EVCLIDIS ELEMENTORVM LIBER XI.

DEFINITIONES.

1. *Solidum* est, quod longitudinem & latitudinem & crassitudinem habet.

2. *Solidi* autem *terminus* est superficies.

3. *Recta linea ad planum rectum* est, quando ad rectas omnes lineas, quae ipsam contingunt & in subiecto plano iacent, rectos angulos efficiat.

4. *Planum ad planum rectum* est, quando rectae lineae, quae communi planorum sectioni ad rectos angulos & in vno plano ducuntur, alteri plano ad angulos rectos fuerint.

5. *Rectae lineae ad planum inclinatio* est, quando a sublimi termino rectae illius lineae ad planum acta perpendiculari, a puncto facto ad terminum lineae, qui est in plano, recta linea iuncta fuerit, angulus nempe acutus, qui iuncta linea & insistente continetur.

6. *Plani ad planum inclinatio* est angulus acutus rectis lineis contentus, quae ad rectos angulos communi planorum sectioni ad vnum ipsius punctum in vtroque planorum ducuntur.

7. *Planum ad planum similiter inclinari* dicitur *atque alterum ad alterum*, quando dicti incli-

inclinationem anguli inter ſe fuerint aequales.

8. *Plana parallela* ſunt, quae inter ſe non conueniunt.

9. *Similes figurae ſolidae* ſunt, quae ſimilibus planis ac multitudine aequalibus continentur.

10. *Aequales* vero *&* *ſimiles figurae ſolidae* ſunt, quae ſimilibus planis, multitudine ſimul & magnitudine aequalibus, continentur.

11 *Solidus angulus* eſt plurium, quam duarum, linearum, quae ſeſe contingant, & non in eadem ſint ſuperficie, ad omnes lineas inclinatio. *Aliter.* Solidus angulus eſt, qui pluribus, quam duobus, planis angulis, in eodem non iacentibus plano, atque ad vnum punctum conſtitutis, comprehenditur.

12. *Pyramis* eſt figura ſolida planis comprehenſa, quae ab vno plano ad vnum punctum conſtituitur.

13. *Priſma* eſt figura ſolida planis comprehenſa, quorum aduerſa duo aequalia & ſimilia parallela ſunt, reliqua vero parallelogramma.

14. *Sphaera* eſt figura quidem comprehenſa, quum circa manentem diametrum ſemicirculus conuertitur, donec, in eundem locum, a quo moueri coeperat, rurſus reſtituatur.

15. *Axis* vero *ſphaerae* eſt manens illa recta linea, circa quam ſemicirculus conuertitur.

16. *Cen-*

16. *Centrum* autem *sphaerae* est idem illud, quod & semicirculi.

17. *Diameter* vero *sphaerae* est recta linea quaedam per centrum ducta, & ex vtraque parte a sphaerae superficie terminata.

18. *Conus* est figura quidem comprehensa, quum rectanguli trianguli manente vno latere eorum, quae circa rectum angulum sunt, triangulum ipsum conuertitur, donec in eundem locum, a quo moueri coeperat, rursus restituatur. Verum si manens recta linea aequalis fuerit reliquo lateri, quod circa rectum angulum conuertitur, *conus orthogonius* erit: si vero minor, *amblygonius*: & si maior, *oxygonius*,

19. *Axis* autem *coni* est manens illa recta linea, circa quam triangulum conuertitur.

20 *Basis* vero circulus a conuersa recta linea descriptus

21. *Cylindrus* est figura comprehensa, quando rectanguli parallelogrammi manente vno latere eorum, quae circa rectum angulum sunt, parallelogrammum ipsum conuertitur, donec in eundem locum, a quo moueri coeperat, rursus restituatur.

22. *Axis* vero *cylindri* est manens illa recta linea, circa quam parallelogrammum conuertitur.

23. *Bases* autem sunt circuli, qui a duobus ex aduerso circumactis lateribus describuntur.

24. *Similes*

24. *Similes coni & cylindri* sunt, quorum & axes & basium diametri proportionales sunt.

25. *Cubus* est figura solida, sex quadratis aequalibus contenta.

26. *Tetraedrum* est figura solida, quatuor triangulis aequalibus & aequilateris comprehensa.

27. *Octaedrum* est figura solida, octo triangulis aequalibus & aequilateris comprehensa.

28 *Dodecaedrum* est figura solida, quae duodecim pentagonis aequalibus & aequilateris & aequiangulis continetur.

29. *Icosaedrum* est figura solida, quae viginti triangulis aequalibus & aequilateris comprehenditur.

* 30. *Parallelepipedum* est figura solida, sex planis, quorum quae ex aduerso parallela sunt, contenta.

* 31. *Solida figura in solida figura* dicitur *inscribi*, quando omnes anguli figurae inscriptae constituuntur vel in angulis, vel in lateribus, vel denique in planis figurae, cui inscribitur.

* 32. *Solida figura solidae figurae* vicissim *circumscribi* dicitur, quando vel anguli, vel latera vel denique plana figurae circumscriptae tangunt omnes angulos figurae, circum quam describitur.

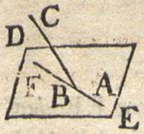
*AXIOMA.

~~Anguli solidi, qui sub aeque multis aequalibus ac eodem ordine positis angulis planis continentur, aequales sunt.~~

PROP.

PROP. I. THEOR.

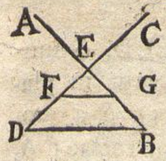
Rectae lineae ABC *pars quaedam non est in subiecto plano* DE, *quaedam vero in sublimi.*



Si enim fieri potest, sit pars AB in plano DE, pars BC autem extra. Iam, quia omnis recta in dato plano in directum continuari potest $^{\alpha}$, sit BF in directum ipsi AB, in plano DE. Ergo rectae ABF, ABC segmentum commune BA $^{\beta}$ habebunt. Q. E. A $^{\beta}$.

α. 2. post. 1.
β. 12. ax. 1.

PROP. II. THEOR.

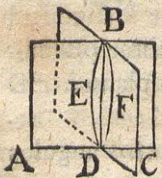


Si duae rectae lineae AB, CD *se inuicem secent, in vno sunt plano. Item, omne triangulum* DEB *in vno plano consistit*

1. Si △ DEB non sit in vno plano: erit pars eius, velut EFG, in alio plano, quam reliqua; ideoque rectarum ED, EB vniuscuiusuis pars erit in plano subiecto, pars in sublimi. Q. E. A $^{\gamma}$. γ. 1. 11.

2. Ergo quum ED, EB sint in eodem plano, CD autem sit $^{\gamma}$ in plano, in quo est ED, & AB in plano $^{\gamma}$ illo, in quo est EB: necesse est, vt AB, CD sint in eodem plano. Q. E. D.

PROP. III. THEOR.

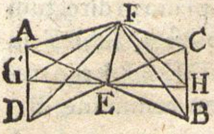


Si duo plana AB, BC *se inuicem secent; communis ipsorum sectio* DB *est linea recta.*

Si enim linea DB, in qua plana se inuicem secant, non sit recta; ducatur a puncto B ad D in

δ. 1. post. 1. in plano AB alia recta [δ] BED, in plano autem BC recta BFD; & recta BFD cum recta BED spatium comprehendet.
ε. 12. ax. 1. Q. E. A [ε].

PROP. IV. THEOR.

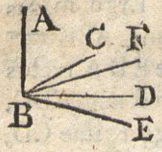


Si recta linea EF *duabus rectis lineis* AB, CD, *se inuicem secantibus, in communi sectione* E *ad rectos angulos insistat, etiam ducto per ipsas* AB, CD *plano ad rectos angulos erit.*

Sumatur AE = EB = CE = ED, & iungantur AD, CB, & per E ducatur in plano ACBD vtcunque recta GEH, & a quouis puncto F in sublimi ducantur rectae FA,
ζ. 4. 1. FG, FD, FB, FH, FC. Iam quia in △is
η. 26. 1. AED, CEB est [ζ] AD = CB, & ang. EAD = EBC: erit [η] in △is AEG, HEB latus AG = HB, & GE = EH. Praeterea quum in △is AEF, BEF sit [ζ] FA = FB, & in △is FED, FEC pari ratione [ζ] FD = FC: erit in △is AFD, BFC ang. FAD = [θ] FBC. Hinc ob AG = HB, & FA = FB, erit [ζ] FG = FH; &
θ. 8. 1. ob id in △is GEF, HFE erunt [θ] anguli ad E aequales, id est recti. Similiter ostenditur EF ad omnes alias rectas in plano ACBD per E ductas angulos rectos efficere. Er-
ι. 3. def. go [ι] FE plano per AB, CD ad rectos angulos
11. est. Q. E. D.

PROP.

PROP. V. THEOR.



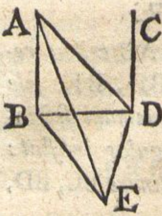
Si recta linea AB *tribus rectis lineis*, BC, BD, BE, *sese tangentibus, in communi sectione* B *ad rectos angulos insistat: tres illae rectae lineae* BC, BD, BE *in uno plano erunt.*

Si fieri potest, sint BD, BE quidem in subiecto plano, BC vero in sublimi. Planum per AB, BC producatur, donec subiectum secet
in [κ] recta BF. Iam quia AB ipsis BD, BE ad κ. 3. 11.
rectos insistit, erit eadem ad planum subie-
ctum recta [λ], ideoque ipsi BF, quae etiam in λ. 4. 11.
plano subiecto est, ad rectum [μ] angulum insi- μ. 3. def. 11.
stet. Sed ponitur quoque ang. ABC rectus. Ergo ang. ABF = ABC. Sed hi anguli sunt in eodem plano per AB, BC. Ergo totus ABF aequalis est parti ABC. Q.E.A.

PROP. VI. THEOR.

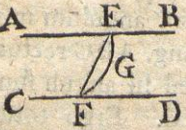
Si duae rectae lineae AB, CD *eidem plano ad rectos angulos fuerint, parallelae erunt ipsae rectae lineae* AB, CD.

Insistant AB, CD subiecto plano in punctis B, D. Iunctae BD ducatur in eodem plano perpendicularis DE, quae fiat = AB, & iungantur BE, AE, AD. Et quia AB est ad planum subiectum recta: erunt ang. ABD, ABE
recti [ν]. Similiter ang. CDB, CDE recti erunt. ν. 3. def. 11.
Quum itaque ang. ABD = [ξ] BDE, & AB = ξ. 10. ax. 1.
DE,



DE, & BD communis: erit
ο. 4. I. AD = ο BE. Ergo in Δis
π. 8. I. BAE, DAE erit ang. ABE = π
EDA; ideoque EDA rectus
erit. Sunt autem & ang. EDC,
EDB recti. Ergo rectae CD,
DA, DB erunt ρ in vno plano.
ρ. 5. II. Sed AB est in eodem plano σ, in quo sunt
σ. 2. II. DA, DB. Ergo AB, CD sunt in eodem plano. Quare, quum ang. ABD, CDB
τ. 28. I. recti sint, ipsae AB, CD parallelae τ sunt. Q. E. D.

PROP. VII. THEOR.



Si duae rectae lineae AB, CD *parallelae sint; sumantur autem in vtraque ipsarum quaelibet puncta* E, F: *quae dicta puncta coniungit recta linea in eodem cum parallelis plano erit.*

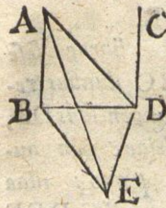
Si fieri potest, sit recta EGF in sublimi. Ducatur per eam planum vtcunque, quod
υ. 3. II. secabit planum subiectum in recta υ EF. Ergo duae rectae EF, EGF spatium comprehendent. Q. E. A.

PROP. VIII. THEOR.

Si fuerint duae rectae lineae AB, CD *parallelae, atque altera earum* AB *plano alicui sit ad rectos angulos: & reliqua* CD *quoque eidem plano ad rectos angulos erit.*

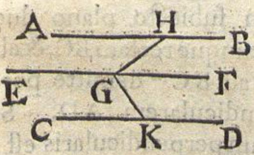
Insistant AB, CD plano subiecto in punctis B, D. Iungatur BD. Ergo AB, BD, DC

erunt



erunt [φ] in vno plano. Duca- φ. 7. 11.
tur in subiecto plano ipsi BD ad
rectos DE, & fiat = AB, iun-
ganturque AD, AE, EB. Quia
AB recta est ad subiectum pla-
num: erunt ang. ABD, ABE re-
cti [χ]. Sed ang. ABD + CDB [ψ] χ. 3. def. 11.
= 2 rectis. Ergo CDB erit rectus. Et quia DE
= AB & BD communis, & ang. EDB = ABD: ψ. 29. 1.
erit BE [ω] = AD. Hinc in △is DAE, EAB ω. 4. 1.
erit ang. EDA [α] = ABE = recto. Sed & α. 8. 1.
ang. EDB rectus est. Ergo [β] ED est ad pla- β. 4. 11.
num per BD, DA recta. Iam quia in plano
per BD, DA sunt ipsae AB [γ], BD: patet, CD γ. 2. 11.
in eodem plano esse. Itaque & ang. EDC
rectus [χ] erit. Sed & ang. CDB rectus erat.
Ergo [β] CD est ad planum subiectum recta.
Q. E. D.

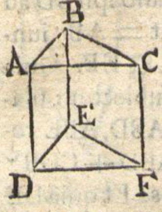
PROP. IX. THEOR.



Quae AB, CD eidem rectae lineae EF sunt parallelae, sed non in eodem cum illa plano, etiam inter se parallelae sunt.

Sume in EF punctum G, ex quo duc ad EF in plano per AB, EF perpendicularem GH, in plano autem per EF, CD perpendicularem GK. Quia ergo ang. EGH, EGK recti sunt: erit [δ] EF ad planum per HG, GK recta. Ita- δ. 4. 11.
que AB, CD ad idem planum rectae [ε] erunt, ε. 8. 11.
ideoque [ζ] parallelae. Q. E. D. ζ. 6. 11.

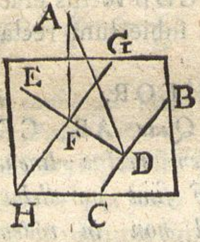
PROP. X. THEOR.



Si duae recta lineae sese tangentes AB, BC *duabus rectis lineis sese tangentibus* DE, EF *sint parallelae, non autem in eodem plano: illae aequales angulos* ABC, DEF *continebunt.*

Sume AB = DE, & BC = EF, & iunge
η. 33. 1. AD, BE, CF, AC, DF. Ergo erunt η AD, CF aequales & parallelae ipsi BE, & ideo
θ. 9. 11. AD, CF inter se aequales & parallelae θ erunt.
ι. 8. 1. Quare & AC = ι DF, & ang. ABC = ι DEF. Q. E. D.

PROP. XI. PROBL.



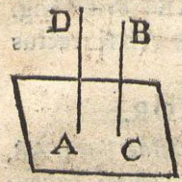
A dato puncto A *in sublimi ad subiectum planum perpendicularem rectam lineam ducere.*

In subiecto plano duc vtcunque rectam BC, & ab
κ. 12. 1. & 2. 11. A ad BC κ demitte perpendicularem AD. Si AD ad planum subiectum perpendicularis est: factum iam erit propositum. Sin minus: duc ex D in subiecto plano ad BC perpendicularem DE, ad quam in plano EDA ex A κ demitte perpendicularem AF. Haec erit desiderata.

Nam in subiecto plano ducatur per F ipsi
λ. constr. BC parallela GH. Et quia λ ang. BDA, BDE
μ. 4. 11. recti sunt, ideoque BC in planum EDA μ recta

cta est: erit & GH ad idem planum [ν] ν. 8. 11.
recta, & ergo ang. GFA [ξ] rectus. Sed est ξ. 3. def. 11.
etiam ang. DFA [λ] rectus. Ergo recta AF est ad planum subiectum [μ] perpendicularis. Q. E. F.

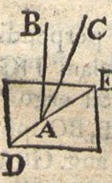
PROP. XII. PROBL.



Dato plano, a puncto A, *quod in ipso datum est, ad rectos angulos rectam lineam constituere.*

Intelligatur punctum B sublime, a quo ad datum planum agatur [ο] perpendicularis BC, & huic ο. 11. 11.
parallela [π] AD ducatur, quae erit plano dato π. 31. 1.
recta [ρ]. Q. E. F. ρ. 8. 11.

PROP. XIII. THEOR.

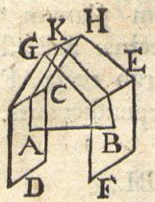


Dato plano a puncto A, *quod in ipso est, duae rectae lineae* AB, AC *ad rectos angulos non constituentur ab eadem parte.*

Si enim AB, AC simul essent perpendiculares plano A: ducto per BA, AC plano, quod planum A secet in recta DAE, forent ang. BAD & CAD [σ] recti, ideoque aequales; pars & to- σ. 3. def. 11.
tum. Q. E. A.

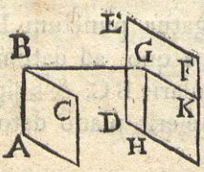
PROP. XIV. THEOR.

Ad quae plana CD, EF *eadem recta linea* AB *est perpendicularis, ea parallella sunt.*



Si negas: pone illa producta se secare in recta G H, in qua sumto puncto K, iunge KA, KB. Ergo KAB erit triangulum. Et quia AB est in planum D H perpendicularis, in
τ.3.def.11. quo ducta est AK: erit [τ] ang. BAK rectus. Similiter ang. ABK rectus
υ.17.1. erit. Q. E. A [υ].

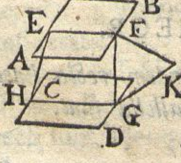
PROP. XV. THEOR.



Si duae rectae lineae AB, BC *sese tangentes duabus rectis lineis* DE, EF *sese tangentibus sint parallelae, non autem in eodem plano: & quae per ipsas transeunt plana* AC, DF *parallela erunt.*

Duc enim ex B in planum DF perpendicularem BG, & per G ipsi ED parallelam HG,
φ.3.def.11. ipsi EF vero parallelam GK. Recti ergo [φ] erunt ang. BGH, BGK. Et quia AB, BC ipsis
χ.9.11. GH, HK sunt [χ] parallelae: erunt & ang. GBA,
ψ.29.1. GBC recti [ψ]. Ergo GB ad planum AC etiam
ω.4.11. [ω] recta erit, & hinc plana AC, DF erunt
α.14.11. parallela [α]. Q. E. D.

PROP. XVI. THEOR.



Si duo plana parallela AB, CD *ab aliquo plano* EFGH *secentur: communes ipsorum sectiones* FE, GH *sunt etiam parallelae.*

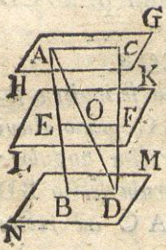
Si

Si non sint parallelae: productae alicubi conuenient, vt in K. Sed quia recta EFK est in [β] plano AB: erit & punctum K in plano AB. Similiter idem K erit & in plano CD. Ergo plana AB, CD producta conuenient, nec ergo parallela [γ] erunt; contra hyp.

β. 1. 11.

γ. 8. def. 11.

PROP. XVII. THEOR.



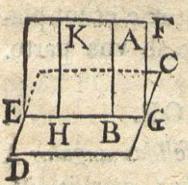
Si duae rectae lineae AB, CD *a parallelis planis* GH, KL, MN *secentur, in eadem ratione secabuntur* (AE : EB = CF : FD).

Iungantur AC, BD, AD. Occurrat autem AD plano KL in O, & iungantur OE, OF. Ergo quia plana parallela KL, MN a plano EODB secantur: erunt [δ] EO, BD parallelae. Eadem ratione OF, AC parallelae erunt. Ergo AE : EB = [ε] AO : OD = [ε] CF : FD. Q. E. D.

δ. 16. 11.

ε. 2. 6.

PROP. XVIII. THEOR.

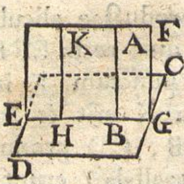


Si recta linea AB *plano alicui* CD *sit ad rectos angulos: & omnia quae per ipsam* AB *transeunt plana* EF *eidem plano* CD *ad rectos angulos erunt.*

Sit planorum CD, EF communis sectio recta EBG, & ex eius puncto quouis H in plano EF ducatur ipsi GE perpendicularis HK. Iam quia & ang. ABH rectus [ζ] est: erunt [η] AB, KH

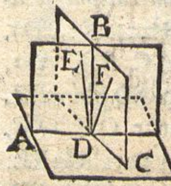
ζ. 3. def. 11.

η. 28. 1.



KH parallelae; & hinc
ϑ. 8. 11. KH erit [ϑ] ad planum CD recta. Sed item & de reliquis ostendetur, quae vt KH in plano EF ad ipsam EG perpendiculares duci possunt. Ergo planum EF plano CD re-
ι. 4. def. 11. ctum [ι] erit. Similiter demonstrabimus, quoduis aliud planum per AB ductum plano CD rectum fore. Q. E. D.

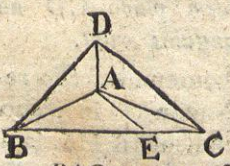
PROP. XIX. THEOR.



Si duo plana se inuicem secantia AB, BC *plano alicui* AC *sint ad rectos angulos: communis ipsorum sectio* BD *eidem plano* AC *ad rectos angulos erit.*

Si negas: duc ex in D plano quidem AB ad AD perpendicularem DE, in plano autem BC perpendicularem DF ad DC. Sunt autem AD, DC communes sectiones planorum AB, BC cum plano AC. Ergo duae rectae ED,
κ. 4. def. 11. FD ad angulos rectos [κ] constitutae erunt plano AC ab vno puncto D & ab vna parte.
λ. 13. 11. Q. E. A [λ].

PROP. XX. THEOR.



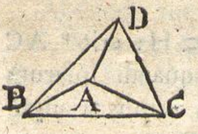
Si solidus angulus A *sub tribus angulis planis* BAC, CAD, BAD *continentur: duo quilibet* CAD, BAD *reliquo* BAC *maiores sunt, quomodocunque sumti.*

Cas. 1.

Cas. 1. Si ang. BAC, CAD, BAD aequales sunt: euidens est propositio.

Cas. 2. Sed si non sint aequales: sit eorum maximus BAC. In plano per BA, AC fiat ang. BAD = BAE & capiatur AE = AD, & per E ducatur recta secans ipsas AB, AC in B, C, & iungantur BD, DC. Erit ergo in △is BAD, BAE basis BD =[μ] BE. μ. 4. 1.
Et quia BD + DC[ν] > BC, erit DC[ξ] > EC, ν. 20. 1.
& ergo in △is ADC, AEC ang. DAC >[ο] ξ. 5. ax. 1.
EAC. Quare DAC + BAD >[π] BAC. ο. 25. 1.
Q. E. D. π. 4. ax. 1.

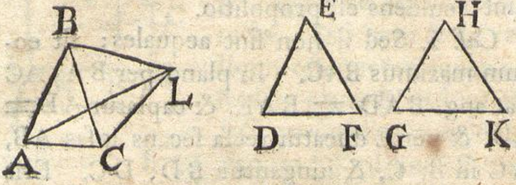
PROP. XXI. THEOR.



Omnis solidus angulus A sub minoribus quam quatuor rectis angulis planis continetur.

In rectis enim, angulos planos BAC, CAD, DAB continentibus, sumtis quibusuis punctis B, C, D, iungantur BC, CD, DB. Quia ergo solidus ang. B continetur sub 3 planis ang. ABC, ABD, DBC: erunt ang. ABC + ABD[ρ] ρ. 20. 11.
> DBC. Eadem ratione in solido ang. C erunt BCA + ACD > BCD, & in solido ang. D erunt CDA + ADB > CDB. Ergo ABC + ABD + BCA + ACD + CDA + ADB > DBC + BCD + CDB id est[σ] 2 rectis. σ. 32. 1.
Sunt autem ang. ABC + ABD + BCA + ACD + CDA + ADB + BAC + CAD + DAB =[σ] 6 rectis. Ergo ang. BAC + CAD + DAB <[τ] 4 rectis. Q. E. D. τ. 5. ax. 1.

PROP. XXII. THEOR.



Si sint tres anguli plani ABC, E, H, *quorum duo reliquo sunt maiores quomodocunque sumti; contineant autem ipsos rectae lineae aequales* AB, BC, DE, EF, GH, HK: *fieri potest, vt ex iis* AC, DF, GK, *quae rectas aequales coniungunt, triangulum constituatur.*

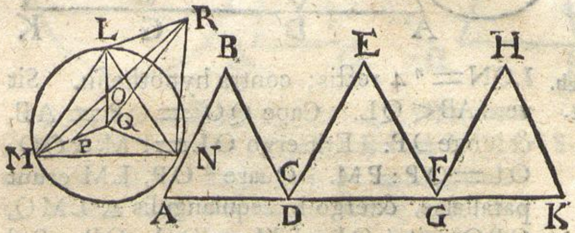
υ. 4. I. *Cas.* 1. Si ang. ABC $=$ E $=$ H: erit υ AC $=$ DF $=$ GK, ideoque duae quaeuis ipsarum tertia maiores erunt vt ergo ex ipsis triangulum constitui queat. Q. E. D.

Cas. 2. Si praedicti anguli non fuerint aequales inter se: fiat ang. CBL $=$ E, & BL $=$ AB, & iungantur AL, LC. Est itaque CL
φ. 24. I. $=$ φ DF, & CL $+$ AC $>$ χ AL. Iam quia
χ. 20. I. ang. E $+$ ABC $>$ H, & E $=$ CBL, patet
ψ. 5. ax. I. esse ψ ang. LBA $>$ H, ideoque AL $>$ φ GK. Ergo DF $+$ AC $>$ AL $>$ GK. Similiter ostendentur AC $+$ GK $>$ DF, & DF $+$ GK $>$ AC. Quum itaque ipsarum AC, DF, GK duae quaeuis tertia sint
ω. 22. I. maiores: triangulum ω ex iisdem construi potest. Q. E. D.

PROP.

PROP. XXIII. PROBL.

Ex tribus angulis planis ABC, DEF, GHK, *quorum duo reliquo ſunt maiores quomodocunque ſumti, ſolidum angulum conſtituere: oportet autem tres angulos quatuor rectis eſſe minores.*



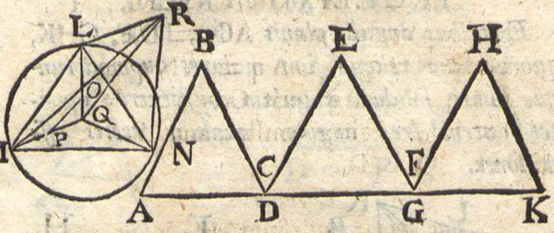
Abſcinde aequales BA, BC, ED, EF, HG, HK, & iunge AC, DF, GK, ex quibus conſtrue △ LMN ita, vt LM = AC, & MN = DF, & LN = GK, quod ſemper [α] fieri poterit. Dein △o LMN circumſcribe [β] circulum, & eius plano ex centro Q ad rectos angulos conſtitue [γ] rectam, in qua cape [δ] QR = √(ABq − LQq), & iunge RL, RM, RN. Factum erit.

α. 22. 11.
β. 5. 4.
γ. 12. 11.
δ. ſch. 47. 1.

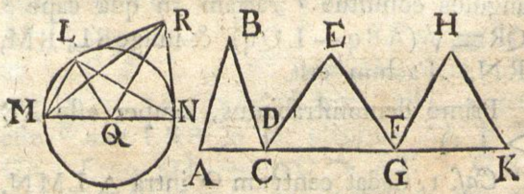
Primo demonſtrabimus, ſemper eſſe AB > LQ.

Caſ. 1. Cadat centrum Q intra △ LMN. Iam ſi non ſit AB > QL: erit AB = QL aut < QL. Sit AB = QL. Iunge QM, QN. Quia ergo BC = AB = QL = QM, & [ε] AC = LM: erit ang. B = [ζ] LQM. Similiter patet, eſſe ang. E = MQN, & ang. H = LQN. Ergo erit B + E + H = LQM + MQN +

ε. conſtr.
ζ. 8. 1.

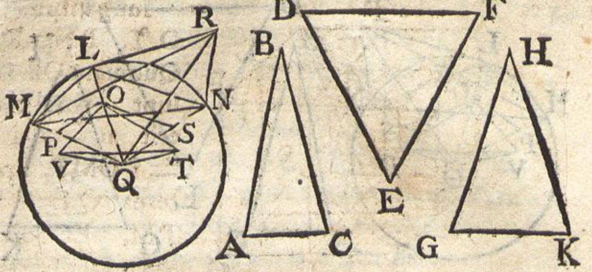


η. 2. sch. $LQN = ^{\eta}$ 4 rectis; contra hypothesin. Sit
15. 1. vero $AB < QL$. Cape $QO = QP = AB$,
& iunge OP. Erit ergo $OL = PM$, & $QO :$
ϑ. 2. 6. $OL = QP : PM$. Quare $^{\vartheta}$ OP, LM erunt
parallelae, & ergo in aequiangulis △ LMQ,
ι. 4. 6. OPQ erit $^{\iota}$ $QL : LM = QO : OP$. Sed
κ. 14. 5. $QL > QO$. Ergo $LM ^{\kappa} > OP$. Quia igi-
λ. 25. 1. tur & $AC > OP$, erit $^{\lambda}$ ang. $B > OQP$. Eadem ratione ang. $E > MQN$, & $H > LQN$. Ergo erit $B + E + H > 4$ rectis; contra hyp. Igitur quia AB nec $=$ nec $< QL$: erit $AB > LQ$.



Cas. 2. Cadat centrum Q in latus MN. Iam si dicas $AB = QL$: erunt $DE = EF = AB$
$= QL = QM = QN$ ideoque $DE + EF$
μ. 20. 1. $= MN = DF$. Q. E. A $^{\mu}$. Si dicas $AB <$
LQ: erunt $DE + EF < DF$. Q. E. A $^{\mu}$.
Ergo $AB > LQ$.

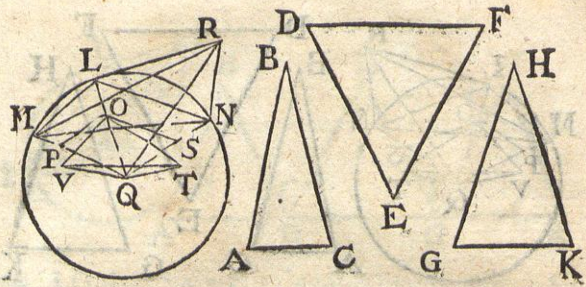
Cas.



Caſ. 3. Sit centrum Q extra △ LMN. Iam ſi dicas AB = LQ: erit ang. B [ζ] = LQM, ζ. 8. 1.
& H = LQN. Ergo B + H = MQN = [ζ] E; contra hyp. Si dicas AB < QL: fac QO = AB, & QP = BC, QS = HK, & iunge OP, OS. Ergo QO = QP = QS, & vti in Caſu 1. demonſtrabitur LM > OP, & LN > OS. Ergo AC > OP, & GK > OS, & ang. B [λ] > OQP, & ang. H > OQS. Fiat ang. λ. 25. 1.
OQT = H, & OQV = B, & QT = QV = QO, & iungantur OV, OT, TV. Erit itaque [ν] OV = AC = LM, & OT = GK = LN. ν. 4. 1.
Sed quia ang. POQ > VOQ, & SOQ > TOQ: erit POT vel MLN > VOT, & hinc [ξ] MN ξ. 24. 1.
> VT, ideoque DF > VT. Quum autem QV = ED & QT = EF, erit ang. E > [λ] VQT, id eſt E > B + H; etiam contra hypotheſin. Itaque BA > LQ.

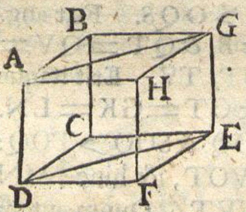
Secundo dico, ang. ſolidum R eſſe ex tribus planis B, E, H conſtitutum. Quia enim QR plano circuli recta eſt: erunt ang. RQL, RQM, RQN recti. Sunt autem aequales LQ, MQ, NQ. Ergo [ν] RL = RM = RN. Et quia

QRq



QRq = ABq − LQq, ac ob id QRq +
•.47. 1. LQq = ABq: erit • LR = AB, & ergo
RM = BC, atque, ob ML = AC, ang.
LRM = B. Eadem ratione ang. LRN =
H, & ang. MRN = E. Quare ex tribus
planis B, E, H conſtitutus eſt ſolidus angulus R. Q. E. F.

PROP. XXIV. THEOR.



Si ſolidum parallelis planis contineatur: oppoſita ipſius plana & aequalia & parallelogramma ſunt.

1. Nam quia plana parallela BH, CF ſe-
π.16. 11. cantur a plano AC in rectis AB, DC: erunt π
AB, CD parallelae. Similiter quia plana AF,
BE parallela ſecantur a plano AC: erunt π
AD, BC parallelae. Ergo AC eſt Pgr. Similiter oſtenditur, reliqua plana AF, HE, BE, BH, FC eſſe Pgra. Q. E. D.

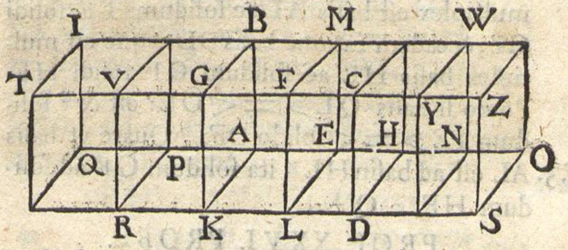
2. Iungantur AG DE. Quia AB, BG
ρ. 10. 11. ipſis DC, CE ſunt parallelae: eſt ang. ABG ρ =
DCE.

DCE. Sed AB[σ] = DC, & BG = CE. Ergo σ. 34. I.
△ AGB = DEC, & igitur Pgr. BH = Pgr. CF.
Similiter ostendetur Pgr. AC = HE, & Pgr.
AF = BE. Q. E. D.

* *Scholium.*

Et quia ostensum est, ang. ABG = DCE, & AB : BG = DC : CE : patet, aequiangula esse Pgra opposita, & latera circum aequales angulos proportionalia habere, ideoque etiam similia esse.

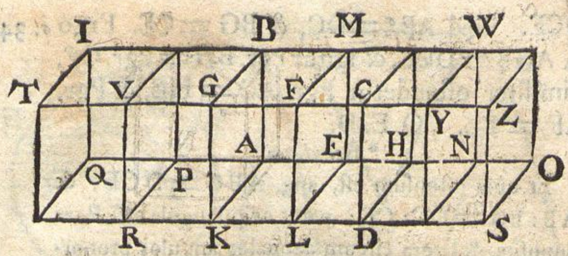
PROP. XXV. THEOR.



Si solidum parallelepipedum ABCD *plano* EF *secetur oppositis planis* AG, CH *parallelo: erit vt basis* AELK *ad basin* EHDL, *ita solidum* ABFL *ad solidum* EMCD.

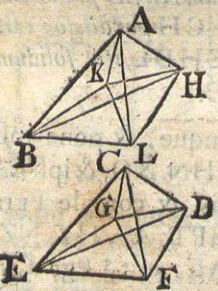
Produc enim AH vtrinque, & pone ipsi EH aequales quotcunque HN, NO, & ipsi EA aequales AP, PQ quotuis, & comple Pgra QR, PK, DN, NS, & Ppda PT, AV, HY, NZ. Iam quia QP = PA = AE : erit [τ] Pgr. QR = PK = AL, & Pgr. PI = PB = BE. Erit quoque Pgr. TQ = [υ] VP = GA. Ergo tria plana solidorum PT, AV, EG tribus planis aequantur. Sed tria tribus oppositis [υ] aequantur. Ergo [φ] tria solida PT, AV, EG aequalia

τ. 38. I.
υ. 24. II.
φ. 10. def. II.



lia sunt. Similiter ostendetur tria solida OY, NC, HF aequalia esse. Ergo basis QL aeque multiplex est basis AL ac solidum TE solidi GE; & eadem ratione basis OL aeque est multiplex basis HL ac solidum OF solidi HF. Porro si basis QL $>=<$ OL: est & φ solidum TE $>=<$ solido OF. Quare vt basis
z. 5. def. 5. AL est ad basin HL χ ita solidum GE ad solidum HF. Q.E.D.

PROP. XXVI. PROBL.



Ad datam rectam lineam AB *& ad datum in ipsa punctum* A *dato angulo solido* C *aequalem angulum solidum constituere.*

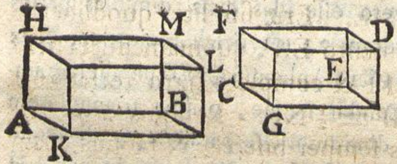
Sint DCE, ECF, FCD anguli plani solidum C continentes. Ex quouis puncto F in recta CF de-
ψ. 11. 11. mitte in planum ECD perpendicularem ψ FG, quae ipsi occurrat in G, & iunge CG. Dein fac ang. BAH $=$ ECD, & ang. BAK $=$ ECG, & AK $=$ CG; atque ex K plano BAH erige per-

perpendicularem $^{\alpha}$ KL, quam fac = GF, & iunge AL. Dico factum. α. 12. 11.

Nam fiat AB = CE, & iungantur KB, BL, GE, EF. Et quia rectae KL, GF planis BAH, ECD perpendiculares sunt: erunt ang. AKL, BKL, CGF, EGF recti. Dein quia KA = GC, & AB = CE, & ang. BAK = ECG: erit $^{\alpha}$ BK = EG. Sed KL = GF. Ergo AL = $^{\alpha}$ CF, & BL = $^{\alpha}$ EF; ac inde ang. BAL $^{\beta}$ = ECF. Similiter, sumta AH = CD & iunctis HK, HL, DG, DF ostendemus ang. LAH = FCD. Ergo tres ang. plani BAH, BAL, LAH anguli solidi A tribus planis ECD, ECF, FCD solidi C aequantur. Hinc ang. solidus A = $^{\gamma}$ C. Q. E. F.

α. 4. 1.
β. 8. 1.
γ. ax. 11.

PROP. XXVII. PROBL.



A data recta linea AB dato solido parallelepipedo CD simile & similiter positum solidum parallelepipedum describere.

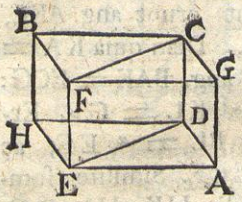
Fac $^{\delta}$ angulo solido C = A, ita vt angulo GCE = KAB, & ang. FCE = HAB, & ang. GCF = KAH. Dein fac EC : CG = $^{\varepsilon}$ BA : AK, ac GC : CF = KA : AH, & comple Pgr. BH, ac solidum AL. δ. 26. 11. ε. 12. 6.

Etenim $^{\zeta}$ Pgr. KB ∾ GE, & Pgr. KH ∾ GF, &, quia ex aequo EC : CF = BA : AH, Pgr. BH ∾ EF. Ergo & tria reliqua Pgra ζ. 1. def. 6. & constr.

HL,

η.sch.24 11. &21.6. HL, LB, LK ∾ [η] tribus reliquis DF, DE, DG. Quare Ppd. AL ∾ [θ] Ppdo CD.
θ. 9. def.11. Q. E. D.

PROP. XXVIII. THEOR.



Si solidum parallelepipedum AB plano CDEF secetur per diagonales CF, DE oppositorum planorum: solidum AB ab ipso plano CDEF bifariam secabitur.

ι. 34. 1. Quia enim △GCF = [ι] △CFB, & △ADE
κ. 24. 11. = △DEH, & Pgr. AC = [κ] BE & Pgr. GE
λ. 10. def. = CH: Prisma GCFEDA = [λ] prismati
11. CFBHDE. Q.E.D.

* *Scholium.*

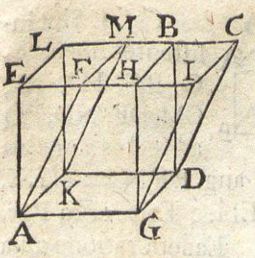
Prismata vero esse illas duas dimidias partes Ppdi AB patet ex 24. 11. & schol eiusdem. & ex eo, quod (per 16. 11.) planum CFED parallelogrammum est. Constat itaque, prisma triangulatem basin habens dimidium esse parallelepipedi aeque alti & in eadem basi GE constituti, vel in basi AH basis triangularis dupla.

PROP. XXIX. THEOR.

Solida parallelepipeda AB, AC in eadem basi AD eademque altitudine, quorum insistentes lineae AE, AF, GH, GI, KL, KM, DB, DC in eisdem rectis lineis EI, LC collocantur, inter se sunt aequalia.

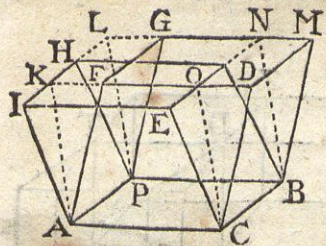
Quia KB & KC sunt Pgra, & inde LB =
μ. 3. ax. 1. KD = MC: erit LM = [μ] BC, & ergo △

LKM



LKM = [ν] BDC, nec ν. 8. 1.
non Pgr. EM = [ξ] HC. ξ. 36. 1.
Eadem ratione △ AEF = GHI. Eſt autem [ο] Pgr. LA = BG, & ο. 24. 11.
Pgr. MA = CG. Ergo Priſma AEFMLK = [π] Priſm. GHICBD. π. 10. def. 11.
Hinc addito communi ſolido AKDGHFMB, tota Ppda AB, AC aequalia erunt. Q. E. D.

PROP. XXX. THEOR.

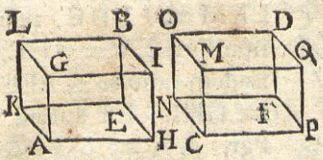


Solida parallelepipeda ABEH, ABDG *in eadem baſi eademque altitudine, quorum lineae inſiſtentes in eiſdem lineis rectis non collocantur, inter ſe aequalia ſunt.*

Producantur enim DF, MG, IH, EO, vt ſe inuicem ſecent in K, L, N, & iungantur KA, LP, OC, NB. Ergo Ppd. ABEH = [ρ] ABNK = [ρ] ABDG. Q. E. D. ρ. 29. 11.

PROP. XXXI. THEOR.

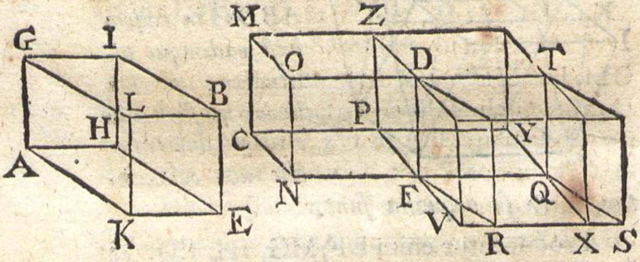
Solida parallelepipeda AB, CD, *quae in aequalibus ſunt baſibus* AE, CF, *& eadem altitudine, inter ſe ſunt aequalia.*



Cas. 1. Sint insistentes lineae AG, BE, HI, KL, CM, DF, NO, PQ ad rectos angulos basibus AE, CF, & sit ang. PFN = HEK, NF = KE, & FP = EH. Erit ergo Pgr.

σ. 1. def. 6. CF = & ∾ σ Pgro AE. Eadem ratione quia altitudines NO, KL aequales, & ang. ONF, ONC, LKE, LKA recti sunt: erit Pgr. ND = & ∾ ipsi KB, & Pgr. CO = & ∾ AL. Quare & reliqua Pgra reliquis aequalia & si-

τ. 10. def. 11. milia erunt, & ergo Ppd. CD = τ ipsi AB. Q. E. D.



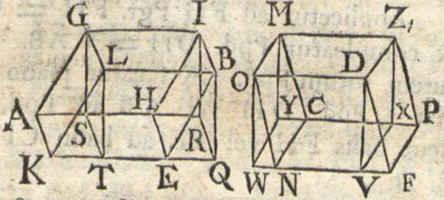
Cas. 2. Sint iterum insistentes perpendiculares, sed ang. PFN non = HEK. Produc NF in Q, & fac ang. QFR = HEK, & FQ = HE, & FR = EK, & comple Pgr. QR ac

υ. cas. 1. solidum TR. Ergo erit Ppd. TR υ = Ppdo AB. Produc PF, SR, quae conueniant in V, & per Q duc ipsi PV parallelam QX, quam produc, donec productae CP occurrat in Y, & comple Ppda TV, TP, quorum bases sunt Pgra

VQ,

VQ, PQ. Iam Ppda TV, TR, eandem basin TF habentia, aequalia φ sunt; & hinc Ppd. TV = AB. Sed quia Pgr. FX = χ FS = ψ AE = ω CF: erit Pgr. FX: FY = ψ CF: FY. Atqui Ppd. TV: TP = α Pgr. FX: FY, nec non Ppd. CD: TP = Pgr. CF: FY. Ergo Ppd. TV: TP = Ppd. CD: TP. Quare Ppd. TV = β CD, ideoque Ppd. CD = AB. Q. E. D.

φ. 29. 11.
χ. 35. 1.
ψ. constr.
ω. hyp.
α. 25. 11.
β. 9. 5.



Cas. 3. Non sint insistentes AG, BE, HI, KL, CM, PZ, FD, NO perpendiculares basibus. Duc a punctis B, I, G, L, D, Z, M, O ad bases perpendiculares BQ, IR, GS, LT, DV, ZX, MY, OW, & iunge ST, QR, TQ, RS, XV, YW, YX, VW. Erit ergo Ppd. MV = γ GQ. Atqui Ppd. CD = δ MV, & Ppd. AB = δ GQ. Ergo Ppd. CD = AB. Q. E. D

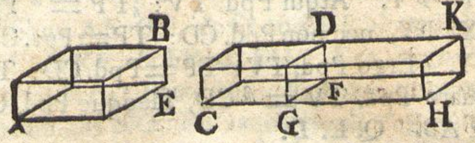
γ. casus praec.
δ. 29. vel 30. 11.

* *Schol.* Itaque Parallelepipeda aequalia AB, CD aequalium basium aeque alta sunt. Nam si alterius AB altitudo maior esset: quia ipsius AB pars capi posset aeque alta ipsi CD, foret pars Ppdi AB = Ppdo CD. Ergo Ppda AB, CD inaequalia forent.

† Reliqui casus demonstrationem Lector facile addet. Similis enim est demonstrationi casus secundi.

PROP. XXXII. THEOR.

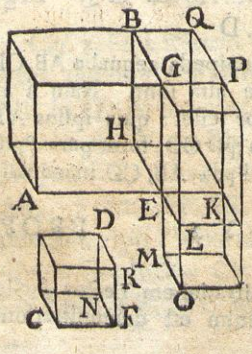
Solida parallelepipeda AB, CD, *quae eandem habent altitudinem, inter ſe ſunt vt baſes* AE, CF.



Applicetur ad FG Pgr. FH = [ε] AE,
ε. 45. I. & compleatur Ppd. DH = [ζ] AB. Quia
ζ. 31. II. autem totum Ppd. CK ſecatur plano DG:
η. 25. II. erit [η] Ppd. DH vel AB ad Ppd. CD ſicut baſis FH vel AE ad baſin CF. Q. E. D.

* *Schol.* Hinc parallelepipedorum aequalium quod maiorem baſin habet, minorem habet altitudinem. Non enim eandem; quia ſic Ppd inaequalia erunt: nec maiorem; quia ſic pars illius Ppdi reliquo aeque alta eodem maior, & a potiori totum eodem maius erit.

PROP. XXXIII. THEOR.



Similia ſolida parallelepipeda AB, CD *inter ſe ſunt in triplicata ratione homologorum laterum* AE, CF.

In productis AE, HE, GE cape EK = CF, EL = FN, EM = FR, Comple Pgr. KL, & Ppd. KO. Iam quia Ppd.

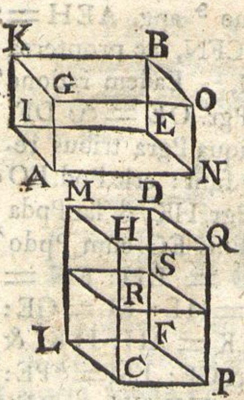
Ppd. AB ∾ CD, ideoque ϑ ang. AEH = ϑ. 9.def.11.
CFN: erit ang. KEL = CFN, ac propterea &1.def.6.
Pgr. KL = & ∾ ι CN. Eadem ratione ι. 4. 1.&34.
Pgr. KM = ∾ CR, & Pgr. OE = ∾ DF. 1.&1.def.
Quoniam ergo & tria reliqua Pgra tribus re- 6.
liquis aequalia & similia κ sunt: erit Ppd. KO κ. sch. 24.
= ∾ λ CD. Comple Pgr. HK, & fac Ppda 11.
HP, PL eiusdem altitudinis EG cum Ppdo λ. 10.def.11.
AB. Et quia ϑ AE : CF = EH : FN =
EG : FR : erit AE : EK = HE : EL = GE :
EM. Est vero μ AE : EK = AH : HK, & μ. 1.6.
HE : EL μ = HK : KL, & GE : EM = μ PE :
KM. Quare AH : HK = HK : KL = PE :
KM. Porro AH : HK = ν Ppd. AB : Ppd. ν. 32. 11.
BK; & HK : KL = ν Ppd. BK : PL; & PE :
KM = Ppd. PL : KO. Ergo ÷ Ppda AB,
BK, PL, KO, ideoque AB : KO = ξ (AB : ξ. 11. def.
BK) 3 = (AH : HK) 3 = (AE : EK) 3 = (AE : 5.
CF) 3. Q. E. D.

Corollarium.

Hinc, si quatuor rectae lineae continue proportionales fuerint, est vt prima ad quartam, ita solidum parallelepipedum, quod fit a prima, ad solidum, a secunda simile & similiter descriptum ξ.

PROP. XXXIV. THEOR.

Aequalium solidorum parallelepipedorum AB, CD *bases* AE, CF *sunt reciproce proportionales altitudinibus* AG, CH. *Et quorum solidorum parallelepipedorum* AB, CD *bases* AE, CF *sunt reciproce proportionales altitudinibus* AG, CH, *ea inter se sunt aequalia.*

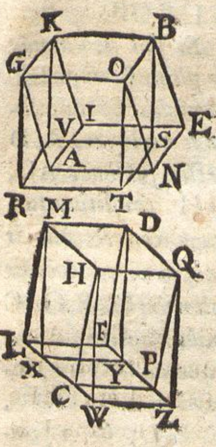


Cas. 1. Si insistentes rectae AG, EB, IK, NO, CH, LM, FD, PQ sunt basibus AE, CF perpendiculares.

Hyp. 1. Si Ppd. AB = CD, & basis AE = CF: erit & alt.
α. sch. 31. 11. AG = α CH. Ergo AE: CF = CH: AG. Sin autem alterutra basis AE > altera CF:
π. sch. 32. 11. quia tunc altitudo AG < π CH, cape CR = AG, & comple Ppd. SC. Iam quia AB = CD: erit AB: CS = CD: CS. Sed
ρ. 32. 11. AB: CS = ρ AE: CF, & CD: CS = ρ
σ. 1. 6. Pgr. CM: Pgr. RL = σ CH: CR = CH: AG. Quare iterum est AE: CF = CH: AG. Q. E. D.

Hyp. 2. Sit AE: CF = CH: AG. Iam si basis AE = CF; erit & AG = CH, ideoque
τ. 31. 11. Ppd. AB = τ CD. Si vero AE > CF: erit
υ. sch. 32. 11. CH > υ AG. Pone rursus CR = AG, & comple Ppd. CS. Ergo AE: CF = CH: CR. Sed AE: CF = ρ AB: CS, & CH: CR = σ CM: RL = ρ CD: CS. Ergo AB: CS
φ. 9. 5. = CD: CS. Igitur iterum AB = φ CD. Q. E. D.

Cas.



Cas. 2. Si insistentes AG, EB, CH &c. basibus AE, CF non sunt perpendiculares: demitte χ in bases perpendiculares GR, BS, OT, KV, HW, MX, DY, QZ, & completa intellige Ppda KT, MZ. χ. 11. 11.

Hyp. 1. Iam si Ppd. AB = CD: quia Ppd. AB = ψ KT, & Ppd. CD = MZ, erit Ppd. KT = MZ. Quum itaque sit ω BG: DH = DY: BS: erit α AE: CF = DY: BS. Q. E. D. ψ. 30. & 29. 11. ω. cas. 1.

Hyp. 2. Deinde si basis AE: CF = alt. DY: BS: erit & α BG: DH = DY: BS. Ergo Ppd. KT = ω MZ, ideoque Ppd. AB = ψ CD. Q. E. D. α. 24. 11.

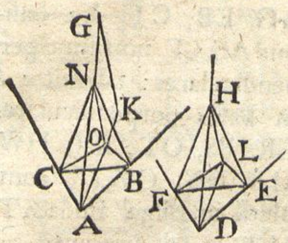
* *Coroll.*

Ostensum est sub hyp. 1. cas. 1. Ppda recta CD, CS aequalium & similium basium esse inter se vt altitudines CH, CR. Et quia his duobus Ppdis quaeuis alia duo aequalia & aeque alta sumi possunt (per 31. 11.): patet in vniuersum, duo quaecunque Ppda aequalium basium esse in ratione altitudinum.

* *Schol.*

Propositiones 31. 32. 33. 34. cum suis scholiis & corollariis valent quoque de Prismatis triangularibus, propter ea, quae ostensa sunt in prop. 28.

PROP. XXXV. THEOR.



Si sint duo anguli plani BAC, EDF *aequales; & in ipsorum verticibus* A, D *rectae sublimes* AG, DH *constituantur, quae cum rectis lineis a principio positis angulos contineant aequales, alterum* GAB, GAC *alteri* HDE, HDF; *in sublimibus autem sumantur quaevis puncta* G, H, *atque ab ipsis ad plana, in quibus sunt angu i primi* BAC, EDF, *perpendiculares ducantur* GK, HL; *& a punctis* K, L, *quae a perpendicularibus fiunt in planis, ad primos angulos iungantur rectae lineae* KA, KD: *cum sublimibus aequales angulos* KAG, LDH *continebunt.*

Pone AN = DH, & in plano AGK duc NO parallelam ad GK, quae ergo plano BAC
β. 8. 11. perpendicularis [β] erit. A punctis O, L duc ad rectas AB, AC, DE, DF perpendiculares OB, OC, LE, LF, & iunge NC, NB, HE, HF,
γ. 47. 1. CB, FE. Iam quia ANq = [γ] NOq + OAq, & OAq = [γ] OCq + ACq, & NOq + OCq = [γ] NCq: erit ANq = NCq + CAq, ideo-
δ. 48. 1. que [δ] ang. NCA rectus. Similiter ostenditur ang. HFD rectus. Quare ang. NCA = HFD. Et quia NAC = HDF, ac AN = DH: erit
ε. 26. 1. AC = [ε] DF. Eadem ratione AB = DE.
ζ. 4. 1. Quare CB = [ζ] FE, & ang. ACB = DFE, &
η. 3. ax. 1. ang. ABC = DEF. Hinc [η] ang. OCB = LFE,

&

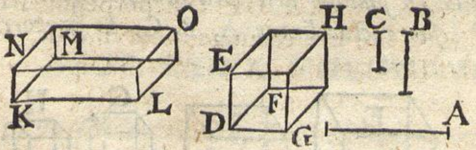
& ang. OBC = LEF, & ob CB = FE, est CO = [s] FL. Unde patet [a] AO = DL. Hinc quoniam NOq + OAq = ANq = DHq = HLq + LDq: erit ONq = HLq, & NO = HL. Igitur constat, ang. KAG = [a] LDH. Q. E. D. a. 8. 1.

Corollar.

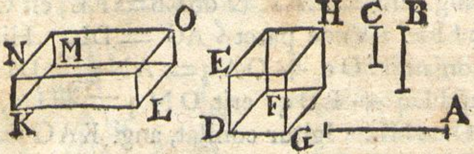
Ex hoc vero manifestum est, si sint duo anguli plani rectilinei aequales, ab ipsis autem constituantur sublimes rectae lineae aequales, quae cum rectis lineis a principio positis aequales contineant angulos, alterum alteri, perpendiculares NO, HL, quae ab ipsis ad plana, in quibus sunt primi anguli, ducuntur, inter se aequales esse.

PROP. XXXVI. THEOR.

Si tres rectae lineae A, B, C *proportionales sint: solidum parallelepipedum, quod a tribus fit, aequale est solido parallelepipedo, quod fit a media* B, *aequilatero quidem, aequiangulo autem antedicto.*



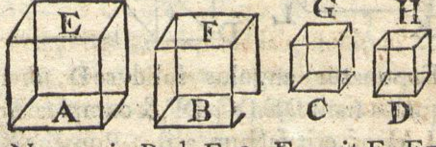
Exponatur angulus solidus D, & ipsi B aequales fiant DE, DG, DF, & compleatur Ppd. DH, quod erit factum a B. Ponatur KL = A, & ad punctum K fiat [t] ang. solidus K = D, ac KM = B, & KN = C, & compleatur Ppd. KO, quod erit factum a tribus A, B, C, & t. 26. 11.



κ. ax. 11. & aequiangulum ipsi DH κ. Et quia λ KL : DG
29. 1. = DE : KN, & ang. LKN = GDE : erit Pgr.
λ. constr. NL = μ EG. Deinde quia & λ ang. MKN
μ. 14. 6. = FDE, & ang. MKL = FDG, & KM = DF:
erunt perpendiculares, a punctis M, F ad
ν. cor. 35. plana NL, EG ductae, aequales ν; id est,
11. Ppda DH, KO, aequales bases habentia EG,
ξ. 4. def. 6. NL, aeque alta ξ erunt, ac ergo aequalia ο.
ο. 31. 11. Q. E. D.

PROP. XXXVII. THEOR.

Si quatuor rectae lineae A, B, C, D *proportionales sint: & quae ab ipsis fiunt solida parallelepipeda* E, F, G, H *similia & similiter descripta proportionalia erunt. Et si quae ab ipsis fiunt solida parallelepipeda* E, F, G, H *similia & similiter descripta proportionalia sint: & ipsae rectae lineae* A, B, C, D *proportionales erunt.*



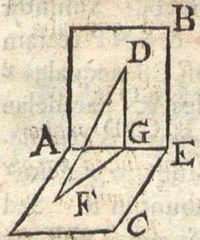
π. 33. 11. 1. Nam quia Ppd. E ∾ F : erit E : F = π $(A : B)^3$. Eodem argumento erit G : H =
ρ. hyp. & 1. $(C : D)^3$. Sed $(A : B)^3$ = ρ $(C : D)^3$. Ergo
sch. 22. 5. E : F = G : H. Q. E. D.

2. Quia,

2. Quia, vt antea, $E : F = (A : B)^3$, & $G : H = (C : D)^3$, atque $E : F = G : H$: erit $(A : B)^3 = (C : D)^3$, ideoque $A : B =$ [σ] $C : D$. Q.E.D. σ. 2. sch. 22. 5.

PROP. XXXVIII. THEOR.

Si planum AB *ad planum* AC *rectum sit, & ab vno puncto* D *eorum, quae sunt in vno plano* AB, *ad alterum planum* AC *perpendicularis ducatur: ea in communem planorum sectionem* AE



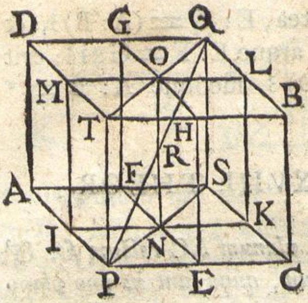
Si negas, cadat extra, vt DF, & a puncto F in plano AC duc ad AE perpendicularem FG, & iunge DG. Iam quia FG perpendicularis [τ] est plano AB: erit ang. FGD rectus [υ]. Sed & ang. DFG rectus [υ] est. Quare in △ GDF duo recti sunt. Q.E.A.

τ. 4. def. 11.
υ. 3. def. 11.

PROP. XXXIX. THEOR.

Si in solido parallelepipedo AB *oppositorum planorum* AC, BD *latera secentur bifariam; per sectiones vero plana ducantur* EFGH, IKLM: *communis planorum sectio* NO *& solidi parallelepipedi diameter* PQ *se mutuo bifariam secabunt.*

Iungan-



Iungantur QO, OT, PN, NS. Quoniam QB, DT sunt parallelae: erit ang. QLO
φ. 29. 1. = φ OMT. Praeterea
χ. 34. 1. QL χ =
ψ. 33. 1. TM. Et quia ψ ML, DQ parallelae sunt, item DT,
ω. constr. GH, QB: erit MO = χ DG = ω GQ = χ
α. 4. 1. OL. Quare α QO = OT, & ang. QOL
β. 3. sch. 15. 1. = MOT, & ob id β QOT recta. Similiter demonstratur, SN = NP, & SNP rectam esse. Et quia PT, SQ, ipsi CB aequales χ & parallelae, ipsae aequales & parallelae sunt: erunt & TQ, PS aequales ψ & parallelae.
γ. 7. 11. Ergo rectae NO, PQ sunt in eodem γ plano TS, & se mutuo secabunt in R. Sed quia φ ang. OQR = RPN, & ang. QOR =
δ. 7. ax. 1. PNR, & QO = δ PN: erit ε OR = RN, &
ε. 26. 1. QR = RP. Q. E. D.

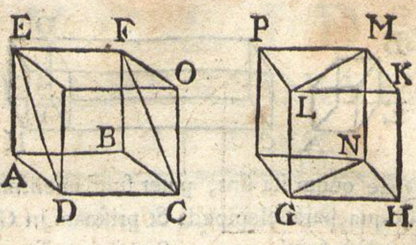
* *Schol.*

Hinc in omni parallelepipedo diametri omnes se mutuo bisecant in vno puncto R.

PROP. XL. THEOR.

Si sint duo prismata ABCDEF, GHKLMN *aeque alta, quorum vnum quidem basin habeat parallelogrammum* ABCD, *alterum vero triangulum* GHN, *&* *parallelogrammum* ABCD *duplum sit trianguli* GHN: *aequalia erunt ipsa prismata.*

Com-



Compleantur enim Ppda AO, HP. Et quoniam Pgr. AC = 2 △ GNH =[ζ] Pgr. GN, atque solida aeque alta sunt: erit Ppd. AO =[η] HP, ideoque Pr. ABCDEF =[θ] Pr. GHKLMN. Q. E. D.

ζ. 34. 1.
η. 31. 11.
θ. 28. 11.

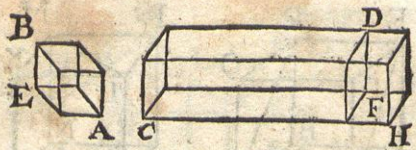
* *Scholium.*

Ex iis, quae hactenus ostensa sunt, demonstrari potest, *parallelepipeda quaeuis* AB, CD, *nec non prismata triangularia, esse in ratione composita basium* AE, CF *&* *altitudinum* BE, DF.

Intelligatur enim aliud Ppd. DH, cuius basis FH = basi AE Ppdi AB, & altitudo DF = altitudini Ppdi CD. Et quoniam est AB : HD =[ι] BE : FD, & HD : CD =[κ] FH : CF = AE : CF: erit AB : CD =[λ] (AE : CF) + (BE : DF). Ergo Parallelepipeda, & triangularia prismata, Parallelepipedorum dimidia, sunt inter se vt bases & altitudines. Q. E. D.

ι. cor. 34. 11.
κ. 32. 11.
λ. 5. def. 6.

Quae



Quae quum ita sint, patet fundamentum methodi, qua parallelepipeda & prismata in Geometria practica metiuntur. Sumunt enim cubum AB, & latus eius BE pro vnitate, qua metiuntur basin Ppdi CD & altitudinem: & ex multiplicatione numerorum, qui basin & altitudinem exprimunt, gignitur numerus, qui soliditatem Ppdi CD exprimit. Sit (per 4. sch. 23. 6.) basis CF $=$ 9 AE, & altitudo DF $=$ 2 BE: & quia CD : AB $=$ (CF : AE) $+$ (DF : BE) $=$ (9 : 1) $+$ (2 : 1) $=$ λ 18 : 1; erit CD $=$ 18 AB.