus H_n(B - B \cap (X_1 \cup \ldots \cup \hat{X}_i \cup \ldots X_d))), \\ H^{n+d-1}(B - B \cap X) &\simeq \mathrm{Coker}(\bigoplus H^n(\Omega_M^{\cdot}[*X_1 \cup \ldots \cup \hat{X}_i \cup \ldots \cup X_d]) \\ &\qquad \to H^n(\Omega_M^{\cdot}[*X_1 \cup \ldots \cup X_d])).\end{aligned}$$

Proof. First observe that (1.1) implies Grothendieck's completeness of rational cohomology, i.e., that for a hypersurface $Y \subset M$ and a small ball B centered at $y \in Y$ we have

$$H^n(B - B \cap Y) \simeq H^n(\Omega^{\cdot}[*Y])_y.$$

Using this result we see that the two statements are dual to each other.

We will prove the second statement. Consider the following resolution of $\mathscr{H}^{d-1}_{[M|X]}(\mathcal{O}_M)$:

$$0 \to \mathcal{O} \to \oplus \mathcal{O}[*X_i] \xrightarrow{\delta} \dots \xrightarrow{\delta} \oplus \mathcal{O}[*X_1 \cup \dots \cup \hat{X}_i \cup \dots \cup X_d] \xrightarrow{\delta} \mathcal{O}[*X_1 \cup \dots \cup X_d] \\ \to \mathscr{H}^{d-1}_{[M|X]}(\mathcal{O}_M) \to 0.$$

Here the maps δ are the obvious maps with appropriate signs. Now the following double complex $D^{\cdot\cdot}$ computes $DR(\mathscr{H}^{d-1}_{[M|X]}(\mathcal{O}_M))$:

$$\begin{array}{ccccccc}
 & & 0 & & 0 & & \\
 & & \uparrow & & \uparrow & & \\
\dots & \xrightarrow{\delta} & \oplus \Omega^n[*X_1 \cup \dots \cup \hat{X}_i \cup \dots \cup X_d] & \xrightarrow{\delta} & \Omega^n[*X_1 \cup \dots \cup X_d] & \longrightarrow & 0 \\
 & & \uparrow d & & \uparrow d & & \\
\dots & \xrightarrow{\delta} & \oplus \Omega^{n-1}[*X_1 \cup \dots \cup \hat{X}_i \cup \dots \cup X_d] & \xrightarrow{\delta} & \Omega^{n-1}[*X_1 \cup \dots \cup X_d] & \longrightarrow & 0 \\
 & & \uparrow d & & \uparrow d & & \\
 & & \vdots & & \vdots & &
\end{array}$$

The natural indexing is such that $\Omega^n[*X_1 \cup \dots \cup X_d]$ is on the $(n+d-1)^{\text{st}}$ diagonal.

Consider the spectral sequence

$$E_2^{pq} \simeq H^p_\delta(H^q_d(D^{\cdot\cdot})) \Rightarrow H^{p+q}(B - B \cap X).$$

An easy computation shows that

$$E_2^{d-1,n} = \operatorname{Coker}(\oplus H^n(\Omega^{\cdot}[*X_1 \cup \dots \cup \hat{X}_i \cup \dots \cup X_d]) \\ \to H^n(\Omega^{\cdot}[*X_1 \cup \dots \cup X_d])).$$

Because no differentials can interfere with this term and it is the only term on its diagonal we get our result. □

Remark. Observe that the above calculation is identical to the calculation of $H^{n+d-1}(B - B \cap X)$ using the Mayer-Vietoris spectral sequence for the cover $B - B \cap X_1, \dots, B - B \cap X_d$.

Let's denote

$$HT_n(B - B \cap X) = \operatorname{Ker}(H_n(B - B \cap (X_1 \cup \dots \cup X_d)) \\ \to \oplus H_n(B - B \cap (X_1 \cup \dots \cup \hat{X}_i \cup \dots \cup X_d)))$$

and let's call τ the isomorphism

$$\tau\colon H^{n-d-1}(L) \xrightarrow[\simeq]{\tilde{\tau}} H_{n+d-1}(B-B\cap X) \simeq HT_n(B-B\cap X).$$

Remark. Geometrically $\tilde{\tau}$ constructs a sphere bundle and τ constructs a torus bundle above the geometric cocycle in $H^{n-d-1}(L)$.

By the completeness of rational cohomology we have for any hypersurface $Y \subset M$ a perfect pairing

$$H^n(\Omega^\cdot[*Y]) \otimes H_n(B-B\cap Y) \to \mathbb{C}$$

by $\omega \otimes \gamma \mapsto \int_\gamma \omega$.

Therefore by the lemma and using example 2 in Sect. 1 we get a pairing

$$H^n(\Omega^\cdot \otimes \mathscr{H}^d_{[X]}(\mathcal{O}_M)) \otimes HT_n(B-B\cap X) \to \mathbb{C}$$

by $\omega \otimes \gamma \mapsto \int_\gamma \omega$.

Finally, we get the perfect pairing

$$H^n(\Omega^\cdot \otimes \mathscr{H}^d_{[X]}(\mathcal{O}_M)) \otimes H^{n-d-1}(L) \to \mathbb{C} \tag{2.1}$$

by $\omega \otimes \gamma \mapsto \int_{\tau(\gamma)} \omega$.

Theorem. *Let X be a normal irreducible local complete intersection of codimension d in M^n with an isolated singularity at x. Then for $F \in \mathscr{H}^d_{[X]}(\mathcal{O}_M)$ we have $F \in \mathscr{L}(X, M)$ if and only if*

$$\int_\gamma F\omega = 0 \tag{2.2}$$

for all $\omega \in \Omega^n_{M,x}$ and for all $\gamma \in HT_n(B-B\cap X)$.

Proof. Let's denote the set of $F \in \mathscr{H}^d_{[X]}(\mathcal{O}_M)$ satisfying (2.2) by $\tilde{\mathscr{L}}$. We first want to show that $\tilde{\mathscr{L}}$ is a $\mathscr{D}$-module. It is clear that if $F \in \tilde{\mathscr{L}}$ and $g \in \mathcal{O}_{M,x}$ then $gF \in \tilde{\mathscr{L}}$. Therefore, it is enough to show that $\tilde{\mathscr{L}}$ is stable under $\frac{\partial}{\partial x_i}$ where $x_1, \ldots, x_n$ are local coordinates at x. Let $F \in \tilde{\mathscr{L}}$ and let $\omega \in \Omega^n_{M,x}$, $\omega = g(x_1, \ldots, x_n)dx_1 \wedge \ldots \wedge dx_n$ where $g \in \mathcal{O}_{M,x}$. Now we have

$$\begin{aligned}\int_\gamma \left(\frac{\partial}{\partial x_i} F\right)\omega &= \int_\gamma \frac{\partial}{\partial x_i}(Fg)dx_1 \wedge \ldots \wedge dx_i \wedge \ldots \wedge dx_n - \int_\gamma F\left(\frac{\partial g}{\partial x_i}\right)dx_1 \wedge \ldots \wedge dx_n \\ &= (-1)^{i-1}\int_\gamma d(Fg\,dx_1 \wedge \ldots \wedge \hat{dx_i} \wedge \ldots \wedge dx_n) - \int_\gamma F\frac{\partial g}{\partial x_i}dx_1 \wedge \ldots \wedge dx_n = 0\end{aligned}$$

because both of the integrals above vanish. Therefore, $\tilde{\mathscr{L}}$ is a $\mathscr{D}$-module. By definition $H^n(DR(\tilde{\mathscr{L}})) = 0$. Because $\mathscr{L}|M-x \simeq \tilde{\mathscr{L}}|M-x$ we get that $\tilde{\mathscr{L}} = \mathscr{L}$.

In [AE] a fundamental class $c_{X/M} \in \Omega^d_M \otimes_{\mathcal{O}_M} \mathscr{H}^d_{[X]}(\mathcal{O}_M)$ is constructed for any subvariety $X \subset M$ of codimension d. It was conjectured by Brylinski and proved by Kashiwara [Br 2] that $c_{X/M}$ generates the $\mathscr{D}$-module $\mathscr{L}(X, M)$.

Kashiwara's proof uses the decomposition theorem [BBD]. As a corollary of our result we will give a proof of this result for local complete intersections with isolated singularities that avoids the use of the decomposition theorem. If a local complete intersection X is given by equations $f_1, \ldots, f_d$ then

$$c_{X/M} = d(\log f_1) \wedge \ldots \wedge d(\log f_d).$$

Corollary. *For X a local complete intersection with isolated singularities the fundamental class $c_{X/M} \in \mathscr{L}(X, M) \otimes \Omega^d_M$.*

Proof. Let $\omega \in \Omega^{n-d}_{M,x}$ and consider

$$\int_{\tau(\gamma)} c_{X/M} \wedge \omega,$$

where $\gamma \in H^{n+d-1}(L) \simeq H_{n-d}(L)$. By taking the Poincare residue along the smooth part we get

$$\int_{\tau(\gamma)} c_{X/M} \omega = \int_\gamma \omega | X - \{x\} = \int_\gamma \omega = 0$$

because γ is homologous to zero on X and ω is regular at x.

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