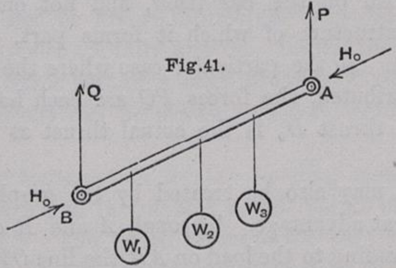
CHAPTER IV.

FRAMEWORK IN GENERAL.

37. *Straining Actions on the Bars of a Frame. General Method of Reduction.*—When the bars of a frame are not straight, or when they carry loads at some intermediate points, the straining action on them is not generally a simple thrust or pull, but includes a shearing and bending action. The present and two following articles will be devoted to some cases of this kind.

First suppose the bars straight, but let one or more be loaded in any way, and in the first instance consider any one bar AB (Fig. 41) apart from the rest of the frame, and suspended by

Fig.41.



strings in an inclined position. Let any weights act on it as shown in the figure, then the tensions of the vertical strings will be just the same as in a beam, AB, supported horizontally at the ends and loaded at the same points with the same weights. Resolve the forces into two sets, one along the bar, the other transverse to the bar. The first set produce shearing and bending just as if applied to a beam in a horizontal position, while the second set produce a longitudinal stress, which will be different in each division of the bar. Let θ be the inclination of the bar to the vertical, then the pulls on the successive divisions are

$$P.\cos\theta : (P - W_3)\cos\theta : (P - W_3 - W_2)\cos\theta : \ldots .$$

the last being a thrust equal to $Q.\cos\theta$, so that the stress varies from $Q.\cos\theta$ to $-P.\cos\theta$. Now observe that we can apply to AB at its ends, in the direction of its length, a thrust, H_0, of any magnitude we please without altering P and Q, but that we cannot apply a force in any other direction, whence it follows that when AB forms one of the bars of a frame, its reaction on the joint A must be a downward force, P, and a force, H_0, which must have the direction BA, while the reaction on B in like manner consists of a downward force, Q, and an equal force, H_0, in the direction AB. The downward forces P, Q, are described as the part of the load on AB carried at the joints A, B, and it is now clear that if these quantities be estimated for each bar and added to the load directly suspended there, we must be able to determine the forces H_0 by exactly the same process as that by which we find the stress on each bar of a frame loaded at the joints. The actual thrust on AB evidently varies between $H_0 - P.\cos\theta$ at the top, to $H_0 + Q.\cos\theta$ at the bottom, so that H_0 may be described as the mean thrust on the bar, while the shearing and bending depend solely on the load on the bar itself, and not on the nature of the framework structure of which it forms part, or on the load on that structure. In the particular case where the load on the bar is uniformly distributed, the forces PQ are each half the weight of the bar, and the thrust H_0 is the actual thrust at the *middle point* of the bar.

This question may also be treated by the graphical method of Art. 35 with great advantage. Through A and B draw a funicular polygon corresponding to the load on AB, the line OV in the diagram of forces will be parallel to AB and may be taken to represent H_0. This funicular polygon will be the curve of bending moment for the bar, and the other straining actions at every point are immediately deducible. It will be seen presently that the bar need not be straight.

For simplicity it has been supposed that the forces acting on the bar are parallel: if they be not, the reduction is not quite so simple. It will then be necessary to resolve the forces into components along the bar and transverse to the bar, the second set can be treated as above, while the total amount of the first set must be considered as part of the force applied to the joints either at A or B. Such cases, however, do not often occur, and it is therefore unnecessary to dwell on them.

The joints have been supposed simple pin joints or their equivalents, but the method used for frames loaded at the joints wil apply even if the real or ideal centres of rotation of the bars are not coincident, provided only the centre lines prolonged pass through the point where the load is applied. The method of reduction just explained then requires modification. Such cases are of frequent occurrence, and the next article will be devoted to them.

38. *Hinged Girders. Virtual Joints.*—The case of a loaded beam, the ends of which overhang the supports on which it rests, has already been considered in Art. 21, where it was shown that the straining actions at any point might be expressed in terms of the bending moments at the points of support, which of course will be determined by the load on the overhanging part. If the overhanging parts be supported, as in the case of a beam continuous over several spans, or with the ends fixed in a wall, the same formula will serve to express the straining actions at any point in terms of the bending moments at the points of support, but those bending moments will not be known unless the material of the beam and the mode of support are fully known. Hence the full consideration of such cases forms part of a later division of our work. Certain general conclusions can be drawn, however, which are of practical interest.

The graphic construction for the bending moment at any point of a beam, CD, which is not free at the points of support, is given in Fig. 28, p. 45. The figure refers to the case where the bending action at C and D is in the opposite direction to the bending action near the centre, as it is easily seen must be the case in general. The points of intersection of the moment line with the curve of moments drawn, as explained in the article cited, on the supposition of the ends being free, show where the negative bending at the ends passes into the positive bending at the centre. Here there is no bending at all, and the central part of the beam (EF in figure) is exactly in the position of a beam supported but otherwise free at its ends. We may therefore treat the case as if E and F were joints, the position of which will be known if the bending moments at the ends are known, and conversely. In some cases there may be actual joints in given positions, while in others there will be "virtual joints," the position of which may be supposed known for the purposes of the investigation.

Fig.42.

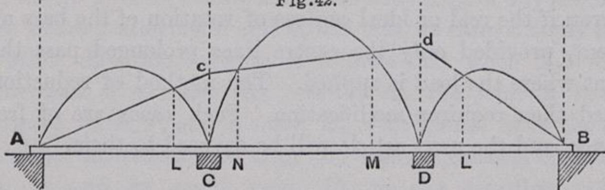


Fig. 42 shows a beam AB continuous over three spans, the moment curves for which will be known when the load resting on each span is known. It is evident from what has been said that the moment line must be the broken line $AcdB$, cutting the moment curve of the centre span in two points, and the moment curves of the end spans each in one point, the others being the ends of the beam. Thus there are four virtual joints, of which two must be supposed known in order to find the straining actions at any point. Their position will depend (1) on whether the supports are on the same level or not, (2) on the material and mode of construction of the beam, (3) on the load. Such a beam is in a condition analogous to that of a frame with redundant bars, considered in Chapter II. Section III.; the straining actions are indeterminate by purely statical considerations, for the same reason as before. We can, however, see that the bending action at each point is in general less than if the beam were not continuous.

In one particular case the position of the virtual joints can be foreseen. Suppose a perfectly straight beam, of uniform transverse section, to be continuous over an indefinite number of equal spans: let the weight of the beam be negligible, and let equal weights be placed at the centre of each span. Then since the pressure on each support must be equal to the weight, the beam is acted on by equal forces at equal distances alternately upwards and downwards, and there being perfect symmetry in the action of the upward and downward forces, the virtual joints must be midway between the centre and the points of support of each span.

In the special case where the beam is uniformly loaded we can further see that the load resting on the supports is not one half the weight of the parts of the beam resting there, as it would be if the beam were not continuous, but must in general be greater for the centre supports and less for the end supports. For if the virtual joints be *LNML′*, as in the figure, it is easily seen that *A* carries half the weight of *AL*, not of *AC*, while *C* carries half the weight of *AL* and *NM*, together with the whole weight of *CL* and *CN*. This observation shows that in trussed beams where, as is usually the case, the loaded beam is continuous through certain joints, the effect of the continuity is generally to transfer a part of the weight from the joints where the ends are free to the joints where the beam is continuous. We shall return to this point hereafter.

The principle of continuity is frequently taken advantage of in the construction of girders of uniform depth by making them continuous over several spans. The virtual joints, then, vary in position for each position of the travelling load, rendering it a complicated matter to determine the maximum straining actions, while there is always an element of uncertainty about the results, for reasons already referred to and afterwards to be stated more fully.

In some structures, however, the joints have a definite position.

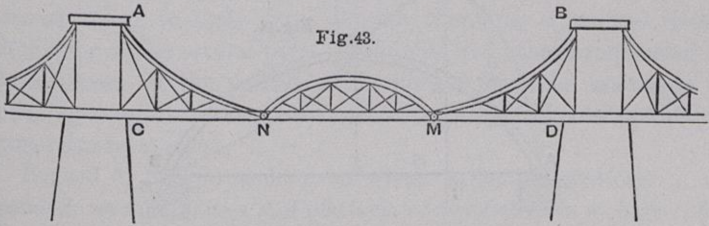


Fig.43.

Figure 43 shows a cantilever bowstring girder, consisting of a central bowstring girder *NM*, the ends of which rest on parts *ACN*, *BDM*, projecting from the piers, technically described as cantilevers. The joints here are at *N* and *M*. In structures of great span, in which the weight of the structure is the principal element, so that the variations in distribution are small, this type of girder is economical in weight. In a bridge over the Forth now in process of execution (1883), the central portion for each of two principal openings consists of a bowstring suspension girder 350 feet span, while the cantilevers are each no less than 675 feet in length, making a total span of 1700 feet. These cantilevers are of great depth near the piers, and, to provide against wind pressure, they are there likewise greatly increased in breadth, and solidly united to them. For a description of this design, which, from its gigantic dimensions and other unusual features, deserves attentive study, the reader is referred to *Engineering* for September, 1882.

39. *Hinged Arches.*—In the second section of Chapter I. certain forms of arches were considered which are simply inverted chains, and require for equilibrium a load of a certain definite intensity at each point. We shall now take the case of an arched rib capable of sustaining a load distributed in any way. We shall suppose the load vertical, and, to take the thrust of the arch, we shall imagine a tie rod introduced so as to convert it into a bowstring girder. If the straining actions at each point of the rib are to be determinate without reference to the relative flexibility of the several parts of the rib, and other circumstances, we must have, as in the case of the continuous beam, joints in some given position. The necessary joints are in this instance three in number, and, we shall suppose, are at the crown C (Fig. 44), and one at each springing A and B.

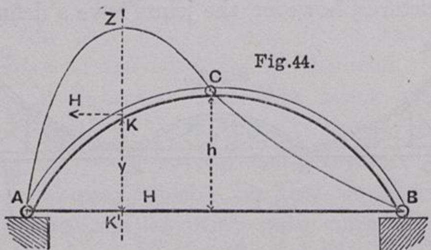


Fig. 44.

Taking a vertical section KK' through the rib and tie, let the bending moment due to the vertical load and supporting forces be M. This bending moment is resisted, *first*, by the horizontal forces called into play; that is to say, the pull of the tie rod H at K', and the equal and opposite horizontal thrust of the rib at K; *secondly*, by the resistance to bending of the rib itself, the moment of which we will call μ. Hence if y be the ordinate of the point considered, we must have

$$M = Hy + \mu.$$

To determine H we have only to notice that at the crown where $y = h$ there is a joint, that is, $\mu = 0$,

$$\therefore \quad M_0 = Hh,$$

where M_0 is the bending moment due to the load for the central section. Thus, to determine μ we have the equation

$$\mu = M_0 - M \cdot \frac{y}{h}.$$

The graphic representation of μ is very simple. Let us imagine the curve of moments drawn for the given vertical load, and let it be so drawn as to pass through A, B, and C, which is evidently always possible. Then, if Y be the ordinate of the curve,

$$M = H \cdot Y.$$

Therefore, by substitution,

$$\mu = H(Y - y).$$

So that the bending moment at each point of the rib is represented graphically by the vertical intercept between the rib and the curve of moments. In the figure, the dotted curve $AZCB$ is the curve of moments, and KZ is the intercept in question.

Arched ribs in practice are rarely, if ever, hinged, and the straining actions on them occasioned by a distribution of the load not corresponding to their form depend, therefore, upon the relative flexibility of the several parts of the rib, and other complicated circumstances. If the position of the virtual joints be known, or the bending moments at any three points, the graphical construction just given can be applied.

Instead of a rigid arch, from which a flexible platform is suspended, we may have a stiff platform suspended from a chain. This is the case where a suspension bridge is adapted to a variable load by means of a stiffening girder. For this case it will be sufficient to refer to Ex. 3, page 97.

40. *Structures of Uniform Strength.*—In any framework structure without redundant bars, the stress on each bar may be determined as in Chapter I., by drawing a diagram of forces for any given load, W, and expressed by the formula

$$H = kW,$$

where k is a co-efficient depending on the distribution of the load. If A be the sectional area of the bar we find by division the stress per sq. inch, which must not exceed a certain limit, depending on the nature of the material as explained in Part IV. of this work. When the structure is completely adapted to the load which it has to carry, the stress per sq. inch is the same for all the bars, and it is then said to be of Uniform Strength. Uniformity of strength cannot be reached exactly in practice, but it is a theoretical condition which is carried out as far as possible in the design of the structure. Other things being equal, the weight of a structure of uniform strength is less than that of any other. Such a structure is therefore less costly, for weight is to a great extent a measure of cost.

Whenever the load is known, the weight of a structure of given type and of uniform strength can be calculated thus. Suppose A the sectional area of a piece, H, the stress on it, f, a co-efficient of strength, then

$$H = fA.$$

Next let w_0 be the weight of a unit of volume, usually a cubic inch, and assume

$$\lambda = \frac{f}{w_0},$$

then λ is a certain length, being in fact the length of a bar of the material which will just carry its own weight. Its value for various materials is given in Chapter XVIII. Then, assuming the piece prismatic and of length s, its weight is

$$w_0 As = \frac{Hs}{\lambda},$$

and therefore the weight of the whole structure must be for the same value of λ,

$$W_0 = \frac{\Sigma Hs}{\lambda},$$

the summation extending to all the pieces in the structure, and being performed by integration in a continuous arch or chain. It will be observed that s is the length of any line in the frame-diagram, and H that of the corresponding line in the diagram of forces; we have only then to take the sum of the products of these lines and divide by λ, the result will be the weight of the structure. It is however generally necessary to find the weights, W_1, W_2, of the parts in compression and in tension separately, because the value of λ is generally different in the two cases.

A remarkable connection was shown by the late Prof. Clerk Maxwell to exist between W_1 and W_2. Let us take a structure of the girder class and suppose the total load upon it G, and the height of the centre of gravity of that load above the points of support h. Imagine this structure to become gradually smaller without altering either its proportions or the magnitude and distribution of the load G, then G descends and does work during the descent in overcoming the resistance (T) of the bars in compression to diminution of length, while at the same time the bars in tension (P) do work during contraction. The values of T and P do not alter, for the diagram of forces remains the same, and therefore if we conceive the process to continue till the structure has shrunk to a point,

$$Gh = \Sigma Ts - \Sigma Ps = \lambda_1 W_1 - \lambda_2 W_2.$$

In particular, if the centre of gravity of the load lies on the line of support, and if the co-efficients be the same, the weights of the parts in compression and tension will be equal. A corresponding formula may be obtained for structures of the arch-class by taking into account the thrust.

The weight of an actual structure is always greater than that found by this method. First, an addition must be made to allow for joints and fastenings. Thus, for example, in ordinary pin joints the eye of the bar weighs more than the corresponding fraction of the length of the bar, and in addition there is the weight of the pin. Secondly, in all structures there is more or less redundant material necessary to provide against accidental strains not comprehended in the useful load. Thirdly, there are local straining actions in the pieces occasioned by their own weight and other causes.

41. *Stress due to the Weight of a Structure.*—The total load on any structure consists partly of external forces applied to it at various points, and partly of its own weight: the total stress on any member is therefore the sum of that due to the external load and of that due to the weight of the structure itself. As that stress cannot exceed a certain limit, depending on the strength of the material, it necessarily follows that the stress due to the weight is so much deducted from the strength. Thus the consideration of the weight of a structure is an essential part of the subject, even if we disregard the question of cost.

The weight of each member is of course distributed over its whole length, and so also may be a part or the whole of the external load. Applying the general method of reduction explained in Art. 37, we suppose an equivalent load applied at each joint, and drawing a diagram of forces we determine the mean stress, H, on the member. If the unsupported length of the bars be not too great, a matter to be considered presently, this stress will be the principal part of the straining action on the bar, and the bending may be neglected as in the preceding article.

Now consider two structures similar in form and loaded with the same total weight, distributed in the same way, so that the only difference in the structures is in size: then the stress on corresponding bars must be the same, for the structures have the same diagram of forces. That is to say, in the formula

$$H = kW,$$

the coefficient k depends on the type of structure and the distribution of the load upon it, but not on its dimensions. Dividing by the sectional area the intensity of the stress is

$$p = k\frac{W}{A}.$$

Next let W_0 be the weight of the structure itself, and suppose the relative sectional areas of the several pieces the same, then

$$W_0 = w_0 \, . \, cAl,$$

where c is a coefficient depending on the type of structure, and l a length depending on the linear dimensions of the structure. For example, in roofs and bridges l may conveniently be taken as the span. Then if k_0 be the value of k, which corresponds to the distribution of the weight of the structure, which will be the same whether the structure be large or small,

$$p_0 = k_0 \, . \, \frac{W_0}{A} = w_0 k_0 cl,$$

will be the stress due to the weight of the structure. In other words, the stress due to the weight of similar structures varies as their linear dimensions.

Since p_0 cannot exceed f it follows at once that there must be a limit to the size of each particular type of structure, beyond which it will not carry its own weight. If L be that limit given by

$$L = \frac{\lambda}{k_0 c},$$

the stress due to the weight of any similar structure of smaller dimensions will be simply

$$p_0 = f \cdot \frac{l}{L},$$

and $$f' = f - p_0 = f \cdot \frac{L - l}{L}$$

is the strength which may be allowed in calculations made irrespectively of weight. If the structure be of uniform strength throughout under its own weight, the value of p_0 will be the same for each member, but this is not necessarily the case, and there may be a different value of f' for each member. The actual limiting dimensions of the structure will of course be the least of the various values corresponding to the various members.

The conclusion here arrived at is obviously of the greatest importance, for it immediately follows that in designing a roof, bridge, or other structure of great size, the weight of the structure is the principal thing to be considered in estimating the straining actions upon it, while a certain limiting span can never be exceeded. On the other hand, in small structures the straining actions due to the weight are unimportant; it is the magnitude and variations of the external load which have the greatest influence. This remark also applies to the local straining actions which produce bending in the pieces, their relative importance increases with the size of the structure, and it is necessary to provide against them by additional trussing. A large structure is therefore generally of more complex construction than a small one, as is illustrated by the various types of roof-trusses considered in Chapter I.

The difference of type of large structures and small ones, as well as the circumstances mentioned at the close of the last article, render tentative processes generally necessary in calculations respecting weight. If the type of structure and the distribution of the total load, W, be supposed known, the value of the coefficients k and c will be known for some given member. By assuming the stress on that member equal to the co-efficient of strength f, we find

$$W_0 = W \cdot ck \cdot \frac{l}{\lambda},$$

a formula which gives the weight of the structure in terms of the load, but the co-efficients will generally vary according to the span. Among the circumstances on which they depend the ratio of the vertical to the horizontal dimensions of the structure is most important. For a given span k diminishes when the depth is increased, while on the other hand c generally increases, so that for a certain ratio of depth to span the weight of the structure is least. In ideal cases c may remain the same (Ex. 10, p. 97), but in actual structures the redundant weight of material necessary to give stiffness and lateral stability increases, so that the most economical ratio of depth to span is generally much less than would be found by neglecting such considerations. These points are illustrated by examples at the end of this Chapter and Chapter XII., where the question is again considered briefly; but for detailed applications to actual structures the reader is referred to works on bridges, in the design of which it is of the greatest importance.

42. *Straining Actions on a Loaded Structure in General.*—The results obtained in the last chapter for the case of parallel forces acting on a structure possessing a plane of symmetry in which the forces lie, may be readily extended to structures which have an axis of symmetry acted on by any forces passing through that axis and perpendicular to it. This is the case, for example, of a beam acted on by a vertical load, and also by some horizontal forces arising say from the thrust of a roof or from wind pressure. We have then only to consider the vertical and horizontal forces separately. Each will produce shearing and bending in its own plane, which may be represented by polygons as before. The total straining action will be simply shearing and bending, and will be as before independent of the particular structure on which the forces operate. The magnitude of the straining action, whether shearing or bending, will be the square root of the sum of the squares of its components, and may therefore be readily found by construction and exhibited graphically by curves. In shafts such cases are common, and some examples will be given hereafter.

Another entirely different kind of straining action sometimes occurs in structures proper (roofs, bridges, etc.), and in machines is one of the principal things to be considered. Imagine a structure of any kind to be divided by an ideal plane section into parts A and B, and to be acted on by forces parallel to that plane.

Let the forces acting on A reduce to a couple the axis of which is perpendicular to the section, the forces on B are equal and opposite, and the two equal and opposite couples tend to cause A and B to rotate relatively to each other. As already stated in Art. 16 this effect is called Twisting, and the magnitude of the twisting action is measured by the magnitude of either of the couples which form its elements.

Simple twisting sometimes occurs in practice, for example, when a capstan is rotated by equal forces applied to all the bars, but it is generally combined with shearing and bending. It is then necessary to know about what axis the twisting moment should be reckoned, which will depend on the nature of the structure. In shafts and other cases to be considered hereafter the geometrical axis is an axis of symmetry which at once determines this.

When twisting exists the shearing and bending are determined by the same method as before, for they are independent of the axis of reference. Should however the structure be subject to a thrust or a pull (Art 16), the axis about which the bending moment should be reckoned must be known, for it will depend on the nature of the structure.

These general observations will be illustrated hereafter, and are only introduced here to show how far straining actions can be regarded as depending solely on the external forces operating on the structure without reference to any other circumstances.

43. *Framework with Redundant Parts.*—In a complete frame, without redundant bars (pp. 13, 56), suppose a link applied to any two bars, one end attached to each. Let the link be provided with a right and left handed screw or other means of altering its length at pleasure, then by screwing up the link a pull may be produced in the link of any magnitude we please, while a corresponding stress will be produced in each bar of the frame which will bear a given ratio to the pull. Such a link may be called a straining link, and by its addition we obtain a frame with one redundant bar. The stress-ratio on the parts of a frame of this kind is completely definite, but the magnitude of the stress may be anything we please. Instead of one straining link we may have any number, and if the stress on each of these links be given, the same thing will be true. Thus it appears that a frame with redundant parts may be in a state of stress even though no external forces act upon it. This is of practical importance on account of the effect of changes of temperature. If all the bars of a frame with redundant parts are equally heated or cooled, the frame expands or contracts as a whole, but no other effect is produced; any inequality, however, causes a stress which may, under certain circumstances, be very great. This (at least theoretically) is one of the reasons why redundant parts are a source of weakness. The necessity of providing against expansion and contraction is well known in large structures resting on supports. The ground connecting the supports suffers little change of temperature, and the structure, therefore, cannot be attached to the supports, but must be enabled to move horizontally by the intervention of rollers. The magnitude of the stress produced when changes of length are forcibly prevented will be considered hereafter (Chapter XII.).

There is no essential difference between a frame the stress on the parts of which is due to the action of straining links, and a frame acted on by external forces; for every force arises from the mutual action between two bodies, and may therefore be represented by a straining link connecting the bodies. Even gravity may be regarded as a number of such links connecting each particle of the heavy body with the earth. Accordingly, if we include in the structure we are considering, the supports and solid ground on which it rests, we may regard it as a frame under no external forces, but including a number of straining links screwed up to a given stress. If the original frame be incomplete, its parts will be capable of motion, and it becomes a machine, as will be explained in Part III. of this work.

44. *Concluding Remarks.*—Various other questions relating to framework remain to be considered, especially with reference to the joints by which the parts are connected, but these, involving other than purely statical considerations, do not come within the present division of our work, but are referred to at a later period.

EXAMPLES.

1. In Ex. 4, page 12, if the weight be supposed uniformly distributed, find the thrust, shearing force, and bending moment at each point of each rafter, and exhibit the results graphically by drawing curves.

Diagrams of shearing force will be sloping lines crossing each rafter at the centre.

Max. shearing for short rafter = 91 lbs.

,, ,, long ,, = 158·5 ,,

Diagrams of bending moment will be parabolas.

Max. moment at centre of short rafter = 117 ft.-lbs.

" " long " = 290 "

2. A triangular frame ABC, supported at A and C, with AC horizontal, is constructed of uniform bars weighing 10 lbs. per foot, the lengths being—$AB = 3$ feet, $BC = 4$ feet, and $AC = 5$ feet. Suppose, further, that AB and BC each carry 50 lbs. in the centre. Draw curves of thrust, shearing force, and bending moment for each bar.

3. The platform of a suspension bridge is stiffened by girders hinged at the centre and at the piers. The chains hang in a parabola, and the weight of the platform, chains and suspending rods may be regarded as uniformly distributed. Find the bending moment at any point of the stiffening girder, and exhibit it graphically by a curve when a single load W is placed (1) at the centre of the bridge, and (2) at quarter span.

First case. On account of W each half of the girder will tend to turn downwards about the ends, and will be supported by the uniform upward pull of the suspending rods. $\therefore$ total upward pull for each $\frac{1}{2}$ girder $= W$, because the centre of action is at $\frac{1}{4}$ span. Thus each $\frac{1}{2}$ girder will be in the state of a beam loaded uniformly with W, and supported at the ends. Max. moment at middle of each half

$$= \tfrac{1}{8}W \times \text{half span.}$$

Second case. The upward pull of the suspending rods will still be uniform, but for each half girder will now be only $\frac{1}{2}$ W, found by assuming an equal action and reaction at the centre joint, and taking moments of each half about the ends. For the half girder which carries the weight the bending moment will be the difference between that due to W concentrated in the centre and $\frac{1}{2}W$ distributed uniformly.

$$\therefore \quad \text{Max.} = \tfrac{3}{16}W \times \text{half span.}$$

On the other half it will be due simply to a distributed load of $\frac{1}{2}W$. Max. $= \frac{1}{16}W \times$ half span.

4. A timber beam 24 feet span is trussed by a pair of struts $8'$ apart, resting on iron tension rods forming a simple queen truss $3'$ deep without a diagonal brace. The beam is loaded with 5 tons placed immediately over one of the vertical struts. Find the shearing force and bending moment at any point of the beam, supposing it jointed at the centre and the centre only.

The thrust on each strut must be $2\frac{1}{2}$ tons; therefore, curves of shearing force and bending moment for each half of the beam are the same as those for a beam 12 feet long loaded at a point 4 feet from one end with $2\frac{1}{2}$ tons.

The problem should also be treated by the method of sections. Results should also be obtained for the case where one half the beam is uniformly loaded.

5. A beam uniformly loaded is fixed horizontally at the two ends, and jointed at two given points. Draw the diagrams of shearing force and bending moment. Show that the beam will be strongest when the distance of each point from centre is rather less than $\frac{3}{8}$ span.

6. The platform of a bowstring bridge of span $2a$ is suspended from parabolic arched ribs hinged at crown and springing. One half the platform only is loaded uniformly with w lbs. per foot run. Show that the greatest bending moment on the ribs is $\frac{1}{16}wa^2$.

7. In the last question, if a weight of W tons travel over the bridge, how great will be the maximum bending moment produced?

8. A girder is continuous over three equal spans, and is hinged at points in the centre span midway between centre and piers. Find the virtual joints in the end spans when uniformly loaded throughout.

9. The weight of the chains, platform and suspension rods of a suspension bridge may be treated as a uniform load per foot-run, which at the centre of the bridge is double the weight of the chain. The dip of the chain is $\frac{1}{13}$th the span. The weight of iron being 480 lbs. per cubic foot, and the safe load per square inch of sectional area of chain being 5 tons, find the limiting span, and deduce the sectional area of chains for a load of $\frac{1}{2}$ ton per foot-run on a similar bridge 300 feet span.

If A = sectional area of chains at centre in sq. ins., then $\frac{20}{3}A$ = weight of bridge per foot-run in lbs.

$$\text{Horizontal tension} = \tfrac{65}{6}AL = 5 \times 2240 \,.\, A.$$

$$\therefore L = 1034 \text{ feet.}$$

If A' = area of one chain of the bridge 300 feet span,

$$\text{Whole load on chain} = \left(\tfrac{20}{3}A' + \tfrac{2240}{4}\right) 300,$$

$$\text{Horizontal tension} \quad = \tfrac{1}{8}\left(\tfrac{20}{3}A' + \tfrac{2240}{4}\right) 300 \times 13 = 5 \times 2240A',$$

$$\therefore A' = 34{\cdot}4 \text{ sq. in. each chain.}$$

Remark.—By the use of steel wire ropes and by lightening the platform and other parts of the structure as much as possible, the limiting span of suspension bridges is much increased, there being several examples of a span of 1250 feet and upwards.

10. In a girder with booms parallel and of uniform transverse section the weight of the web is equal to the weight of the booms. Assuming a co-efficient of strength of 9000 lbs per sq. inch, and the weight of a cubic inch $\frac{5}{18}$th of a lb., show that the limiting span in feet is

$$L = 5400N,$$

where N is the ratio of depth to span.

11. The weight of a rib of parabolic form, span l, rise nl, with transverse section varying for uniform strength under a uniformly distributed load W, is

$$W_0 = \left(\frac{1}{8n} + \tfrac{2}{3}n\right) W \frac{l}{\lambda}.$$

This is least when $n = \dfrac{\sqrt{3}}{4} = {\cdot}433$, then $W_0 = {\cdot}577\, W \dfrac{l}{\lambda}$.

The formula fails if W_0 be nearly equal to W, for the external load would then have to be partly acting upwards to secure uniform distribution of the total load.