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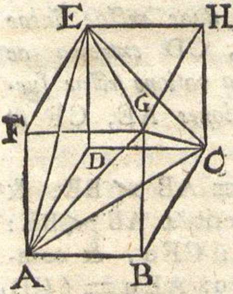
EVCLIDIS ELEMENTORVM LIBER XV.

PROP. I. PROBL.



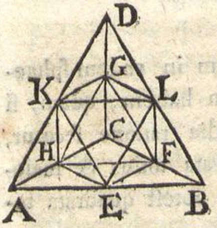
In dato cubo ABCDEFGH *pyramidem describere.*

Ducantur diametri quadratorum GA, GE, GC, EA, EC, CA, quae omnes α inter se aequales sunt. Ergo triangula EGC, EAG, AGC, EAC sunt aequilatera, & aequalia. Proinde EGCA tetraedrum est, cubi angulis insistens, & ergo ipsi inscriptum β. Q.E.F.

α. 4. 1.

β. 31. def. 11.

PROP. II. PROBL.



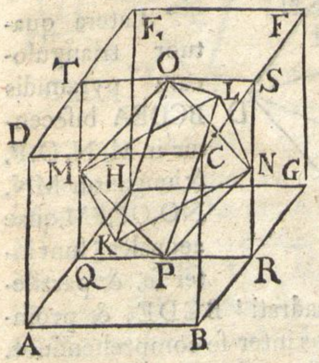
In data pyramide ABCD *octaedrum describere.*

* Bisecentur latera tetraedri in punctis E, F, G, H, K, L. Haec puncta connectantur 12 rectis, quae omnes inter se γ aequales erunt. Quare octo triangula, quae bases habent rectas HG, GL, LE, EH & vertices K, F, aequilatera erunt & aequalia; & solidum sub ipsis com-

γ. 4. 1.

comprehensum octaedrum erit, dato tetraedro inscriptum. Q. E. F.

PROP. III. PROBL.

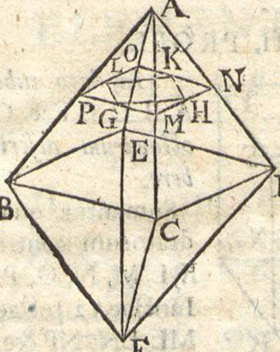


In dato cubo ABCDEFG *octaedrum describere.*

Sumantur [δ] qua- δ. 8. 4. dratorum centra K, L, M, N, O, P. Iunctae 12 rectae ML, LN, NP &c. constituent octaedrum. Nam per P, N, O, M ducantur lateribus quadrati AC parallelae QR, RS, ST, MQ, quae iisdem lateribus, & ergo inter se aequales erunt. (*Patet vero, has rectas se mutuo tangere; quia [δ] QT, ST eandem ED, & QR, SR eandem GB, & NR, QR eandem AH &c. bisecant). Ergo anguli MQP, NRP [ε] sunt recti. Hinc ε. 10. 11. quia MQ, QP, PR, NR, quippe aequalium TQ, QR, RS dimidia, aequantur: erit MP = [ζ] PN. Similiter ostenditur, MP, OM, ζ. 4. 1. NK, NL & reliquas aequari. Ergo 8 triangula, quorum vertices L, K, bases latera quadrati MONP, sunt aequilatera & aequalia, & constituunt ergo octaedrum, cubo inscriptum. Q. E. F.

PROP.

PROP. IV. PROBL.



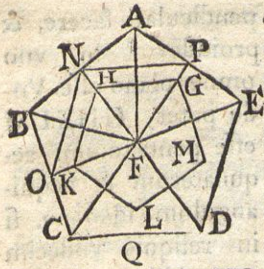
In dato octaedro ABCDEF *cubum describere.*

* Latera quatuor triangulorum pyramidis BCDEA bisecentur in M, N, O, P, & iungantur MN, NO, OP, PM, quae aequales ν sunt in-
η. 4. 1. ter se, & paralle-
ϑ. 2. 6. lae ϑ lateribus quadrati ι BCDE, & proin-
ι. 14. 13. de κ angulos rectos inter se comprehendunt.
κ. 10. 11. Quare MNOP est quadratum. Deinde bisectis lateribus huius quadrati in G, H, K, L, iungantur GH, HK, KL, LG, quae ν sunt aequa-
λ. 2. sch. les, & angulos rectos λ comprehendunt; quia
13. 1. anguli, quos cum rectis MN, NO, OP, PM
μ. 4. sch. faciunt, semirecti μ sunt. Ergo GHKL est
32. 1. quadratum. Si in reliquis 5 pyramidibus octaedri eadem fiant: constituentur 5 alia quadrata ipsi GHKL aequalia, & cum ipso cubum terminantia, dato octaedro inscriptum. Q. E. F.

PROP. V. PROBL.

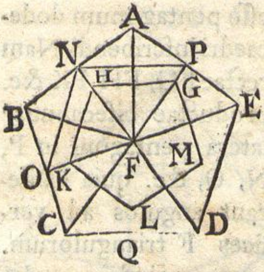
In dato icosaedro dodecaedrum describere.

Sit ABCDEF pyramis icosaedri, cuius basis pentagonum ABCDE. Iungantur centra circulorum, in triangulis AFB &c. inscriptorum, rectis GH, HK, KL, LM. Dico, GHKLM esse



eſſe pentagonum dodecaedri inſcribendi. Nam rectae FG, FH, FK &c. productae biſecabunt υ latera pentagoni in P, N, O, &c. quia ξ biſecant angulos ad vertices F triangulorum. Iungantur PN, NO, quae proinde aequales erunt υ. Iam quia υ FP = FN = FO, ac ο FG = FH = FK: erunt GH, HK ipſis PN, NO π parallelae, ac inde erit ρ PN : GH = NF : FH = NO : HK, ideoque GH = HK. Similiter HK = KL &c. Porro quia ang. GHK σ = PNO, ac HKL = NOQ &c. ang. autem PNO = τ NOQ, quia ambo ſunt complementa aequalium υ angulorum in N, O ad duos rectos: erit ang. GHK = HKL &c. Denique ex puncto ſublimi F in planum ABCDE ductum intelligatur perpendiculum, & a puncto, in quo plano occurrit, ductae ſint rectae ad puncta P, N, O, Q, quae cum perpendiculari angulos υ rectos facient. Illi, quae per P ducta eſt, parallela intelligatur alia per G, & a puncto, in quo haec dictae perpendiculari occurrit, ducantur rectae ad H, K, L, &c. Iam quia π perpendicularis illa a recta per G ducta ſecatur in ratione FP : FG = FN : FH = FO : FK &c. patet, reliquas rectas, a punctis H, K, L ad perpendicularem ductas, parallelas π eſſe illis, quae in plano ABCDE ad eandem ductae ſunt, ac ob id angulos rectos cum perpendi-

υ. 4. 1.
ξ. 4. 3.
ο. 26. 1. & 4. 3.
π. 2. 6.
ρ. 2. ſch. 4. 6.
σ. 10. 11.
τ. 2. ſch. 13. 1.
υ. 4. 11.



p. 5. 11. pendiculari facere, & proinde p in vno omnes plano esse. Vnde patet, GHKLM esse pentagonum aequilaterum & aequiangulum, ideoque, si in reliquis vndecim pyramidibus icosaedri eadem construxerimus, proditura esse 12 pentagona huiusmodi, quae constituent dodecaedrum, icosaedro inscriptum. Q. E. F.

PROP. VI. PROBL.

Quinque figurarum latera & angulos inuenire.

1. Quia icosaedrum continetur 20 triangulis, & vnum triangulum 3 lateribus; singula vero latera bis sumuntur: numerus laterum erit dimidius facti ex 20 & 3, qui est 30. Similiter dodecaedri laterum numerus est dimidius facti ex 12 & 5, qui est 30. Et sic porro in cubo, & reliquis inueniemus numerum laterum, sumentes dimidium facti ex numero planorum & numero laterum vniuscuiusque plani.

2. Numerum autem augulorum solidorum in his figuris habebimus, factum ex numero figurarum planarum & numero angulorum planorum in vnaqualibet diuidentes per numerum angulorum planorum in quolibet solido angulo. Sic in icosaedro factum ex 20

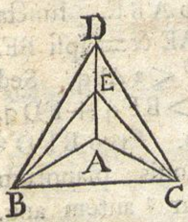
&

& 3, quod eſt 60, partientes per 5, habebimus 12 angulos ſolidos. Q. E. F.

PROP. VII. PROBL.

Planorum, quae ſingulas quinque figuras continent, inclinationem inuenire.

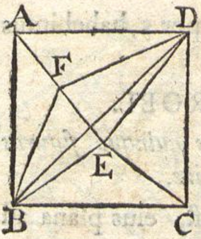
1. De cubo manifeſtum eſt, eius plana ad ſe inuicem recta eſſe.



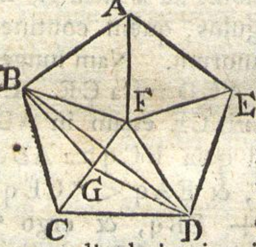
2. Sit tetraedri ABCD expoſitum vnum triangulum ABD, in quo a vertice B ad latus AD ducta ſit perpendicularis BE. Si centris A, D, interuallo BE deſcribantur duo circuli, & a puncto ſectionis ad centra A, D iungantur rectae: angulus, quem continebunt, erit inclinatio planorum. Nam iungatur in altero triangulo ACD recta CE. Et quia ϰ DE = EA: erit CE etiam in AD perpendicularis. Sed quia BCq = ABq = ψ AEq + EBq, & EAq < ω CEq: erit BCq < CEq + EBq, & ergo ang. CEB acutus. Quare CEB erit inclinatio β planorum tetraedri. Hinc quum ſit CE = EB, & BC = AD, manifeſtum eſt, praedicta conſtructione inueniri angulum γ = BEC = inclinationi planorum. Q. E. F.

ϰ. ſch. 3. 3.
ψ. 47. 1.
ω. 2. ſch. 12. 13.
α. 13. 2.
β. 6. def. 11.
γ. 22. & 8. 1.

3. A latere octaedri deſcribatur quadratum, ducatur eius diameter BD, & centris B, D, interuallo perpendiculari, quae a vertice ad baſin

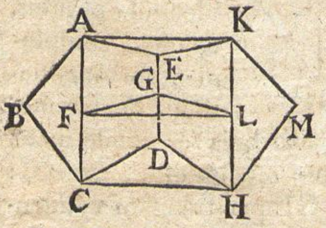


basin trianguli in octaedro ducitur, describantur duo circuli. Rectae a sectione circulorum ad B, D iunctae continebunt angulum aequalem complemento inclinationis ad 2 rectos. Sit enim ABCDE pyramis octaedri, & BF perpendicularis in △o ABE. Iuncta DF erit perpendicularis in AE & = ipsi BF.
δ. 19. 1. Hinc BFq + FDq = 2 BFq [δ] < 2 ABq. Sed BDq = 2 ABq. Ergo BDq > BFq + FDq,
ε. 12. 2. ac ob id [ε] ang. DFB obtusus. Ergo BFD [ζ]
ζ. 6. def. 11. = complemento inclinationis planorum
η. 22. & 8. 1. octaedri ad 2 rectos. Datur [η] autem ang. BFD dicta constructione. Q. E. F.

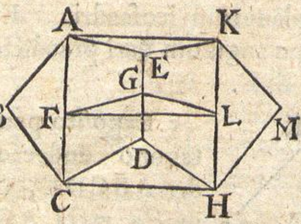


4. A latere icosaedri, descripto pentagono aequilatero & aequiangulo ABCDE, ducatur recta BD, angulum pentagoni C subtendens, & centris B, D, interuallo perpendiculari cuiusuis e triangulis icosaedri, describantur duo circuli, a quorum sectione ad B, D iunctae rectae continebunt complementum inclinationis planorum ad 2 rectos. Sit enim ABCDEF pyramis icosaedri, & BG perpendicularis vnius trianguli: erit DG per-
θ. 19. 1. pendicularis proximi trianguli. Et quia BG < [θ]
ι. 21. 1. BC: erit [ι] ang. BGD > obtuso BCD, ideoque ipse obtusus. Quare BGD complementum in-

erit inclinationis planorum icoſaedri. Et manifeſtum eſt, hunc angulum dari praedicta conſtructione. Q. E. F.



5. Expoſito pentagono dodecaedri ABCDE, & iuncta recta AC, angulum pentagoni ſubtendente; centris A, C, interuallo FG rectae a puncto F bipartitae ſectionis ipſius AC in latus pentagoni parallelum ED perpendicularis deſcribantur duo circuli, & rectae a ſectione ad terminos A, C ductae comprehendent complementum inclinationis planorum dodecaedri. Nam quia κ AC eſt κ. 17. 13.
latus cubi, a quo dodecaedrum deſcribitur: ponatur, ACHK eſſe vnum quadratorum illius cubi. Ergo erit KH recta, ſubtendens angulum in pentagono adiacente, quod ſit EKMHD. Ex G ad ED ducatur perpendicularis GL, & iungatur FL. Et quia ED, AC ſunt parallelae: erit GF in AC perpendicularis, ergo per centrum circuli, pentagono ABCDE circumſcripti, tranſibit λ, & ED biſecabit μ in λ. cor. 1. 3.
G. Hinc ſimiliter GL biſecabit ipſam KH. μ. 3. 3.
Quare FL = ν AK = AC. Et quia per- ν. 33. 1.
pendicularis ex G in FL cadens = κ ½ AE, & ½ FL = ½ AC ξ > ½ AE: erit ½ FL maior ξ. 8. 13.
perpendiculari ex G in FL ducta, & ergo angulus, quem ea cum GF continet, maior ipſo GFL. Hinc quia κ haec perpendicularis biſecat



e. 4. 1. ſecat ipſam FL, erit ang. LGF $>$ᵉ GFL $+$ GLF, ideoque obtuſus, & ob id complementum inclinationis pentagonorum dodecaedri. Sed quia ex modo dictis eſt FG $=$ GL, atque oſtenſa eſt FL $=$ AC: patet, dari ang. FGL per traditam conſtructionem. Q.E.F.

F I N I S

ELEMENTORVM EVCLIDIS

CORRIGENDA.

Pag. 6. lin. 17. *vni* leg. *vno*.
p. 11. lin. vltima poſt *habentes* interſerantur verba, *cum rectis AC, BC initio ductis*.
p. 63. lin. antepenult. *connexam* l. *conuexam*.
p. 68 lin. 7. AKC leg. ABC.

p. 235 lin. 5 ab ultimo