PART I.—STATICS OF STRUCTURES.

CHAPTER I.

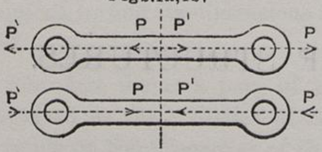
FRAMEWORK LOADED AT THE JOINTS.

1. *Preliminary Explanations and Definitions.*—A frame is a structure composed of bars, united at their extremities by joints, which offer no resistance to rotation. In the first instance we may suppose the centre lines of the bars all in one plane, and in that case the joints may consist simply of smooth pins passing through holes at the ends of the bars, which are to be imagined forked, if necessary, so as to allow the centre lines to meet in a point. A large and important class of structures, known to engineers as "trusses," approach so closely to frames that calculations respecting them may be conducted by treating them as if they were frames. The differences between a truss and a frame will appear as we proceed.

The frame may be acted on by forces applied at points in one or more of its bars, or at the joints which unite the bars together. An important simplification, however, is effected by supposing, in the first instance, that the joints only are loaded, an assumption which will be made throughout this chapter, except in a few simple examples. It will be shown hereafter that all other cases may be derived from this by means of a preliminary reduction (see Chapter IV.).

Assuming, then, that the frame is acted on by forces at the joints, due either to weights or other external causes, or to the reaction of supports on which the frame rests, the problem to be solved is to find the forces called into play on each of the bars of which it is constructed. These forces are caused by the pressure of the pins on the sides of the holes through which they pass, and it at once follows, since no other forces act on the bar, that for each bar these pressures must be equal and opposite, their common line of action being the line joining the centres of the holes. There are two possible cases shown in Figs. 1a, 1b; in the first the bar is acted on by a pair of equal and opposite forces tending to lengthen it, and in the second to shorten it. The pairs of forces are called a Pull and a Thrust respectively, while the bars subjected to their action are called Ties or Struts respectively. Between a pull and a thrust there is no statical difference but that of sign; the constructive difference, however, between a tie and a strut is great. The first may theoretically be a rope or chain, and the second may be made up of pieces simply butting against one another without fastening, while a rigid bar will serve either purpose, though its powers of resistance are generally entirely different in the two cases.

Figs.1a,1b.



It often happens that it is unknown whether a bar be a strut or a tie, and the pair of forces are then called a STRESS on the bar. This word "stress" was introduced by Rankine to denote the mutual action between any two bodies, or parts of a body, and here means, in the first instance, the mutual action between the parts of the frame united by the bar we are considering. If, however, we imagine the bar cut into two parts, A and B, by any transverse section, as shown in Figs. 1a, 1b, those parts are held together in the case of a pull, or thrust away from each other in the case of a thrust, by internal molecular forces called into play at each point of the transverse section, and acting one way on A and the other way on B. As A and B must both be in equilibrium, it is obvious that these internal forces must be exactly equal to the original forces, and thus it appears that the stress on the bar may also be regarded as the internal molecular action between any two parts into which it may be imagined to be divided. Stress, regarded in this way, will be fully considered in a subsequent division of this work; it will be here sufficient to say that its intensity is measured by dividing the total amount by the sectional area of the bar, and is limited to a certain amount, depending on the nature of the material of which the bar is constructed.

It is further manifest from what has been said, that the stress on a bar may likewise be regarded as a mutual action between the bar and either of the pins at its ends which are pulled towards the middle of the bar in the case of a pull, or thrust away from it in the case of a thrust; each pin is therefore acted on, in addition to any load which may be suspended from it, by forces, the directions of which are the lines joining the centres of the pins, from which it follows at once that *every joint may be regarded as a point kept in equilibrium by the load at that joint and by forces of which the bars of the frame are the lines of application.* This principle enables us to find the stress on each bar of a frame loaded at the joints, whenever such stress can be determined by statical considerations alone, without reference to the material or mode of construction, that is to say, in all cases which properly belong to the present division of our work.

Forces are measured in pounds-weight or, when large, in tons of 2240 lbs. They are often distributed over an area or along a line, and are then reckoned per square foot or per "running" foot, the last expression being commonly abbreviated to "foot-run."

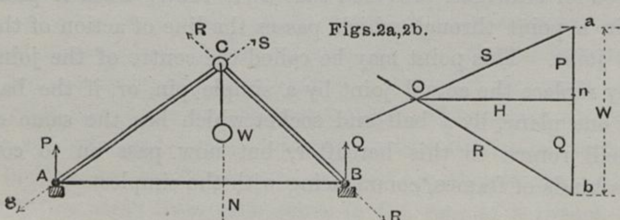
The bars need not be connected by simple pin joints as has been supposed for clearness, provided that their centre lines if prolonged meet in a point through which passes the line of action of the load on the joint. This point may be called the centre of the joint, and we may replace the actual joint by a simple pin, or, if the bars are not in one plane, by a ball and socket which has the same centre. We shall return to this hereafter, but now pass on to consider various kinds of frames, commencing with the simplest.

SECTION I.—TRIANGULAR FRAMES.

2. *Diagram of Forces for a Simple Triangular Frame.*—The simplest kind of frame is a triangle.

In Fig. 2a ACB is such a triangle; it is supported at AB so that AB is horizontal, and loaded at C with a weight W. Then evidently the effect of the weight is to compress AC, BC, and to stretch AB, which is conveniently indicated by drawing AC, BC in double lines, and AB in a single line. Also the weight produces certain vertical pressures on the supports A, B, which will be balanced by corresponding reactions P and Q.

To find the magnitude of the thrust on AC, BC, the pull on AB, and the reactions, the diagram of forces Fig. 2b is drawn: ab is a vertical line representing W on any convenient scale, while aO, bO are lines drawn through a, b respectively, parallel to AC, BC, to meet in O, and finally On is drawn parallel to AB, or, what is the same thing, perpendicular to ab. Now, applying the fundamental principle laid down above, we observe that C is a point kept in equilibrium by three forces, the load at C, namely W, the thrust of AC which we will call S, and the thrust of BC which we will call R. In the second figure the triangle Oab has its sides parallel to these forces, and hence it follows that Oa, Ob represent S, R on the same scale that ab represents W. Again A is a point kept in equilibrium by three forces, the thrust of AC, the pull of the tie AB, which we will call H, and the upward reaction P of the support A. But referring to the figure 2b, On, an, are respectively parallel to the two last forces, so that, by the triangle of forces, they represent H, P on the same scale that Oa represents S. The same reasoning applies to the point B, and therefore bn represents the other supporting force Q, as is also obvious from the consideration that $P + Q = W$. We thus see that all the forces acting upon and within the triangular frame ACB are represented by corresponding lines in Fig. 2b, which is thence called



the "diagram of forces" for the triangular frame. Such a diagram can be drawn for any frame, however complicated, and its construction to scale is the best method of actually determining the stresses on the several parts of the frame.

The force H requires special notice: it is called the "*thrust*" of the frame. In the present case the thrust is taken by the tension of the third side of the triangle, but this may be omitted, and the supports A and B must then be solid and stable abutments capable of resisting a horizontal force H. In many structures such a horizontal thrust exists; and its amount and the mode of providing against it are among the first things to be considered in designing the structure. Besides the graphical representation just given, which enables us to obtain the thrust of a triangular frame by constructing a simple diagram, it may also be calculated by a formula which is often convenient. Let AC be denoted by b and BC by a, as is usual in works on trigonometry, and let AN, BN their projections on AB be called b', a', and let the height of the triangle be h and its span l, then by similar triangles,

$$\frac{P}{H}=\frac{an}{On}=\frac{CN}{AN}=\frac{h}{b'},$$

$$\frac{Q}{H}=\frac{bn}{On}=\frac{CN}{BN}=\frac{h}{a'}.$$

Therefore, by addition,

$$\frac{W}{H}=\frac{ab}{On}=h\left(\frac{1}{b'}+\frac{1}{a'}\right)$$

$$\text{or}\quad H=W\frac{a'b'}{lh}.$$

In practical questions it often happens that a', b', h are known by the nature of the question, whence H is readily determined. The case when the load bisects the span may be specially noticed; then $a'=b'=\frac{1}{2}l$ and

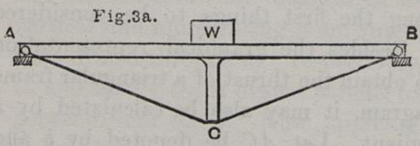
$$H=\frac{Wl}{4h}.$$

When the height of the frame is small compared with the span, this calculation is to be preferred to the diagram, which cannot then be constructed with sufficient accuracy.

The simple frame here considered may be inverted, in which case the diagram of forces and the numerical results are unaltered, the only change being that the two struts have become ties and the tie a strut.

3. *Triangular Trusses.*—Triangular frames are common in practice, and the rest of this section will be devoted to some of the commonest forms in which they appear.

Fig. 3a shows a simple triangular truss consisting of a beam, AB, supported by a strut at the centre, the lower extremity of which is carried by tie rods, AC, BC, attached to the ends of the beams. If now a weight, W, be placed at the centre, immediately over the strut, it does not bend the beam (sensibly) as it would do if there were no strut, but is transmitted by the strut to the joint C, so that the truss is equivalent to the simple triangular frame of the last article. This, however, supposes that the strut has exactly the proper length to prevent any bending of the beam; if it be too short or too long the load on the frame will be less or greater than W, a point which will be further considered presently. It should be noticed that D is not necessarily at the centre.



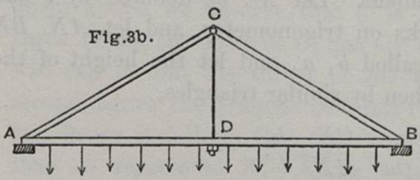


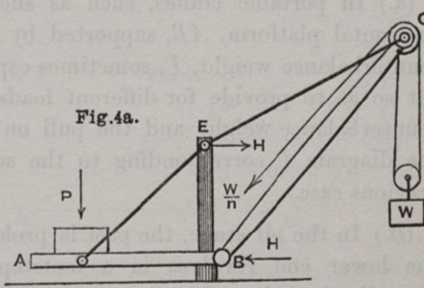
Fig. 3b shows the same construction inverted. CD is a tie by which D is suspended from C; we will suppose this rod to pass through AB and a nut applied below, by means of which D may be raised or lowered. Let AB now be uniformly loaded with a given weight, then the bending of AB is resisted by CD, which supports it and carries a part of the load, which may be made greater or less by turning the nut. If, however, we imagine AB, instead of being continuous through D, to be jointed at D, then the tie CD necessarily carries half the weight of AD and half the weight of BD, that is to say, half the whole load, whatever be its exact length. This simple example illustrates very well the most important difference between a truss and a mathematical frame; namely, that in the truss one or more of the bars is very often continuous through a joint. Such cases can only be dealt with on the principles of the present division of our work, by making the supposition that the bar in question, instead of being continuous, is jointed like the rest. The error of such a supposition will be considered hereafter; it is sufficient now to say that in order that it may be exact in the particular case we are considering, the nut must be somewhat slackened out so that D may be below the straight line AB, and that being dependent on accuracy of construction, temperature, and other varying circumstances, such errors cannot be precisely stated, but must be allowed for in designing the structure by the use of a factor of safety. The supposition is one which is usual in practical calculations, and will be made throughout this division of our work.

The foregoing is one of the simplest cases where, as is very common in practice, the bars of the frame are loaded and not the joints alone. When such bars are horizontal and uniformly loaded, the effect is evidently the same as if half the load on each division of the loaded bar were carried at each of the joints through which it passes. This is also true if the loaded bars be not horizontal, but the question then requires a much more full discussion, which is reserved for a later chapter (see Ch. IV.).

When one of the joints of the loaded bar is a point of support, like A in Fig. 3, the supporting force is due partly to the half weight of one or more divisions of the loaded bar, and partly to the downward pull or thrust of other bars meeting there: the first of these causes does not affect the stress on the different parts of the truss, and the calculations are therefore made without any regard to it. The explanations given in this article should be carefully considered, as they apply to many of the examples subsequently given.

The triangular truss in both the forms given in this article is frequently employed in roofs and bridges of small spans, as well as for other purposes.

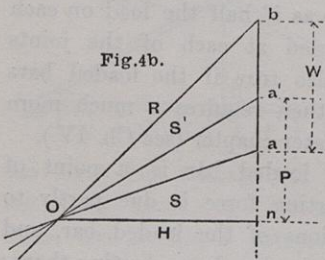
4. *Cranes.*—The arrangements adopted for raising and moving weights furnish many interesting examples of triangular frames. Fig. 4a shows one of the forms of the common crane, a machine the essential members of which are the jib, BC, supported by a stay, CE, attached to the crane-post, BE, which is vertical. In cranes proper this third member rotates, carrying BC and CE with it, but in the sailors' derrick a fixed mast plays the part of a crane-post,



and the stay, CE, is a lashing of rope frequently capable of being lengthened and shortened by suitable tackle, so as to raise and lower the jib, a motion very common in cranes and hence called a derrick motion. The weight is generally also capable of being raised and lowered directly by blocks and tackle, but for the present will be supposed directly suspended from C.

The diagram of forces now assumes the form shown in Fig. 4b, in which the lettering is the same as in Fig. 2b, page 4, the only difference in the diagrams being that in the present case AC, which is now a tie, is divided into two parts, AE and EC, inclined at an angle. The stress on AE is therefore not the same as on EC, but is got by drawing a third line, Oa', parallel to AE. The perpendicular On gives us in this instance not only the stress on AB and the horizontal thrust of CB at B, but also the horizontal pull of CE at E—we may call this H as before. There is an upsetting moment on the structure as a whole which is equal to the product of the weight W by its horizontal distance from B (often called the radius of the crane) and also to the force H, multiplied by the length of the crane-post, BE. One principal difference between different types of cranes lies in the way in which this upsetting moment is provided against.

Fig.4b.

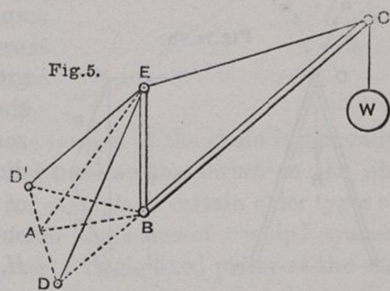


(α.) In portable cranes, such as shown in Fig. 4a, there is a horizontal platform, AB, supported by a stay, AE, and carrying a counterbalance weight, P, sometimes capable of being moved in and out so as to provide for different loads. The right magnitude of counterbalance weight and the pull on the stay AE are shown by the diagram P corresponding to the supporting force at A in the previous case.

(β.) In the pit crane, the post is prolonged below into a well and the lower end revolves in a footstep, the upper bearing being immediately below B. In this instance the post has to be made strong enough to resist a bending action at B, equal to the upsetting moment, and the bearings have to resist a horizontal force equal to H multiplied by the ratio of the length of the crane-post, BE, to that of its prolongation below the ground.

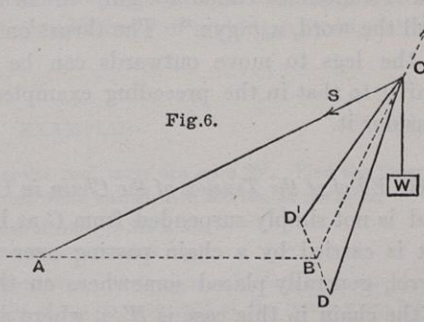
(γ.) The upper end of the crane-post may revolve in a headpiece, which is supported by a pair of stays anchored to fixed points in the ground. The upright mast of a derrick frequently requiring support in the same way, this arrangement is known as a derrick-crane. It is shown in Fig. 5, ED, ED' being the stays. To find the stress on the stays it is necessary to prolong the vertical plane through EC, to intersect the line DD', joining the feet of the stays in the point A, and imagine the two stays, ED, ED', replaced by a single stay EA: then a diagram of forces, drawn as in the previous case, determines S', the pull on this stay. But it is clear that S' must be the resultant pull on the two original stays, and may be considered as a force applied at E in the direction of AE to the simple triangular frame DED'. A second diagram of forces therefore will determine the pull on each stay, just as in the next following case.

Fig.5.



5. *Sheer Legs and Tripods.*—Instead of employing an upright post to give the necessary lateral stability to the triangle, one of its members may be separated into two. Thus in moving very heavy weights sheer legs are used, the name being said to be derived from their resemblance to a gigantic pair of scissors (shears) partly opened and standing on their points. In Fig. 6, CD, CD' are spars, or tubular struts, often of great length, resting on the ground at DD' and united at C, so as to be capable of turning together about DD' as an axis. The load is carried at C and the legs are supported by a stay, CA,

Fig.6.



which is sometimes replaced by a rope and tackle, capable of being lengthened or shortened so as to raise or lower the sheers. Drawing AB to the middle point of DD', the pair of legs are to be imagined replaced by a single one, CB, then the diagram of forces may be constructed just as in Fig. 4b, and we shall obtain the tension of the rope S and the resultant thrust on the pair of legs R. Now draw the triangle CDD', as in Fig. 7a, and imagine it loaded at C with a weight, R, then drawing the diagram of forces, Fig. 7*b*, we get R' the thrust on each leg. The horizontal force, H', in this second diagram represents the tendency of the feet of the legs to spread outwards laterally, while the force, H, of the original diagram represents their tendency to move inwards perpendicular to DD'. In some cases the guy rope and tackle CA are replaced by a third leg called the back leg, and the sheers are then raised and lowered by moving A by a large screw; the force H is then also the force to be overcome in turning the screw.

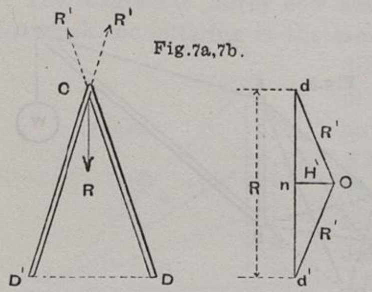


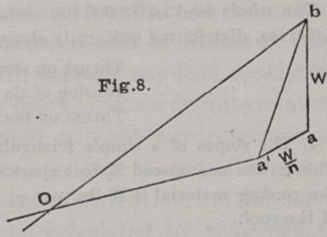
Fig. 7a, 7b.

Instead of having only two legs, as in sheers, we may have three forming a tripod. This arrangement is frequently used to obtain a fixed point of attachment for the tackle required to raise a weight, and is sometimes called a "gin," or as military engineers prefer to spell the word, a "gyn." The thrust on each leg and the tendency of the legs to move outwards can be obtained by a process so similar to that in the preceding examples that we need not further consider it.

6. *Effect of the Tension of the Chain in Cranes.*—In most cases the load is not simply suspended from C as has been hitherto supposed, but is carried by a chain passing over pullies and led to a chain barrel, generally placed somewhere on the crane-post. The tension of the chain in this case is W/n, where n is a number depending on the nature of the tackle, and this tension is to be considered as an additional force applied at C to be compounded with the load W, the effect of which has been previously considered. Fig. 8 shows the

form the diagram of forces assumes in this case. Drawing ba as before to represent W, and aa' parallel to the direction in which the chain is led off from the pulley at C and equal to the tension W/n, the third side of the triangle, ba' must be the resultant force at C due to both forces, whence drawing $a'O$ parallel to the stay and bO parallel to the jib, and reasoning as before as to the equilibrium of the forces at C, we see that these lines must be the tension of the stay and the thrust on the jib. The effect of the tension of the chain is generally to diminish the pull on the stay and increase the thrust on the jib, sometimes very considerably, as for example in certain older types of crane still used for light loads under the name of "whip" cranes. In these cranes the chain passes over a single fixed pulley at the end of the jib, and is attached directly to the weight, so that the tension of the chain is equal to the weight. The other end of the chain is led off along a horizontal stay to a wheel and axle at the top of the crane post, a chain from the wheel of which passes to a windlass below. This arrangement, the double windlass of which facilitates changes in the lifting power corresponding to the load to be raised, is a development of the primitive machine in which the wheel was a tread wheel worked by men or animal power. In this case the pull on the stay is diminished by the whole weight lifted, and is thus reduced very much. Where a crane has to be constructed of timber only, this is a considerable advantage, from the difficulty of making a strong tension joint in this material.

Fig. 8.



EXAMPLES

1. The slopes of a simple triangular roof truss are each 30°. Find the thrust of the roof and the stress on each rafter when loaded with 250 lbs. at the apex.

Thrust of roof = 216·5 lbs.
Stress on rafters = 250 ,,

2. A beam 15 feet long is trussed with iron tension rods, forming a simple triangular truss 2 feet deep. Find the stress on each part of the frame when loaded with 2 tons in the middle.

Thrust on strut = 2 tons.
Pull of tension rods = 3·88 ,,
Thrust on beam = 3·75 ,,

3. The platform of a foot bridge is 20 feet span, and 6 feet broad, and carries a load of 100 lbs. per sq. ft. of platform. It is supported by a pair of triangular trusses each 3 feet deep, one on each side of the bridge. Find the stress on each part of one of the trusses.

The whole load of 12,000 lbs. rests equally on the two trusses, there is therefore 6,000 lbs. distributed uniformly along the horizontal beam of each truss.

Thrust on strut	= 3,000 lbs.
Tension of tie rods	= 5,220 ,,
Thrust on horizontal beam	= 5,000 ,,

4. The slopes of a simple triangular roof truss are 30° and 45° and span 10 feet. The rafters are spaced $2\frac{1}{2}$ feet apart along the length of the wall, and the weight of the roofing material is 20 lbs. per sq. ft. Find by graphical construction the thrust of the roof.

Each rafter carries a strip of roof $2\frac{1}{2}$ feet wide, the load on rafter = 50 lbs. per foot length of rafter. Find the lengths by construction or otherwise. The virtual load at apex = $\frac{1}{2}$ weight on the two rafters = 311 lbs.

Thrust of roof = 198 lbs.

5. The jib AC of a ten-ton crane is inclined at 45° to the vertical, and the tension rod BC at an angle of 60°. Find the thrust of the jib, and the pull of the tie rod when fully loaded, the tension of the chain being neglected. If a back stay BD be added inclined at 45°, and attached to the end of a horizontal strut AD, find the counterbalance weight required at D to balance the load on the crane, and find also the tension of the back stay.

Thrust on jib AC	= 33·5 tons.
Tension of tie rod	= 27·5 ,,
Counterbalance weight	= 23·5 ,,
Tension of back stay	= 33·5 ,,

6. A pair of sheer legs are 40 feet high when standing upright, the lower extremities rest on the ground 20 feet apart, the legs stand 12 feet out of the perpendicular, and are supported by a guy rope attached to a point 60 feet distant from the middle point of the feet. Find the thrust on each leg, and the tension of the guy rope under a load of 30 tons.

Thrust on each leg	= 19·5 tons.
Tension of guy rope	= 12·8 ,,

7. In example 5 the tension of the chain is half the load, and the chain barrel is so placed that the chain bisects the crane post AB. Find the stress on the jib and tie rod.

Thrust of jib	= 36 tons.
Pull of tie rod	= 25 ,,

8. In a derrick crane the projections of the stays on the ground form a right-angled triangle, each of the equal sides of which is equal to the crane post. The jib is inclined at 45° and the stay at 60° to the vertical. Find the stress on all the parts (1) when the plane of the jib bisects the angle between the stays; (2) when it is moved through 90° from its first position. Load 3 tons.

9. A load of 7 tons is suspended from a tripod, the legs of which are of equal length and inclined at 60° to the horizontal. Find the thrust on each leg. If a horizontal force of 5 tons be applied at the summit of the tripod in such a way as to produce the greatest possible thrust on one leg, find that thrust and determine the stress on the other two legs.

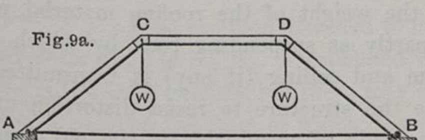
SECTION II.—INCOMPLETE FRAMES.

7. *Preliminary Remarks.*—A frame may have just enough bars and no more to enable it to preserve its shape under all circumstances, or the number of bars may be insufficient or there may be redundant bars. The distinction between these three classes of frames is very important: in the first the structure will support any load consistent with strength, and the stress on each bar bears a certain definite relation to the load, so that it can be calculated without any reference to the material or mode of construction; in the second, the frame assumes different forms according to the distribution of the load, but the stress on each bar can still be calculated by reference to statical considerations alone; in the third, where the frame has redundant bars, the stress on some or all of the bars depends on the relative yielding of the several bars of the frame. It is to the second class, which may be called incomplete frames, that the present section will be devoted.

In incomplete frames the structure changes its form for every distribution of the load, and, strictly speaking, therefore such constructions cannot be employed in practice, because the distribution of the load is always variable to a greater or less extent. But when the greater part of the load is distributed in some definite way the principal part of the structure may consist of an incomplete frame, designed for the particular distribution in question, and subsequent moderate variations of distribution may be provided for either by stiffening the joints or by subsidiary bracing. Such cases are common in practice, and investigations relating to incomplete frames are therefore of much importance.

8. *Simple Trapezoidal or Queen Truss.*—We will first consider a frame which is composed of four bars. The most common case

Fig.9a.



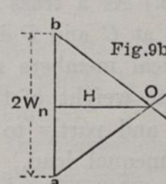


Fig.9b.

is that in which two of the bars are horizontal and the other two equal to one another, thus forming a trapezoid. The structure is called a *trapezoidal frame.*

It is suitable for carrying weights applied at the joints CD, either directly or by transmission through vertical suspending rods from the beam AB. From the symmetry of the figure it is evidently necessary for stability that the loads at C and D should be equal. This fact will also appear from the investigation. Consider first the joint C, and draw the triangle of forces, Oan, for that point; an being taken to represent W, aO will represent the thrust on AC and On that along CD. The triangle Obn will represent the forces at the joint D, Ob representing the thrust of BD; bn will represent the load at D, and from the symmetry of the figure must equal an, and hence weight at D must for equilibrium equal that at C. Now let us proceed to joint A, where there are also three forces acting, one along AC is now known and represented by aO, thus On will represent the tension of AB, and nb will be the necessary supporting force at A equal to W, as might be expected. The tension of AB is equal to the thrust on CD. We observe that the diagram of forces is the same as that of a triangular frame, carrying $2W$ at the vertex and of span equal to the difference between AB and CD.

Trapezoidal frames are employed in practice for various purposes.

(α.) A beam, AB (Fig. 10a), loaded throughout its length may be strengthened by suspending pieces, CN, OM, transmitting a part

Fig. 10a.

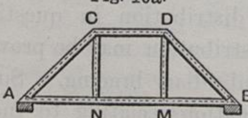
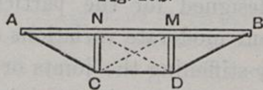


Fig. 10b.



of the weight to the arch of bars AC, CD, BD, an arrangement common in small bridges.

(β.) As a truss for roofs, in which case there will be a direct load at C and D due to the weight of the roofing material, while vertical members serve partly as suspending rods by which part of the weight of tie beam and ceiling (if any) is transmitted to CD, and partly to enable the structure to resist distortion under an unequal load. When made of wood this is the old form of roof called by carpenters a "Queen Truss," CN, DM, being the "queen posts" (see Section III. of this chapter). This name is constantly used for all forms of trapezoidal truss erect or inverted which include the vertical "queens."

(γ.) Not less common is the inverted form, Fig. 10b, applied to the beams carrying a traversing crane, the cross girders which rest on the main girders of a railway bridge and carry the roadway, and many other purposes. The bars AC, CD, BD are now iron tie rods. In this case also if the two halves of the beam are unequally loaded there will be a tendency to distortion, to resist which completely, diagonal braces, CM, DN, must be provided, as shown in the figure by dotted lines. Such diagonal bars occur continually in framework, and their function will be fully considered in the next chapter. But in the present case they are quite as often omitted, the heavy half of the beam then bends downwards and the light half bends upwards (see Ex. 4, p. 97), but the resistance of the beam to bending is found to give sufficient stiffness.

9. *General case of a Funicular Polygon under a Vertical Load. Example of Mansard Roof.*—We next take a general case. In Fig. 11a

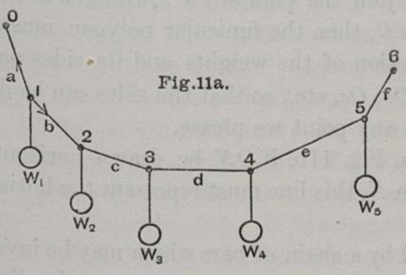


Fig.11a.

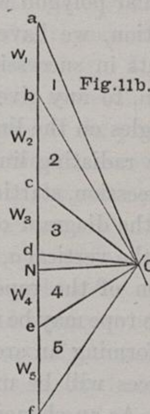


Fig.11b.

0 1 2 3 ... 6 is a rope or chain attached to fixed points at its ends and loaded with weights W_1 W_2... suspended from the points 1, 2, etc. The figure shows 5 weights, but there may be any number. The rope hangs in a polygon the form of which depends on the proportions between the weights. It is often called a "funicular polygon" and possesses very important properties. We shall find it convenient to distinguish the sides of this polygon by letters a, b, c, etc. We are about to determine the proportions between the weights when the

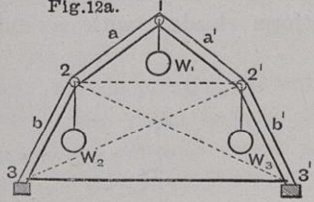
rope hangs in a given form, and, conversely, the form of the rope when the weights are given. In Fig. 11b draw ab vertical to represent W_1, the load suspended at the angle of the polygon where the sides a and b meet, then draw aO, bO parallel to a, b respectively to meet in O, thus forming a triangle Oab, which we distinguish by the number 1, which represents the forces acting on the point 1, so that the tensions of the sides a, b are thus determined. Now draw Oc parallel to the side c to meet the vertical in c; we thus obtain a triangle distinguished by the number 2, which represents the forces acting at that point, and as Ob is already known to be the tension of b it follows that bc must be the weight W_2, and Oc the tension of the side c. Proceeding in this way we get as many triangles as there are weights, and the sides of these triangles must represent the weights and the tensions of the parts of the rope to which they are respectively parallel. Thus, if the form of the rope is known and one of the weights, all the rest can be determined. Conversely, to find the form of the funicular polygon when the weights are given in magnitude and line of action, we have only to set downwards on a vertical line the weights in succession and join the points a b...,which will now be known, to any given point O, then the funicular polygon must have its angles on the lines of action of the weights and its sides parallel to the radiating lines Oa, Ob, Oc, etc., so that the sides can be drawn in succession, starting from any point we please.

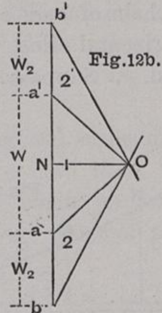
In the diagram of forces, Fig. 11b, if ON be drawn horizontal to meet the vertical a, b, c... in N, this line must represent the horizontal tension of the rope.

The rope may be replaced by a chain of bars which may be inverted, thus forming an arch resting on fixed points of support, the diagram of forces will be unaltered, and ON will represent the thrust of the arch. As an elementary example of an arch of bars we will consider a truss used for supporting a roof of double slope called a Mansard roof. We will take the usual case in which the truss is symmetrical about the centre. Suppose it is loaded at the joints. There is one proportion of load which the truss is able to carry without any bracing bars being added.

From symmetry the weights at 2 and 2′ (See Fig. 12a) must be equal. To find the proportion between the weights at 1, and at 2, 2′, together with the stresses on the bars of the frame, in Fig. 12b set down aa' to represent W at 1, and draw aO and $a'O$ parallel to a and

a', the thrusts along these bars will be determined. Then, considering the equilibrium of either 2 or 2′, say 2, one of the three forces acting at the joint, namely aO, along the bar a being known, the





other two forces may be determined by drawing ab and Ob parallel to them, ba parallel to W_2, and Ob to the bar b. If ON be drawn horizontally it will give the amount of the horizontal thrust of the roof or the tension of a tie bar 3 3′, if there is such a bar. If the proportion of W_2 to W_1 is greater than ab to aa^1 the structure will give way by collapsing, 2 and 2′ coming together; and if the proportion is less, the structure will give way by 2 and 2′ moving outwards and 1 falling down between. In practice it is impossible to secure the necessary proportion of loads, on account of variation of wind pressure and other forces, and therefore stiffening of some kind is always needed. If bracing bars be placed as shown by the dotted lines 2 3′, 2′3, 2 2′, the structure will stand whatever be the proportion between the loads. The truss may be partially braced by the horizontal bar 2 2′ only. Then the proportion between the loads W_1 and W_2 may be anything we please, but the loads at 2 and 2′ must be equal, at least theoretically, but in practice the stiffness of the joints will generally be sufficient for stability, especially if vertical pieces be added connecting these points to the tie beam as in a queen truss.

10. *Suspension Chains. Arches. Bowstring Girders.*—We now go on to consider another important example, in which the number of bars composing the frame is very much increased, as found in the common suspension bridge.

Let AB (Fig. 13a) be the platform of a bridge of some considerable span, which has little strength to resist bending. Suppose it divided into a number of equal parts, an odd number for convenience, say nine, and each point suspended by a vertical rod from a chain of bars secured at the ends to fixed points, D and E, in a horizontal line. In the figure only half the structure is shown. Suppose the platform loaded with a uniformly

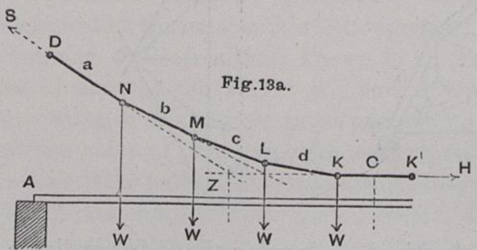


Fig. 13a.

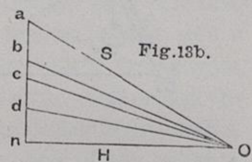


Fig. 13b.

distributed weight; we require to know the stress on each bar and the form in which the chain will hang. Equal weights on each division of the platform will produce equal tensions in the vertical suspending rods, and if we neglect the differences of weight of the rods and bars themselves, the load at each joint of the chain of bars will be the same. (Compare Art. 11.) Let W = load at each joint. Now the centre link, KK', since there is an odd number and the chain is symmetrical, will be horizontal. Let us consider the equilibrium of the half chain between C and D. The four weights, W, hanging at K, L, M, N, are sustained in equilibrium by the tensions of the bars KK' and ND.

The resultant of the four W's will act at the middle of the third division from the left end, and since this resultant load together with the tensions of the middle and extreme links maintain the half chain in equilibrium, the three forces must meet in a point, the point Z shown in the figure. Thus the direction of the extreme link DN may be drawn. The direction and position of the other links may be found also. Considering the portion of the chain NC carrying three weights, the resultant of which is in the line through L, the link NM must be in such a direction as to pass through the point where this resultant cuts KK' produced. Having drawn NM, ML may be drawn in a similar way, and then LK. Returning to the consideration of the half chain, the three forces which keep it in equilibrium may be represented by the three sides of a triangle. Set down an (Fig. 13b) to represent $4W$, and draw aO and nO parallel to DZ and ZC; aO will be the tension of DN and nO of KK'. If an be divided into 4 equal parts, and the points b, c, d, joined to O, these lines will represent the tensions of links NM, ML, and LK. It may be easily shown that they will be parallel to those links. We see that the tension increases as we pass from link to link, from the centre to the ends.

In many cases in practice, the number of vertical suspending rods and links in the chain is very great. We may then, in what follows, without sensible error, regard the chain as forming a continuous curve. In such a case, C, the lowest point of the chain

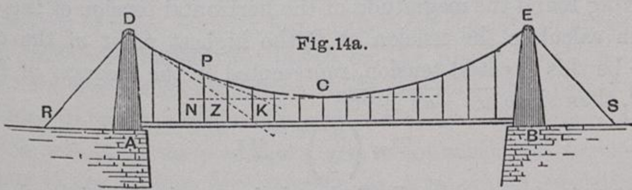


Fig.14a.

(Fig. 14a), is over the middle of the platform. The tangent at C, which is horizontal, will meet the tangent to the chain at D, in a point Z, which will be over the middle of the half platform, for that will be a point in the line of action of the resultant load on the half chain. We can now draw a triangle of forces anO, for the half chain as before; On will represent the tension of the chain at the lowest point, or the horizontal component of the tension of the chain at any point. We can easily obtain a convenient expression for this horizontal tension thus:—Let l = span of the bridge, and w = load per foot run. Then $\frac{1}{2}wl$ = weight on the half chain represented by an. Let H = horizontal tension, then

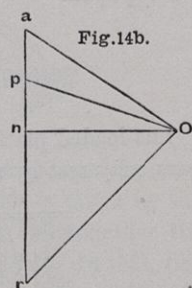


Fig.14b.

$$\frac{H}{\frac{1}{2}wl} = \frac{On}{an}.$$

But if we drop a perpendicular from D to cut the horizontal tangent in a point V (not shown in the figure), DV will be the dip of the chain d, and comparing the triangles DVZ, aOn,

$$\frac{On}{an} = \frac{VZ}{DV} = \frac{\frac{1}{4}l}{d} = \frac{\frac{1}{2}wl}{H}$$

$$\therefore H = \frac{1}{8} wl \frac{l}{d},$$

which, since wl = total load on chain, may be written

$$H = \frac{1}{8} \text{ load on chain } \frac{\text{span}}{\text{dip}},$$

This is the same as the horizontal thrust of a triangular frame of the same height which carries a uniformly distributed load of the same intensity.

Having found the magnitude of the horizontal tension of the chain we can calculate the tension at D, the highest point of the chain. Let S be this greatest tension, represented in the diagram of forces by aO, then since $\overline{aO}^2 = \overline{an}^2 + \overline{nO}^2$

$$S^2 = \left(\frac{W}{2}\right)^2 + H^2.$$

The tension at any point P of the chain may be found by drawing from O a line op parallel to the tangent to the chain at P. It will cut an in a point p such that $np : na ::$ length of platform below $PC : \frac{1}{2}$ span.

$$\text{Since } \overline{Op}^2 = \overline{np}^2 + \overline{On}^2$$

$$\text{Tension at } P = \sqrt{\left(\frac{np}{na}\frac{W}{2}\right)^2 + H^2}.$$

The loaded platform, instead of being suspended from the chain of bars, may rest by means of struts on an arch of bars as in the figure.

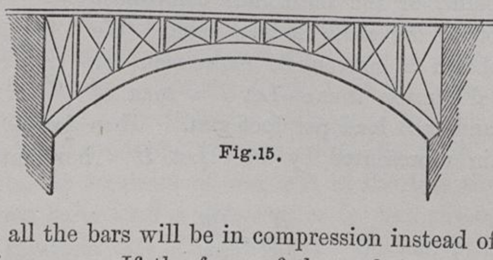


Fig.15.

In this case all the bars will be in compression instead of tension, as in the previous case. If the form of the arch is similar to that in which the chain hung, it will have no tendency to change its form under the load. There will be simple thrust of varying amount at different parts of the arch. The horizontal thrust at the top of the arch is given by the same expression as for the horizontal tension of the chain, and the thrust of any bar of the arch may be determined in a manner similar to that for finding the tension of any link of a chain. We shall show presently that the proper form of the arch and chain under a uniform load is a parabola. Hence, the structure just described is called a Parabolic Arch. In iron bridges the platform is not unfrequently carried by a number of ribs placed side by side. Each rib is approximately parabolic in form, usually of I. section, of depth from $\frac{1}{40}$th to $\frac{1}{60}$th the span at the crown, increasing somewhat towards the abutments. The roadway is supported sometimes by simple vertical struts, as in the ideal case just considered, sometimes by spandrils of more complex form, chiefly for the sake of appearance. When uniformly loaded, the stress on the ribs is nearly as found above: for resistance to variation in the load reliance is placed on the resistance to bending of the ribs and platform. The case of a stone or brick arch is far more complex, and is not considered here.

There is yet another very common structure whose construction is founded on the same principles as those just described. In this the

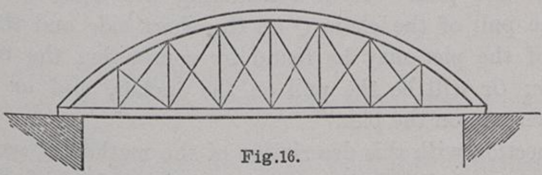


Fig.16.

platform, instead of resting on an arch below it, is suspended from an arch above it. In this case the thrust of the arch is taken by the platform, which serves as a tie, just as the string ties together the ends of a bow. Hence it is called a *Bowstring Girder*. In this, like the others, the loading proper to the parabolic form is a uniformly distributed one, and any variation of the loading will tend to distort the bow. The structure may, however, be enabled to sustain a varying load by the addition of bracing bars as shown by the diagonal lines. When the bridge is heavily loaded it will almost always happen that the greater part of the weight is uniformly distributed, and is sustained by simple thrust of the arch, so that the bracing is only a subsidiary part of the structure.

11. *Suspension Chains (continued). Bowstring Suspension Girder.*—In describing the suspension bridge we spoke of the chain as being secured at the ends to fixed points. In practice the securing of the ends is effected thus. The chain is led to the top of a pier of cast-iron or masonry, and instead of being simply attached to the top of the pier, and thus producing an enormous tendency to overturn the pier, the chain is secured to a saddle which rests on rollers on the top of the pier, and on the other side the chain is prolonged to the ground, passes through a tunnel for some little distance, and is finally secured by means of anchors to a heavy block of masonry. By this arrangement the only force acting on the pier is a purely vertical one, and a comparatively slender pier will be sufficient to sustain it. It is not necessary that the tension of the chain should be the same on each side of the pier, or that it should be inclined at the same angle. What is necessary is that the horizontal component of the tension on each side should be the same If an (Fig. 14b, page 19) = half weight on chain as before, and $On = H$, the horizontal tension (which may either be calculated from the formula just obtained, or found by construction), then aO will be the pull of the chain S at the top of the pier. Then considering the equilibrium of the saddle, the pull of the chain Q on the short side and the upward reaction of the pier may be found by completing the triangle of forces aOr; Or will be the pull on the anchor, and ar the total vertical pressure on the pier.

In connection with this description of the method of securing the ends of the suspension chain, we may mention a form of structure in which the arch and chain are combined, a good example of which occurs in the railway bridge at Saltash. The horizontal pull of the chain is here balanced by the thrust of an arch, so that the combined effect is to produce simply a vertical pressure on the piers. The suspending rods are secured to the chains and prolonged to the arch above, so that a portion of the load is carried by the arch, producing a thrust, and a portion by the chain, causing a pull. To prevent any tendency to overturn the piers, (this is insured by means of saddles resting on rollers) the horizontal component of the thrust of the arch must equal the horizontal component of the pull of the chain. The proportion between the loads on arch and chain will depend on the proportion between the rise of the arch and dip of the chain.

If W_1 = load on arch, and W_2 = load on chain,

d_1 = rise of arch, and d_2 = dip of chain,

then
$$H = \frac{W_1 l}{8d_1} = \frac{W_2 l}{8d_2}; \quad \therefore \frac{W_1}{W_2} = \frac{d_1}{d_2};$$

also
$$W_1 + W_2 = \text{total load on bridge}:$$

from which the stresses on the structure may be determined. It is known as a Bowstring Suspension Girder (pp. 47, 79).

We shall next show that the form of the curve of a chain carrying a uniformly loaded platform is a parabola. Referring to Fig. 14*a*, let P be any point in the chain, drop a perpendicular PN to meet the tangent at C, and bisect CN in K. Then KP must be the direction of the pull of the chain at P in order that the portion PC may be kept in equilibrium. The triangle PNK has its sides parallel to the three forces which act on PC, and the sides are therefore proportional to the forces. Let $CN = x$ so that the load hanging on $PC = Wx$, also let $PN = y$.

Then
$$\frac{H}{wx} = \frac{NK}{PN} = \frac{\frac{1}{2}x}{y}.$$

$$\therefore x^2 = \frac{2H}{w}y; \text{ or, since } H = \frac{wl^2}{8d},$$

$$x^2 = \frac{l^2}{4d}y;$$

therefore x^2 is proportional to y.

Now the curve whose co-ordinates have this relation one to another is called a *parabola*.

If the load, instead of being uniformly distributed on a horizontal platform, were simply due to the weight of the chain itself, then the curve in which the chain would hang would deviate somewhat from the parabola; for in that case, since the slope increases as we approach the piers, the load also, per horizontal foot, would increase as we approach the piers, causing the chain near the piers to sink and become more rounded, and at the centre to rise and become more flattened. The curve in which the chain hangs by its own weight is called the *catenary*. In the catenary, as in the parabola, the tension increases as we approach the piers. This may be taken account of by proportioning the section of the chain to the tension at the various points; this would tend still more to make the weight of chain, per horizontal foot, increase as we approach the piers, and cause the chain to deviate still further from the parabolic form. Such a curve is called the catenary of uniform strength.

In an actual suspension bridge, where there is a uniformly loaded platform, as well as a heavy chain, the true curve in which it hangs will lie somewhere between the parabola and the catenary; but since in most cases the deviation from uniformity of the weight of chain is small compared with the load it carries, the deviation from the parabola is not great. The error involved in assuming the curve to be parabolic is generally greatest in bridges of large span; in such cases a preliminary calculation of approximate weights may be necessary so as to be able to apply the general process of article 9.

EXAMPLES.

1. A trapezoidal truss is 16 feet span and 4 feet deep, the length of the upper bar is 6 feet. Find the stress on each part when loaded with 2 tons at each joint.

Stress on sloping bars = 3·2 tons,
,, horizontal ,, = 2·5 ,,

2. The platform of a bridge, 8 feet broad and 27 feet span, is loaded with 150 pounds per square foot. It is supported on each side by an inverted queen truss placed below, the queen posts, each 3 feet deep, dividing the span into three equal portions. Find the stress on each part.

Load on each truss = half whole load on platform = 162,000.
$\frac{1}{3}$ 16,200 = 5,400 is the load at each of the two joints of one of the queen trusses.
Tension of sloping bars = 17,074 lbs.
Tension and thrust of horizontal bars = 16,200.

3. The height of a mansard roof without bracing is 10 feet and span 14 feet. The height of the triangular upper portion is 4 feet and span 8 feet. The load being 1 ton at the ridge, find the necessary load at each intermediate joint and the thrust of the roof.

By the construction described in the text, load at each intermediate joint = $\frac{1}{2}$ ton, and the thrust of the roof = $\frac{1}{2}$ ton.

4. If the roof in the last question be partly braced by a bar joining the intermediate joints, find the stress on the bar when the load at each intermediate joint is 1 ton.

Thrust on bar = $\frac{1}{4}$ ton.

5. The load on the platform of a suspension bridge, 600 feet span, is $\frac{1}{2}$ ton per foot run, inclusive of chains and suspending roads. The dip is $\frac{1}{13}$th the span. Find the greatest and least tensions of one of the chains.

Least tension = horizontal tension = $243\frac{3}{4}$ tons.
Greatest tension = 255 tons.

6. The load on a simple parabolic arch, 200 feet span and 20 feet rise, is 360 tons, determine the thrust and greatest stress on the arch.

Thrust = 450 tons; greatest stress = 484 tons.

7. The rise of a bowstring bridge is 15 feet and span 120 feet, find the thrust when loaded with 2,000 lbs. per foot run.

Thrust 240,000 lbs. = $107\frac{1}{7}$ tons.

8. In example 5 the ends of the chain are attached to saddles resting on rollers on the tops of piers 50 feet high, and prolonged to reach the ground at points 50 feet distant from the bottoms of the piers, where they are anchored. Find the load on the piers and the pull on the anchors.

Load on the pier = $637\frac{1}{2}$ tons;
Pull on each anchor = 344·6 tons.

9. A light suspension bridge is to be constructed to carry a path 8 feet broad over a channel 63 feet wide by means of 6 equidistant suspending rods, the dip to be 7 feet. Find the lengths of the successive links of the chain. Supposing a load of 1 cwt. per square foot of platform, find the sectional areas of the links of the chain, allowing a stress of 4 tons per square inch.

$\frac{6}{7}$ of the whole load is carried by the chains and the remaining portion by the piers directly. Tension of each suspending rod = 36 cwt.

Links.	Tensions.	Areas.	Lengths.
Centre	277·7	3·47	9·
2nd	280·	3·5	9·08
3rd	287·	3·6	9·3
4th	298·	3·72	9·66

10. Construct a parabolic arch, the thrust of which is half the total load.

Span = four times the rise.

11. If the weight of a uniformly loaded platform be suspended from a chain by vertical rods, show that the corners of the funicular polygon lie on a parabola.

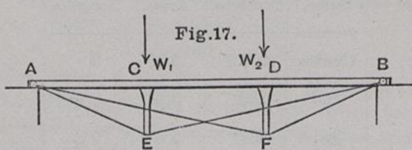
SECTION III.—COMPOUND FRAMES.

12. *Compound Triangular Frames for Bridge Trusses.* By a compound frame is meant a frame formed from two or more simple frames by uniting two or more bars. Many frames of common occurrence in practice may conveniently be considered as combinations of the simpler examples already described. They are generally dealt with by use of what we may call the principle of superposition, which may be thus stated:—*The stress on any bar due to any total load is the algebraical sum of the stresses due to the several parts of the load.*

We will now consider some examples of compound frames, which are used in bridge trusses. In these structures the object is to carry a distributed load by means of a comparatively slender beam. A prop in the centre may still leave the halves too weak to carry the weight on them, and the beam may be strengthened by supporting it in more than one point.

(1) Suppose the beam supported by a number of equidistant struts, the lower ends of which are carried by tension rods attached to the ends of the beam, we then have a structure called a *Bollman truss*. There may be any number of struts—2, 3, 4, or more; the structure has been used for bridges of comparatively large span. If the actual load is distributed in some manner over the beam, we must first reduce the case to that of a structure loaded at the joints only. The loads on the struts are due to the weights resting on the adjacent divisions of the beam, and may be determined by supposing the beam broken or jointed at the points where the struts are applied.

Fig.17.



Let us suppose the beam has three divisions, and that the load on the two struts are W_1 and W_2. These loads will be transmitted down the struts to the apices (Fig. 17) E and F, and will be independently supported, each by its own pair of tension rods. We may thus separately determine the stress on each part of either of the elementary triangular frames AEB or AFB. AB will be in compression on account both of the load at E and also at F. On account of W_1, using the formula previously obtained, the horizontal thrust $H_E = W_1 \dfrac{a'b'}{lh}$, and on account of W_2 at F, $H_F = W_2 \dfrac{a'b'}{lh}$.

$$\text{Tension of } AE,\ T_{AE} = H_E \sec EAB, \qquad T_{FB} = H_F \sec FBA;$$
$$\text{,,} \qquad EB,\ T_{EB} = H_E \sec EBC, \qquad T_{AF} = H_F \sec FAD.$$

The actual tensions of the sloping rods are simply as written, but since AB is a part of both triangular frames, the total thrust along it is found by summing the thrusts due to each; so

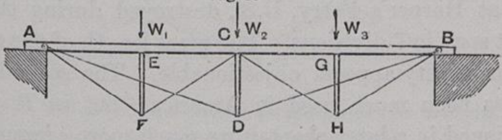
$$H = H_E + H_F.$$

This is an example of the *principle of superposition* stated above.

(2) Suppose the beam which carries the distributed load to be supported by a central strut forming a simple triangular truss, and further let the halves of the beam, not being strong enough to carry the load on them, each be subdivided and trussed by a simple triangular truss, the tension rods from the bottom of the subdividing struts proceeding only to the ends of each half beam. If the quarter spans are still too great, they may each of them be trussed in a similar way, and so on. Such a structure is called a *Finck truss.*

Suppose, for example, we have three struts. (Fig. 18.) We must first determine the load at the joints—that is, in this case the load on the struts due to the distributed load on the beam. Suppose that on account of the weights on the *adjacent* subdivisions those loads are W_1, W_2, W_3. If the load is uniformly distributed over the beam the W's are each of them equal to $\frac{1}{4}$ total weight on beam.

Fig. 18



We may now separately consider the triangular frame AFC carrying the load W_1. On account of it there will be a thrust on AC

$$H_F = W_1 \frac{AC}{4h} = W_1 \frac{l}{8h}.$$

The tensions of AF and FC are each $= H_F \sec FAE$. We get similar results from the triangle CHB. Just in the same way we may consider the principal triangular frame ADB, but in this case the thrust down the strut CD, which is the load at D, is not simply W_2, but greater by the amount of the downward pull of the two tension rods CF and CH. The vertical components of these tensions are $\frac{1}{2}W_1$ and $\frac{1}{2}W_3$, so that the total thrust down the strut $= W_2 + \frac{1}{2}(W_1 + W_3)$. This is the load which must be taken to act at D in determining the stresses on the members of ADB.

Thus $H_D = (W_2 + \frac{1}{2}W_1 + \frac{1}{2}W_3)\,\frac{l}{4h}$, and the tensions of AD and DB are each $= H_D \sec DAB$.

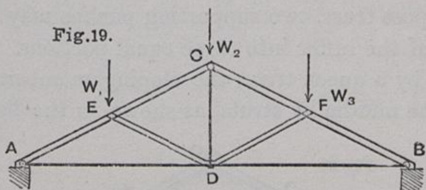
It will be seen that the thrust on the central strut and tensions of the longer rods are the same as if the secondary trusses had not been introduced. For example, if the W's each $= \frac{1}{4}$ whole load on beam, then the virtual load at $D = \frac{1}{2}$ weight on beam. The mere strengthening of each half the beam by trussing it can no more relieve the central strut of the load it has to carry, than the fact of strengthening a structure of any kind can relieve the two points of support from the duty each must have of bearing its own proper share of the weight. In stating the thrust on the beam we must divide it into two portions AC and CB. The portion AC is subjected to the thrust of the triangles AFC and ADB; $\therefore H_{AC} = H_F + H_D$, and CB being a portion of the triangles CHB and ADB, $H_{CB} = H_H + H_D$. When W_3 is not equal to W_1, the thrusts on the two portions will be different. This is quite possible although the beam AB may be a continuous one.

Both these simple forms of truss have been used for bridges of considerable span. As an example of the first may be mentioned the bridge at Harper's Ferry, U.S., destroyed during the war. It was 124 feet span in 7 divisions. The great length of the tension rods and their inequality appears objectionable. The second in 8 or 16 divisions has been much used in America; but in England other forms mentioned in a later chapter are much more common.

13. *Roof Trusses in Timber.*—In roofs of small span, 10 or 12 feet only, the roofing material, slates or tiles, rests on a number of laths set lengthways to the roof, and these laths rest on sloping rafters spaced 1 or 2 feet apart, with their feet resting on the walls of the building; the stability of the walls being depended on for taking the thrust.

When we come to larger and more important roofs we find additional members added for strength and security. The closely spaced rafters just mentioned are called common rafters. These being too long and slender to carry the weight of the roofing material and transmit it to the walls, are supported, not only at the ends by the walls and ridge piece, but also at the middle by a longitudinal beam of wood called a *purlin*, and the purlin is supported at intervals of its length by principal rafters. The principal rafters again are supported by struts at their central points, immediately below the purlins. To carry the lower ends of the struts, a vertical tension piece is introduced, by which they are suspended from the apex of the principals, while the thrust is taken by a tie beam connecting the feet of the rafters. In such a roof, a ceiling or floor may frequently be required to be supported by the tie beam, and to prevent it from sagging under the weight an additional tension will come on the vertical suspending rod. This rod is then a very important member of the structure, and is called the king post, and the whole structure, consisting of the principal rafters, king post, &c., is called a *king post truss*. This truss is often constructed entirely of wood. The sloping struts then for constructive reasons (Ch. xv.) butt on an enlarged part at the bottom of the king post above the point where the horizontal tie beam is attached, but for calculation purposes may be regarded as meeting at that point as shown in Fig. 19.

Fig.19.



By means of the purlins and the ridge piece the weight of the roofing material will produce loads at the joints $ECF = W_1 W_2 W_3$ suppose.

Now treat the structure as made up of three simple triangular frames AED, DFB, and ACB. First consider AED with the load W_1 at vertex E. The horizontal thrust of this frame $H_E = W_1 \dfrac{AD}{4h}$ where h is the height of point E above AD. Also the thrust along AE and ED due to the load at $E = H_E \sec EAD$. In an exactly similar manner we may consider the triangle DFB; the results for this will be to those for AED in the proportion of W_3 to W_1. Next as to the primary triangle ACB. There is at C a direct load of W_2 due to the weight between E and C, and F and C. But besides this, the king post pulls the point C downwards, so that the total load at $C = W_2 +$ tension of king post. In addition to a portion of the weight of the ceiling (if any) the post has to support D against the downward thrust of the two struts ED and FD. The vertical components of these thrusts are $\frac{1}{2}W_1$ and $\frac{1}{2}W_3$, therefore, neglecting the weight of ceiling, the virtual load at $C = W_2 + \frac{1}{2}(W_1 + W_3)$. Let us call the total load W, then H_C the horizontal thrust of $ACB = W\dfrac{AB}{4CD}$ and the thrusts along AC and CB due to load at $C = H_C \sec A$.

Now in the complete structure, since AD is a member both of the triangular frame AED and ACB, the total tension of $AD = H_E + H_C$. For the same reason tension of $DB = H_F + H_C$,

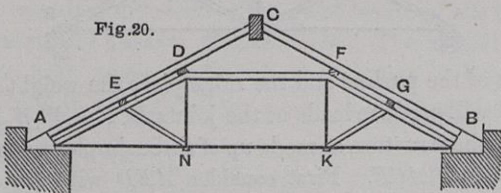
and thrust of $AE = (H_E + H_C) \sec A$,

" " $FB = (H_F + H_C) \sec A$.

The other members of the structure are portions of one elementary frame only, and the stress is due only to the load at the apex of that frame.

The king post truss serves for roofs of spans under 30 feet, but for spans greater than this trusses of more complicated construction are required. If the span is from 30 to 50 feet, then instead of supporting the common rafters by a purlin at the centre of its length only, as in the king post truss, two supporting purlins may be used, dividing the length of the rafter into three equal portions. These purlins may be carried by a queen truss, the sloping members of which are supported in the middle by struts, as shown in the figure (Fig. 20).

Fig.20.



The vertical queen posts DN and FK serve to sustain the downward thrust of the struts EN and GK, and also to support the weight of a ceiling, if there is one. Supposing the weight of the ceiling omitted, let W be the weight of roofing material on one side for a length of roof equal to the spacing of the trusses, then $\frac{1}{3}W$ will, through the common rafters and purlins, act at E, and $\frac{1}{3}$ at D; and similarly for the other side. At the ridge C there will also be $\frac{1}{3}W$ acting; but this will be distributed equally amongst the common rafters which are carried by the truss, and will produce compression in those rafters without directly affecting the truss. The part of the thrust of the roof arising from this will, however, generally, like the rest, ultimately come on the principal tie beams.

To find the stresses on the different members of the truss. Consider first the small triangles AEN and BGK, each carrying $\frac{1}{3}W$ at the vertex. We then consider the trapezoidal truss $ADFB$. The loads at D and F will be $\frac{1}{3}W$ + tension of queen post. Since the tension of the queen post DN = the vertical component of the thrust along EN it will equal $\frac{1}{2} \cdot \frac{1}{3}W = \frac{1}{6}W$, and the total load at each joint of the trapezoidal truss will be $\frac{1}{3}W + \frac{1}{6}W = \frac{1}{2}W$, the same as would have acted if there had been no purlin at E and no strut EN. After having determined the respective stresses due to the triangles and trapezoid separately, we must add the results for any bar which is a part of both. Were it not for the friction at the joints and the power of resistance of the continuous rafters AC, CB to bending, this structure would be stable only under a symmetrical load. In practice, however, it is able to sustain an unsymmetrical load, such as roofs are frequently subjected to.

14. *Queen Truss for large Iron Roofs.*—As the span of the roof is still further increased we find other kinds of trusses employed to support them. A common form in iron roofs is constructed, as shown in Fig. 21. It is in reality a further development of

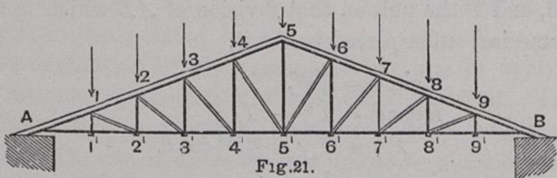


Fig. 21.

the wooden queen truss, and is known by the same name. AC and CB are divided into a number of equal parts, and sloping struts and vertical suspending rods are applied as shown. Suppose the load the same at each joint on one side of the roof, the load on the right, however, not being necessarily equal to that on the left. Let the upward supporting force at $A = P$. P will be $\frac{1}{2}$ total weight if the loading is symmetrical, but in any other case it may be found by taking moments of the loads about B. We might solve the problem of finding the stress on each member of the structure by treating separately each elementary triangle into which the structure may be divided, and summing the stresses for any bar which may form a part of two or more triangular frames. But we will describe another method.

First, to find the tension of the vertical suspending rods consider $A12'$ as an independent triangle, carrying a load W at its vertex. The slope of $12'$ being the same as that of $A1$, the tension rod $22'$

must supply a supporting force to the joint $2' = \frac{1}{2}W$. Considering next the triangle $A23'$ and its equilibrium about the point A. The forces along 23 and $3'4'$ have no moment about A, so that the moment of the two weights W at 1 and 2 about A must be balanced by the upward pull of the tension rod $33'$. $\therefore$ tension of $33' = W$.

In a similar way we can see that the tension of $44' = \frac{3}{2}W$. However many divisions of the roof there may be, the tensions of the vertical suspending rods will increase in arithmetical progression, with the same difference between each. The rod $11'$, except so far as may be due to the weight of the rod $A2'$, will have no tension on it. Calling this the 1^{st} tension rod, the tension of the n^{th} = $\frac{n-1}{2}W$. We must notice that the rod $55'$ is common to both sides of the roof, and we must add the two tensions to get the total. Now consider any joint, say $4'$ in the tie bar AB, and resolve vertically and horizontally. If R = thrust of $34'$, θ its inclination to the horizontal, and T the pull on that division of AB which is indicated by the numerical suffix placed below it,

$$R \sin\theta = \tfrac{3}{2}W,$$
$$R\cos\theta = T_{3'4'} - T_{4'5'};$$
$$\therefore\ T_{3'4'} - T_{4'5'} = \tfrac{3}{2}W\cot\theta.$$

But from figure $$\cot\theta = \tfrac{1}{3}\cot A;$$
$$\therefore\ T_{3'4'} - T_{4'5'} = \tfrac{1}{2}W\cot A.$$

Whichever joint we select we should find the same result—namely, that the difference between the tensions of two consecutive portions of the tie rod is a constant quantity = $\frac{1}{2}W\cot A$. So that these tensions are in arithmetical progression diminishing towards the centre.

If we call $A2'$ the 1^{st} division of tie rod, then for the joint between the $n-1^{th}$ and n^{th} we have

$$R\sin\theta = \frac{n-1}{2}W.$$

$$R\cos\theta = T_{n-1} - T_n, \text{ and } \cot\theta = \frac{1}{n-1}\cot A;$$

$$\therefore\ T_{n-1} - T_n = \tfrac{1}{2}W\cot A.$$

If $A1$ is the 1^{st} division of the rafter, then the thrust on the n^{th} division $= T_n \sec A$.

Now, the tension of the tie rod in the

$$1^{\text{st}} \text{ division} = P \cot A,$$
$$2^{\text{nd}} \quad ,, \quad = (P - \tfrac{1}{2}W) \cot A,$$
$$n^{\text{th}} \quad ,, \quad = \left(P - \frac{n-1}{2} W\right) \cot A.$$

The thrust on the n^{th} division of rafter $= \left(P - \dfrac{n-1}{2} W\right) \operatorname{cosec} A.$

The thrust on any strut may best be found by squaring and adding the two equations of equilibrium of the lower joint of it. We get

$$\text{Thrust of } n^{\text{th}} \text{ strut} = \frac{W}{2} \sqrt{n^2 + \cot^2 A}.$$

15. *Concluding Remarks—General Method of Constructing Diagrams of Forces.*—Cases of framework often occur which are much more complicated than those which we have hitherto considered, but if there are no redundant bars the stress on each part depends on statical principles only, without reference to the relative yielding of the several parts of the structure. Such cases may always be treated by use of the general principle stated in Art. 1, and we shall conclude this chapter by explaining briefly a graphical method of applying that principle invented by the late Professor Clerk Maxwell. The forces will be supposed all in one plane, and each of them will be supposed known, that is to say, if there be any unknown reactions at points of support they will be supposed previously found by a graphical or other process, from the consideration that the whole must form a set of forces in equilibrium. In Fig. 22a a frame is shown acted on by known forces PQR..., an ideal example is chosen which is better suited for the purpose of explaining the method than any case of common occurrence in practice. First seek out a joint where only two bars meet: there will usually be two such joints if there be no redundant bars in the frame, and in the present instance we will choose the joint where P acts. Distinguish all the triangles, making up the frame by letters A, B, C, &c., and place numbers or letters outside the frame, one for each bar. In Fig. 22b draw 18 parallel to the force P and representing it in magnitude, 8a parallel to 8, 1a parallel to 1, to intersect in the point a; then, as in previous examples, 8a, 1a represent the stress on the two bars to which they are parallel. Pass now to the joint where Q acts: this joint is chosen because only three bars meet there, on one of which we have just determined the stress; draw 12 parallel to Q and representing it, then ab parallel to the bar lying between the triangles A and B, and $2b$ parallel to the bar 2; we thus get a polygon $12ba$, the sides of which are parallel to the four forces acting at the joint where Q acts, while two of them represent two forces

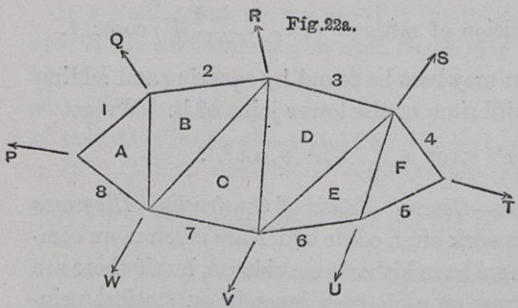


Fig. 22a.

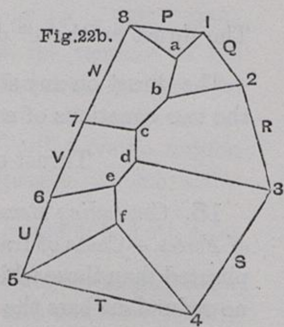


Fig. 22b.

already known, the other two, therefore, will represent the remaining two forces. Proceed now to the joint where W acts and complete in the same way the polygon $8abc7$, then to the joint where R acts, and so on. We at length arrive at the triangle $4f5$, the third side of which, if we have performed the construction accurately, and if the forces be really in equilibrium, must be parallel to the last force T. On examination of the diagram of forces (Fig. 22b) it will be seen that to every joint of the frame corresponds a polygon representing the forces at that joint, while each line, such as ab or $7c$, gives the stress on the bars separating those letters or numbers in the frame-diagram. The polygon 12...8 is the polygon of external forces, each side representing the force to which it is parallel.

The method here described is easy to understand in the general case we have considered, and with a little practice the transformations the diagram of forces undergoes will offer no difficulty. Some joints are usually unloaded, and the corresponding lines in the polygon of external forces vanish; the forces may be parallel, in which case the polygon becomes a straight line, while not unfrequently the sides of two of the polygons representing the forces at the joints coincide. The figure, however, always possesses the same properties.

In Mr. Bow's excellent work referred to at the end of this chapter over 200 examples will be found of the application of this method, including almost all known forms of bridge and roof trusses.

EXAMPLES.

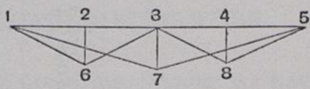
1. A Bollman truss of three divisions is 21 feet span, and is loaded uniformly with 1 ton per foot. The depth of the truss is $3\frac{1}{2}$ feet. Find the stress on each part.

Load on each strut $= 7$ tons,
Tension of short rods $= 10{\cdot}4$,,
,, longer ,, $= 9{\cdot}6$,,
Total thrust on beam $= 18\frac{2}{3}$,,

being $9\frac{1}{3}$ due to each triangle.

2. A Finck truss of 4 divisions, 20 feet span and 3 feet deep, is loaded with 1 ton per foot, find the stress on each part.

Thrust on 26 and 48 $= 5$ tons.
,, 37 $= 10$,,
Tensions of 16, 63, 38, and 85 $= 4{\cdot}86$,,
,, 17 and 75 $= 17{\cdot}4$,,
Thrust on 13 and 35 $= 4\frac{1}{6} + 16\frac{2}{3} = 20\frac{5}{6}$ tons.



3. In the last question suppose one half the truss loaded with an additional 1 ton per foot. Find the stress on each part.

Suppose the additional load on the right-hand side.

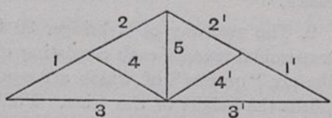
Thrusts.
On 26 = 5 tons.
,, 37 = 15 ,,
,, 48 = 10 ,,
,, $13 = 4\frac{1}{6} + 25 = 29\frac{1}{6}$.
,, $35 = 8\frac{1}{3} + 25 = 33\frac{1}{3}$.

Tensions.
On 16 and 63 $= 4{\cdot}86$ tons.
,, 38 ,, 85 $= 9{\cdot}72$,,
,, 17 ,, 75 $= 26{\cdot}1$,,

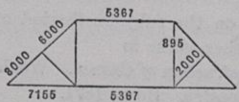
4. A roof 28 feet span, height 7 feet, rests on king-post trusses spaced 10 feet apart. The weight of roof is 20 lbs. per square foot. Find the stress on each part. Also obtain results when an additional load of 40 lbs. per square foot rests on one side.

Load at each joint. 1st case $= 1566{\cdot}6$ lbs.

Bars.	Stress in lbs.		Bars.	Stress.	
	Equal Load.	Additional Load.		Equal Load.	Additional Load.
1	5254	8756	1′	5254	12261
2	3503	7006	2′	3503	7006
3	4700	7833	3′	4700	10966
4	1752	1752	4′	1752	5255
5	1566·6	3113			



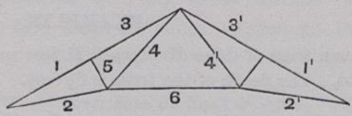
5. A roof 48 feet span, 12 feet high, rests on queen trusses 8 feet high, spaced 10 feet apart. Find the stresses for a load of 20 lbs. per square foot.



6. An A roof, braced as in the figure, is 40 feet span, and 10 feet high; the horizontal tie bar is 8 feet below the vertex. Find the stresses on each part

when loaded with 2 tons at each joint by constructing a diagram of forces or otherwise.

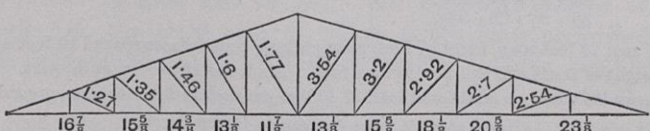
Bars.	Stress.
1	10·4
2	9·4
3	9·4
4	4·7
5	1·8
6	5·



7. In the last question suppose an accumulation of snow on one side equivalent to an additional load of 2 tons at the middle of the rafter, and 1 ton at the ridge. Find the stress on each part.

Bars.	Stress.	Bars.	Stress.
1	13·9	1′	17·3
2	12·5	2′	15·7
3	12·8	3′	15·4
4	5·5	4′	8·6
5	1·8	5′	3·6
6	7·5		

8. Suppose there are 11 suspending rods in iron roof shown in the figure, the height of which is $\frac{1}{10}$th the span. Find the stress on each part—1st, when loaded with $\frac{1}{2}$ ton at each joint on both sides, and, 2nd, when loaded with an additional $\frac{1}{2}$ ton at each joint on one side, not including the ridge.



Additional load is on right-hand side, and the figures on the diagram refer to case 2.

9. The roadway of a bridge, 80 feet span, is carried by a pair of compound trapezoidal trusses, each consisting of three simple trapezoids of the same height, the six "queens" of which are equidistant, forming six divisions of length four thirds the height of the truss. Find the stress on all the bars due to $\frac{1}{4}$ ton per foot run on the bridge.

10. Find the stress on each part of a "straight-link suspension" bridge formed by inverting the truss of the last question, assuming the pull at the centre of the platform zero.

REFERENCES.

For further information on the subjects treated of in the present chapter the reader may refer amongst other works to

GLYNN—*Construction of Cranes*. Weale's series.
HURST—*Carpentry*. Spon, 1871.
BOW—*Economics of Construction*. Spon, 1873.

CHAPTER II.

STRAINING ACTIONS ON A LOADED STRUCTURE.

16. *Preliminary Explanations.*—In the preceding chapter we have considered only those structures in which the parts are subject to compression and tension alone, except by way of anticipation in a few special cases. But the parts of a structure are generally subject to much more complex forces, and besides, although the forces acting on each bar have been determined, we should, if we stopped here, have a most imperfect idea of the way in which the load affects the structure as a whole.

If we imagine a structure to be made up of any two parts, *A* and *B*, united by joints, or distinguished by an ideal surface cutting through the structure in any direction, the whole of the forces acting on the structure may be separated into two sets, one of which acts on *A*, the other on *B*. Since the structure is in equilibrium as a whole, the two sets of forces must balance one another, and must therefore produce equal and opposite effects on *A* and *B*, effects which are counteracted by the union existing between the parts. The two sets of forces taken together constitute a STRAINING ACTION of which each set is an element, and the object of this and the next two chapters is to consider the straining actions to which loaded structures and parts of structures are subject.

Straining actions differ in kind, according to the nature of the effects which they tend to produce. Four simple cases may be distinguished :—

(1) The parts *A* and *B* may tend to move towards each other or away from each other perpendicular to a given plane. This effect is