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CHAPTER III.

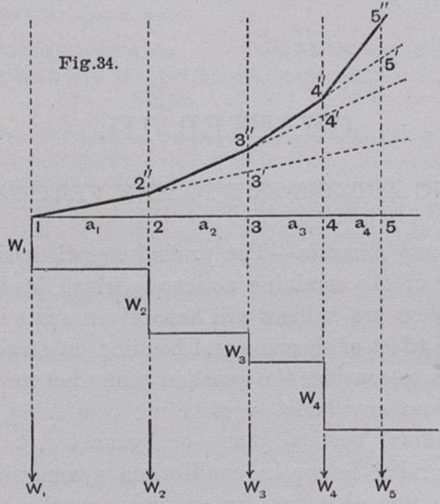
STRAINING ACTIONS DUE TO ANY VERTICAL LOAD.

28. *Preliminary Remarks.*—The preliminary discussion in the preceding chapter of the straining actions to which loaded beams and framework girders are subject will have given some idea of the importance of the effect of shearing and bending on structures, and we shall now go on to consider the question somewhat more generally.

Let us suppose any body or structure possessing, as it usually will, a longitudinal vertical plane of symmetry, to be acted on by a set of parallel forces in equilibrium symmetrically disposed with respect to this plane, as, for example, gravity combined with suitable vertical supporting forces. Then these forces will be equivalent to a set of parallel forces in the plane of symmetry in question. Let the structure now be divided into two parts, A and B, by an ideal plane section, parallel to the forces and perpendicular to their plane. Then the forces acting on A may be reduced to a single force F lying very near the section considered and a couple M, while the forces acting on B may be reduced to an equal and opposite force F lying very near the section and an equal and opposite couple M. The pair of forces are the elements of the shearing action on the section, and the pair of couples are the elements of the bending action on the section. As the nature of the structure is immaterial, we may consider these straining actions for a given vertical section quite independently of any particular structure, and describe them as the Shearing Force and Bending Moment *due* to the given Vertical Load. We shall first consider the connection which exists between the two kinds of straining action and the method of determining them for any possible load.

CONNECTION BETWEEN SHEARING AND BENDING.

29. *Relation between the Shearing Force and the Bending Moment.*— Figure 34 shows the lines of action of weights W_1, W_2, &c., placed at the successive intervals a_1, a_2, &c.



In the first division the shearing force is

$$F_1 = W_1;$$

in the second $\quad F_2 = W_1 + W_2 = F_1 + W_2,$

$$\therefore \quad F_2 - F_1 = W_2;$$

in the third $\quad F_3 = W_1 + W_2 + W_3 = F_2 + W_3,$

$$\therefore \quad F_3 - F_2 = W_3;$$

and so on for all the divisions, so that in the n^{th} division

$$F_n - F_{n-1} = W_n.$$

We express this in words by saying that *the difference between the shearing forces on two consecutive intervals is equal to the load applied at the point between the two intervals;* or it may be written

$$\Delta F = W.$$

By setting down ordinates to a horizontal base line we obtain the stepped figure as the graphical representation of the shearing force at any point of the beam. It is drawn by first setting downwards at 1 an ordinate for the shearing force on the 1st interval, and then passing along the beam to the other end, on meeting the lines of action of the successive weights the length of the ordinates is increased by the amount of the weights. In so doing we make use of the proposition which has just been proved.

This is called the *Polygon of Shearing Force*, or more generally, when the loads are continuous, the *Curve of Shearing Force*.

Next as to the bending moment. At the first point where W_1 is applied $M_1 = 0$,

at the second point $M_2 = W_1 a_1 = F_1 a_1$;

„ third „ $M_3 = W_1(a_1 + a_2) + W_2 a_2 = W_1 a_1 + (W_1 + W_2) a_2$
$= M_2 + F_2 a_2$,

$\therefore \quad M_3 - M_2 = F_2 a_2$;

„ fourth point $M_4 = W_1(a_1 + a_2 + a_3) + W_2(a_2 + a_3) + W_3 a_3$,
$= W_1(a_1 + a_2) + W_2 a_2 + (W_1 + W_2 + W_3) a_3$,
$= M_3 + F_3 a_3$,

$M_4 - M_3 = F_3 a_3$;

and generally, $M_n - M_{n-1} = F_{n-1} a_{n-1}$.

We may express this in words by saying that the *difference between the bending moments at the two ends of an interval is equal to the shearing force, multiplied by the length of the interval.* Or the result may be written

$$\Delta M = Fa.$$

We will now take a numerical example and see how we may make use of this property to determine a series of bending moments.

Let AB be a beam fixed at one end, and loaded with weights of 2, 3, 5, 11, 13, 7 tons, placed at intervals of 3, 2, 3, 5, 4, 6 feet,

W.	F.	a.	Fa.	M.
2				0
	2	3	6	
3				6
	5	2	10	
5				16
	10	3	30	
11				46
	21	5	105	
13				151
	34	4	136	
7				287
	41	6	246	
				533

commencing from the free end. We adopt a tabular method of carrying out the work of calculation.

First set down a column of weights applied, as shown by the figures in the column headed W. In the next column write the shearing forces. Since the shearing forces are uniform over the intervals between the weights, it will be best to write the F's opposite the spaces between the weights. Any F is found by adding to the F above it the adjacent W. In the third column we set down the lengths of the intervals. Then multiplying the F's and corresponding a's together, set the results in column 4. Lastly, we can write down the column of bending moments by the repeated addition of the Fa's—the bending moment at any point being found by adding to the bending moment at the point above the value of Fa between the points.

If instead of all the forces acting one way some of them act upwards, a minus sign should be set opposite, and all the operations performed algebraically.

The method is equally applicable however the beam is supported.

For example, let a beam 23 feet long be supported at the ends and loaded with 3, 2, 7, 8, 9 tons, placed at intervals of 2, 2, 3, 4, 5, 7 feet, reckoning from one end.

First calculate one supporting force, say at the left-hand end

W.	F.	a.	Fa.	M.
16·17				0
	16·17	2	32·34	
−3				32·34
	13·17	2	26·34	
−2				58·68
	11·17	3	33·51	
−7				92·19
	4·17	4	16·68	
−8				108·87
	−3·83	5	−19·15	
−9				89·72
	−12·83	7	−89·81	
12·83				0

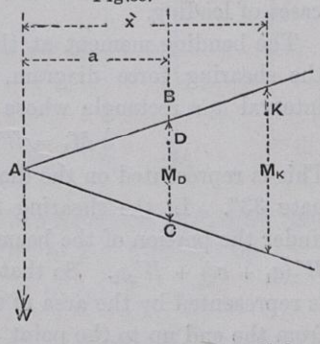
by taking moments about the other end. In the column of W's set this for the first force, and since all the loads act in the contrary direction, put negative signs opposite them, and in writing down the next column of F's add algebraically. We shall at the bottom of the column determine the supporting force at the right hand end. At the bottom of the column of M's, that is at the point where the right hand supporting force acts, we ought to get a zero moment. The obtaining of this will be a test of the accuracy of the work. In this example the small difference between 89·72 and 89·81 is due to our having taken the supporting force only to two places of decimals.

Observation of the process of calculation leads us to a very important proposition, viz., *where the shearing force changes sign, the bending moment is at that point a maximum.* This will be true for all important practical cases, but exceptional cases may be imagined in which, where the shearing force changes sign, the bending moment is a minimum. Since $\Delta M = Fa$, then, so long as F is positive, M will be an increasing quantity as we pass from point to point. But where F changes to negative there M commences to diminish.

We will now explain the construction of a diagram of bending moment for a system of loads: and first let us consider how the moment of a force about any point or succession of points may be graphically expressed.

Let W be a force and D any point, and suppose the numerical magnitude of the moment of W about D known. Draw a line through D parallel to the force at a distance a (Fig. 35), and anywhere in this line take a length BC to represent on some convenient scale the moment, $M_D = Wa$, of W about D. The scale must be so many inch-tons, foot-lbs., or similar units to the inch. Then choose any point A in the line of action of the force, join AB and AC, and produce these lines indefinitely. The moment of W about any point whatever is represented by the intercept by the radiating lines AB, AC of a line drawn through the point parallel to the force. For example, the moment about $K = M_K = Wx$, where x is the perpendicular distance of K from the line of action of W.

Fig.35.



$$\therefore \quad \frac{M_K}{M_D} = \frac{Wx}{Wa} = \frac{x}{a}.$$

By similar triangles the intercepts are to one another in the ratio $x : a$, so that they correctly represent the moments.

We will first draw the diagram of bending moments for a beam fixed at one end and loaded at intervals along its length.

Returning to Fig. 34, take a line representing the length of the beam as base line. Produce upwards the lines of action of the loads. Commence by setting up at the point where W_2 acts a line to represent the moment of W_1 about that point, that is, take $2'\,2$ to represent W_1a_1. If $1\,2'$ be joined and produced, then the intercept between this line and base line $1\,5$ will represent on the same scale the moment of W_1 about any point in the beam. Next at the point $3'$, where $12'$ cuts $W_3\,2'\,3'$, set up $3'\,3''$ to represent W_2a_2, join $2''\,3''$ and produce it. The intercept between $2'\,3'$ and $2''\,3''$ will represent the moment of W_2 about any point in the beam. Then at the point $4'$, where $2''\,3''$ cuts W_4, set up $4'\,4''$ to represent W_3a_3. Join $3''\,4''$, produce it, and so on with all the weights. The polygon $1, 2'', 3'', 4'', 5''$ ... will be obtained, the ordinates of which measured from the base line AB will represent the bending moment at any point, due to all the weights on the beam. This is called the *Polygon of Bending Moment.* In the case of a continuous distribution of load it is called the curve of bending moment.

There is a very important relation between the polygons of shearing force and bending moment which have been drawn in all cases of loading.

The bending moment at the point $2 = W_1a_1$. Now referring to the shearing force diagram, we observe standing underneath the interval a_1 a rectangle whose area $= W_1a_1$. Next, for the point 3,

$$M_2 = W_1(a_1 + a_2) + W_2a_2.$$

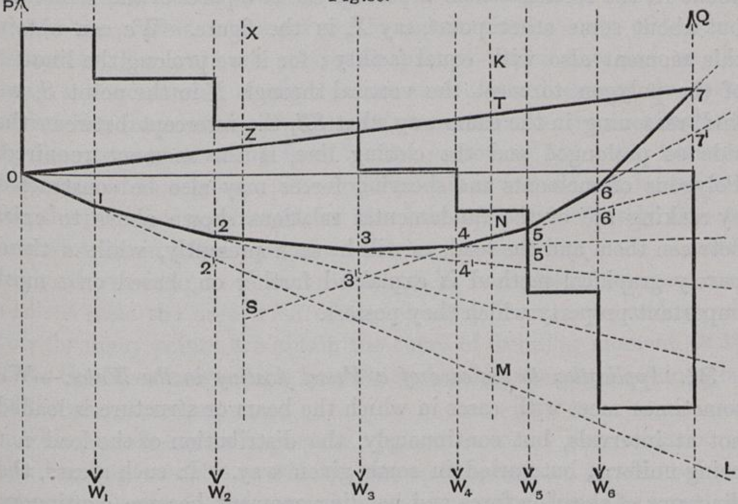
This is represented on the diagram of bending moment by the ordinate $33''$. In the shearing force diagram we notice that the area under the portion of the beam from 1 to 3 consists of two rectangles, $W_1(a_1 + a_2) + W_2a_2$. So that at this point also the bending moment is represented by the area of the polygon of shearing force, reckoned from the end up to the point 3. And so on for every point. This important deduction may be stated generally thus :—*The ordinate of the curve of bending moment at any point is proportional to the area of the curve of shearing force reckoned from one end of the beam up to that point.*

30. *Application to the case of a Loaded Beam.*—We will next take the case of a beam supported at the two ends.

First, calculate the supporting force P, set it up at the end of the base line as an ordinate, and draw the stepped polygon by continually subtracting the W's. At some point in the beam we shall cross the base line. At that point the shearing force changes sign, and there the bending moment is a maximum. The shearing force on the last interval will give the magnitude of the supporting force Q. The polygon thus drawn will be the polygon of shearing force.

The polygon of bending moment may be drawn without previously determining the supporting force at either end thus :—

Fig. 36.



Commencing at O (Fig. 36), the point of application of P, draw any sloping line $0\,1\,2'$ cutting W_1 in 1, and W_2 in $2'$. Then set up

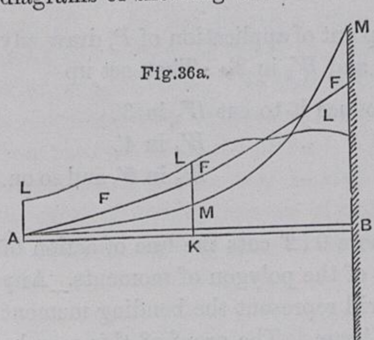
$2'\,2$ to represent W_1a_1, join 1 2, produce it to cut W_3 in $3'$.
$3'\,3$,, W_2a_2, ,, 2 3, ,, W_4 in $4'$.
$4'\,4$,, W_3a_3, ,, 3 4, ,, W_5 in $5'$, and so on.
$7'7$ will represent W_6a_6.

Now join 7 with the point 0, where $0\,1\,2'$ cuts the line of action of P. This is called the Closing Line of the polygon of moments. Any vertical intercept of this polygon will represent the bending moment at the corresponding point of the beam. The proof of this may be stated shortly thus :—If we produce 0 1 to meet the line of action of Q in L, then $L7$ will, from what has been said before, represent the sum of the moments of all the weights W about the end of the beam where Q acts. And from the conditions of equilibrium this must equal the moment of P about that end. Accordingly, if we take any point K, the vertical intercept MT below it will represent the moment of P about K. This is an upward moment. The four weights which lie to the left of K will together have a downward moment about K represented by MN. Therefore, the difference NT will represent the actual bending moment at the point K.

It sometimes happens that we want the moment of the forces not about K, the section which separates the two parts of the structure, but about some other point, say X, in the figure. We can obtain this moment also with equal facility; for if we prolong the line 4 5 of the polygon to meet the vertical through X in the point S, we find, reasoning in the same way, that SZ, the intercept between the side so prolonged and the closing line, is the moment required. Polygons of moments and shearing forces may also be constructed by making use of the fundamental relations shown above to exist between them and the load, as will be seen presently, while a third purely graphical method is explained farther on, based on a most important property which they possess.

31. *Application to the case of a Vessel floating in the Water.*—We sometimes meet with cases in which the beam or structure is loaded not at intervals, but continuously, the distribution of the load not being uniform, but varied in some given way. In such a case, the diagrams of shearing force and bending moment become continuous curves. The most convenient way of expressing how the load is distributed is by means of a curve, the ordinate of which at any point represents the intensity of the load at that point. Such a curve is called a *curve of loads.* It may be regarded as the profile of the upper surface of a mass of earth or other material resting on the beam.

Fig.36a.



We will consider, first, the case of a beam fixed at one end and loaded continuously throughout, in a manner expressed by a curve of loads LL. (Fig. 36a.) The total area inclosed by the curve of loads will represent the total load on the beam, and between the two ordinates of any two points will be the load on the beam between the two points. Now, the area of the curve of loads, reckoned from the end A up to any point, K say, since it represents the total load to the left of K, will be the shearing force at K. If at K we erect an ordinate, KF, to represent on some convenient scale the area ALK, and do this for many points of the beam, we shall obtain a second curve FF, the curve of shearing force. Having done this, we may repeat the process on the curve FF, and obtain the curve of bending moment. For we have previously proved that if the load on the beam is concentrated at given points, then the ordinate of the curve of bending moments is at any point proportional to the area enclosed by the curve of shearing force for the portion of the beam between the end and that point. The truth of this is not affected by supposing the points of application of the load to be indefinitely close to one another, in which case the load becomes continuous. Accordingly, if we set up at K an ordinate, KM, to represent on some convenient scale the area AFK of the shearing force curve, and repeat this for many points, we obtain the curve of bending moment, MM. Thus the three curves form a series, each being the graphical integral of the one preceding.

This process has an important application in the determination of the bending moment to which a ship is subjected on account of the unequal distribution of her weight and buoyancy along the length of the ship. On the whole, the upward pressure of the water, called the buoyancy, must be equal to the downward weight of the ship; and the lines of action of these two equal and opposite forces must be in the same vertical. But for any portion of the length, the upward pressure and the downward weight will not, in general, balance one another; so, on account of the difference, shearing and bending of the ship will be induced. In the case of a rectangular block of wood floating in water, the upward pressure of the water will, for every portion of its length, equal the downward weight, and there will be no shearing and bending action on it. But, in actual ships, the disposition of weight and buoyancy is not so simple. Taking any small portion of the length of the ship, the difference between the weight of that portion of the ship and the weight of the water displaced by that portion of the ship, will be a force which acts on the vessel sometimes upwards and sometimes downwards, according to which is the greater, just in the same way as forces act on a loaded beam producing shearing and bending. In the construction of the vessel, strength must be provided to resist these straining actions, and it is a matter of great practical importance to determine accurately the magnitude of them for all points of the length of the ship. We will select an example of very frequent occurrence, that in which at the ends of the ship the weight exceeds the buoyancy, whilst at the centre the buoyancy exceeds the weight. If the ship were very bluff ended, and carried a cargo of very heavy material in the centre hold, the distribution of weight and buoyancy would probably be the reverse of this.

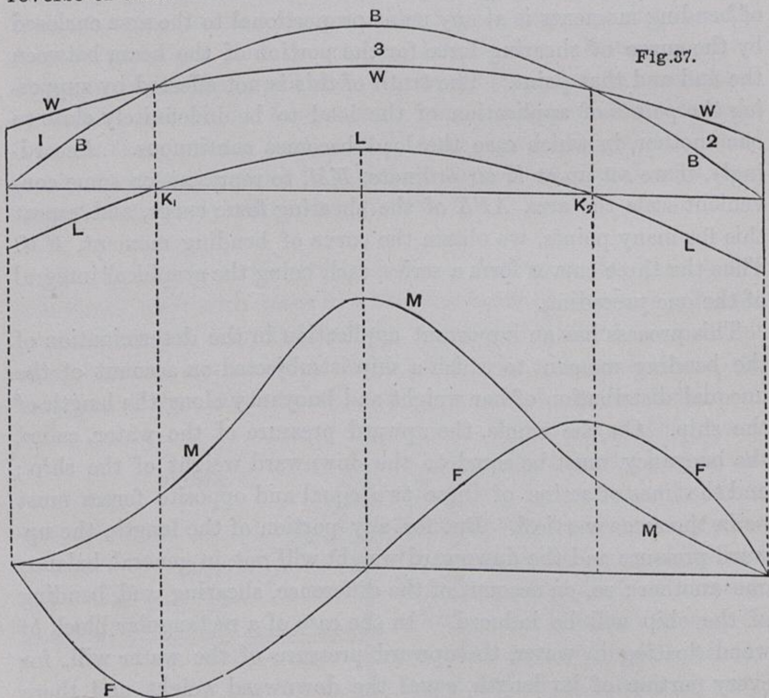


Fig.37.

In the example the ship is supposed to be divided into any number of equal parts, and the weight of water displaced by each of those parts determined; ordinates are set up to represent those weights, and so, what is called a curve of buoyancy *BBB* (Fig. 37) is drawn. The whole area enclosed by the curve will represent the total buoyancy or displacement of the vessel, and is the same thing as the total weight of the vessel. Next we suppose that the weights of the different portions of the ship are estimated, and ordinates set up to represent these weights, then what is called a curve of weight, WWW, is obtained. In the figure it is set up from the same base line. The total area enclosed by this curve will also be the total weight of the ship, and must therefore equal the area enclosed by the curve of buoyancy. Thus the sum of the two areas marked 1 and 2 must equal the area marked 3. Not only must this be true, but also the centres of gravity must lie on the same ordinate. The difference at any point between the ordinates of the two curves will express by how much at the ends the weight exceeds the buoyancy, and in the middle portion by how much the buoyancy exceeds the weight, representing, in the first case, the intensity of the downward force, and, in the second, the intensity of the upward force. Where the curves cross one another and the ordinates are the same height, as at K_1 and K_2, the *sections are said to be water-borne.* If now we set off from the base line ordinates equal to the difference between the ordinates of the two curves BBB and WWW, we obtain the curve of loads LLL; some portions where the weight is in excess will lie below the base line, and the rest, where the buoyancy exceeds the weight, will lie above the base line. From what has been said before, the area above the base line must equal the area below. Having obtained the curve of loads, the curve of shearing force is to be obtained from it in the manner previously described, by setting up, at any point, an ordinate to represent the area of the curve LLL between the end of the ship and that point. In performing the operation, due regard must be paid to the fact that the loads on different parts of the ship act in different directions, and for one direction they must be treated as negative, and the corresponding area of the curve as a negative area.

Having thus determined the curve of shearing force FFF, the same operation must be repeated on that curve to determine the curve of bending moment. In drawing the curve of shearing force it will be found that at the further end of the ship we return again to the base line from which we started at first, for the shearing force at the end must be zero. Also the bending moment at the end must be zero. This gives us tests of the accuracy of our work.

In this example the bending is wholly in one direction, tending to make the ends of the ship droop or the ship to "hog" in the technical language of the naval architect, but in some examples the direction of bending changes once or more times. Curves of shearing force and bending moment were first explained in relation to a vessel floating in the water by the late Professor Rankine in his work on shipbuilding. It does not, however, appear that any such curves were ever constructed in any actual example until 1869, when some were drawn for vessels of war by direction of Mr. (now Sir E.) Reed, at that time chief constructor of the Navy. The results obtained by him are described in a paper read before the Royal Society (Phil. Trans. for 1871, part 2). They now form part of the ordinary calculations of a vessel.

Since the water exerts on the vessel not only vertical but also horizontal forces, the straining actions upon her do not consist solely of shearing and bending, but include also a thrust. The horizontal pressure also produces bending in a manner which we shall hereafter explain.

32. *Maximum Straining Actions.*—The set of forces we are considering are in equilibrium, and must therefore be partly upwards and partly downwards. The downward force is the total weight W, and is generally more or less distributed, the upward force is of equal magnitude, and is usually concentrated near two or more points. In the case of the vessel, however, the upward force is distributed like the weight, though not according to the same law. In any case the greatest shearing force must be some fraction of the weight, and the greatest bending moment must be some fraction of the weight multiplied by the length l over which the weight is distributed. We may therefore express the maximum straining actions by the formulae

$$F_0 = k \,.\, W\,;\; M_0 = m \,.\, Wl,$$

where k, m are numerical quantities depending on the distribution of the load and the mode of support. Thus for a uniformly loaded beam supported at the ends $k = \frac{1}{2}$, $m = \frac{1}{8}$. The greatest value m can have in a beam resting on supports without attachment is $\frac{1}{4}$; this occurs when the beam is supported at the ends and the load concentrated in the middle or conversely. In vessels where the supporting force is distributed m is much less; its maximum value is estimated by Mr. White at $\frac{1}{35}$ in ordinary merchant steamers.

EXAMPLES.

1. A Warren girder with 12 divisions in the lower boom is supported at the ends and loaded with 250 tons, which may be supposed to be equally distributed among all the 25 joints. Find the stress on each bar by calculating the series of shearing force and bending moments.

RESULTS FOR LEFT-HAND HALF OF GIRDER.

F	115	105	95	85	75	65	55	45	35	25	15	5	5
S	132·2	120·7	109·2	97·7	86·2	74·7	63·2	51·7	40·2	28·7	17·2	5·7	
$\Delta H = F\frac{1}{\sqrt{3}}$	66·1	60·3	54·6	48·8	43·1	37·3	31·6	25·8	20·1	14·3	8·6	2·8	
H	66·1	126·4	181	229·8	272·9	310·2	341·8	367·6	387·7	402	410·6		413·4

2. The buoyancy of a vessel is 0 at the ends and increases uniformly to the centre, while the weight is 0 at the centre and increases uniformly to the ends. Draw the curves of shearing force and bending moment, and find the maximum values of these quantities in terms of the displacement and length of the vessel.

Answer—$k=\frac{1}{4}$; $m=\frac{1}{12}$.

3. A beam, 48 feet span, is supported at the ends and loaded with weights of 6, 9, 10, 13, 5, and 7 tons, placed at intervals of 4, 5, 9, 7, 13, and 8 feet respectively, commencing at one end. Calculate the shearing force in each interval and the series of bending moments.

4. In the last question construct the polygons of shearing force and bending moment.

5. In the case of a uniformly loaded beam supported at the ends, verify the principle that the area of the curve of shearing force is proportional to the ordinate of the curve of bending moment.

6. When a beam is supported at the ends and loaded in any way, show that an ordinate at the point of maximum moment divides the area of the curve of loads into parts, which are equal to the supporting forces. Further, if a b are the distances of the centres of gravity of these parts from the ends of the beam, and l the span, show that the maximum moment is mWl where

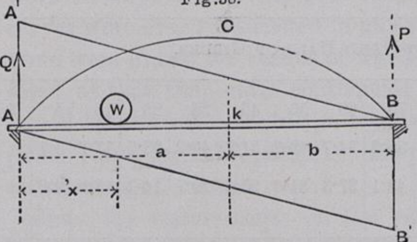
$$\frac{1}{m}=\frac{l}{a}+\frac{l}{b}.$$

TRAVELLING LOADS.

33. We have hitherto been investigating the effect of a permanent fixed load on a structure in producing straining actions on it. We next examine the effect of a load which is not permanent, but which at different times takes up different positions on the structure, and we require to know what position of the load will produce the greatest straining action at any particular part of the structure, and also the amount of that maximum straining action.

This question arises principally in the design of bridges across which a *travelling load*, such as a train, may proceed. We will take first the simple case of a beam of span l, supported at the ends and suppose a single concentrated load W to travel across it in the direction of the arrow. Let us consider any point K (Fig. 38) in the beam, distant a and b from the ends. As the load traverses the beam, each position of the load will produce a certain shearing force and bending moment at the point K. To find their greatest value let x = distance of W from A, then the supporting force at $B = P = W\frac{x}{l}$. So long as the weight lies between A and K the shearing force at K will be simply P.

Fig. 38.



$$F_K = W\frac{x}{l},$$

consequently the shearing force will increase as x increases, until the load reaches the point K. So long as the weight lies to the left of K, the tendency will be for the portion KB to slide upwards relatively to the portion AK. This we will call a positive shearing force. Therefore, putting $x = a$,

$$\text{Max. positive shearing force at } K = W\frac{a}{l}.$$

Now, supposing the weight to move onward, it will in the next instant have passed to the other side of K, and the shearing force will have undergone a sudden change. It will now be equal to the supporting force at the end B,

$$Q = W\frac{b}{l}.$$

But not only is the magnitude of the shearing force suddenly changed, but the tendency to slide is now in the other direction, and the shearing force is negative. As the weight moves farther to the right of K the shearing force diminishes, thus

$$\text{Max. negative shearing force at } K = W\frac{b}{l}.$$

Wherever we take the point K it will always be true that the maximum positive shearing force will occur when the weight lies immediately to the left of K, and the maximum negative when the weight lies immediately to the right. The maximum positive shearing force for every point in the beam may be represented by the ordinates of a sloping line AB' below the beam, the length BB' being taken to represent W. And similarly the maximum negative shearing force at any point by the ordinates of the sloping line $A'B$ above, AA' also being taken to represent W.

Next as to the bending moment. When the weight lies to the left of K, and is at a distance from A equal to $-x$, the bending moment at K is given by

$$Pb = W\frac{b}{l}x.$$

This goes on increasing as x increases until the weight reaches the point K. After having passed K the bending moment at K must be differently expressed, being then

$$\frac{W(l-x)}{l}a,$$

which becomes smaller as x increases; so that the greatest bending moment at K occurs when the load is immediately over K, and then the

$$\text{Max. bending Moment at } K = \frac{Wab}{l}.$$

If the point K is taken in the centre of the beam,

$$\text{Max. Moment at centre} = \tfrac{1}{4}Wl \quad \text{as before.}$$

If ordinates be set up at all points to represent the maximum bending moments at these points, a parabola (ACB) will be obtained. For the expression for the maximum bending moment is just twice that previously obtained for the same weight distributed uniformly.

If there are more weights, W_1, W_2, &c., on the beam, and W_1 lie to the right of K, the shearing force at $K = P - W_1$, where P is the right-hand supporting force. Now, suppose we shift W_1 to the left of K, we shall diminish the supporting force to P' say, and this will be the new shearing force at K. The difference between P and P' will be less than W_1, and the shearing force will be increased by passing W_1 to the left of K. If we were to remove W_1 altogether the diminution of P will be less than the whole of W_1, and so the shearing force at K will be increased by so doing. We obtain the greatest positive shearing force at K when all the weights are to the left of K, but as near to K as possible. The greatest negative shearing force will occur when all the weights lie to the right of K, as near to K as possible.

The maximum bending moment at K will occur when the weights are as near K as possible, whether to the right or left. Any addition to the load, on whichever side of K it is placed, will cause an addition to the bending moment.

There is another important case, that in which we have a continuous load of uniform intensity passing over the beam, as when a long train passes on to a bridge. We observe that as the train approaches K, the supporting force at B, and therefore the shearing force at K, increases. When any portion of the weight lies to the right of K, the supporting force will be increased by a part of the weight lying to the right of K; but when we have subtracted the whole of that weight, the difference, which will be the shearing force at K, will be less than before; thus the maximum positive shearing force at K will occur when the portion AK is fully loaded, and no part of the load is on KB. To find its value we have only to determine the supporting force at B, by taking moments about it; then

$$F_K = wa\,\frac{\frac{1}{2}a}{l} = \frac{1}{2}\frac{wa^2}{l},$$

that is, the magnitude is proportional to the square of the distance of the point from the end A. It will be graphically represented by the ordinates of a parabola which has its vertex at A and axis vertical, cutting the vertical through B in a point B' such that $BB' = \frac{1}{2}wl$, that is, half the weight on the beam when fully loaded. As the load travels onward the shearing force diminishes at last to zero, and then changes sign, becoming negative, the numerical magnitude increasing as the rear of the load approaches K. The maximum negative shearing force will occur when the portion KB only is loaded. The ordinates of a parabola set below the line of the beam having its vertex at B and axis vertical, will represent the maximum negative shearing force.

The question of maximum bending moment is more simple. It will occur at any point when the beam is fully loaded; for at any point the bending moment is the sum of the bending moments due to all the small portions into which the load may be divided, and the removal of any one of them will cause a diminution of bending action throughout the whole length of the beam. A parabola, with its highest ordinate at the centre = $\frac{1}{8}wl^2$, will represent it at any point.

34. *Counter Bracing of Girders.*—In the design of a framework girder it is very important to take account of the maximum positive and negative shearing forces due to a travelling load.

In such a structure the shearing force is resisted by the diagonal bars, and in general these bars are so placed as to be in tension, for the bar may then be made lighter than if subject to a compressive force of the same amount. Suppose the diagonal bars so arranged as to be all in tension when the girder is fully loaded, or when there is only the dead weight of the girder itself to be taken account of. There may be ample provision made for withstanding the tensile forces, and yet it will be important to examine if there may not be some disposition of the travelling load which would cause a thrust on some of the diagonals. If so, the maximum amount of this must be calculated, and the structure made capable of withstanding it. If the shearing force at any section of the girder is what we have called a positive shearing force, that in which the right-hand portion tends to slide upwards relatively to the left, then, in order that it may be withstood by the tension of a diagonal bar, the bar must slope upwards to the right. If the bar so slopes, and by the movement of the travelling load the shearing force becomes negative, then the bar will be subjected to compression. Now, it will frequently happen that in the central divisions of a girder the positive or negative shearing forces due to the dead load are less than the negative or positive shearing forces due to the travelling load, so that if those bars are arranged to be in tension under the dead load, then, on the passage of the travelling load, the stress will be changed to compression. In some cases the bars are slender and not suited to sustain compression; the shearing force is then provided for by the addition of a second diagonal, sloping in the opposite direction, which, by its tension, will perform the duty the first bar would otherwise have to perform by compression. Such a bar is called a counter-brace. We frequently see such additional bars fitted to the middle divisions of framework girders.

Again, the powers of resistance of a piece of material to a given maximum load are greater the smaller the fluctuation in the stress to which it is exposed; and therefore, in determining its dimensions, it is important to know not only the maximum but also the minimum stress to which it is exposed. This can be done on the principles which have just been explained.

EXAMPLES.

1. A single load of 50 tons traverses a bridge of 100 feet span. Draw the curves of maximum shearing force and bending moment, and give the values of these quantities for the quarter and half span.

2. A train weighing one ton per foot run, and more than 100 feet long, traverses a bridge 100 feet span. Draw the curves of maximum shearing force and bending moment, and give the values of these quantities at the quarter and half span.

3. In the last question, suppose the permanent load $\frac{3}{4}$ths ton per foot run. Find within what limits counterbracing will be required.

4. In Ex. 5, page 55, the maximum rolling load is estimated at 1 ton per foot run. Determine which of the diagonals will be in compression, and the amount of that compression, assuming a complete number of divisions to be loaded.

The two centre diagonals are the only ones which can be in compression, the maximum amount of which will be $= (3{\cdot}2 - 2)\sqrt{2} = 1{\cdot}7$. It will occur when the rolling load occupies 4 divisions only of the bridge.

5. In the last question, suppose a single load of 20 tons to traverse the bridge. Find the maximum stress, both tension and compression, on each part of the girder.

Divisions.	1	2	3	4	5
Max. tension, bottom boom, -	0	27	48	63	72
Max. compression, upper boom, -	27	48	63	72	75
Max. tension of diagonals, - -	38·1	31·1	24	17	9·8
Max. compression of diagonals, -	—	—	—	0	2·8

6. In the two preceding questions, find the fluctuation of stress on each part of the girder.

METHOD OF SECTIONS.

35. *Method of Sections applied to Incomplete Frames. Culmann's Theorem.*—The straining actions due to a vertical load may either be wholly resisted by internal forces called into play within the structure itself, or also in part by the horizontal reaction of fixed abutments: the supporting forces being in the first case vertical, and in the second having a horizontal component. The distinction is one of the greatest importance in the theory of structures, which are thus divided into two classes, Girders and Arches, including under the last head also chains. It is the first class alone which we consider in this chapter.

The general consideration of internal forces is outside the limits of this part of our work, and we shall here merely consider some cases of framework structures, commencing with that of an incomplete frame.

Incomplete frames are in general, as in Chapter I., structures of the arch and chain class, but by a slight modification we can readily convert such a frame into a girder and thus obtain very interesting results.

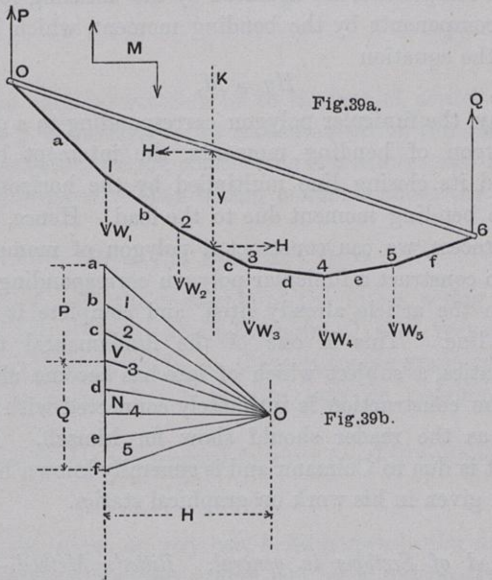


Fig. 39a.

Fig. 39b.

Fig. 39a shows a funicular polygon such as that in Fig. 11, page 15, except that the supports are removed and replaced by a strut $O6$. By this addition the polygon becomes a closed figure, and $O6$ is therefore called its "closing line." The structure is carried by suspending rods at the joints $O6$, and loaded as shown. The construction of the diagram of forces, Fig. 39b, has been sufficiently explained in the article referred to, and it only remains to observe that the supporting forces PQ are immediately derived from the diagram by drawing OV parallel to the closing line, which is not necessarily horizontal. The horizontal thrust of the strut and tension of the rope is found as before by drawing ON horizontal.

This structure may now be regarded as a girder, the load on which, together with the vertical supporting forces, produce definite straining actions M and F on any section. Let the section be KK' in the figure cutting one of the parts of the rope and the strut as shown in the figure: let the intercept be y. Consider the forces acting at the section on the left-hand half of the girder, the horizontal components of these forces are equal and opposite, acting as shown in the figure, each being H or ON in the diagram of forces. The vertical components are balanced by the shearing force, and the horizontal components by the bending moment, which last fact we express by the equation

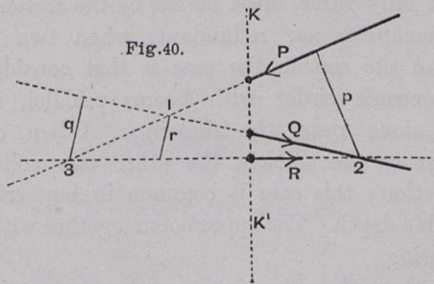
$$Hy = M,$$

that is to say, the funicular polygon corresponding to a given load is also a polygon of bending moments, the intercept between the polygon and its closing line multiplied by the horizontal force is equal to the bending moment due to the load. Hence, by a purely graphical process, we can construct a polygon of moments, for we have only to construct a funicular polygon corresponding to the load as shown in the article already cited, and complete it by drawing its closing line. This is one of the fundamental theorems of graphical statics, a subject which of late has become almost a new science. The construction is intimately connected with the process of Art. 29 as the reader should show for himself. In its complete form it is due to Culmann and is generally known by his name, having been given in his work on graphical statics.

36. *Method of Sections in general. Ritter's Method.*—In frames which are complete the number of bars cut by the section, instead of being two only, as in the preceding case, is in general three at least.

In Fig. 40 let KK' be the section cutting the three bars in three points which may be considered as the points of application of three forces PQR due to the reaction of the bars, which balance the shearing and bending actions to which the section is subject. Resolving horizontally and vertically, and taking moments, we should —remembering that the load being wholly vertical the sum of the horizontal components must be zero—obtain three equations which would determine P, Q, R. It is, however, simpler to employ a method introduced by Ritter which enables us to obtain the value of each force at once. Let the lines of action of P, Q intersect in the point 1, Q and R in 2, P and R in 3, and let the perpendicular

Fig. 40.



dropped from each intersection on to the line of action of the third force be r, p, q, respectively: by measurement on the drawing of the framework structure we are considering it is always easy to determine these perpendiculars. Then taking moments about the three points we get

$$Rr = L_1\,;\ Pp = L_2\,;\ Qq = L_3,$$

where L_1, L_2, L_3, are the moments of the forces acting on the left-hand half of the structure about the points 1, 2, 3, respectively. At page 68 it was shown how to get these moments graphically from the polygon of moments, but they also may be obtained by direct calculation.

We may write down a general formula for this method, thus—

$$Hh = L,$$

where H is the stress on any bar, h its perpendicular distance from the intersection of the two others cut by a section, and L is the moment of the forces about that intersection. The special case in which the intersection lies on the section considered so that the moment L becomes the bending moment (M) on the section, has already been considered in Chapter II. When the stress on a single bar is required as a verification of results obtained by graphical methods, or where the maximum stress due to a travelling load has to be determined, this method is often serviceable, but as a general method it is inconvenient from the amount of arithmetical labour involved, The shearing action on the section is resisted by the components parallel to the section of the stress on the several bars. In the case of the incomplete frame of Fig. 39, p. 79, these components are given at once by the diagram of forces. In general, however, three bars, and only three, must be cut by the section if the frame be neither incomplete nor redundant; when two of these are perpendicular to the section the case is that considered in Chap. III. of a framework girder with booms parallel, in which the diagonal bars alone resist the shearing. When one bar only is perpendicular to the section, the other two collectively resist the shearing action: this case is common in bowstring and other girders of variable depth. The upper boom together with the web here resists the shearing.

When more than three bars are cut by the section, the stress in each is generally indeterminate on account of the number of bars being redundant. On this question it will be sufficient for the present to refer to Chapter II., Section II.

EXAMPLES.

1. In example 3, page 73, construct the polygon of bending moments by Culmann's method.

2. In example 6, page 36, find the stress on each part of the roof by Ritter's method.

3. In example 7, page 36, find the stress on each bar by Ritter's method.

4. If a parabolic bowstring girder be subject to a uniform travelling load, represented by the application of equal weights to some or all of the verticals, show that the horizontal component of the maximum stress on each diagonal is the same for all.

REFERENCES.

For further information on subjects connected with the present chapter the reader may refer to

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Graphical Statics. LIEUT. CLARKE, R.E. Spon.

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