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PART II.—KINEMATICS OF MACHINES.

45. *Introductory Remarks.*—The object of a machine is to enable the forces of nature to do work of various kinds. In this operation some given resistance is overcome, which is accompanied by a given motion, while the driving force is accompanied by some other given motion, often at a distant place. Hence a machine may be regarded as an instrument for converting and transmitting motion. When considered under this aspect it is called a Mechanism, or sometimes a Movement, a Motion, or a Gear, the first being the scientific term, and the others occurring in practical applications.

Every mechanism consists of a set of pieces possessing one degree of freedom, that is to say they are so connected together that when one changes its position all the rest do so too in a way precisely defined by the nature of the mechanism. Thus, for example, when the piston of a steam engine moves through any fraction of a stroke, the connecting rod, crank shaft, and the parts of any machine which it may be driving all shift their position in such a way that the connection between the various changes is completely determinate, and can be studied without reference to the work which the engine is doing, or the speed at which it is running. This branch of study is called the Kinematics of Machines.

The changes of position may be of any magnitude we please, and if they are very small are proportional to the velocities of the moving parts, hence a part of the subject, and generally an important part, is the consideration of the comparative velocities, or, as they are usually called, the velocity-ratios, of the moving parts. Further, since the comparative velocities are fixed by the nature of the machine, the same must be true of the rates of change of these velocities, that is to say the accelerations. Hence the general question is to study completely the comparative motions of the several parts of a machine, so that when the position, velocity, and acceleration of any piece are given, those quantities may be known for every other piece. It is the positions and velocities which are chiefly considered.

The converse problem is to discover the mechanisms by which any required motion may be obtained, and for this purpose the connection which exists between different mechanisms is considered. The subject therefore forms an introduction to the science of Descriptive Mechanism in which existing machines in all their vast variety are classified and studied systematically.

AUTHORITIES.

The principal treatises on the theory of mechanism are—

WILLIS. *Principles of Mechanism.* Longman.
RANKINE. *Millwork and Machinery.* Griffin.
REULEAUX. *Kinematics of Machinery.* Macmillan.

The modern form of the theory is due to Professor Reuleaux, whose nomenclature and methods are followed, with some modifications, in the present work. The treatise referred to is a translation from the German by Professor A. B. Kennedy.

CHAPTER V.

LOWER PAIRING.

46. *Definition of Lower Pairs.*—Each piece of a machine is in direct connection with at least one other, and constitutes with it what is called a PAIR, of which the two pieces are said to be the Elements. The whole machine may be regarded as made up of pairs, and the nature of the mechanism depends on the nature of the pairs of which it is constructed.

In the present chapter we consider exclusively mechanism composed of pairs of rigid elements which are in contact with each other, not merely at certain points or along certain lines, but throughout the whole or part of the area of certain surfaces. Such pairs are of peculiar importance from the simplicity of the relative movement of their elements, from their resistance to wear when transmitting heavy pressures, and from their tightness under steam and water pressure. They are called Lower Pairs, and in many cases this kind of pairing is alone admissible.

In order that two rigid surfaces may be capable of moving over each other while continuing to fit, they must either be cylindrical, including under that head all surfaces generated by the motion of a straight line parallel to itself, or surfaces of revolution, or screw surfaces. In the first case the relative motion of the elements is one of translation along the line, in the second of rotation about the axis of revolution, in the third the motion of translation and rotation are combined in a fixed proportion. Hence there are three kinds of lower pairs, known as Sliding Pairs, Turning Pairs, and Screw Pairs. In each case one of the surfaces is hollow, and wholly or partly encloses the other which is solid, and the motion depends on the surfaces only, and not on the other parts of the elements which assume very various forms, according to the purpose of the mechanism. Either element may be fixed and the other move, or both elements may move in any way whatever, the relative motion is still of the same kind.

As an example of a sliding pair may be taken a piston and cylinder, in which either the cylinder may be fixed and the piston move, or the piston be fixed and the cylinder move, as in some steam hammers, or both cylinder and piston move, as in the oscillating engine. The relative motion is always a simple translation. Velocities of translation are most conveniently measured in feet per 1″ or feet per 1′, but miles per hour and knots per hour are also used, as to which it is convenient to remember that one mile per hour is 88 feet per 1′, and one knot per hour approximately 101 feet per 1′.

As examples of turning pairs may be taken a cart and its wheel, a shaft and its bearing, or a connecting rod and crank pin. The relative motion here is one of simple rotation, which may be measured by the number of revolutions (n) per unit of time, or by the speed of periphery (V) of a circle of given radius (r), or by the angle (A) turned through per unit of time. The first two modes of measurement are common in practice, the third is used for scientific purposes only. When employed the angle is always expressed in circular measure, and the three methods are therefore connected by the equations

$$V = Ar = 2\pi nr.$$

When angular velocity is used as a measure of speed of rotation, the unit of time is always 1″, but the minute and hour are common in other cases.

A screw pair consists of a screw and its nut, and the relative motion consists of a motion of translation along the axis of the screw combined with a rotation about that axis. The motion of translation is often called the "speed of the screw," and is equal to np, where p is the pitch, that is to say, the space traversed in one revolution, and n the revolutions in the unit of time. Strictly speaking, the two first lower pairs are limiting cases of the screw pair: in the turning pair the pitch is zero, and in the sliding pair infinite.

In all three cases the motion of either element relatively to the other is identically the same, and the rate of that motion may properly be called the Velocity of the Pair, whether the movement considered be translation or rotation. When the velocity of a sliding pair and a turning pair are compared, rotation may be measured by the speed of periphery of a circle of given diameter; it is the velocity with which bearing surfaces of that diameter would rub each other. The radius of this circle may be called the "radius of reference." The velocity of a screw pair may be measured by the rate either of its translation or its rotation.

In these three simple pairs the motion of one element relatively to the other is completely defined, each point describing a definite curve. Such a pair is called a "complete" or "closed" pair, but we may have pairs in which the motion is not defined unless further constraint be applied, and the pair is then said to be "incomplete." An incomplete pair cannot be used in mechanism without employing such constraint, and this process is called "closing" the pair. A pair may be incomplete, because there is nothing to prevent the disunion of its elements, as, for example, a shaft and its bearing when the cap is removed, but it also may be incomplete in itself. Lower pairing is sometimes, though not very frequently, incomplete in this latter sense; there are three possible cases, first, when the surfaces are spherical, as in a ball and socket joint; second, where a rod fits into a hole, and is free to move endways as well as rotate; third, where a block fits in between parallel plane surfaces. The methods of producing closure will be considered hereafter.

It may be here remarked, in anticipation of what will be said hereafter, that cases of lower pairing may be imagined in which the elements are not in contact over an area but along a line. For example, a rod may fit into a square hole. It is the simplicity of the relative motion which is the essential characteristic.

The motion of the elements of a pair may be prevented by a pin key or other fastening removable at pleasure: the pair is then said to be "locked." In capstans and windlasses, provided with ratchet wheel and pawls, we have examples in which a pair is locked in one direction only.

47. *Definition of a Kinematic Chain.*—It has been already said that a machine consists of a number of parts so connected together as to be capable of moving relatively to one another in a way completely defined by the nature of the machine. Each part forms an element of two consecutive pairs, and serves to connect the pairs so that the whole mechanism may be described as a chain, of which the parts form the links. Such a series of connected pieces is called a Kinematic Chain.

The motion of any piece may be considered either relatively to one of the pieces with which it pairs, or with reference to any other piece which we may choose to regard as fixed. In the first case the rate of movement has already been defined as the Velocity of the Pair. In the second, the fixed piece is usually the frame of the machine, which unites the rest of the pieces, and is commonly attached to the earth or some structure of large size, such as a vessel. For pieces which pair with the frame the velocity of the pair is the same as the velocity of the moving element, and this element alone need be mentioned. In some common practical cases the speed of an element means the speed of one of the pairs of which it forms part. For example, the speed of piston of an oscillating engine would be understood to mean its velocity relatively to the cylinder, in other words, the speed of the "cylinder-piston pair." In the present chapter we consider exclusively chains of closed lower pairs, so that the motion of the pairs is a simple translation, rotation, or screw motion. The motion of some of the pieces relatively to the frame may be much more complex, but this is a subject for subsequent investigation; it is the motion of the pairs alone we now consider. We shall first direct our attention to the very common and important piece of mechanism employed in direct-acting steam engines. An example is shown in Fig. 1, Plate I., p. 118, which represents a direct-acting engine of the vertical inverted cylinder type which is common in marine engines and often occurs in other cases.

Let us consider the pairs of which this mechanism is constructed. We have, *first*, a cylinder, to which are rigidly attached guides for the crosshead, and bearings for carrying the crank shaft. The cylinder-guide bars and crank-shaft bearings all form one part rigidly connected together, and must be considered as being one piece or link of the kinematic chain. It may conveniently be called the frame. *Secondly*, there is a piston, which fits and slides in the cylinder. To the piston a rod and crosshead are rigidly attached, forming practically one piece. Not only is the piston guided in the cylinder, but the crosshead also between the guide bars, and the piston rod in the stuffing box; but yet, since there are practically two pieces only which move relatively to one another, we must look on the cylinder, stuffing box, and guide bars as altogether forming the hollow element of a sliding pair, and the piston, rod, and crosshead as together forming the solid element of the pair. *Thirdly*, there is a connecting rod which is attached by a gudgeon or crosshead pin to the piston-rod head. These two parts will together compose a turning pair. At the other end the connecting rod embraces the crank pin, forming a second turning pair with it. The crank pin is one of the elements contained in the *fourth* piece of the mechanism. This piece consists of the crank pin, crank arms, and shaft with its journals. The journals turn in the bearings of the fixed frame of the machine, the first link mentioned, and so form a third turning pair. Thus the chain is complete. It consists of four links forming one sliding pair and three successive turning pairs.

The same mechanism, in a different form, is shown in Fig. 2 of the same plate which represents the air-pump of a marine engine worked, as is not unusual, by a large eccentric keyed on the crank shaft. The crank pin is here enlarged so as to become an eccentric; and to save room, the piston rod is replaced by a trunk within which the eccentric rod vibrates. We have, however, exactly the same pairs arranged in the same way, and the difference between the mechanisms is therefore merely constructive, the motions of the parts being identical.

48. *Mechanism of Direct-Acting Engine—Position of Piston.*—This is such an important piece of mechanism that we will examine its motion somewhat fully.

First as to the relative positions of the crank in its revolution and the piston in its stroke. The position of the piston in its stroke will compare exactly with the position of the crosshead, so instead of

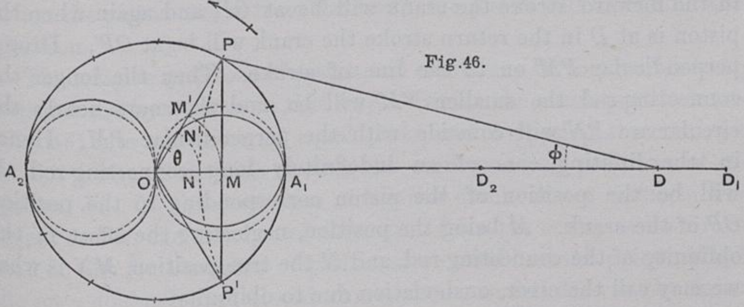


Fig. 46.

introducing the length of the piston rod into the diagram, we may just as well determine the relative positions of the point D (Fig. 46) in its straight-line path, and P in its circular path.

Suppose the line of stroke to pass through the centre of the crank-pin circle. Let OP = length of the crank arm, and PD the length of the connecting rod. When the crank arm is in the line of stroke, away from the piston, the piston will be in one extreme position, and when the crank is in the line of stroke towards D, the piston will be in its other extreme position. The points A_1 A_2 on the crank-pin circle are called the dead points. If we take distances A_1D_1 A_2D_2 $= PD$, the length of the connecting rod, the points D_1 D_2 represent the ends of the stroke of the piston. If now we place the crank in any position OP we obtain the corresponding position of the piston by cutting the line of stroke with a circular arc of radius $= PD$ and with centre P. DD_1 DD_2 will be the distances of the piston from the ends of its stroke. Since $A_1A_2 = D_1D_2$, the length of the stroke, it will be convenient to find the point in A_1A_2 which corresponds to the position of the piston in its stroke. This may be readily done by striking a circular arc PN with centre D. N will be the point for $A_1D_1 = PD = ND$, therefore $A_1N = D_1D$, and the point N is the same distance from A_1 and A_2 as the piston is from the ends of its stroke.

We may just as easily solve the converse problem of finding the position of crank corresponding to any given position of the piston in its stroke. Let D be any position, cut the crank-pin circle by a circular arc of which D is the centre and DP the radius, then OP or OP' will be the corresponding position of the crank. Let the direction A_1 PA_2 be the ahead direction of the crank, and let us call the motion D_1D_2 towards the crank the forward stroke, and D_2D_1 the back or return stroke of the piston, then when the piston is at D in the forward stroke the crank will be at OP, and again when the piston is at D in the return stroke the crank will be at OP'. Drop a perpendicular PM on to the line of stroke. Then the longer the connecting-rod the smaller NM will be, and the more nearly the circular arc PN will coincide with the perpendicular PM. Hence in the limiting case of an indefinitely long connecting-rod, M will be the position of the piston corresponding to the position OP of the crank. M being the position, neglecting the effect of the obliquity of the connecting-rod, and N the true position, MN is what we may call the error, or deviation due to obliquity.

In general the slide valve is worked by an eccentric, the radius of which is set at a particular angle on the shaft, so that the cut-off takes place when the crank occupies a certain angular position in its revolution, and it consequently follows that the fraction of stroke completed before cut-off takes place will be affected by the obliquity of the connecting-rod, so that in the ordinary setting of the slide valve the rates of cut-off will be different in the two strokes. This is well illustrated by Ex. 4, page 112.

We may obtain a convenient approximate expression for MN, the error due to obliquity. Referring to Fig. 46.

$$NM = DN - DM = DN(1 - \cos\phi).$$

Now the length of the connecting-rod may be conveniently expressed as a multiple of the length of the crank radius a or stroke s.

$$DN = na \text{ suppose} = \tfrac{1}{2}ns.$$

$$\therefore \quad NM = n\frac{s}{2}(1 - \cos\phi) = ns\sin^2\frac{\phi}{2}.$$

In the triangle POD, the sides being proportional to the sines of the opposite angles,

$$\text{Sin}\,\phi = \frac{OP}{DP}\sin\theta = \frac{1}{n}\sin\theta.$$

Now, the angle ϕ is in all practical cases a small angle, so we may write approximately

$$2\sin\frac{\phi}{2} = \sin\phi,$$

$$\therefore \quad NM = n\,.\,s\,.\,\frac{\sin^2\theta}{4n^2} = \frac{s}{4n}\sin^2\theta.$$

This is greatest when $\theta = 90$. $NM_{\text{max.}} = \dfrac{s}{4n}$.

If the connecting-rod is four times the crank, the greatest error due to obliquity $= \frac{1}{16}$ stroke.

We see that, in the forward stroke, the effect of the obliquity of the connecting-rod is to put the piston in advance of the position due to an indefinitely long connecting-rod, and, in the return stroke when the piston moves from the crank, the piston will be behind that position.

The relative positions of piston and crank may be very conveniently represented by a curve in this way. Divide the crank-pin circle (see Fig. 46) into a number of equal parts, and supposing the crank-pin at the points of division P, find the corresponding positions of the piston N. If then we take along the crank arm a distance ON' equal to ON, the distance of the piston from the centre of its stroke, and do this for a number of positions, we shall find the points N' will lie on a double-looped closed curve, shown in full lines in the figure. This may be called a curve of position of the piston. If we had supposed the connecting-rod to be indefinitely long, and had taken a distance OM' along $OP = OM$, the curve of position in such a case would have been a pair of circles, dotted in the figure, on OA_2 and OA_1 as diameters. The true curves of position will deviate from these circles more the shorter the connecting-rod. For the half stroke nearer the crank the curve will lie outside the dotted circle, and for the further half stroke inside. In Zeuner's valve diagram the obliquity of the eccentric rod is neglected, and the circles employed to show the position of the slide valve.

49. *Velocity of Piston.*—We will now pass on to the question of the relative velocity of the piston and crank-pin.

We will suppose the crank to turn uniformly at so many revolutions in the unit of time. If n = number of revolutions and a = length of crank-arm, s = stroke.

$$\text{Velocity of crank-pin } V_0 = 2\pi an = n\pi s.$$

Now, as the crank-pin moves with uniform velocity, the piston undergoes continual changes of velocity, from being zero at the ends to a maximum at about the centre of the stroke. What is commonly spoken of as the speed of piston is the mean speed. If in the unit of time a complete number of revolutions are performed at a uniform rate, the mean speed will be the actual distance traversed by the piston in the unit of time. In each revolution the piston will complete a double stroke, so that speed of piston $= \overline{V} = 2ns$. This may be compared with the speed of crank-pin V_0,

$$\frac{V_0}{\overline{V}} = \frac{\pi ns}{2ns} = \frac{\pi}{2}.$$

Next, as to the actual velocity of the piston at any point of its stroke.

Fig.47a.

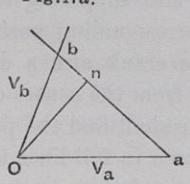
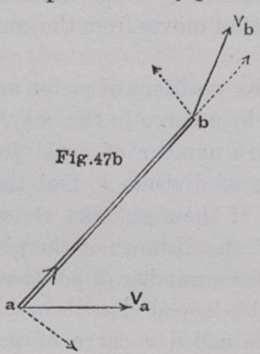
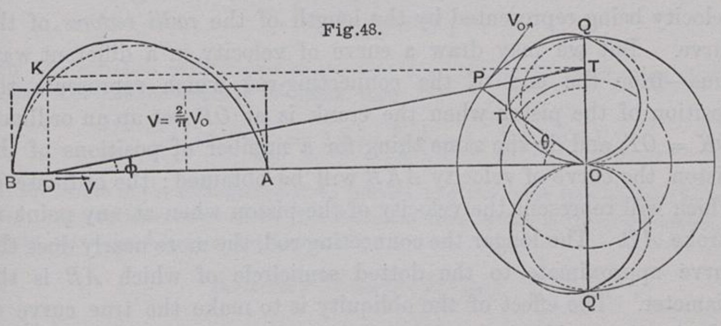


Fig.47b



The piston and crank-pin are joined together by a connecting-rod of invariable length; one end of this rod has the velocity of the piston and the other that of the crank-pin. In Fig. 47*b* let *ab* be a rod, the ends of which move with velocities V_a, V_b in given directions. If one of these velocities be given, the other can be determined. For in Fig. 47*a* draw *Oa* parallel and equal to V_a and *Ob* parallel to V_b to meet a line *ab* which is perpendicular to the line *ab* of the first figure; then, if we drop a perpendicular *On* on *ab*, this will be parallel to *ab* of the first figure, and must represent the resolved part of the velocity V_a along the rod. But the velocities of *a* and *b* resolved along the rod must be equal, because the length *ab* of the rod is invariable; hence *On* also represents the resolved part of V_b along the rod, and consequently *Ob* must represent that velocity in magnitude as well as in direction. The figure *Oab* is called the Diagram of Velocities of the rod, and from it we can find the velocity of any point we please either in, or rigidly connected with, the rod. We shall return to the properties of this diagram frequently hereafter: it will be sufficient now to remark that the triangle *Oab* determines the velocity-ratio of the two ends. In drawing the triangle it is generally convenient to turn it through 90°, so that the lines *ab* in the two figures become parallel, while the sides *Oa*, *Ob* become perpendicular to the velocities they represent.

Fig.48.



In Fig. 48 *OP* is the crank arm, *PD* the connecting-rod; through *O* draw *OT* at right angles to the line of stroke to meet the connecting rod produced in *T*, then *P* moves perpendicular to *OP*, and *D* to *OT*, therefore *OPT* is a triangle of velocities, so that if *V* be the velocity of the piston, V_0 that of the crank pin,

$$\frac{V}{V_0} = \frac{OT}{OP}.$$

This simple construction enables us very conveniently to draw a curve of piston velocity. In the first place, set off along OP a length $OT' = OT$, and do this for a number of positions of the crank. The points T' will be found to lie on a pair of closed curves, shown in full lines in the figure, passing through O and also through Q, Q', the upper and lower ends of the vertical diameter of the crank circle. Had the connecting-rod been indefinitely long, the points T' would have been found to lie on a pair of circles, of which the diameters are OQ and OQ', shown in dotted lines. On account of the obliquity of the connecting-rod, the curve of actual velocity lies outside the circle on the cylinder side of the crank, and inside the circle when the crank lies away from the cylinder.

When the crank is at right angles to the line of dead centres, the velocity of the piston is the same as that of the crank-pin, and neglecting the obliquity of the connecting-rod this will be the maximum velocity of the piston. If the obliquity is taken into account, the greatest velocity of piston occurs when the crank is inclined a little towards the cylinder, it is very approximately when the crank is at right angles to the connecting-rod, and the maximum velocity will a little exceed the velocity of the crank-pin.

The curve just described is a polar curve, the magnitude of the velocity being represented by the length of the *radii vectores* of the curve. But we may draw a curve of velocity in a different way, thus—from the end of the connecting-rod which represents the position of the piston when the crank is at OP, set up an ordinate $DK = OT'$, and do the same thing for a number of positions of the piston, the curve of velocity AKB will be obtained; the ordinate of which will represent the velocity of the piston when at any point of stroke AB. The longer the connecting-rod, the more nearly does the curve approximate to the dotted semicircle of which AB is the diameter. The effect of the obliquity is to make the true curve of velocity lie outside the semicircle in the first half of the stroke of the piston towards the crank, and inside for the second half of the stroke.

The mean velocity of the piston may be conveniently represented by an addition to the diagram, thus:—On the same scale that OP, the length of the arm, represents the velocity V_0 of the crank-pin, take a length to represent

$$\overline{V} = \frac{2}{\pi} V_0.$$

In the polar diagram draw a circle with O as centre and radius of this length. Where this circle cuts the polar curve of velocity the positions of the crank are given at which the actual speed of the piston is equal to its mean speed. In the second diagram of velocity, set up an ordinate to represent $\overline{V}$, and draw a line parallel to the line of stroke. It will cut the curve of piston velocity in two points.

An approximate expression for the velocity of the piston may be determined thus:

$$V = V_0 \frac{\sin OPT}{\sin OTP} = V_0 \frac{\sin(\theta+\phi)}{\cos\phi};$$

or expanding the numerator,

$$V = V_0\{\sin\theta + \cos\theta\tan\phi\}.$$

Since ϕ is in all practical cases a small angle, $\tan\phi$ may be written $= \sin\phi$ without sensible error.

$$\therefore V = V_0\{\sin\theta + \cos\theta\sin\phi\}.$$

$$\text{Now} \quad \frac{\sin\phi}{\sin\theta} = \frac{OP}{PD} = \frac{1}{n}.$$

$$\therefore V = V_0\left\{\sin\theta + \frac{1}{n}\sin\theta\cos\theta\right\}$$

$$= V_0\left\{\sin\theta + \frac{1}{2n}\sin 2\theta\right\}.$$

By differentiation with respect to the time t we obtain the acceleration of the piston. Let a be the length of the crank, then

$$\frac{dV}{dt} = \frac{dV_0}{dt}\left\{\sin\theta + \frac{1}{2n}\sin 2\theta\right\} + \frac{V_0^2}{a}\left\{\cos\theta + \frac{1}{n}\cos 2\theta\right\}.$$

If the length of the connecting-rod be infinite, and the crank turn uniformly, we obtain a simple harmonic motion, the deviation from which is therefore, approximately, assuming n large and dV_0/dt small,

$$\textit{Deviation} = \frac{V_0^2}{na}\cos 2\theta + \frac{dV_0}{dt}\sin\theta.$$

The graphical construction for the acceleration when the crank turns uniformly will be found in Ch. IX.

EXAMPLES.

1. The driving wheels of a locomotive are 6 feet in diameter, find the number of revolutions per minute and the angular velocity, when running at 50 miles per hour. If the stroke is 2 feet, find also the speed of piston.

Revolutions per minute,	=	$233\frac{1}{2}$.
Angular velocity,	=	$24\frac{4}{9}$ per second.
Speed of piston,	=	933·6 feet per minute.

2. The pitch of a screw is 24 feet, and revolutions 70 per minute. Find the speed in knots per hour. If the stroke is 4 feet, find also the speed of piston in feet per minute.

Speed of screw = 16·58 knots per hour.
,, piston = 560 feet per minute.

3. The stroke of a piston is 4 feet, and the connecting-rod is 9 feet long. Find the position of the crank, when the piston has completed the first quarter of the forward and backward strokes respectively. Also find the position of the piston when the crank is upright.

Ans. The crank will make, with the line of dead centres, the angles 55° and 66°. When the crank is upright the piston will be $2\frac{3}{4}$ inches from the middle of its stroke.

4. The valve gear is so arranged in the last question as to cut off theste am when the crank is 45° from the dead points both in the forward and backward strokes. Find the point at which steam will be cut off in the two strokes. Also when the obliquity of the connecting rod is neglected.

Ans. Fraction of stroke at which steam is cut off is—

·175 in forward stroke,
·118 in backward ,,
·146 neglecting obliquity.

5. Obtain the results of the two last questions for the case of an oscillating engine, 6 feet stroke, the distance from the centre of the trunnions to the centre of the shaft being 9 feet.

Ans. Angles 68° and 51° : Cut off ·2 and ·115.

6. In Ex. 3 construct both curves of piston velocity. If the revolutions be 70 per minute, find the absolute velocity of the piston in the positions given. Find also the maximum velocity of the piston.

Ans. $\frac{1}{4}$ stroke forward, velocity = 810 feet per 1′.
$\frac{1}{4}$,, back, ,, = 730 ,,
Maximum, ,, = 900 ,,

Find also the points in the stroke at which the actual speed of piston is equal to the mean speed.

Ans. $4\frac{3}{4}$ in. from commencement of forward stroke.
$6\frac{3}{4}$ in. ,, end ,, ,,

7. The travel of a slide valve is 6 in., outside lap 1 in. Find, in feet per second, the velocity with which the port commences to open when the revolutions are 70 per minute.

Ans. Port commences to open when the valve is 1 in. from the centre of its stroke. Neglecting the obliquity of the eccentric rod, velocity of valve is then 1·72 feet per second.

8. Show that the maximum velocity of the piston occurs when the crank is nearly at right angles to the connecting rod, the difference being a small angle, the sine of which is $\frac{1}{n(n^2+2)}$ nearly, where n is the ratio of connecting rod to crank.

50. *Mechanisms Derived from the Slider-Crank Chain.*—In the investigation just given it has been supposed, for simplicity, that the crank turns uniformly, but if this be not the case the curve constructed will show the ratio of the velocities of the piston and crank pin. In all cases it is the velocity-ratio of two parts, not the velocities themselves, which are determined by the nature of the mechanism. The velocities are of course reckoned relatively to the frame, but as both piston and crank pair with the frame, they are also the velocities of the piston-frame pair and the crank-frame pair (see p. 104), the crank being the radius of reference. The velocities of the other pairs will be determined presently, but in this mechanism are of less importance. We will now direct our attention to other examples of the simple chain of lower pairs, of which the direct-acting engine is only a particular case. In Fig. 49, *D* is a

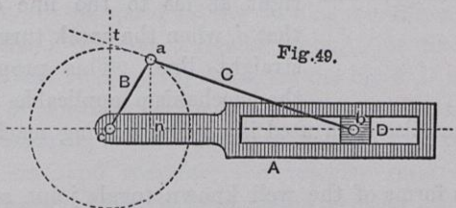


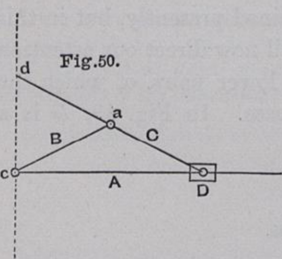
Fig. 49.

block capable of sliding in the slot of the piece *A*. By means of a pin this block is connected with one end of the link *C*. *B* is a crank capable of rotating about a pin attached to the piece *A*, and united to *C* by another pin. Each of the four pieces of which this mechanism is composed, together with either of the adjacent pieces, constitutes a "pair," of which there are *four*, viz., three turning pairs, *AB*, *BC*, *CD*, and a sliding pair *DA*. This simple combination of pairs is known, in the modern theory of machines, as a Slider-Crank Chain.

Since the relative motions of the parts depend solely on the form of the bearing surfaces of the pairs and the position of their centres, not on the size and shape of the pieces in other respects, we may vary these at pleasure, and thus adapt the same chain to a variety of purposes. Especially we may interchange the hollow and solid elements of the pairs, a process which occurs constantly in kinematic analysis, and is called "inversion of the pair."

Again, any one of the four pieces may be fixed and the others move, so that we can obtain four distinct mechanisms from the same chain, simply by altering the link which we regard as fixed, a process called "inversion of the chain."

(1.) Let A be fixed, then we obtain the mechanism of the direct-acting engine already fully considered. In this, however, the connecting link C is much longer than the crank B; by supposing them equal we obtain a mechanism well known in various forms. In Fig. 50 C is prolonged beyond the crank pin a to a point d, such that $ad = ac$, a circle struck with centre a then passes through c, d, and the centre of the block, thus cd is at right angles to the line of stroke, so that d, when the crank turns, describes a straight line. This property renders the mechanism applicable as a parallel motion. It has also been used in air-compressing machinery. (See page 129.)



The various forms of the well known toggle joint, some of which will be given hereafter, are examples of the same mechanism with different proportions of C to B.

(2.) Instead of A, let us suppose C to be the fixed link, so that A and the other pieces have to take a corresponding motion. With this, by a change in the shape of the pieces, we are able to derive a mechanism well known in two forms. C being fixed, and B caused to rotate, A will have given to it an oscillating motion about the block D, and, at the same time, will slide to and fro on the block, the block itself having a vibrating motion about the other end of the piece C. Now, the relative movement of the parts of this mechanism is identical with that of the oscillating steam engine, and by a suitable alteration in the shape of the pieces, that mechanism may be derived. Thus, suppose, in the first place, the hollow element of A to become the solid one, in the shape of a piston-rod and piston, whilst the block D is enlarged into a cylinder to surround the piston, and so becomes the hollow element of the pair. The cylinder D will oscillate on trunnions, in bearings in the fixed piece C, which must be so constructed as to be a suitable frame for carrying the engine, and have bearings in which the crank shaft and crank B can turn.

The oscillating cylinder is in general mounted on bearings, the centre line of which coincides with the centre of the stroke of the piston, so that the distance apart of the shaft and trunnion bearings is equal to the length of the piston-rod. An example is shown in Fig. 4, Plate I.

Next let us consider the relative motions of the parts. Returning to Fig. 49 above, suppose a, b, c, to be the centres of the turning pairs, and draw ct, an perpendicular to the line of centres bc, to meet C and A in t and n, then it was shown above (page 109) that the velocity-ratio of the pairs DA, BA in the direct-acting mechanism was ct/ac, and as fixing a link makes no difference in the relative motions, this must also be the ratio of the speed of the piston of the oscillator in its cylinder, to the speed of the turning movement of the crank *relatively* to the piston-rod. Again, when C is fixed, as in the oscillator, the link A (Fig. 49) slides on the block D with a velocity the direction of which is perpendicular to an, while the point c in it moves perpendicular to ac. Hence it follows that the triangle of velocities is acn, and therefore the velocity ratio of piston and crank pin is an/ac. The curve of piston velocity can be drawn as before; it differs little in form from that of the direct actor, but the maximum velocity of the piston is equal to that of the crank pin, instead of being somewhat greater. Once more, remembering that fixing a link does not alter the relative motions, it appears that, in all cases, the velocity-ratio of the pairs BC, DA must be an/ac, so that we have determined the ratio of the speed of piston in the direct actor to the speed of the turning movement of the crank *relatively* to the connecting rod.

Comparing our results, we see that the velocity-ratio of the turning pairs BC, BA must be $ct : an$, or what is the same thing $bt : ab$. Since the three angles of the triangle abc are always together equal to 180°, it is clear that the sum of the speeds of the three turning pairs must be zero, due regard being taken of the direction of rotation, and it follows, therefore, that in any slider-crank chain the speeds of the three turning pairs are as $at : ab : bt$. By the introduction of a suitable radius of reference, we may compare these velocities with that of the sliding pair. The most convenient radius to take is that of the crank, then assuming, as before, $ab = n \cdot ac$, the velocities of the pairs are shown by the annexed table :—

VELOCITY RATIOS IN A SLIDER-CRANK CHAIN.				
Pair,	DA	BA	BC	DC
Velocity,	ct	ac	$\frac{bt}{n}$	$\frac{at}{n}$

In the oscillator the angular velocity-ratio of the cylinder and crank is the velocity-ratio of the pairs CD, CB and is therefore $at : bt$ or $cn : bc$. This can readily be constructed by drawing nz parallel to ac, then, since ab is constant, the angular velocity of the cylinder is proportional to az, as may be verified by an independent investigation. The result may be exhibited by a polar curve similar to the curve of piston velocity already drawn. In Fig. 51, c is the crank pin, $TST'S'$ the crank circle, az' is set off along the crank radius equal to az, then a curve with two unequal loops is obtained, which shows the law of vibration of the cylinder.

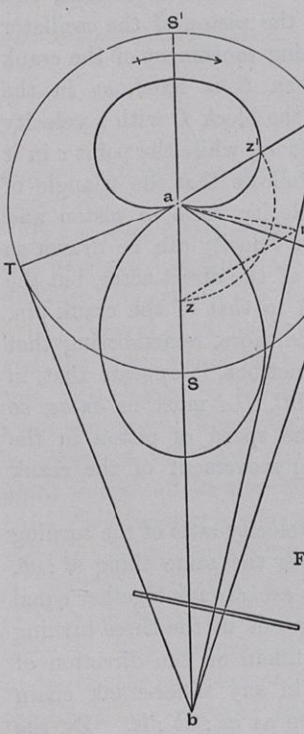
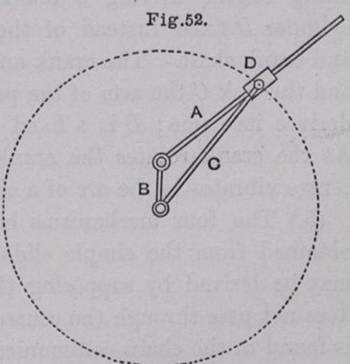


Fig.51.

The motion of the cylinder is such that, in the swing to the left, whilst the crank pin moves along the arc $T'ST$, the angular velocity is much greater than in the return swing to the right, whilst the crank pin moves along the arc $TS'T'$. Supposing the crank to revolve uniformly, the times occupied by the forward and return swings are as the arcs $T'ST$ and $TS'T'$, which are proportional to the angles subtended by them. By measuring or otherwise estimating these angles, the mean angular velocities in the forward and backward oscillation may be determined. This peculiar vibration, rapid one way and comparatively slow the other, has been made use of to obtain a quick return motion of a cutting tool in a shaping machine. The velocity with which a tool will make a smooth cut in metal is limited, and since in general the tool is made to cut in one direction only, time is saved by causing the return stroke to be made more quickly. One construction of such a quick return motion may be thus described. A slotted lever D vibrates on a fixed centre in the frame piece C, its motion being derived from the revolution of a crank B on another fixed centre in the same frame piece C. The crank pin of B turns in the block A, which slides in the slotted lever D. There is in addition a connecting rod, by means of which a to-and-fro motion of a headstock carrying the cutting tool is communicated from the oscillating lever, the headstock sliding in a guide.

Omitting the connecting rod, we have the same kinematic chain, with the same fixed link C, as in the oscillating engine. There has been a change made only in the form of some of the pieces. What was the oscillating cylinder is now the slotted lever, and instead of a piston and rod, we have here the simple block A sliding in the slot. The crank B and frame-link C remain practically unaltered. The slotted lever will vibrate according to the same law which we have investigated for the oscillating cylinder, and thus with a uniform rotation of the crank, a quick return motion of the tool will be obtained. This mechanism is shown in Fig. 5, Plate I., in a form employed for giving motion to the table of small planing machines.

Fig.52.



(3.) Let us next take an example in which B is the fixed link, and becomes the frame, its form being of course modified to suit the new conditions.

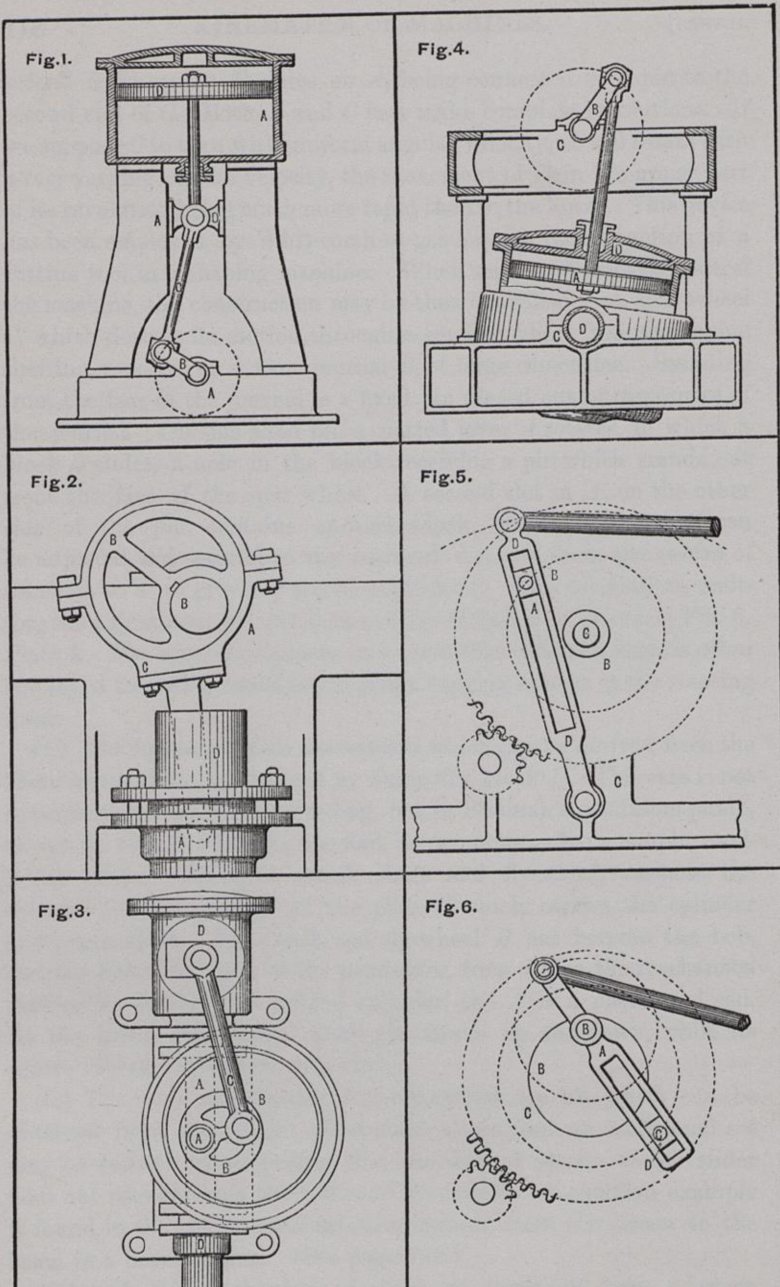
A crank arm C (Fig. 52) turns on a fixed centre in the frame piece B; so also does another arm A on a second fixed centre, D slides on A, being connected by a pin to the second end of C. Both A and C may make complete revolutions. If we suppose C to turn with uniform angular velocity, A will rotate with a very varying angular velocity, the movement of A in the upper part of its revolution being much more rapid than in the lower. This device has been employed by Whitworth to get a quick return motion of a cutting tool in a shaping machine. When separated from the rest of the machine, the construction may be thus described :—A spur wheel C, which derives its motion through a smaller wheel from the engine shafting, revolves on a fixed journal B, of large dimension. Standing from the face of the journal is a fixed pin placed out of the centre of the journal. On this fixed pin a slotted lever A rotates, in which a block D slides, a hole in the block receiving a pin which stands out from the face of the spur wheel. A second slot in A, on the other side of the pin, contains another block, which, by a screw, can be adjusted and secured at any required distance from the centre of rotation, so as to give any stroke at pleasure. This mechanism, omitting the adjustment by which the stroke is varied, is shown in Fig. 6, Plate I. The same mechanism in a somewhat different form is often employed in sewing machines to give a varying motion to the rotating hook.

(4.) The fourth possible mechanism which can be derived from the slider-crank chain is obtained by fixing the block D. This case is not so common as the three preceding, but in Stannah's pendulum pump, shown in Fig. 3, Plate I., we find an example. In a simple oscillating engine driving a crank shaft and fly-wheel, suppose the cylinder D fixed instead of the piece C which carries the cylinder and crank shaft. The crank and fly-wheel B has become the bob, and the link C the arm of the pendulum, from which the mechanism derives its name; D is a fixed cylinder, and A is a piston and rod. As the crank rotates the crank pin moves up and down, while its centre vibrates in the arc of a circle.

(5.) The four mechanisms here described are all which can be obtained from the simple slider-crank chain, but an additional set may be derived by supposing that the line of stroke of the slider does not pass through the centre of the crank. A common example is found in the chain communicating motion from the piston to the beam in a beam engine. (See page 129.)

Although the mechanisms derived by inversion from a given

Plate. I.



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kinematic chain may be described as distinct, it must be carefully observed that there is in reality no kinematic difference between them, the distinction consisting merely in a different link being chosen to reckon velocities from. If we consider the velocities of the pairs which constitute the chain, those velocities are always related to each other in the same way, and the same machine may be regarded sometimes as one mechanism and sometimes another. For example, suppose a direct-acting engine working on board ship; the ship may be imagined to roll so that the connecting rod of the engine is at rest relatively to the earth, and the engine becomes an oscillator to an observer outside the ship. Dynamically and constructively, however, there is a great difference, for the fixed link is the frame, and is attached to the earth or other large body, the predominating mass of which controls the movements of all bodies connected with it. To illustrate and explain the inversion of a slider crank-chain, Plate I. has been drawn. The six examples which have just been described are here placed side by side with the same letters $ABCD$ attached to corresponding links, so that they may readily be recognized. It will be seen that each link assumes very various forms; thus, for example, the link A is the frame and cylinder in Figs. 1 and 2, a piston and rod in Figs. 3 and 4, a block in Fig. 5, and a rotating arm in Fig. 6. The relative motions of corresponding parts are, however, always the same.

51. *Double Slider-Crank Chains.*—We now pass on to the consideration of a kinematic chain consisting of two turning pairs and two sliding pairs. We will commence by showing how this chain may be derived from that previously described. Suppose the piece D, instead of being simply a block, is a sector shaped as shown in Fig. 1, Plate II., having a slot curved to the arc of a circle of centre O, while the piece C, which was before the connecting rod, is compressed into a block sliding in the curved slot. The law of relative motion of the parts of this mechanism will be precisely the same as in the direct-acting engine, for the block C will move just as if it were attached by a link, shown by the dotted line, to a point O, a fixed point in the piece D. The piece D will slide in A, just as if there were a connecting link from C to O and no sector—that is, it will slide just as the piston does in the cylinder of a direct-acting engine. Moreover, there are, in reality, exactly the same pairs in this as in the mechanism of the direct-acting engine, for C and D together make a turning pair, although only portions of the surfaces of the cylindric elements are employed.

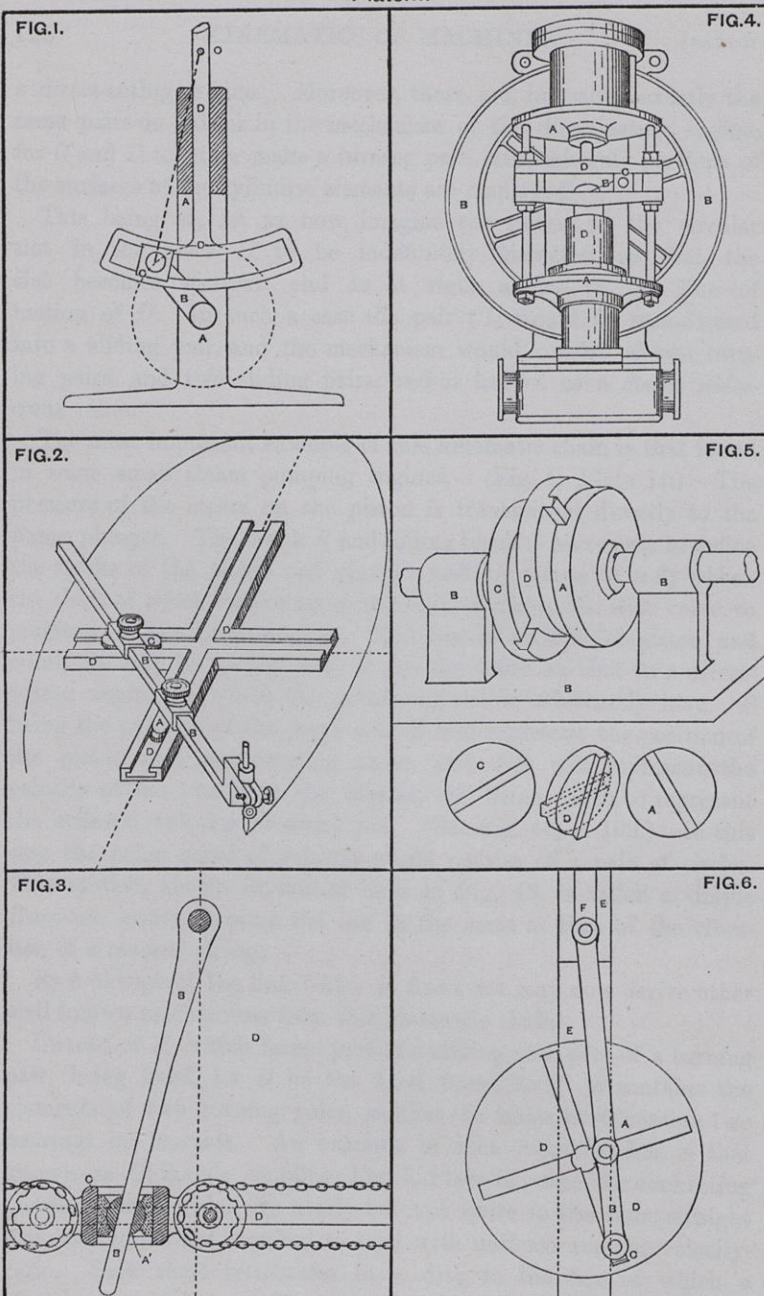
This being so, let us now imagine the radius of the circular slot in the piece D to be indefinitely increased, so that the slot becomes straight, and is at right angles to the line of motion of D. In such a case the pair CD would be transformed into a sliding pair, and the mechanism would consist of two turning pairs, and two sliding pairs, and is known as a *double slider-crank chain.*

The most important example of this kinematic chain is that found in some small steam pumping engines. (Fig. 4, Plate II.) The pressure of the steam on the piston is transmitted directly to the pump plunger. The crank B and sliding block C serve only to define the stroke of the piston and plunger, and, by means of a fly-wheel, the shaft of which carries an eccentric for working the slide valve, to maintain a continual motion. The law of motion of piston and crank pin may be readily seen to be the same as that in a direct-acting engine, in which the connecting rod is indefinitely long. P being the position of the crank pin, M will represent the position of the piston and reciprocating piece, and PM will represent the velocity of the piston at the instant, OP being taken to represent the uniform velocity of crank pin. (See Fig. 48, p. 109.) In this case the polar curve of velocity would consist of a pair of circles. This motion, shown in dotted lines in Fig. 48, is called a simple *Harmonic motion*, because the law is the same as that of the vibration of a musical string.

By a change of the link which is fixed, we may now derive other well known mechanisms from this kinematic chain.

Instead of A, which forms part of a sliding and part of a turning pair, being fixed, let B be the fixed frame link. B contains the elements of two turning pairs, so that the frame must contain two bearings or journals. An example of such a mechanism is that known as Oldham's coupling, Fig. 5, Plate II., used for connecting parallel shafts, which are nearly but not quite in the same straight line, and which are required to turn with uniform angular velocity-ratio. Each shaft terminates in a disc, in the face of which a straight groove is cut. The two discs, A and C in the figure, with

Plate. II.



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the grooves face each other, and are placed a little distance apart, with the grooves at right angles to each other. Filling up the space between them is placed a disc *D*, on the two faces of which are straight projections at right angles to one another, which fit into the grooves in the shaft discs. In the revolution of the shafts each of these projections slides in the groove in which it lies, and rotates with it. The two grooves are, therefore, maintained always at right angles to one another, and the two shafts rotate one exactly with the other.

Next, let the fixed link of the chain contain the elements of two sliding pairs, which would be obtained if we made *D* the frame piece. An interesting example of this is the instrument sometimes employed in drawing ellipses. (Fig. 2, Plate II.) Two blocks slide in a pair of right-angled grooves. By means of clamp screws a rod unites them at a constant distance from one another. Pins fitting in holes in the blocks allow the rod to rotate relatively to the blocks. Any point in the rod will describe an ellipse, as indicated in the figure.

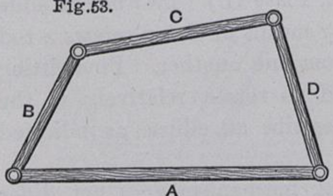
If the link *C* be fixed, the resulting mechanism does not differ from that derived by fixing *A*, and the three mechanisms just described are therefore all which can be obtained by inversion of a double-slider chain. In Figs. 2, 4, 5 of the plate referred to, they are shown side by side with the same letters attached to corresponding links, as in Plate I.

The directions of motion of the two sliding pairs have been supposed at right angles, but any other angle may be assumed, and mechanisms obtained which we need not stop to examine. A more important change is to suppose that the sliding pairs and turning pairs alternate, so that each link forms an element of one sliding and one turning pair. A mechanism known as "Rapson's Slide," employed as a steering gear in large ships, will furnish an example. Fig. 3, Plate II., shows one way in which it is applied. *A′* is an enlarged pin made in two pieces between which the tiller *B* slides while turning about an axis fixed in the ship *D*. *A′* is carried by the piece *C*, which slides in a groove fixed transversely to the ship being drawn to port or starboard by the tiller chains passing round pullies mounted on *C*, as shown in the figure. The further the tiller is put over the slower it moves (Ex. 8, p. 133), and therefore the greater the turning moment (Ch. VIII.), a property of considerable practical value. In this kinematic chain the same mechanism is obtained whichever link is fixed.

The mechanism shown in Fig. 6 of this Plate is a compound chain, to be referred to hereafter.

52. *Crank Chains in General.*—Instead of having a chain of turning pairs connected by one or two sliding pairs, we may have turning pairs alone. The number will be four, and their axes must meet in a point or be parallel. Taking the second case, the chain in its most elementary form consists of 4 bars united by pin joints at their extremities, as in Fig. 53. It is called a crank or four-bar chain, and from it may be derived the slider-crank chain already considered, in the same way as from that chain we derived the double slider chain. All the mechanisms hitherto considered may therefore be regarded as particular cases of it. In its present form, however, many new mechanisms are included, some of which will be briefly indicated, referring for descriptions and figures to works specially devoted to mechanism.

Fig.53.



Assuming A the fixed link, B and D which pair with it are called for distinction cranks or levers, according as they are or are not capable of continuous rotation, while C the connecting link is called for shortness the *coupler.*

(1.) Let B be a crank and D a lever, then the mechanism is a "lever-crank," an example of which occurs in the common beam engine, D being the beam, B the crank, C the connecting rod, and A the entablature, foundation, and all other parts connected therewith.

(2.) The links B and D may be equal, and C may be equal to A. This may be called "parallel cranks" when B and D are set parallel, as in the coupled outside cranks found in locomotives, or "anti-parallel cranks" when they are set crosswise, a case to be hereafter referred to. (Page 178.)

(3.) The links D and B may still both be cranks if C be greater than A, provided that the difference between B and D be not too great. The mechanism is called "double cranks," and occurs in the common drag-link coupling, and also in the mechanism of feathering paddles.

(4.) If the coupling link be too short, neither B nor D will be capable of a complete rotation. The mechanism is then a "double lever," and an example occurs in the common parallel motion to be considered hereafter.

(5.) A number of additional mechanisms may be derived by supposing the axes of the four turning pairs to meet in a point, instead of being parallel; we thus obtain a "conic crank chain." Hooke's joint is a particular case of this, but in general these mechanisms are of less importance.

53. *Diagram of Velocities in Linkwork.*—A simple construction has already been given, by means of which the velocity-ratios of the parts of a slider-crank chain are determined, and we will now consider this question for any case of linkwork in which the axes of the pairs are parallel.

Figure 54*a* represents a chain of links *zOabcd*... united by pins so as to form a succession of turning pairs. The first link *Oz* is fixed,

Fig.54a.

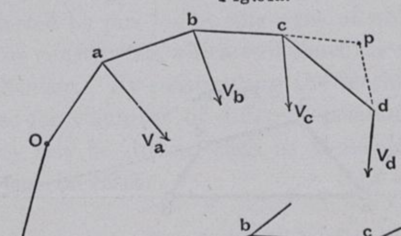
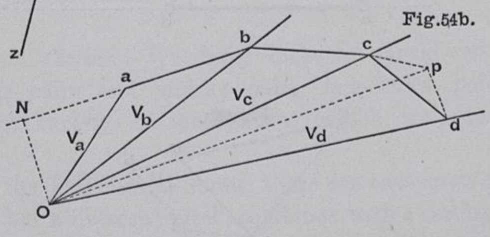


Fig.54b.



so that the second turns about a fixed O point as centre, and therefore a moves perpendicularly to Oa, with a velocity V_a, which we may suppose known. The other points b, c, d... move in directions which we suppose given, and with these data it is required to find the magnitudes of the velocities. In Figure 54*b*, from a pole O draw radiating lines perpendicular to the given directions, and set off on the first Oa to represent V_a, then draw ab, bc, cd... parallel to the links of the chain to meet the corresponding rays, then the lengths of those rays represent the velocities.

For drop a perpendicular ON from O on to ab, or ab produced, then ON represents the component of V_a in the direction of the second link, but this must also be the component of V_b in that direction, since ab is of invariable length; that is, Ob must represent V_a. Similarly all the other rays must represent the velocities of the corresponding points.

The figure thus drawn may be called the Diagram of Velocities of the chain. It may be constructed equally well, if the magnitudes of the velocities be given instead of their directions, also any of the turning pairs may be changed into sliding pairs. If both ends of the chain be attached to fixed points, the diagram will evidently be a closed polygon. Its sides, when divided by the lengths of the corresponding links of the chain, represent their angular velocities, for each side is the algebraical difference of the velocities of the ends of the link perpendicular to the link.

In the four-link chain (Fig. 55*a*), consisting of two links turning

Fig. 55a.

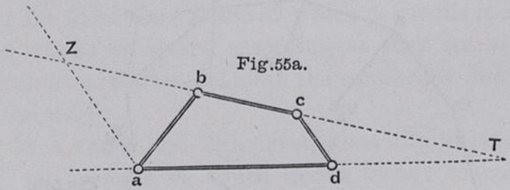
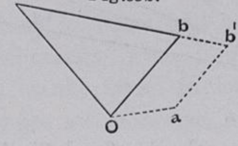


Fig. 55b.



about fixed centres a, d, coupled by a link bc, the diagram of velocities is a simple triangle Obc (Fig. 55*b*), the sides of which, when divided by the lengths of the links to which they are parallel, represent the angular velocities of the links. Through a draw aZ parallel to cd, and prolong bc to meet it in Z, and the line of centres in T, then, since the triangle Zab is similar to the triangle of velocities, the angular velocities of the levers cd, ab will be proportional to Za/cd and ab/ab. The last fraction is unity, and therefore we have

$$\text{Angular Velocity-Ratio} = \frac{Za}{cd} = \frac{aT}{dT},$$

showing that the ratio in question is the inverse of the ratio of the distance of T from the centres.

If, instead of the link ad being fixed, the chain of four bars be imagined to turn about one joint such as d, the diagram of velocities would be a quadrilateral $Oab'c$, with sides parallel to $abcd$.

Returning to the general case, let p be any point rigidly connected with one of the links of the chain, say cd, in the figure; then if we lay down on the diagram of velocities a point p, similarly situated with respect to the corresponding line cd of that diagram, it follows at once, by the same reasoning, that the ray Op, drawn from the pole O, must represent the velocity of p in the same way that the other rays represent the velocities of the points a, b... Thus it appears that for any linkwork mechanism, consisting of pieces of any size and shape connected by pin joints, the axes of which are parallel, a diagram may be constructed which will show the velocities of all points of the mechanism. By constructing the mechanism and its diagram of velocities for a number of different positions, curves of position and velocity may be drawn, such as those described in preceding articles for special cases.

54. *Screw Chains.*—We have hitherto considered only chains of turning pairs and sliding pairs, but screw pairs also occur in a great variety of mechanisms which we can only briefly indicate.

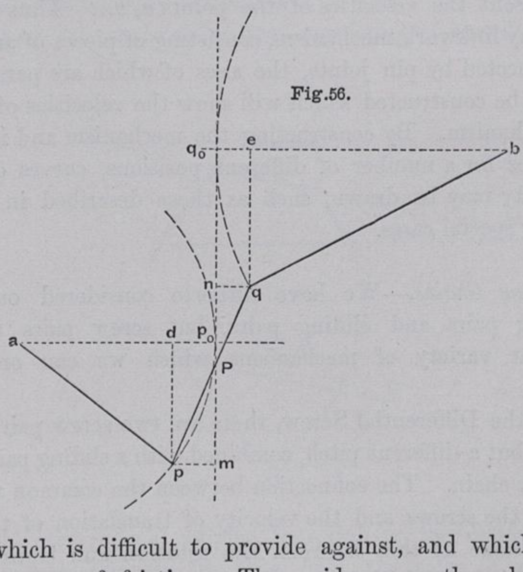
(1.) In the Differential Screw, there are two screw pairs with the same axes but a different pitch, combined with a sliding pair, forming a three-link chain. The connection between the common velocity of rotation of the screws and the velocity of translation of the sliding pair is the same as that between the rotation and translation of a screw, the pitch of which is the difference between the pitches of the actual screws. The arrangement has often been proposed for screw presses, a mechanical advantage being obtained, at least theoretically, with screws of coarse pitch, which would otherwise require a thread so fine as to be of insufficient strength. The right and left-handed screw is an example in common use.

(2.) In the Slide Rests of lathes and other machine tools, the traversing motion of planing machines, and many other cases we find a three-link chain, consisting of a screw pair, a turning pair, and a sliding pair. This may be regarded as a particular case of the preceding, the pitch of one of the screws being zero.

(3.) In presses, steering gear, and many other kinds of machinery we find a simple screw chain employed to work a slider-crank chain. Some examples will be given hereafter.

55. *Parallel Motions Derived from Crank Chains.*—In beam engines the connecting rod by which the reciprocating motion of the piston is communicated to the vibrating beam is necessarily short, in order to diminish the height of the machine, and therefore, if guides are employed to retain the end of the piston-rod in a straight line, there will be considerable lateral

Fig. 56.



pressure which is difficult to provide against, and which involves a large amount of friction. The guides may then be replaced with advantage by some linkwork or other mechanism. Such a mechanism is called a Parallel Motion, and in the early days of engineering was employed more extensively than at the present time. In its most simple form it consists of two levers capable of turning about the fixed centre a and b. (Fig. 56.) The ends of the levers are connected by a coupling link pq, then, so long as the angular movement of the levers is not too great, there is a point in the link pq which will describe very approximately a straight line. In the first instance let us suppose the links so set that when ap_0 and bq_0 are parallel, p_0q_0 is at right angles to them. Let $apqb$ be the extreme downward movement of the levers, then p lying to the left and q to the right, there will be some point P in pq which in this extreme position lies in the straight line p_0q_0. In the upward extreme position the same point of pq will, approximately, also lie in this line. If, then, p_0q_0 be the line of stroke, and the point P be selected for the point of attachment of the piston-rod head, then this point will be exactly in the line at the middle and bottom of the stroke, and at other points will deviate but little from it.

To find the point where pq intersects p_0q_0, we must first obtain expressions for the amount that the point p deviates to the left of p_0 and q to the right of q_0; these amounts being the versines of the arcs in which the points move, and shown by dp_0 and eq_0, where pd and qe are drawn perpendicular to ap_0 and bq_0. By supposing the circle of which a is the centre to be completed, it is easy to see that

$$(ad + ap_0)dp_0 = pd^2,$$

$$\therefore \quad dp_0 = \frac{pd^2}{ad + ap_0}.$$

If the angle p_0ap is not greater than 20°, we may write

$$dp_0 = \frac{pd^2}{2.ap_0},$$

the error not being greater than 1 per cent. Now, neglecting the small effect due to the obliquity of the connecting link when in the extreme positions, $pd = \frac{1}{2}$ stroke; therefore, supposing $ap = r_a$ and $bq = r_b$,

$$pm = dp_0 = \frac{(\text{stroke})^2}{r_a},$$

and

$$qn = eq_0 = \frac{(\text{stroke})^2}{r_b}.$$

Now P being the point where pq intersects p_0q_0, we have similar triangles in which

$$\frac{pP}{qP} = \frac{pm}{qn} \text{ and } \therefore = \frac{r_b}{r_a}.$$

Thus the point P, which has most correctly the straight-line motion, is such that it divides the coupling link into segments which are inversely proportional to the lengths of the levers. If the levers be placed into all possible positions, then in the motion the connecting link will be inverted and the point P will trace a closed curve resembling a figure of 8. There are two limited portions of this curve which deviate very little from a straight line.

We may approximate still more nearly to a straight line by a little alteration in the setting of the levers. Suppose the centres of vibration a, b, are brought a little nearer together so that the line of stroke bisects the two versines dp_0 and eg_0. Then when the levers are parallel, the link slopes to the left upwards, whereas at the ends of the stroke the link will slope to the right upwards. At two intermediate positions about quarter stroke from the ends, the link will be vertical. If we choose the point P as previously described, the maximum deviation will be only about one fifth of its former amount. In practice, the final adjustment of the centres of motion is performed by trial.

The use of parallel motions is almost exclusively confined to beam engines. In that case bq will be the half length of the beam of the engine, and in order that the angle through which the beam vibrates should not exceed 20° above and below the horizontal, the length of the beam should not be less than three times the stroke. The radius rod may be somewhat shorter than the half beam, but should not be less than the stroke, or the error in the motion of P will be too great. This mechanism will, therefore, occupy a considerable space. To economise space, and also to provide a second straight-line path to guide the air-pump rod, a modification of the mechanism is made use of.

In Fig. 57, be being the half length of beam, a point q is chosen so that

$$\frac{bq}{be} = \frac{\text{stroke of air pump}}{\text{stroke of piston}},$$

and a parallelogram of bars $qeQp$ provided, united by pins. The point p is jointed to the end of the radius rod ap, vibrating on the fixed centre a. Consequently there will be some point P in

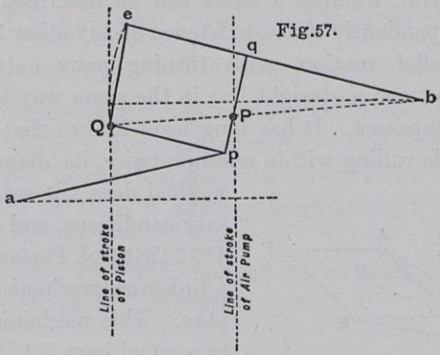


Fig.57.

the back link qp which will describe very nearly a straight line. This point is such that

$$\frac{Pp}{Pq} = \frac{bq}{ap}.$$

Now, if the proportions of the links are such that bPQ is a straight line, bQ/bP will be constant, and therefore the path described by Q will be an enlarged copy of the path described by P. That is to say if P moves approximately in a straight line, then Q will do so also. If then the radius rod is of suitable length we provide a point Q for the attachment of the piston rod, and also a point P for the attachment of the air-pump rod. To find this length we have

$$\frac{bq}{pQ} = \frac{qP}{pP},$$

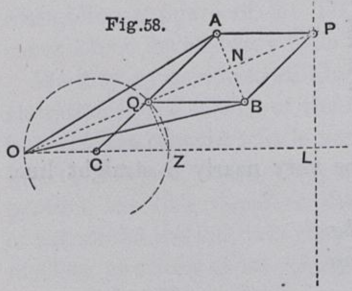
whence multiplying by the preceding equation

$$bq^2 = pQ \times ap,$$

$$\text{or Length of radius rod} = \frac{(bq)^2}{eq}.$$

The parallel motion just described which was introduced by Watt is the only one much used in practice, but there is another form which possesses great theoretical interest because it is exact and yet involves only turning pairs. Scott Russell's parallel motion (Fig. 50, page 114) is exact, but as it involves a sliding pair its accuracy depends on the exactness with which the guides of the slides are constructed. Now, a straight edge or a plane surface can only be constructed by a process of copying from some given plane surface or by trial and error, whereas a circle can be described by a pair of compasses independently of the existence of any other circle. Hence an exact parallel motion, with turning pairs only, enables us theoretically to trace a straight line in the same way that a circle is traced with compasses. It has long been known that this could be done by a circle rolling within another twice its diameter, but this method does not satisfy the necessary conditions, and it was not till 1872 that Col. Peaucellier invented a linkwork mechanism for the purpose. This mechanism consists of two equal bars OA, OB, jointed to each other at O, and at A, B to a parallelogram of equal bars $AOBQ$, so that OQP are in a straight line (Fig. 58). This being so then, however the bars are placed there will always be some fixed relation existing between OQ and OP. Thus drop a perpendicular AN on OP, then $OQ = ON - QN$ and $OP = ON + NP$. Also since $AQ = AP$, $QN = NP$,

Fig.58.



$$\therefore OQ \,.\, OP = ON^2 - QN^2.$$

But $ON^2 = OA^2 - AN^2$ and $QN^2 = QA^2 - AN^2$, therefore $OQ \,.\, OP = OA^2 - QA^2$, and is a constant quantity for all positions; that is to say, if we cause Q to move over any curve, then P will describe its reciprocal.

We can now show how this mechanism may be employed to draw a straight line. Let O be a fixed centre and PL be the straight line which it is required to describe. Draw the perpendicular OL on PL. Then the mechanism being placed in any position with P at any point on the line to be drawn, draw QZ at right angles to OQ. Bisect OZ in C and attach Q to C by means of a jointed rod which can turn on the fixed centre C. The circle which Q describes during the motion of the bars will have OZ as a diameter, for OQZ is a right angle, and therefore the angle in a semicircle. We observe now that we have similar triangles OQZ and OLP.

$$\therefore OL = \frac{OP.OQ}{OZ};$$

but $OZ = 2.OC$ is a constant quantity and so is the product OP, OQ.

$$\therefore OL \text{ is constant.}$$

That is to say, wherever P is, the length of the projection of OP on the perpendicular OL is a constant quantity. This can be true only so long as P lies in the perpendicular line PL. Thus, by the constrained motion of Q in a circle passing through O, P is caused to move perfectly in a straight line.

This mechanism has been applied to a small engine used for ventilating the House of Commons.

56. *Closure of Kinematic Chains. Dead Points in Linkwork.*—A kinematic chain, like a pair (p. 103), may be "incomplete," that is, the relative movements of its links may not be completely defined. It then cannot be used as a mechanism without employing some additional constraint, a process called "closing" the chain. In order that a chain may be closed it must be endless, and the number of links must not be too great; for example, in a simple chain of turning pairs with parallel axes we cannot have more than 4 links. If there be 5 the motion of any one link relatively to the rest will not be definite, but may be varied at pleasure.

So also a chain may be "locked" either by locking one of the pairs of which it is constructed; or by rigidly connecting two links not forming a pair; it then becomes a frame, such as was considered in a previous part of this book.

A chain is often incomplete or locked for special positions of its links, though closed and free to move in all other positions; this, for example, is the case at the dead points which occur in most linkwork mechanisms. A well known instance is that of the mechanism of the steam engine, in which the chain is locked and the direction of motion of the crank indeterminate when the connecting rod and crank are in the same straight line. This instance further shows that it is necessary to distinguish between the two directions in which motion may be transmitted through the mechanism, for the dead points in question would not occur if the crank moved the piston instead of the piston the crank. A piece, then, which transmits motion is called a "driver," in relation to the piece to which motion is transmitted, which is called a "follower," terms which will be frequently used hereafter in cases where both pair with the fixed link. (Compare Arts. 47, 86.) The dead points in a mechanism may be passed either by the union of two similar mechanisms, with dead points in different positions, as in a steam engine with a pair of cranks at right angles, or by aid of the inertia of the moving parts. This last method involves what is called "force-closure," a term which will be explained presently.

EXAMPLES.

1. The stroke of an oscillating engine is 6 feet, and the distance between the centre line of the trunnions and the centre of shaft is 9 feet. Find the maximum and mean angular velocity of the cylinder in each of its two oscillations as compared with that of the crank. Find also the velocity of the piston at half stroke as compared with the speed of piston.

Ans. Maximum angular velocity-ratios of cylinder and crank, $\frac{1}{2}$ and $\frac{1}{4}$.
Mean ,, ,, ·277 and ·178.

$$\frac{\text{Velocity of piston at } \frac{1}{2} \text{ stroke}}{\text{Mean speed of piston}} = \frac{1 \cdot 54}{1}.$$

2. The travel of the tool of a shaping machine is to be 9 inches, and the maximum return, three times the maximum cutting, velocity. Neglecting the obliquity of the connecting link, find the proper proportions of the quick return motion (Fig. 5, Plate I.). Find also the revolutions per minute for a maximum cutting velocity of 6 inches per second, and compare the times of cutting and return.

Ans. The length of slotted lever = 9 inches.

Distance apart of centres = twice length of crank.
Revs. per minute of crank = 19·1.

$$\frac{\text{Time of cutting}}{\text{Time of return}} = \frac{2}{1}.$$

Remark.—The lever vibrates through 60°, an inconveniently large angle. Whitworth's mechanism would be preferable, as it readily gives an even greater inequality.

3. In Whitworth's quick return motion find the proportions that the maximum return may be three times the maximum cutting velocity, and compare the times of cutting and return.

4. In example 1, draw curves showing the angular velocity and position of the piston for any position of the crank.

5. In example 2, draw a diagram of velocities for any position of the mechanism, and hence trace curves showing the velocity of the tool at any point of the cutting and return stroke.

6. In question 1, p. 111, supposing two pairs of driving wheels coupled, the lengths of cranks 1 foot, find the velocity of the coupling-rod in any position. First, relatively to the locomotive; second, relatively to the earth.

7. In Ex. 5, p. 112, find in feet per second the maximum and minimum velocity of rubbing of the crank pin, assuming its diameter 12 in. Draw a curve showing this velocity in any position of the crank.

8. In Rapson's Slide (p. 121), if the tiller be put over through an angle θ, show that the velocity-ratio of tiller and slide varies as $\cos^2\theta$, and draw a curve of velocity.

9. In a drag-link-coupling the shafts are 6 in. apart, the drag-link 1 foot long, and the cranks each 3 feet long. By construction, determine the four positions of the following crank when the leading crank is on the line of centres, and at right angles to the line of centres.

10. The length of the beam of an engine is three times the stroke. Supposing the end of the beam when horizontal is vertically over the centre of the crank shaft at a height equal twice the stroke, and the crank also is then horizontal, find the length of connecting rod and the extreme angles through which the beam will sway. Adjust the crank centre so that the beam may sway through 20° above and below the horizontal.

Length of rod = 2·06 stroke. The beam sways $22\frac{1}{2}°$ above the horizontal, and 17° below.

11. The depth of the floats of a feathering paddle wheel is $\frac{1}{8}$th the diameter of the wheel, and the immersion of the upper edge in the lowest position $\frac{1}{4}$th the depth of the float. Assuming the stem levers $\frac{2}{3}$ths the depth of the floats, find the position of the centre of the collar to which the guide rods are attached. Determine the length of the rods, and draw the float in its highest position.

If O be centre of wheel, K centre of collar, OK = ·054 of diameter of wheel, and is horizontal (very approximately).

Length of guide rods = 1·01 radius of wheel.

12. In Ex. 9, find the angular velocity-ratio of the shafts when the cranks are in the positions mentioned. Find also the maximum and minimum angular velocity ratio.

13. In Oldham's coupling, show that the centre of the coupling disk revolves twice as fast as the shafts, and hence show how to give two strokes of a sliding piece for one revolution of a shaft.

14. In a simple parallel motion the lengths of the levers are 3 feet and 4 feet respectively, and the length of the connecting link is $2\frac{1}{2}$ feet. Find the point in the link which most nearly moves in a straight line, and trace the complete curve described by this point as the levers move into all possible positions, the motion being set so that, when the levers are horizontal, the link is vertical.

Ans. The required point in link is $17\frac{1}{7}$ in. from the 3-feet lever,
and $12\frac{6}{7}$ in. ,, 4-feet ,,

15. In a beam engine the stroke of piston is 8 feet, of air pump $4\frac{1}{2}$ feet, length of beam 24 feet, the front and back links of the parallel motion being 4 feet. Find the proper length of radius rod, and the point in the back link where the air-pump rod should be attached.

Ans. Length of radius rod = 8 feet $8\frac{1}{8}$ inches.
Point of attachment of air-pump rod = 3 ,, 3 ,, below beam.

16. Suppose in last question the parallel motion set for least deviation from a straight line, find the correct positions of the centre lines of air pump and piston, and the position of the centre of motion of the radius rod.

Ans. Horizontal distances from centre of beam—

Line of stroke of piston, -	11 feet	8 inches.	
,, air pump,	6 ,,	$6\frac{3}{4}$	,,
Centre of motion of radius rods,	15 ,,	$1\frac{1}{8}$	,,

REFERENCE.

A good collection of linkwork and other mechanisms, some of which do not occur in the larger works cited on page 100, will be found in the 4th edition (1880) of Professor Goodeve's *Elements of Mechanism.* Much valuable information on the details of machine design is contained in a treatise on Machine Design by Professor W. C. Unwin, M.I.C.E. (Longman.)

CHAPTER VI.

CONNECTION OF TWO LOWER PAIRS BY HIGHER PAIRING.

SECTION I.—TENSION AND PRESSURE ELEMENTS.

57. *Preliminary Remarks—Tension Elements.*—If one of the elements of a pair be not rigid, or if the contact be not of the simple kind considered in the preceding chapter, the pairing is said to be "higher," because the relative motion of the elements is more complex. Higher pairing is seldom employed alone; it is generally found in combination with lower pairs, the elements of which it serves to connect. The most important case is that where a chain of two lower pairs is completed by contact between their elements or by means of a link which is flexible or fluid. Motions may thus be produced in a simple way which are impossible or difficult to obtain by the use of lower pairing alone. The present chapter will be devoted to mechanisms derived from chains of this kind, the fixed link being generally a frame common to the two lower pairs. The velocities of each of the pairs are thus the same as those of their moving elements. We commence with the case of non-rigid elements.

A body which was incapable of resistance to any kind of change of form and size would of course be incapable of being used as part of a machine, for it could not furnish any constraining force whereby the motion of other pieces could be affected, but if it resists any particular kind of change it will supply a corresponding partial constraint which may be supplemented by other means. The first case we take is that of a flexible inextensible body, such as is furnished approximately by a rope, belt, or chain. This is called a Tension Element, being capable of resisting tension only, and it is plain that when any two points are connected by it, their distance apart, measured along the element itself, must be invariable so long as the rope remains tight. If the rope be straight, it may be replaced by a link, and we obtain the mechanisms already considered, but we now suppose it to pass over a surface of any form.

Fig. 59a.

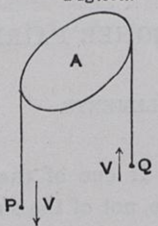


Fig. 59b.

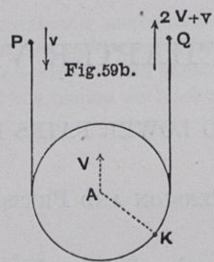
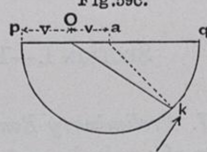


Fig. 59c.

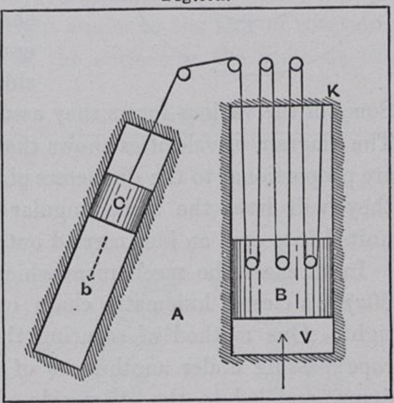


In Fig. 59*a*, let A be a fixed body of any shape, round which an inextensible rope PQ passes, the ends hanging down. If P moves downwards with velocity V, Q moves upwards with the same velocity, the rope slipping over A at all points with velocity V. In practice A is generally circular, and is mounted on an axis, upon which it revolves. We have then a "pulley block," of which A is the "pulley" or "sheave," and the rope causes it to rotate instead of slipping over it, but this makes no difference in the motion, and the only object of the arrangement is to diminish friction and wear.

Next suppose the pulley moveable (Fig. 59*b*), and imagine P attached to a fixed point, while Q moves upwards with the same velocity V relatively to A as before. Then A must move upwards with velocity V, because its motion relatively to the fixed point P is unaltered, and hence Q moves with velocity $2V$. More generally, if P, instead of being fixed, moves downwards with velocity v, Q must move upwards with a velocity $2V+v$, or to express the same thing otherwise — *the difference of velocities of the two sides of the rope is twice the velocity of lifting* — a principle applicable to all questions relating to pullies. The velocity of rotation of the pulley is $V+v$, its radius being the "radius of reference" (Art. 46). The motion of rope and pulley may be represented by a diagram of velocity. Thus, in Fig. 59*c*, describe a semi-circle with radius equal to $V+v$, then the radius of that circle represents the velocity of rotation or the velocity of any point in the rope relatively to the centre of the pulley. The actual velocity of any point K in the rope is found by compounding this with V, the velocity of the centre of the pulley. The pole of the diagram is therefore a point O, distant V from the centre of the circle, so that if k be the point in the diagram corresponding to the point K of the rope, Ok represents the velocity of K.

58. *Simple Pulley Chain—Blocks and Tackle.*—We have now a simple means of solving one of the most important problems in mechanism—namely, to connect two sliding pieces with a constant velocity ratio.

Fig.60a.



In Fig. 60*a*, B, C are pieces sliding in guides attached to a frame-piece A, thus forming two sliding pairs with one link common. In B a number of pins are fixed, and in A an equal number placed as in the figure, so that a rope passing round them as shown may form a number of plies parallel to B's motion.*

The rope is attached at one end to C, and led to the nearest fixed pin, over a guide pin placed so that this part of the rope may be parallel to C's motion, while the other end is attached to a fixed point K. The effect of this arrangement is that when C moves in the direction of the arrow, B also must move with a velocity which is readily found by the principle just explained, for the difference of velocities of the two parts of each ply must be the same, being twice the velocity of B. Thus reckoning from the fixed end, if B's velocity

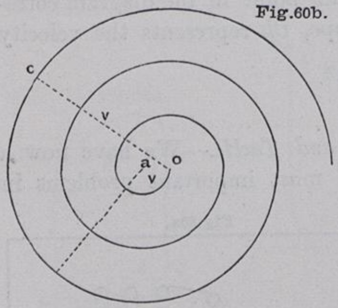
* This Figure is taken, with some modifications, from the 2nd edition (1870) of Willis's Mechanism.

be V, the velocities of the several parts of the rope must be

$$0,\ 2V,\ 2V,\ 4V,\ 4V,\ 6V,\ 6V\ldots\ldots,$$

so that if there are n pins in B, the velocity of the other end of the rope must be $2nV$, and the velocity ratio $2n:1$. The diagram of velocities consists of a number of semi-circles (Fig. 60b), the lower set struck with centre a and the upper with centre O, where O is the pole and Oa the velocity of lifting.

Fig. 60b.



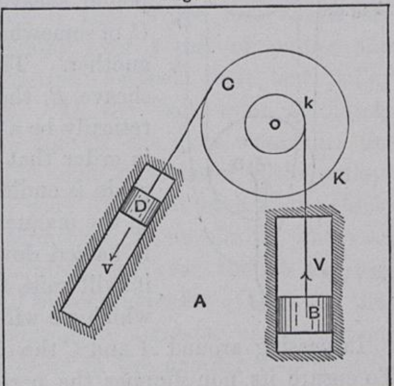
The simple kinematic chain here described may be inverted, by fixing B or C instead of A. In the blocks and tackle so common in practice, the pins are replaced by moveable sheaves, usually, but not always, of equal diameters, and placed side by side so as to rotate on the same axis. Some of the various forms they assume will be illustrated hereafter. The diagram of velocities shows that, if the diameters of the sheaves are proportional to the diameters of the circles shown in the diagram, they will have the same angular velocity, and may therefore be united into one, an idea carried out in White's Pulleys.

In all cases the mechanism which we have been considering (Fig. 60a) is a closed kinematic chain only so long as the rope remains tight. One method of securing this would be to supply a second rope passing under another set of pins below B (not shown in the figure) and led to the other side of C by a suitably placed guiding pulley; we should then, by tightening up the ropes, have a self-closed chain similar to those considered in the preceding chapter. In practice, however, forces are applied to B and C which produce tension in the rope; thus, for example, when employed for hoisting purposes the weight which is being lifted keeps the rope tight. This is the simplest example of what is called force closure, where a kinematic chain, which is not in all respects closed, is made so by external forces applied during the action of the mechanism. In practical applications the principle of force-closure is carried still further, for the guides which compel the pieces B and C to move in straight lines are usually omitted. In the case of B the weight and inertia of the load which is being raised or lowered supply sufficiently the necessary closure, while in the case of C the end of the rope may be guided by the hand.

59. *Wheel and Axle.*—When mechanical power is employed for hoisting purposes, the end of a rope is frequently wound round an axle, the rotation of which raises or lowers the weight, and this leads us at once to a different and equally important method of employing tension elements — namely, by attaching one end to a fixed point in the cylindrical surface of an element of a turning pair. The rope in this case passes over the surface and is guided by it, but does not slip over it as it does over the pins of the previous arrangement. The most useful case is that where the transverse section of the surface is a circle, and the direction of the rope always at right angles to the axis of rotation; then it is clear that the motion of the surface is the same as the motion of the rope.

The well known Wheel and Axle is a combination of two chains of this kind. In its complete ideal form it consists of two sliding pairs AB, AD, with planes parallel and one link A (Fig. 61) common. A rope is attached to D and, passing partly round a wheel, is attached to it at a fixed point K in its circumference; a second rope is attached to B, and passing partly round an axle, is attached to a fixed point k in its circumference, the two ropes lying in parallel planes. The wheel and axle are fixed together, and form with A the turning pair AC. We have thus a second means of connecting two sliding pieces so that their velocity-ratio may be uniform, for the velocities of B and D must be inversely as the radii of the wheel and the axle. As before, the ropes must be kept tight, also the guides of the pieces B and D may be omitted and replaced by force-closure,

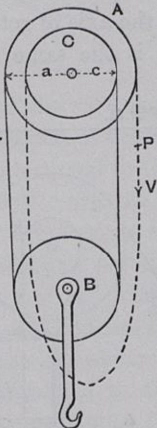
Fig.61.



and this will be necessary if the wheel is to make more than one revolution, for then a lateral movement is required to enable the rope to coil itself on the surfaces.

In practical applications the second rope is generally omitted and the wheel turned by other means; the lateral movement is sometimes provided for by permitting the axle to move endways in its bearings, but more often, in cases where the load is not free to move laterally, the effect of a moderate inclination of the rope to the axis is disregarded. We may, however, escape this difficulty by the use of force-closure of a different kind. Instead of attaching the rope to a fixed point in the surface, let it be stretched over it by a force at each end, there will then be friction between the rope and the surface, which will be sufficient to prevent slipping if the tendency to slip be not too great.

Fig. 62.



The Differential Pulley is a good example of the application of these principles. As is shown in Fig. 62, there are two blocks, of which the upper, which is fixed, carries a compound sheave, consisting of two pullies A and C, of somewhat different diameters, fixed to one another. The lower block carries a single sheave B, the diameter of which should theoretically be a mean between those of A and C, in order that the chain may be vertical. The chain is endless, and passes round the pullies in the manner shown, so that when the side P is hauled downwards with a given velocity V, it will raise the lower block B with a velocity which we will now determine.

In passing around A and C the chain is not capable of slipping. To ensure its non-slipping the periphery may be recessed to fit the links of the chain. In passing around B the slipping is immaterial; the raising of B would take place with the same velocity, whether there were an actual slipping of the chain round the circumference, or whether B were a rotating pulley.

When the point P is hauled downwards with velocity V, it necessitates the rotation of A, and with it of C. Thus the left hand portion of the chain passing round B will be hauled upwards with the same velocity as the point P downwards, and the right hand will descend with a velocity which is less in the ratio of the radii, c, a, of the united pullies, and thus on the whole there will be an ascending motion given to B. Now, since the upward velocity of B is $\frac{1}{2}$ the difference between the velocities of the two portions of the chain,

$$v = \tfrac{1}{2}\left(V - \frac{c}{a}V\right) = \frac{V}{2}\left(1 - \frac{c}{a}\right) = \frac{a-c}{2a}V.$$

Thus, by making the difference between a and c small, the relative velocity of B to P may be made as small as we please.

This apparatus, in a somewhat modified form, is much employed. It is called Weston's Differential Pulley Block, and possesses the valuable property that the weight will not descend when the hauling force is removed, for reasons which will be explained hereafter (Ch. X.).

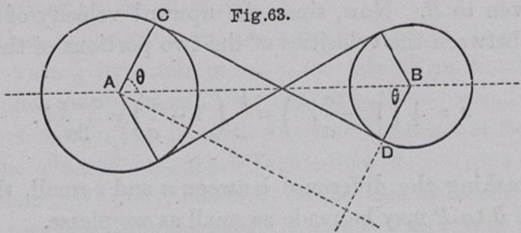
60. *Pulley Chains with Friction-Closure. Belts.*—A tension element may also be employed to connect the elements of two turning pairs. The most important case is that where two shafts are connected by an endless belt passing over a pair of pullies and stretched so tightly that the friction between belt and pulley is sufficient to prevent slipping. If the belt were absolutely inextensible the speed of centre line of the belt would be the same at all points, and therefore the angular velocities of the pulleys would be inversely as their radii each increased by half the thickness of the belt. This mode of connection is unsuitable where an exact angular velocity-ratio is required, for even though the belt may not slip as a whole, yet it will be seen hereafter (Chap. X.) that its extensibility causes a virtual slipping to a greater or less extent. In the case of leather belts, the error in the angular velocity-ratio due to this cause is said to be about 2 per cent.

There are two ways in which the belt may be wrapped around the pullies, being either *crossed or open.* If the belt is crossed, the pullies will run in opposite directions of rotation. The crossed belt embraces a larger portion of the circumference of the pullies than the open belt, and there is thus less liability to slip.

There is a proposition of some importance connected with the length of a crossed belt, which it will be useful to give here.

AC and BD (Fig. 63) being radii, each drawn at right angles to the straight portion of the belt CD, will each make the same angle θ with the line of centres. Thus

Fig.63.



the portion of the belt in contact with the pulley $A = (2\pi - 2\theta)\, r_A$ and that in contact with the pulley $B = (2\pi - 2\theta) r_B$.

The length not in contact $= 2 . CD = 2(r_A + r_B) \tan\theta$.

Thus whole length of belt $= 2(\pi - \theta + \tan\theta)(r_A + r_B)$.

$$\text{Now} \quad \cos\theta = \frac{r_A + r_B}{AC}.$$

Thus, if the distance AB between the centres is a constant quantity, and if, further, the sum of the radii $r_A + r_B$ is constant, then the angle θ will be constant. That being so, the total length of the belt will be a constant quantity.

This property is made use of when it is desired to connect two parallel shafts with an angular velocity-ratio, which may be altered at pleasure. A set of stepped pullies, such as are shown in Fig. 1, Plate III., are keyed to each shaft, and the belt being shifted from one pair to another of the pullies, the angular velocity-ratio is altered at will. If the belt is crossed, then the same belt will be tight on any pair of pullies, if the sum of the radii is the same for each pair. This does not hold good for open belts. The actual length of belt required in any given example is best found by construction.

The tightness of the belt necessary to effect closure by friction of this kinematic chain may be produced simply by stretching the belt over the pullies so as to call into play its elasticity, but the axis of rotation of one pulley is sometimes made moveable, so that the belt may be tightened by increasing the distance apart of the shafts, while in other cases an additional straining pulley is provided. The belt may then be tightened and slackened at pleasure, a method frequently used in starting and stopping machines.

In order that the belt may remain on the pulleys they must be provided with flanges, or, as is more common in practice, they must be slightly swelled in the middle, for when the shafts are properly in line, a belt always tends to shift towards the greater diameter. Great care, however, is necessary in lining the shafts that each side of the belt lies exactly in the plane of the pulley on to which it is advancing. Thus, for example, if the shafts be in the same plane, they must be exactly parallel, otherwise the belt will shift towards the point of intersection. This remark, however, does not apply to the receding side of the belt, and the shafts may make a considerable angle with each other, subject to the above restriction.

Friction-closure is always imperfect, because the magnitude of the friction is limited, but this is often a great advantage, since it permits the chain to open when the machine encounters some unusual resistance, which would otherwise produce fracture. By the use of grooved pullies provided with clips the friction may be increased to any extent, so that great forces may be transmitted, but these devices are only suitable for low speeds, as in steam-ploughing machinery. Slipping may be avoided altogether by the employment of gearing chains, the links of which fit on to projections on the pullies; force-closure is here replaced by chain-closure, and the action is in other respects analogous to toothed gearing. The speed is limited, as will be seen hereafter.

61. *Shifting of Belts. Fusee Chain.* — By the use of drums of considerable length as pullies, the belt may be shifted laterally at pleasure. This principle is much employed in practice, as for example—

(1.) To stop and set in motion a machine.—The drum on one of the shafts is divided into two pullies, one fast and the other loose on the shaft.

(2.) To reverse the direction of motion.—The drum is divided into three pullies, the centre one fast, the two end ones loose on the shaft. Two belts, one crossed and the other open, are placed side by side. By shifting the belt either is made to work on the fast pulley at pleasure.

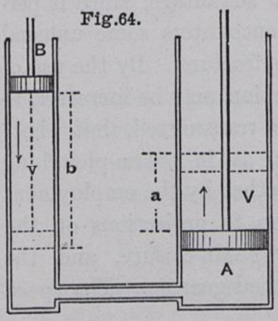
(3.) To produce a varying angular velocity-ratio.—The drums are made conical instead of cylindrical. The fusee employed in watches to equalize the force of the main spring is a common example.

The kinematic character of these devices will be considered in the next chapter.

62. *Simple Hydraulic Chain. Employment of Springs.* — Incompressible fluids may be employed to connect together two or more

rigid pieces forming a class of elements which may be called "pressure-elements," since they are capable of resisting pressure only. The pressure must be applied in all directions, and the fluid must therefore be enclosed in a chamber which pairs with the different pieces to be connected. For constructive reasons lower pairing must generally be adopted, and almost all cases are included in the following investigation.

Fig. 64.



Suppose two cylinders, each fitted with a piston (A and B in Fig. 64), to be connected by a pipe, the space intervening between the pistons being filled with fluid. Then when the piston B moves downwards with velocity v, the piston A will rise with velocity V, which is easily found by considering the spaces traversed by the two pistons in a given time. Let A, B be the areas of the pistons, a, b the spaces traversed, then, since the volume of the fluid remains the same, we must have $Aa = Bb$, and therefore

$$\frac{V}{v} = \frac{a}{b} = \frac{B}{A}.$$

The chain here considered, in which the elements of two sliding pairs are connected by a fluid, is kinematically identical with the arrangement of Fig. 60, p. 137, the replacement of a tension-element by a pressure-element constituting merely a constructive difference between the mechanisms. In the hydraulic press, in pumps, in water-pressure engines driven from an accumulator, and in other cases this kinematic chain is of constant occurrence, and will be frequently referred to hereafter. Combinations of an hydraulic chain with blocks and tackle are common in hydraulic machinery. (See Part V.)

Springs, compressible fluids, and even living agents, are employed in mechanism, not only in a manner to be explained hereafter as a source of energy, by means of which the machine does work, but also in force-closure, and especially for the purpose of supplying the force necessary to shift pieces which open and close, or lock and unlock kinematic chains, and so produce changes in the laws of motion of the mechanism. The force of gravity, which, as has already been shown, frequently produces closure, should be regarded as the tension of a link of indefinite length connecting the frame-link of the mechanism with the link we are considering. The inertia of moving parts likewise gives rise to forces which are not unfrequently applied to similar purposes. Examples will be given in a later section.

EXAMPLES.

1. A shaft making 90 revolutions per minute carries a driving pulley 3 feet in diameter, communicating motion by means of a belt to a parallel shaft, 6 feet off, carrying a pulley 13 inches diameter. Find the speed of belt and its length—1st, when crossed, and 2nd, when open. Find also the revolutions of the driven shaft, allowing a slip of two per cent.

Speed of belt	=	847·8 feet per minute.
Length when crossed	=	19 feet 2 inches.
,, open	=	18 ,, 8 ,,
Revolutions of the follower	=	$244\frac{1}{4}$

2. Construct a pair of speed pullies to give two extreme velocity-ratios of 7 to 1 and 3 to 1, and two intermediate values. The belt is to be crossed and the least admissible diameter is 5 inches.

$$\text{Velocity-ratios} \quad - \quad - \quad \frac{21}{3} \quad \frac{17}{3} \quad \frac{13}{3} \quad \frac{9}{3},$$

$$\text{Diameter of pullies} \quad \begin{cases} 5 & 6 & 7\frac{1}{2} & 10 \\ 35 & 34 & 32\frac{1}{2} & 30. \end{cases}$$

3. The diameters of the compound sheave of a differential pulley block are 8 inches and 7 inches respectively; compare the velocities of hauling and lifting.

Velocity-ratio = 16 to 1.

4. In a pair of ordinary three-sheaved blocks compare the velocity of each part of the rope with the velocity of lifting.

5. In a hydraulic press the diameter of the pump plunger is 2 inches and that of the ram 12 inches, determine the velocity-ratio.

SECTION II.—WHEELS IN GENERAL.

63. *Higher Pairing of Rigid Elements.*—We next consider pairs of rigid elements in which the relative motion is not consistent with continuous contact over an area. The elements then touch each other at a point or along a line which is not fixed in either surface, but continually shifts its position. The form of the surfaces is not then limited as in lower pairing, but may be infinitely varied, with a corresponding variety in the motion produced.

This kind of pairing occurs when a chain of two lower pairs is completed by simple contact between their elements. In the double slider-crank chain shown in Fig. 4, Plate II., of the last chapter, let us omit the block C and enlarge the crank pin so as just to fill the slot. By so doing the relative motions of the remaining parts will be unaltered, but we shall have three pairs instead of four, the turning pair BC and sliding pair CD being replaced by a single higher pair BD. This process is called *Reduction* of the chain, and when higher pairing is admissible the reduced chain serves the same purpose as the original, but with fewer pieces. The crank pin and slot are in contact along a line only which during the motion continually shifts its position. In practice, the elements not being perfectly rigid, the contact extends over an area, but this area is of very small breadth, and consequently if heavy pressures are to be transmitted at high velocities the wear is excessive. If we trace the development of pieces of mechanism we observe that in the earlier stages higher pairing is much employed for the sake of simplicity of construction, but is gradually replaced by lower pairing. Nevertheless, where the object of the machine is mainly to transmit and convert motion rather than to do work, or where the velocity of rubbing is low, higher pairing may be employed. In many cases it is necessary, because the required motion cannot be produced by any simple combination of lower pairs.

Higher pairing of rigid elements may be divided into two classes according as the surfaces in contact do or do not slip over one another, just as in the case of tension elements considered in the last section. In the first case the contact is spoken of as Sliding Contact and in the second as Rolling Contact. In rolling contact the difficulty of wear does not occur, and hence it is always used when possible. The relative motion of the two elements is determined by considering that as the surfaces do not slip, the space moved through by the line of contact along each surface must be the same. Or as we may otherwise express it, if A, B be a pair of elements in rolling contact, the velocity of the surface of A must be the same as that of the surface of B.

64. *Rolling Contact.*—Rolling contact may be employed for the communication of motion between two shafts, the centre lines of which are either parallel or intersect, by means of surfaces rigidly attached to the shafts. In the first case the surfaces are cylindrical and in the second conical, the apex of the cone being the intersection of the shafts. By far the most important case, and the only one we shall here consider, is that in which the transverse sections of the surfaces are circular. Portions of the surfaces are used, as in Figs. 64*a*, 64*b*,

Fig.64a.

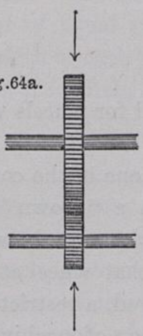
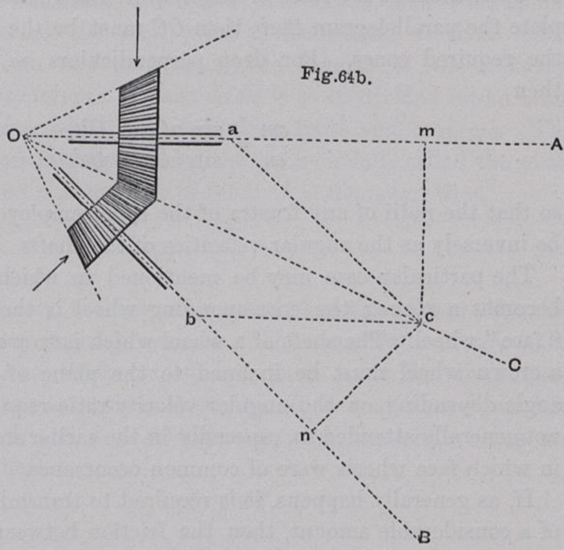


Fig.64b.



and are pressed together by external forces, so that sufficient friction is produced to prevent the slipping of the surfaces. In other words, force-closure is necessary, as in the case of connection by a belt. This being supposed, it will immediately follow that the velocity of the two surfaces at the points of contact is the same, and hence, as before, the angular velocity-ratio of the shafts is inversely proportional to the radii of the wheels. In the case of intersecting shafts, the surfaces are frustra of cones called "bevel," or, if the semi-angle of the cone be 45°, "mitre wheels," and their radii may be reckoned as the mean of that at the inner and outer periphery. The shafts revolve in opposite directions, unless one of the surfaces be hollow so that the other may be inside it, in which case the corresponding wheel is said to be "annular." When it is inconvenient to use an annular wheel, the same result may be obtained by transmitting the motion through an intermediate or "idle" wheel. If the radius of a wheel be infinite, it becomes a "rack," and the surface a plane.

In the case of bevel wheels the corresponding cones may be found, when the centre lines of the shafts and the angular velocity-ratio are given, by a simple construction. In Fig. 64*b*, let OA, OB be the centre lines of the shafts, and let distances Oa, Ob be marked off upon them in the ratio of the required angular velocities. Complete the parallelogram $Oacb$, then OC must be the line of contact of the required cones. For drop perpendiculars cm, cn, on OA, OB, then

$$\frac{cm}{cn} = \frac{\sin aOc}{\sin bOc} = \frac{Ob}{Oa},$$

so that the radii of any frustra of the cones employed for wheels will be inversely as the angular velocities of the shafts.

The particular case may be mentioned in which one of the cones becomes a plane; the corresponding wheel is then a "crown" or "face" wheel. The shaft of a wheel which is to work correctly with a crown wheel must be inclined to the plane of that wheel at an angle depending on the angular velocity-ratio required, a restriction not generally attended to, especially in the earlier stages of machinery in which face wheels were of common occurrence.

If, as generally happens, it is required to transmit a working force of a considerable amount, then the friction between the two circumferences will be found not to be sufficient to prevent slipping taking place, unless a considerable pressure to force the shafts together is employed, which involves an excessive friction on the bearings. In what is known as "frictional gearing," this is partially avoided by the use of wheels with triangular grooves fitting each other as the thread of a screw fits into its nut; but, in general, to prevent slipping, teeth are cut on the two peripheries, and the motion is transmitted by the gearing together of the teeth. Since this is a substitution for the rolling contact of two surfaces, it is required to so design the number and form of the teeth that the wheels on which they are cut shall turn one another with the same constant angular velocity-ratio as that due to the two original surfaces. If recesses are cut in each wheel, and projections be added between the recesses so as to fit into the corresponding recesses of the other wheel, then the two wheels may be placed to gear together at such a distance that the two original surfaces would have been in contact and would have rolled together. In the case of a pair of toothed wheels, such a pair of imaginary surfaces which will roll together with the same angular velocity-ratio as that obtained from the toothed wheels, are called *pitch surfaces.* Considering first the case of parallel shafts, the transverse sections of these surfaces are called *pitch circles,* and their point of contact is called the *pitch point.* The radii of these pitch circles must be to one another in the inverse of the velocity-ratio. The circumference of each circle is to be divided into a number of equal parts, which will include a tooth and a recess. The length of each part measured along the pitch circle is called the *pitch.* Let p = pitch, and n = number of teeth, d = diameter, then

$$p = \frac{\pi d}{n}.$$

The thickness of each tooth is made a little less than $\frac{1}{2}p$ to allow the clearance necessary for easy working. The magnitude of the pitch which governs the thickness of the teeth must be determined from considerations as to their strength. If n' = number of teeth in the second wheel, and d' = its diameter, then the pitch being the same for each wheel

$$p = \frac{\pi d}{n} = \frac{\pi d'}{n'}.$$

The distance apart of the shafts is generally adjusted to allow the pitch to be some exact number of inches, half, or quarter inches. The pitch is to be measured along the pitch circle, and is not the chord of the arc, as is sometimes stated.

In some small wheels used for spinning machinery, another kind of *pitch* is referred to. The diameter of the pitch circle is divided by the number of teeth, and the result is called the *diametral pitch.* In the smallest class of wheelwork used in clocks, the dimensions of the teeth are stated as so many to the inch. The proper form of teeth will be considered farther on.

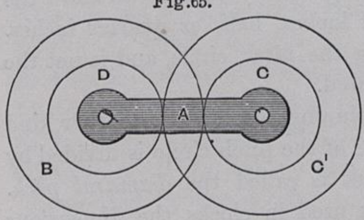
65. *Augmentation of a Kinematic Chain. Trains of Wheels.*—Another important application of rolling contact is to diminish friction by the intervention of rollers, hence called Friction Rollers. Thus

the friction between the elements of a sliding pair, subject to heavy pressure, will be so great as to require a great force to overcome it, but if rollers be placed between the elements the friction is greatly reduced, as will be seen hereafter. In this case sliding friction is wholly replaced by rolling friction; in carriage wheels the sliding velocity which, without the wheel, would be the actual velocity of the carriage, is reduced to that at the periphery of the axle, that is to say, in the ratio of the diameters of the axle and the wheel. The sheaves of an ordinary pulley block are examples of the same principle. In all these cases where additional pieces are added to a kinematic chain, in order to reduce friction or to serve some other non-kinematical purpose, the chain is said to be "augmented."

Chains are frequently augmented for purely constructive reasons; thus, if the velocity-ratio of a pair of shafts is great, the diameters of a single pair of wheels necessary in order to obtain it will be inconveniently large or small. A train of wheels is then resorted to. This is also the case where the shafts to be connected are too near or too far apart; in the latter case bevel wheels and an intermediate transverse shaft may be employed.

When, however, the shafts to be connected are in the same straight line, a train of wheels is kinematically necessary, and forms virtually a new mechanism. This is a common case in practice when a pulley or wheel is loose on a shaft, and it is required to connect the wheel and the shaft so as to revolve with different velocities. Such a train is shown in Fig. 65 in a simple ideal form. B and D are two wheels turning on the same centre but disconnected. C, C' are two wheels gearing with B and D and turning about another centre but united. The two centres are connected by the frame-link A. When B revolves it drives C, and C' drives D. If the numbers of teeth in these wheels be denoted by the letters which distinguish them, and the velocity of B be unity, the velocity of C or C' will be B/C, and that of D will be BC'/DC. Let it now be observed that the wheels B and D form a pair, the velocity of which will be the difference between the velocities of these wheels.

Fig.65.



We have then altogether four turning pairs in this train of wheels, the relative velocities of which are—

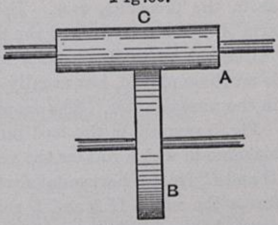
Pair, -	BA	CA	DA	DB
Velocity,	Unity	$-\frac{B}{C}$	$\frac{BC'}{CD}$	$1-\frac{BC'}{CD}$

One of the wheels in this train may be annular, and all may be bevel; in either case the wheels C, C' may be equal, and the train reduced to three wheels, though the number of simple pairs remains as before four. Examples are given in the figures of Plate III.

Either this or any other train of wheels may be inverted by fixing one of the wheels instead of the frame-link, the resulting mechanism is then called an Epicyclic Train; the velocity-ratios of the various pairs are unaltered, and are therefore shown by a table similar to that given above. Should the angular velocity of any wheel be required relatively to the fixed wheel, we have only to add to the velocity of the corresponding pair the velocity of the frame-link. Some examples of epicyclic trains are shown in the figures, but for detailed descriptions we must refer to a work on mechanism. Their use in compound chains will be further referred to in the next chapter.

66. *Wheel Chains involving Screw Pairs.*—In a simple wheel chain (Fig. 66) consisting of a wheel B, a pinion C, and a frame-link A, not shown on the figure, suppose C to be of considerable length, then there will be nothing to prevent the endways movement of B in its bearings if they be supposed cylindrical. This circumstance is often taken advantage of in machinery in shifting wheels in and out of gear, but the case to be examined here is that in which the endways movement is given by independent means during the action of the mechanism. The simplest example is a three-link chain derived from the train of wheels just considered by changing the turning pair

Fig. 66.



BA into a screw pair; B then travels endways through the pitch of the screw in each revolution. The pinion C sometimes slides on the shaft which carries it, but quite as often it is made long enough to permit the necessary traverse of B. A well known example of this mechanism is that of the feed motion common in drilling and boring machines, in which the train of wheels of the last article is used with B and D nearly equal, so that the velocity of the pair BD is very small. B is attached to the nut and D to the screw, so that BD is a screw pair. D then traverses through B by a space each revolution which may be made very small.

To illustrate and explain preceding articles Plate III. has been drawn, giving examples of trains of wheels, especially of the differential trains of Fig. 65.

Fig. 1 shows the slow motion of a lathe. D is a wheel keyed on the mandrel and connected with B, the driving pulley, when the motion is not in use. B rides loose on the mandrel, and by means of a pinion gears with C, a wheel on the same shaft with C', which gears with D. CD being large compared with BC', the speed of the mandrel is much less than that of the pulley. For lighter work CC' are thrown out of gear by an endways movement of the shaft.

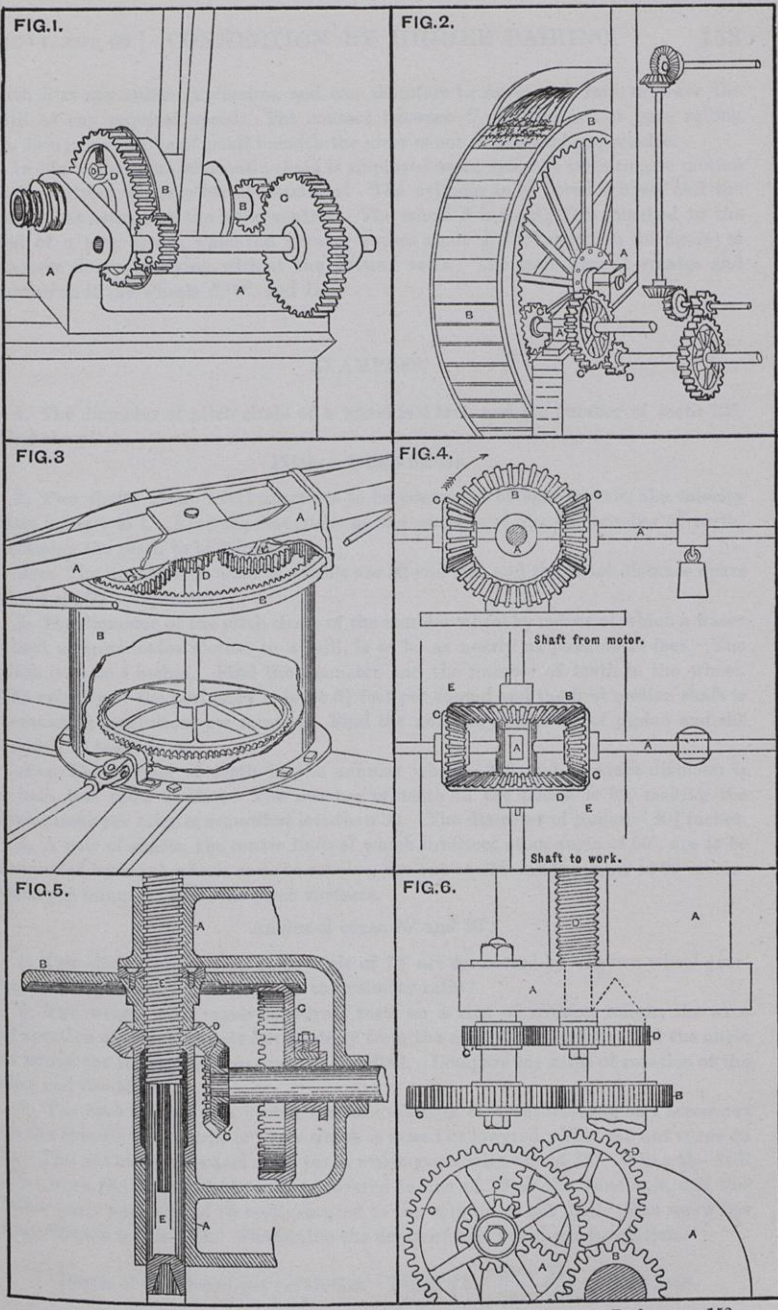
Fig. 2 represents the train of wheels by which the slow movement of a water-wheel is multiplied and transmitted to all parts of a factory. B is now an annular wheel attached to the water-wheel gearing with C, C' with D, and so on. A vertical shaft F with bevel wheels transmits the motion to the upper floors. The bearings of the secondary shafting are omitted for clearness, but they all form part of a frame-link A, which is fixed.

In Fig. 3 the kinematic chain is inverted. B is a fixed annular wheel, CC' are of equal diameter and reduce to one wheel, which, however, is in duplicate, in order to balance the driving forces. This epicyclic train is applied to many purposes. In the example shown the frame-link is a long arm, at the end of which a horse is attached, and a rapid motion thus given to the central pinion D. The motion is further multiplied by the bevel gear shown below, and applied to drive a thrashing machine or some similar purpose. The same mechanism is employed as a purchase in capstans and tricycles.

In Fig. 4 the train consists of three bevel wheels, BCD, C and C' reducing to one, as in the preceding case. The simple chain consists of these wheels and the train arm A. When A is fixed the wheels B and D turn in opposite directions with equal velocities; when B is fixed A revolves with half the velocity of D. The mechanism is much employed, but usually as a compound chain, and as such will be considered in the next chapter. The example shown is a dynamometer.

Fig. 5 represents the feed motion of a drilling machine. A is the frame of the machine in which rotates the vertical drill spindle E driven by a pair of mitre wheels D and C' from a horizontal shaft. A screw thread is cut on the spindle, of which B forms the nut. If B and D rotate at the same speed the drill moves neither up nor down, but any difference will result in a motion of the screw pair BE, and will thus give the necessary feed or raise the drill out of the hole. In the example chosen B is driven by a flat disc gearing by friction with a wheel C' turning with D (Naish's patent). This wheel, by means of a lever, can be moved along the shaft so as to gear

Plate.III.



To face page 152.

with B at any radius at pleasure, and can therefore be set so as to raise or lower the drill at any required speed. The contact between C, D here is not pure rolling (p. 148); but as C is of small breadth the error is not of practical importance.

In Fig. 6 the same kinematic chain is employed as an epicyclic train to give motion to the cutters of large boring machines. The cylinder to be bored is fixed, and the boring bar rotates on the lathe centres. The wheel B is fixed; D is attached to the end of a long screw, which on turning causes a nut E (not shown in the figure) to traverse slowly, carrying with it the cutting tools. The train arm A rotates and carries on it the wheels C, C', and D.

EXAMPLES.

1. The diameter of pitch circle of a wheel is 4 feet, and the number of teeth 120. Find the pitch.

Pitch = 1·2566 inches.

2. Two shafts about 4 feet apart are to be connected by spur wheels, the velocity ratio being 4 to 1. Find the diameters of the wheels and also the number of teeth, assuming the pitch to be 2 inches.

Ans. The numbers of teeth in wheels are 30 and 120, and the exact distance apart of the shafts = $47\frac{3}{4}$ inches.

3. The diameter of the pitch circle of the annular wheel by means of which a water wheel communicates motion to a mill, is to be as nearly as possible 24 feet. The pitch is to be 4 inches. Find the diameter and the number of teeth in the wheel. The velocity of the periphery is to be $5\frac{1}{4}$ feet per second and the first motion shaft is to make 30 revolutions per minute. Find the necessary diameter of pinion and the number of teeth in it.

Ans. The number of teeth in the annular wheel = 226, and its exact diameter is $\frac{1}{4}$ inch less than 24 feet. The number of teeth in the pinion is 32, making the revolutions per minute somewhat less than 30. The diameter of pinion = $40\frac{3}{4}$ inches.

4. A pair of shafts, the centre lines of which intersect at an angle of 60°, are to be connected by bevel wheels so as to revolve, the one at 250 and the other at 90 revolutions per minute. Find the pitch surfaces.

Angles of cones 90° and 30°.

5. Two shafts intersecting at an angle of 75° are connected by a crown wheel gearing with a pinion. What must be the velocity-ratio?

6. The weight of a revolving turret rests on a ring of friction rollers, the axes of rotation of which radiate horizontally from the axis of the turret: find the angle at which the rolling surfaces must be bevelled. Compare the rates of rotation of the ring and the turret.

7. The feed motion of a boring machine consists of a nut working on a screw cut on the spindle of the drill or borer which is raised or lowered whilst the nut turns on it. The nut carries a wheel of 96 teeth which gears with one of 35. When the drill is at work the wheel of 35 teeth is secured to one of 36 on the same axis, and this latter gears with one of 95 teeth secured to the spindle of the drill. The screw has four threads to the inch. Determine the depth of hole bored per revolution.

$$\text{Depth of hole bored per revolution} = \tfrac{1}{4}\text{ inch}\left(1 - \frac{35 \times 95}{36 \times 96}\right) = \ 0095\text{ inch.}$$

8. The train of wheels in the preceding question is used as an epicyclic train by fixing the wheel of 96 teeth. Find the direction and number of revolutions of the train arm for each revolution of the spindle.

Ans. For each revolution of 95 wheel forwards, the arm turns backwards through

$$\frac{95 \times 35}{96 \times 36 - 95 \times 35} = 25{\cdot}4 \text{ revolutions.}$$

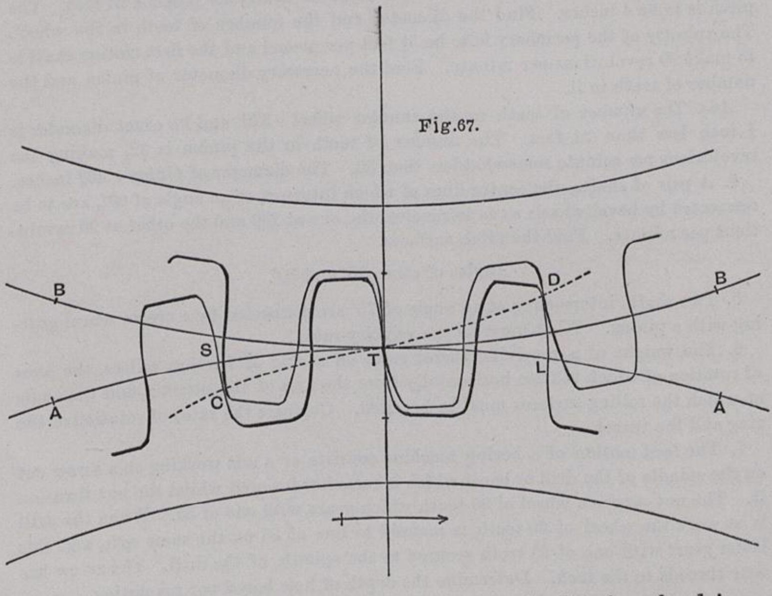
SECTION III.—TEETH OF WHEELS.

67. *Preliminary Explanations.*—Even though the number of teeth in a pair of wheels be such as to give the correct mean angular velocity-ratio due to the rolling together of the pitch circles, yet if they be of improper form they will jam or work roughly.

Theoretically the form of the teeth of one of a pair of wheels may be chosen at pleasure if a proper corresponding form be given to the teeth of the other; the problem of rightly determining the form is therefore one which admits of many solutions. We commence with some general explanations applicable to all forms of teeth.

The diagram (Fig. 67) shows a section of a pair of spur wheels in

Fig.67.



gear, with three teeth in action, the lower wheel being the driver. *BTB*, *ATA* are the pitch circles in contact at the pitch point *T*.

$ST = TL$ is the pitch, being the distance of a point in one tooth from the corresponding point in the next consecutive measured along the pitch circle. The teeth as shown in the figure partly project beyond the pitch circle and fit into corresponding recesses in the other wheel, so that each tooth is divided into two parts, a part within and a part without the pitch circle. The corresponding acting surfaces are called the Flank and the Face of the tooth respectively. In annular wheels the flank is outside and the face inside the pitch circle. The teeth commence action before reaching the line of centres by the flank of a tooth of the driver A coming into contact with the face of a tooth of the follower B, as shown at C in the diagram, and gradually approach that line till after the wheels have turned through a certain arc, which measured on the pitch circle is called the Arc of Approach; they are then in contact at T the pitch point. After passing the line of centres they remain in contact till the wheels have turned through a second arc called the Arc of Recess and then cease contact as shown at D, the face of a tooth of the driver being always in contact with the flank of a tooth of the follower. The sum of these arcs is called the Arc of Action, and must be great enough to permit at least two teeth to be in contact at once. Their magnitudes depend on the projection of the teeth beyond the pitch circle, a quantity which is called the Addendum of the corresponding wheel, the arc of approach depending on the addendum of the follower, and the arc of recess on the addendum of the driver.

68. *Involute Teeth.*—The question of the form of the teeth requires much explanation to render it completely intelligible; we shall only give a brief sketch, referring for full details to the works cited on page 100. Some points will be further considered at a later period. We commence with what are known as Involute Teeth.

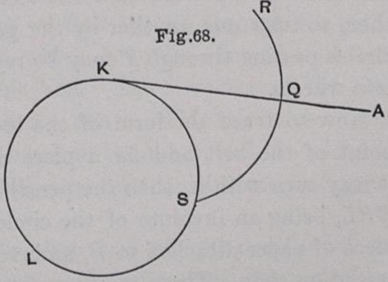


Fig.68.

Imagine a string AKL wound on a cylinder (Fig. 68). If the string be gradually unwound, the string being kept tight all the time, a point Q of the string will trace out a curve SQR called the *Involute* *of the Circle.* Instead of causing the string to be unwound around the fixed circle we may if we please move A in a fixed straight line and cause the unwinding to take place by the revolution of the circle. If now a piece of paper be fixed to and revolve with the circle, the same involute curve will be traced on it as before.

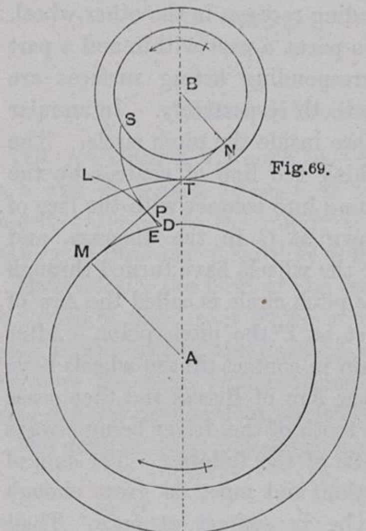


Fig. 69.

Now let A and B (Fig. 69) be two circles not in contact which are each capable of revolution about its centre. If we connect them by a crossed belt, of which one half is shown in the diagram by the line MTN, each will be capable of driving the other with a constant angular velocity ratio, namely, the inverse ratio of the radii. If, therefore, T be the point where the belt crosses the line of centres,

$$\frac{A_A}{A_B} = \frac{r_B}{r_A} = \frac{BT}{AT}.$$

Now, with centres A and B and radii AT and BT, describe circles which touch one another. These two circles would turn one another by rolling contact with the same angular velocity-ratio as that due to the belt. If we were to form teeth on the two wheels and cause them to turn one another by the gearing of the teeth, then the two circles passing through T may be regarded as the pitch circles of the two wheels.

Now to trace the form of the teeth. Attach a pencil (P) to any point of the belt and fix a piece of paper to the wheel A so that it may turn with it, then the pencil will trace on the paper the curve EPL, being an involute of the circle A. Similarly, if we imagine a piece of paper attached to B, an involute DPS of the circle B will be traced on that. These two curves will be in contact at the tracing point P, and will always remain in contact as the circles turn. If, therefore, we construct teeth of this form with any given pitch, and then remove the belt, the two toothed wheels will drive one another with the constant angular velocity required. In this form of tooth the face and flank are one continuous curve, which is a property practically confined to involute teeth. From this fact a practical advantage follows. By the continual action of the teeth together they wear and cause a looseness of fit, which may be remedied by bringing the centres of the wheels more nearly together, and this without altering the smooth action of the teeth or the exact uniformity of the angular velocity-ratio. In no other form of tooth occurring in practice is this possible.

The line of action of the mutual pressure between the teeth is always along the tangent line to the two base circles, from which the teeth are generated, thus tending always to force the axles apart. If the angle between this line and the common tangent to the two pitch circles, or as it is called, the "obliquity," be large, much friction in the bearings would result. On this account the obliquity is made as small as possible, not being allowed to exceed $14\frac{1}{2}°$ or $15°$. With this a limit is introduced to the smallness of the number of teeth which may be used. The action of the teeth must always be along the line MTN, and hence cannot extend beyond the point N. If it is essential that when two teeth are in contact at the pitch point another pair of teeth should just be coming into action whilst a third pair are just ceasing action, then the length of the arc of the pitch circle which corresponds to an arc on the base circle equal to TN will be the greatest length that can be given to the pitch of the teeth, and when the obliquity is $14\frac{1}{2}°$ there will be about twenty-five such pitches on the pitch circle, and hence the number of teeth cannot be less than twenty-five.

Having given the pitch circles we first lay off, through the pitch point, the line of oblique action which is to be allowed, and then draw the base circles touching this line. The involutes of the base circles will give us the form of the teeth. The thickness of the tooth is to be taken a little less than half the pitch, and the addenda of the teeth such as to give a sufficient number of teeth in contact at the same time. (Art. 71.)

All involute teeth of the same pitch and obliquity will work together; they have never been much used in practice, although there appears to be no reason why they should not be in cases where it is not necessary to have less than twenty-five teeth. Their wear is said to be greater than that of teeth of other kinds.

69. *Path of Contact the Pitch Circle.*—In involute teeth the tracing point is attached to a belt stretched over pulleys, and therefore describes a straight line on paper, which is fixed to the line of centres so as not to revolve with either wheel. Now, the tracing point is also the point of contact of the two teeth, and therefore the path of this point, or, as it is conveniently called, the "path of contact," is a straight line. Teeth of any shape may be traced by this method if, instead of simply stretching the belt over the pulleys, we pass it over a fixed curve between the pulleys, so that the tracing point describes the curve in question instead of a straight line, provided the fixed curve be such that the curves traced on the rotating circles touch one another. In other words, we may assume various "paths of contact" at pleasure and obtain teeth which will work together correctly. We shall next suppose the tracing point attached to the circumference of a rotating wheel, in which case the path of contact is a circle.

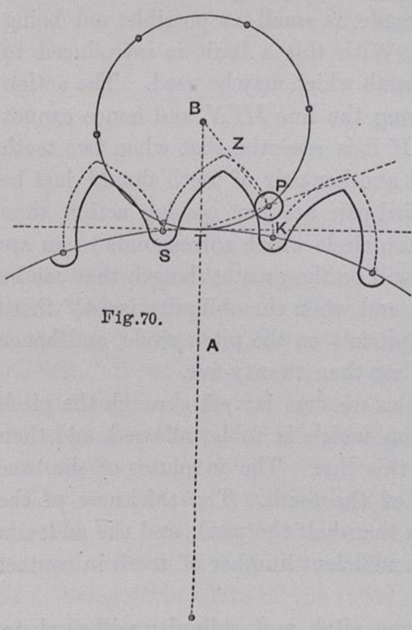


Fig.70.

In the use of toothed wheels the earliest idea was, for simplicity of construction, to form the smallest wheel of a number of cylindrical pins projecting from a disc. Supposing one of a pair of wheels to be so constructed, it is required to determine the proper form of the teeth for the other wheel.

On the wheel B (Fig. 70) let pins be placed at equal distances, with their centres on the pitch circle, and in the first place suppose the pins indefinitely small, being mere points. Now, if at one of the points P a pencil be attached, then if B be caused to roll without slipping over the surface of A kept fixed, the pencil P will trace a curve on a piece of paper attached to the wheel A. The same curve will be drawn if we cause one wheel to drive the other without slipping, the centres A and B being fixed, while the paper is attached to A and turns with it. If the tracing point started from the pitch point T, then the curve KP will have been drawn on the paper, which, by the further rotation of the circles, will be produced to Z. This curve is called an Epicycloid, and will be the proper form of teeth for the wheel A to drive the pinion B. For the pin P will be always in contact with the tooth KZ as the wheels revolve with uniform angular velocity-ratio. We complete the form of the teeth by drawing a similar curve ZS for the other face, SK being the pitch, in order to enable the wheels to be turned in the opposite direction if necessary. Placing a number of such teeth on the pitch circle A, we see they all touch one another at the roots on the pitch circle. The reason is because we have imagined the pins of B to have no definite dimensions, but to be mere mathematical points. In practice some definite dimensions must be given to the pins of B. In such a case the proper form for the teeth of A is derived from the previous construction by drawing a curve which at all points shall be at a distance from the epicycloid, when measured along the normal, equal to the radius of the pin. Below the pitch circle A a semi-circular recess must be formed, as shown by the full curve in figure.

These teeth possess the peculiar property of having faces but no flanks. The consequence is that, the toothed wheel A being the driver, the action of the teeth is wholly after the line of centres; there is no arc of approach, but only an arc of recess. On this account the pin-wheel must always be the follower, for if it be the driver the action of the teeth would be wholly before the line of centres, in consequence of which the friction is said to be more injurious.

The angle which PT makes with the common tangent is, as in the case of involute teeth, called the "obliquity"; it is now no longer constant, but varies from zero, when P passes the line of centres at T, to a maximum value when P escapes. It is easily seen that this angle is always one-half the angle PBT, which PT subtends at the centre of the pin-wheel, and hence the obliquity increases uniformly as the wheels turn; its mean value may be taken at half the maximum, and is limited in the same way as in involute teeth to about 15°, so that the greatest value of the angle *PBT* may be taken as 60°.

If the two sides of the teeth are alike, as in the figure, the pin then comes to the point of the tooth at *Z*. This circumstance determines the smallest number of pins which can be used, for one pin must not escape before the next comes to the line of centres; that is to say, *PT* cannot be greater than the pitch, the pitch then must not be greater than one-sixth the circumference of the pin-wheel, whence it appears that the least number of pins is six.

Pins are now rarely employed unless in clock and watch work; they have the great practical disadvantage that the toothed wheel to work with them must be specially designed, as it will work with only one diameter of pinion.

If we imagine a pin-wheel to work with an annular wheel, the teeth may be traced in the same manner as shown in Fig. 71 (below), to which the same letters are attached. The point *P* now traces out a curve called a Hypocycloid, the general character of which may be seen by joining *P* to *F*, the other extremity of the diameter *TF* of the circle *B*; for since the angle *FPT* must be a right angle, the angle *APT* will be greater than a right angle if, as in the figure, *F* lies between *A* and *T*, and less than a right angle if *F* lies beyond *A*. Thus the hypocycloid must reduce to the radius *AK* if *F* coincides with *A*, that is, if the diameter of the pin-wheel be half the diameter of the annular wheel; while, for smaller diameters, it forms a curve always concave towards *T*. Hence it appears that to work with a pin-wheel of half its diameter the teeth of the annular wheel should be constructed simply by drawing radii of the pitch circle. With a larger diameter of pin-wheel the teeth would be undercut, and therefore weak; the annular wheel must be the driver as before.

In all epicycloids and hypocycloids the normal to the curve at the tracing point *P* passes through the point of contact *T* of the circles considered—an important geometrical property, which we shall presently make use of, and hereafter prove.

70. *Path of Contact any Circle.*—Teeth traced in the way just described are wholly within the pitch circle, and this circumstance suggests that by a combination with the preceding case, where they were wholly without, a form may be found which may be more suitable for practical use.

In Fig. 71 a third circle C is shown, touching the two others

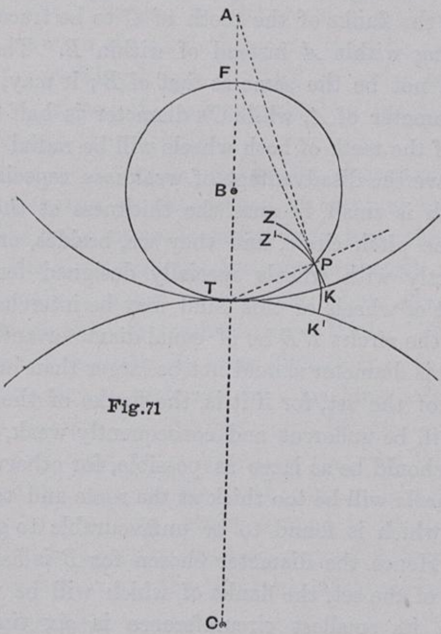


Fig. 71

at the same pitch point T. The three circles ABC turn each about its own centre without slipping. Imagine paper attached to A and C and rotating with them, while a pencil P is attached to B as before; then P will trace out two curves as in the case of involute teeth, one outside the circle C, the other inside the circle A. A's curve will be an hypocycloid KPZ, starting from K in the circle A, while C's curve is an epicycloid $K'PZ'$, starting from K' in the circle C'. Now these curves will, as in involute teeth, touch one another, having a common normal PT, and hence it follows that, while the circles turn with uniform angular velocity-ratio, the curves will always be in contact, and may be taken as face and flank of a pair of teeth. Thus it appears that we can obtain the faces of the teeth of C, and the flanks of the teeth of A, by causing a third circle B of any diameter to rotate within the circle C. If the diameter of B be half the diameter of C, the flanks for C will be simply radial lines, but if it be less the teeth will be concave towards T, the effect of which is that the teeth will spread out at the root, which is desirable on the score of strength. We can now imagine the faces of the teeth of A and the flanks of the teeth of C to be traced by another circle B' rotating within A instead of within B. The diameter of this circle need not be the same as that of B; it may, for example, be half the diameter of A, while C's diameter is half that of B; if so, the flanks of the teeth of both wheels will be radial. Teeth with radial flanks have the disadvantage of weakness, especially when the number of teeth is small, because the thickness at the root is less than that at the pitch circle, and they are, besides, only capable of working correctly with wheels specially designed for them. In order that a set of wheels of this kind may be interchangeable, it is necessary that the circles $B'B$ be of equal diameter and the same for all the set. This diameter should not be larger than half that of the smallest wheel of the set, for if it is, the flanks of the teeth of the small wheels will be undercut and consequently weak, while, on the other hand, it should be as large as possible, for otherwise the teeth of the large wheels will be too thick at the roots and too thin at the points, a form which is found to be unfavourable to good wearing. (See p. 184.) Hence the diameter chosen for B is half that of the smallest wheel of the set, the flanks of which will be radial. As B is a pin-wheel, its smallest circumference is six times the pitch (Art. 69), and the smallest wheel of the set has consequently 12 teeth; but if no wheel is required with so small a number of teeth as this, it will be better, for the reason stated above, to take a larger describing circle.

71. *Addendum and Clearance of Teeth.*—In any form of teeth it is clear from what has been said that the point of contact travels along the path of contact DT (Fig. 67, page 154) from the pitch point T to the end of the tooth at D, where the contact ceases. The length of the path of contact thus traversed is equal to the arc of recess in all kinds of cycloidal teeth, and less than that arc in a given ratio in involute teeth. By stepping off a suitable length on the path of contact then, we can find the end of the tooth for any given arc of recess, and the distance of this point from the pitch circle A of the driver is what we have already defined as the "addendum" of that wheel. The position of this point on the flank of the tooth of the follower B gives the working length of flank necessary. Similarly the length of face in the follower and flank in the driver depend on the arc of approach. The depth of the recesses between the teeth, however, must be made greater than is necessary for working length of flank, in order to allow the ends of the teeth to clear; the amount usual in practice appears to be about one-fifteenth the pitch.

The allowance necessary in practice for clearance in the thickness of the teeth depends on the degree of accuracy attainable in construction. The value formerly employed for teeth shaped by hand was one-eleventh the pitch, but the best modern teeth are machine cut, and a much smaller amount is sufficient. Less clearance is required for involute teeth than in teeth of other kinds. The setting out of bevel teeth is not theoretically more difficult than in the case of spur gear, but their accurate execution by a machine is far from easy. If the machine operate by straight cuts like an ordinary shaping machine, the tool must be mounted so that the line of cut always passes through the apex of the pitch cone. Gear cutting machines generally employ revolving cutters formed to fit the space between two teeth. Much ingenuity has been expended on giving the cutter a lateral movement to suit the bevel, but an exact bevel tooth cannot be formed in this way.

72. *Endless Screw and Worm Wheel.*—When two shafts are to be connected which are not parallel, and the centre lines of which do not intersect, it is necessary to resort to skew bevel, or screw, teeth. Only one case of this kind need be mentioned here as being of common occurrence, namely, the endless screw and worm wheel employed when the shafts are at right angles, and a slow motion of one of them is desired. In a common screw let the thread be so formed that the longitudinal section of the screw thread shows a range of teeth like those of a rack which would gear with a given spur wheel. Let the teeth of the wheel be set obliquely at an angle equal to the pitch angle of the screw; strictly speaking they also are screw threads, the pitch angle of which is the complement of the pitch angle of the screw. Then the screw and wheel will gear together, and the wheel moves through one tooth for each revolution of the screw. Like screws in general, this combination is non-reversible unless the pitch of the screw be coarse (Ch. X.), and for this reason, and on account of its simplicity, is much employed in practice. The method of constructing the teeth of a worm wheel is explained in a work by Prof. Unwin, cited on page 134.

1. A pair of wheels have 25 and 120 involute teeth respectively, and the addendum of each is $\frac{3}{10}$ths the pitch. Find the arcs of approach and recess in terms of the pitch, assuming the obliquity $14\frac{1}{2}°$, the large wheel being the driver. (See Art. 71.)

Ans.—Arc of approach = 1·07 × pitch.
Arc of recess = ·88 × pitch.

2. If the arcs of approach and recess in involute teeth are each to be equal to the pitch, show that the addenda of the wheels should be calculated by the approximate formula

$$\text{Addendum} = \left(\frac{1}{4} + \frac{3}{n}\right) \times \text{pitch},$$

where n is the number of teeth.

3. A pair of wheels have 25 and 120 teeth respectively, the flanks being in each case radial. Find the addendum of each wheel that the arcs of approach and recess may each be equal to the pitch.

Ans.—Addendum of driver = ·283 × pitch.
Addendum of follower = ·178 × pitch.

4. Connect two shafts which are not parallel and which do not intersect by bevel wheels, an intermediate idle wheel being admissible.

SECTION IV.—CAMS AND RATCHETS.

73. *Reduction of a Crank Chain by Omission of the Coupling Link.*—A pair of spur wheels in gear form a particular case of a three-link kinematic chain consisting of two lower pairs with parallel axes, two elements of which are united and generally form the frame-link, while the other two pair by contact.

Such a chain may be derived from the four-link crank chain of Art. 52, page 122, by omission of the coupling link, a process of reduction which has already been employed on page 146.

In Figure 72*a*, *ab*, *dc* are levers turning about fixed centres and connected by a coupling link *bc*, all three links being in one plane as in the article referred to. Imagine now the crank pins at *b* and *c* enlarged until they touch one another as shown by the dotted circles and then remove the coupling link. Suitable forces being applied to close the chain by keeping the surfaces in contact, the link bc may be removed without in any way altering the motion, and therefore the angular velocity-ratio will still be as before $aT : dT$, where T is now

Fig. 72b.

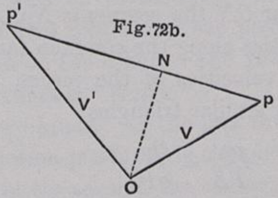
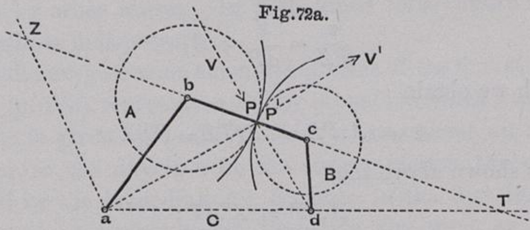


Fig. 72a.



the intersection of the common normal at the point of contact with the line of centres. Now the instantaneous motion of the levers cannot be affected by the shape of the pins except at the point of contact, and it therefore follows that if we replace the pins by any surfaces such as those indicated by the full lines in the figure, which have the same common normal at the point of contact, the result will be the same.

We may reach this conclusion directly by constructing a diagram of velocities for the two pieces in question. For let P, P' be points in the profiles which at the instant considered coincide by becoming the point of contact. Then P's velocity in the direction of the normal must be the same as that of P', for otherwise the surfaces would interpenetrate or move out of contact. If then from a given point O (Fig. 72b) we draw Op, Op' parallel to the lines aP, dP', to meet a parallel to the normal in pp', it follows by the same reasoning as in the case of linkwork that Opp' is a triangle of velocities of which the sides Op, Op' represent the velocities of P, P'. Hence drawing aZ parallel to dP' it appears as before that the angular velocity-ratio of the lines aP, dP' is dT / aT, and these lines are fixed in the rotating pieces so as to have the same velocity-ratio.

The third side pp' of the triangle of velocities represents in this case the velocity with which the surfaces rub against one another, for dropping the perpendicular ON the segments Np, Np' represent the resolved part of the velocities along the common tangent. Suppose A, A' to be the angular velocities of the pieces, V, V' the actual velocities of P, P', then by similar triangles

$$\frac{pp'}{PZ} = \frac{Op}{aP},$$

that is, if v be the velocity of rubbing,

$$\frac{v}{PZ} = \frac{V}{aP} = A,$$

from which we obtain

$$v = A.PZ = A(TZ-PT).$$

But it was shown above that

$$A.aT = A'.dT;$$
$$\therefore A.TZ = A'.PT;$$

hence

$$v = (A' - A)PT.$$

This formula suppose the pieces to turn in the same direction, as in the figure. If they turn in opposite directions, as in a pair of toothed wheels,

$$v = (A + A')\,PT,$$

a simple and important result which we shall hereafter verify.

It follows at once that for rolling contact the point of contact must lie on the line of centres, and that for a constant angular velocity ratio T must be a fixed point. Thus in all forms of teeth for wheels the common normal at the points of contact of the teeth must always pass through a fixed point on the line of centres, as is easily seen to be the case in the examples already considered. The velocity with which the teeth slide over one another is given by the above formula.

The diagram of velocities may when necessary be completed by laying down on it the velocities of all points rigidly connected with either rotating piece as explained before in the case of linkwork.

74. *Cams with Continuous Action.*—In toothed wheels the revolution of one wheel is always accompanied by that of the other in the same or in opposite directions, according as the gearing is inside or outside, or, in other words, the directional relation is always the same. We now pass on to cases in which the directional relation varies, the continuous rotation of one piece being accompanied by an oscillating motion on the other. The rotating piece is then called a "Cam," or sometimes a "Wiper."

Cams are of two kinds. In the first the contact is continuous, and the oscillating motion produced is completely defined by the form of the cam; while, in the second, the contact is only during the forward vibration of the oscillating piece, while the backward vibration is produced by other causes. In both kinds force-closure is common, and sometimes indispensable.

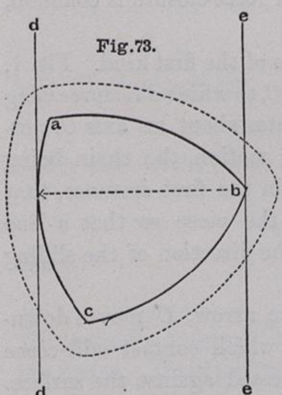
We shall now give some examples of cams of the first kind. Fig. 1, Plate IV. (p. 173), represents a sliding piece C, to which a reciprocating movement is given by a cam B, which rotates about an axis O, perpendicular to the direction of the sliding motion, the chain being completed by the frame-link A. Suppose, in the first instance, that the cam presses against a pin placed in the piece so that a line joining it to the centre of rotation gives the direction of the sliding motion.

As the cam turns in the direction of the arrow, C moves downwards to a certain limiting position, after which contact will cease unless some force be applied to keep it pressed against the surface. With suitable force-closure, however, supplied by the spring shown in the figure, C will return upwards to a second limiting position, and so on, continuously oscillating to and fro.

By properly taking the shape of the cam, any required relation may be obtained between the motions of the cam and slider; we have, in fact, only to draw a curve of position such as that constructed in Fig. 46, page 105, showing the position of the sliding piece for each position of the rotating piece. This curve will be the proper profile for the cam. In practice the chain is usually augmented by the addition of a friction roller, and the shape of the cam is modified by cutting away its surface to a depth equal to the radius of the friction roller, as was done in the case of the teeth of a wheel which drives a pin-wheel.

Force-closure, though common, is not necessary for the action of a cam chain of this kind; it may be avoided in two ways, both of which occur frequently in practice, though the mechanism would not always be described as a cam. First, the pin of the last example may be made to work in a slot cut in the face of a cam-plate, the centre line of the slot being formed to the profile of the original cam. Secondly, a slot may be cut in the sliding piece at right angles to the direction of sliding, and the cam may fit into the slot. Thus, for example, the cam may be a pin or an eccentric of any size; the chain is then merely a reduced double-slider crank motion, as explained on page 146. With other forms of cam other kinds of motion may be obtained; a common example is the Triangular Eccentric formed by three circular arcs (Fig. 73), each struck from one of the corners of an equilateral triangle *abc*. Such a curved triangle will fit between the sides *dd*, *ee* of a rectangular slot, and may therefore be used as an eccentric by fixing it to an axis passing through any point in it. In practice a figure would be used with rounded off corners, derived by striking small circular arcs with centres *a*, *b*, *c*, and uniting them by larger arcs having the same centres, thus obtaining a profile shown by dotted lines in the diagram, possessing the same essential property of uniform breadth, so that it will fit a rectangular slot of somewhat larger size. The mechanism is shown in Fig. 3, Plate IV.; it is sometimes used for a valve motion, the opening and closing of the valve taking place more rapidly than with a common eccentric. It has also been used in the "man engine" employed in mines to enable the miners to reach the surface without the fatigue of ascending ladders.

Fig. 73.



In these, as well as all other cam motions, a triangle of velocities can be constructed by the general method explained in Art. 73, and hence curves can be drawn showing the comparative velocities of the cam, the slider, and the rubbing between the two.

75. *Mechanisms with Intermittent Action.*—In all cases of higher pairing by contact between rigid elements, the closure of the chain is imperfect in the absence of external forces, for an exact fit between the surfaces, even if it exist originally, is soon destroyed by wear during the action of the mechanism. Thus, for example, when a pin works in a groove, as in the last article, the smallest looseness of fit will prevent the grooved piece from exactly following the movement of the pin when the contact passes from one side to the other of the groove. The same effect is produced by the clearance necessary for the safe action of the teeth of a wheel. In cam mechanisms, where the contact is continually changing from one side to the other, the chain opens for a short interval at every change unless force-closure be employed as described above. The pair, of which the oscillating piece forms an element, is locked by friction during the interval.

Suppose now that the groove is purposely made of much greater dimensions than the pin, the oscillating piece will remain at rest for a considerable interval, and will thus have an intermittent motion. The same thing occurs in wheels which work by the successive action of a number of teeth when some of the teeth in one of the wheels are removed. The pair which moves intermittently may be locked during the interval of rest either by friction or by the special means described in the next article.

Intermittent motions of both the cam and wheel class occur frequently in mechanism. Two common examples may be mentioned.

(1.) A wheel with one tooth may be employed to turn another wheel with any number of teeth through a small space at each revolution.

(2.) A wheel with one or more teeth may move a sliding piece alternately backwards and forwards.

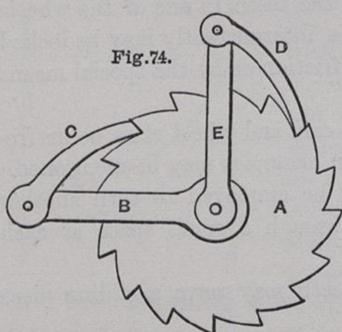
In all cases, during the interval of motion, we have a chain of the kind already described which closes at the commencement of the interval. The closure is accompanied by a shock which renders such mechanisms unfit for the transmission of considerable forces, and limits the speed at which they can be run. (See Ch. XI.)

76. *Ratchets.*—The oscillating motion of the piece C may be a turning instead of a sliding motion, as is often the case in shearing machines for example, but no new principle is here involved, and we now proceed to the second class of cam motions in which the forward vibration alone is subject to the action of the cam, while the backward vibration is effected by independent causes, generally by means of springs or of gravity. In such cases the forward vibration follows the same laws as in cams of the first kind, but during the backward vibration the oscillating piece forms a distinct machine by itself, working by means of energy supplied by the cam during the forward movement. In tilt hammers and stampers the work of the machine is done in this way and we need not here further consider them; but the object may be merely to shift the position of the piece and so to lock or unlock a pair, to open or close a kinematic chain. The piece is then called in general a Ratchet, though it may receive other names according to circumstances, and a chain in which it occurs is thus known as a Ratchet Chain.

(1.) The shifting piece may lock a turning or a sliding pair in one or both directions. A common latch for example rises to permit a gate to close and then drops into its place and fastens the gate until again raised by external means.

The piece C (Fig. 74) forming a turning pair with a fixed piece B, fits in the hollows of the teeth of a wheel A which also pairs with B. The teeth are formed as in the figure so as to permit A to move in one direction by raising C till it drops by the action of a spring or by gravity into the next hollow. In the other direction the pair AB is locked. C is then called a pawl, and the arrangement is the ordinary one employed in windlasses, capstans, and lifts to prevent the machine reversing when the hauling power is removed.

Fig.74.



(2.) Two shifting pieces may be employed to lock alternately two pairs which have a common element. This is the ratchet mechanism proper from which the name of the class is derived.

Returning to Fig. 74, A, B, C are the same as in the previous case, E is an additional piece which pairs with B: in the figure the axis of the pair has been supposed concentric with A, but this is not necessary: D is the ratchet pairing with E and at the same time fitting like C into the teeth or the ratchet wheel. If now an oscillating movement be communicated to E, the ratchet wheel A will be locked alternately with B and E according to the direction of motion of E. Accordingly A has an intermittent movement moving with E in its forward oscillation and resting in the backward. Instead of a pawl C, friction may be relied on to lock AB in the backward movement as in the common ratchet brace, but the nature of the mechanism is the same always. It sometimes happens that the pairs AB, BE are not concentric; the chain $ABED$ is then an ordinary four-link chain which opens when moved in one direction and closes when moved in the other, while the pair CA unlocks and locks as before, so as to permit A to move intermittently. In both cases the movement is single acting, but two such chains may be employed which move in opposite directions and open and close alternately; the movement may then be described as double acting. The well known "Levers of Lagourousse" (Fig. 6, Plate IV.) is a double acting ratchet mechanism in which the two chains have all the links common except the ratchets. The ratchet wheel then moves continuously in one direction, and the locking pawl C may be omitted. The ratchet wheel employed in the case of a turning pair may of course be replaced by a rack when a sliding pair is required, but no new principle is here involved.

(3.) The shifting piece may be connected with a pendulum or balance wheel which vibrates in equal times. Time may be thus measured by unlocking a kinematic chain at intervals. In clocks and watches a tooth of the ratchet wheel escapes from the action of the ratchet at each vibration or semi-vibration; the mechanism is therefore called an escapement.

(4.) In pumps various kinds of ratchet mechanisms are universal. The common reciprocating pump is a true ratchet mechanism, the column of water being locked and pairing with the plunger alternately; it may be single or double acting. It is needless to say that the ratchet is here called a "valve."

77. *Other Forms of Ratchet Mechanism.*—In all the examples of the preceding article the shifting piece is not subject to the action of the rest of the mechanism during its return oscillation, but it may also be worked by a cam movement of the first kind, or by linkwork mechanism: the slide valves of a steam engine are a familiar instance. Also it may be worked by external agency instead of by the machine itself, as in all kinds of starting and reversing gear. The ratchet chains form a large and interesting class of mechanical combinations, but their discussion would be out of place here.

78. *Screw Cams.*—The three-linked chain of Art. 72 may have the axes of its lower pairs inclined at an angle instead of parallel, and a number of mechanisms of the cam class may thus be derived which are analogous to those already considered. Some of these may also be derived from a screw chain, and may here be briefly mentioned.

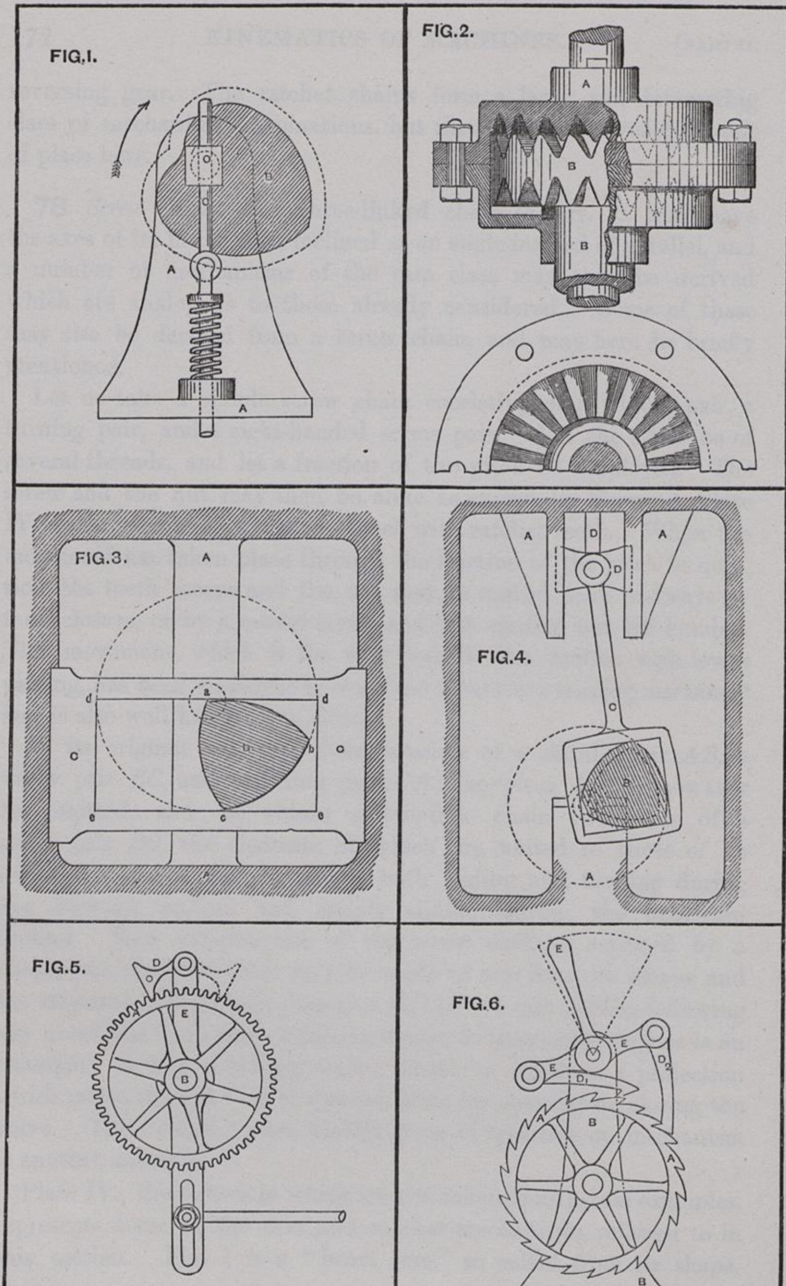
Let us take a simple screw chain consisting of a sliding pair, a turning pair, and a right-handed screw pair. Let the screw be of several threads, and let a fraction of the pitch be employed. The screw and the nut may then be alike as shown in Figure 2, Plate IV., each resembling a crown wheel with ratchet teeth. When the movement has taken place through the fraction of the pitch in question, the teeth escape and the nut may be moved back endways by force closure, or by a second screw and nut similar, but left-handed. This movement, which is the only possible cam motion with lower pairing, has been employed to work the shears in a reaping machine,* and is also well known as a clutch.

In its original form the chain consists of a sliding pair AB, a screw pair BC, and a turning pair CA; the piece A may however be omitted, and we obtain a two-link chain consisting of a screw pair BC, the elements of which are united to those of an incomplete lower pair, B and C both sliding and turning during the forward motion and simply sliding during the backward motion. Now imagine one of the screw surfaces replaced by a simple pin, then the other may be made of any form we please, and the elements of the incomplete pair will have a cam motion following any given law. A valve motion common in stationary engines is an example. B is a revolving crown wheel on which is a projection which raises the rod C at the proper time for opening or closing the valve. The "swash" plate usually given in treatises on mechanism is another example.

Plate IV., the figures in which are not taken from actual examples, represents some of the cam and ratchet mechanisms referred to in this section. Fig. 1 is a "heart cam," so called from its shape,

* Journal of the Franklin Institute for March, 1880.

Plate.IV.



To face page 173.

in which the sliding and rotating pieces are connected with uniform velocity-ratio. Fig. 2 is the screw cam just described. Figs. 3 and 4 are two forms of the triangular eccentric motion (p. 181). Fig. 5 shows a ratchet motion (p. 171) in a form common in the feed motions of machine tools: the direction of movement of the ratchet wheel A is here reversible by putting over the ratchet D into the dotted position. Fig. 6 is referred to on page 171.

EXAMPLES.

1. A reciprocating piece moves in guides under the action of a cam attached to a shaft which rotates uniformly, and the centre of which lies in the line of motion. Trace the form of the cam that the piece may slide uniformly and make one complete movement in each revolution. Suppose a friction roller used of diameter equal to $\frac{1}{6}$ stroke, and suppose also that the least radius of the cam is $\frac{1}{4}$ the stroke.

2. A stamper is raised by a cam such that the rise takes place uniformly during a part of the revolution of a shaft which is distant from the stamper half the rise. Trace the proper form of cam, and find the fraction of the revolution in which the rise takes place.

The best solution is that in which the profile of the cam has the form of the involute of the dotted circle, whose radius is half the lift of the stamper; for then the pressure of the cam on the pin is always in the vertical direction. The rise takes place whilst the cam turns through an angle, the arc of which is equal to twice the radius, or $\frac{1}{\pi}$ of a revolution.

3. Draw a curve of velocity for a reciprocating piece moved by a uniformly rotating triangular eccentric.

CHAPTER VII.

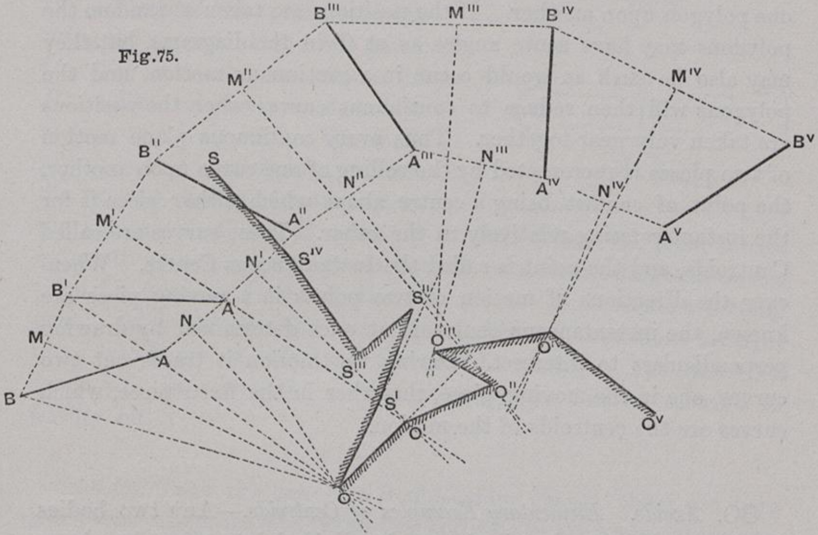
MECHANISM IN GENERAL.

79. *Plane Motion in General. Centroids.*—In the two preceding chapters the mechanisms considered have been composed either wholly of lower pairs or else of two lower pairs connected by higher pairing. The velocity-ratios of the various lower pairs have been considered, and diagrams of velocity have been drawn for the complete mechanism, but without attempting to form any idea of the comparative motion of pieces which do not pair with each other, or which form the elements of a higher pair. It will now be necessary to consider the comparative motion of two pieces more generally.

First, suppose the two pieces to move in such a way that any point in one moves parallel to a plane fixed in the other. The motion is then the same as that of a plane area which slides on a fixed plane, and may be called for brevity "plane motion." If the position of any two points in the moving area be given, all the rest can be found, and the motion is therefore completely defined by the movement of the straight line joining these points.

Let AB, $A'B'$, $A''B''$ ... (Fig. 75) be successive positions of such a line. Join AA', BB', and from the middle points of these lines draw perpendiculars NO, MO to meet in O, then $OA = OA'$ and $OB = OB'$, from which it can be proved that $AOB = A'OB'$, so that AB might be moved to $A'B'$ by attaching it to a plane area, and rotating that area about O as a centre. Obtain similar centres O', O'', O''' ... for the succeeding changes of position, then it is clear that the motion of AB, and therefore of the plane area to which it is attached, may be completely represented by the rotation of the area about the centres O, O', O''... in succession through certain angles which are given,

Fig. 75.



being the inclinations to each other of the successive positions of AB.

Next, through O draw OS', making it equal to OO' and inclined to OO' at the first angle of rotation, $S'S''$ equal to $O'O''$ and inclined to it at an angle equal to the sum of the first and the second angle of rotation, and so on; we thus obtain a second polygon $OS'S''$..., the sides of which are equal to those of the original polygon $OO'O''$... Imagine this polygon rigidly attached to AB so as to move with it, then during the motion the polygon will rotate about O till S' reaches O', then about O' till S'' reaches O'', and so on in succession; that is to say, the changes of position of AB may be produced by the rolling of one polygon upon the other. Thus, by properly determining the polygons, any given set of changes of position of a plane area may be produced at pleasure by rolling the moveable polygon on the fixed one.

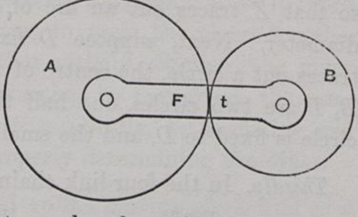
Now imagine the moving area to become fixed in its original position, and let the originally fixed area move by rolling the polygon $OO'O''$... which is attached to it upon the polygon $OS'S''$ which is now fixed. Evidently the two areas take up the same relative positions, and we obtain the very important proposition that any set of changes of relative position of two areas may be obtained by the rolling of one polygon upon another. If the positions are taken at random the polygons may have acute angles as at O'' in the diagrams, but they may also be such as would occur in a continuous motion, and the polygons will then reduce to continuous curves when the positions are taken very near together. Thus every continuous plane motion of two pieces is represented by the rolling of one curve upon another, the point of contact being a centre about which either piece is for the instant rotating relatively to the other. These curves are called Centroids, and the point is called the Instantaneous Centre. Whenever the directions of motion of two points in a moving piece are known, the instantaneous centre is at once determined by drawing perpendiculars to intersect. During the motion it traces out two curves, one in the moving piece, the other in the fixed piece, which curves are the centroids of the motion.

80. *Axoids. Elementary Examples of Centroids.*—Any two bodies moving in the way described may be divided into slices by planes parallel to the plane of motion, the centroids of which will of course be all similar and equal, so that we may regard them as the transverse sections of cylindrical surfaces in contact with each other along a generating line. The surfaces are called Axoids, and the line the Instantaneous Axis. The relative motion of the bodies is represented by the rolling of the axoids upon one another, endways motion being supposed prevented.

Any two parts of a mechanism have a relative motion which is completely defined by the nature of the mechanism, as has been sufficiently explained already; and it follows, therefore, that they must have given axoids, the nature of which completely defines the motion of the pieces. In every kinematic chain there are as many sets of axoids as there are sets of two pieces, and these surfaces are the same for all the mechanisms derived from that chain by inversion. These remarks apply even when the motion is not plane, as will be seen further on.

First. Take the case of a pair of spur wheels AB in gear, F being the frame-link (Fig. 76), forming the three-link chain considered in the last chapter. Let the pitch circles touch at the pitch point t, then, as before explained, those circles roll together without slipping, and therefore must themselves be the centroids, the pitch surfaces being the axoids. Hence the point t is the instantaneous centre of B's motion relatively to A, or A's motion relatively to B. We shall return to this immediately, but for the present merely remark that if the centres of A and B move up to each other, the pitch circles reduce to points, and the axoids become coincident straight lines, the point t is fixed in A and B, the two pieces then become a turning pair. In lower pairing, then, the axoids are coincident straight lines, which are at infinity if the pair be sliding. The case of a screw pair in which the motion is not plane will be referred to farther on.

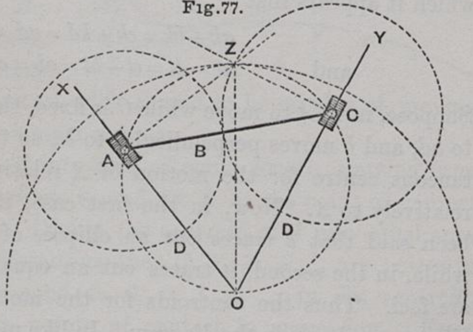
Fig.76.



Secondly. Take the case of a double-slider chain; there are here four pieces which may be taken two and two in six ways; there are, therefore, six sets of axoids. Four of these, however, are only the four axes of the four lower pairs, and it remains to determine the other two.

In Fig. 77 the blocks A, C are connected by a link B and slide on a piece D along lines OX, OY, forming the chain described fully in a former chapter. The blocks A, C form two turning pairs with the link B, and the velocities of these pairs are equal because B makes angles with OX, OY, the difference of which is constant. The centroids for A, C are therefore equal circles, the centres of which are the centres of A, C (see note, p. 191). Next, to find the centroids of B, D, through those centres draw perpendiculars to OX, OY to

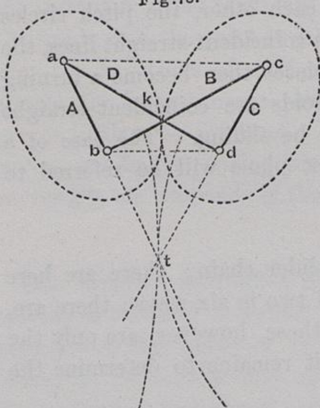
Fig.77.



meet in Z, then Z is the instantaneous centre for B when D is fixed, and for D when B is fixed. First, suppose B fixed, then the angle at Z is the supplement of the angle at O, and is therefore constant, so that Z traces out an arc of a fixed circle, of which OZ is the diameter. Next, suppose D fixed, then, since OZ is constant, Z traces out a circle, the centre of which is O. Thus the centroids of B, D are two circles, one half the diameter of the other; the large circle is fixed to D, and the small circle to B.

Thirdly. In the four-link chain A, B, C, D, consisting of four turning pairs with parallel axes, the sections of which are represented by the points a, b, c, d (Fig. 78); suppose opposite links equal, but set so as not to be parallel. This is the case referred to already (page 122) as "anti-parallel" cranks.

Fig.78.



Joining ac, bd by the dotted lines in the figure, the quadrilateral $abdc$ has two sides and two diagonals equal, hence the triangles bac, cda must be equal in every respect, so that bd is parallel to ac. Hence if k be the intersection of the diagonals, and t the intersection of the sides, $ak = ck : bk = dk : bt = dt$, from which it appears that

$$ak + bk = ck + kd = ad = bc$$

and $$at - dt = ct - bt = ab = cd.$$

Suppose, now, A to move while C is fixed, then a moves perpendicular to ad, and b moves perpendicular to bc, so that k must be the instantaneous centre for the motion of A relatively to C, or for that of C relatively to A. Now, in the first case, it appears from what has been said that k traces out an ellipse, of which c and d are foci, while, in the second, it traces out an equal ellipse, of which a and b are foci. Thus the centroids for the motion of A and C are equal ellipses, as shown in the diagram. In like manner the centroids for the motion of B and D are the equal hyperbolæ traced out by the point t.

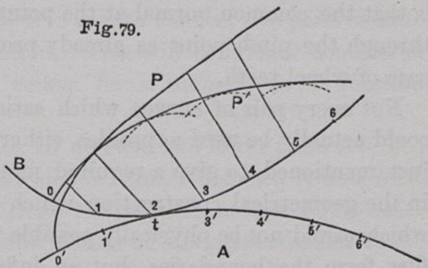
The four other pairs of centroids are the points ab cd, which are the centres of motion of the four turning pairs.

81. *Profiles for given Centroids.*—Any given motion of one piece relatively to another may be produced in an infinite number of ways. One way of doing this is by rolling contact, for if the motion is given the centroids will be given, and by forming the profiles so as to represent the centroids, and applying forces to press the pieces together and cause them to roll on one another without slipping, the given motion may be produced. But if slipping be permitted, the same motion may be produced, at least theoretically, by assuming any form whatever for one profile and properly determining the other.

(1.) Let a given profile be attached to the moving piece, and as it rolls into different positions let that profile be traced on paper attached to the fixed piece. If the positions be taken near enough together, a curve may be drawn through their ultimate intersections which will envelope them all, and if a profile formed to that envelope be attached to the fixed piece, the two pieces will fit one another and yet be capable of relative motion of the prescribed kind.

(2.) To apply the foregoing process a model would be necessary, but by a simple modification, a geometrical construction may be obtained. In Fig. 79, A and B are the pieces, which move so that the centroids are 0, 1, 2, 3 ..., 0′, 1′, 2′, 3′ ..., curves which are shown touching at the point t. P is a profile of given form attached to B; it is required to find a profile attached to A, which will always remain in contact with P, and so be capable of moving it in the required way by simple contact.

Fig. 79.



Divide the centroid of B into arcs of equal length, starting from O, the point where P intersects it, and let 2 be the point of contact at the instant considered. Divide the centroid of A into equal arcs, stepping from 2 in both directions, then 0′, 1′, 3′, 4′ ... are points in A centroid, which correspond to 0, 1, 3, 4 ... in B's centroid, being during the motion points of contact in succession. From 1, 2, 3 ... drop normals on to the curve P, and with these normals as radii trace circular arcs with centres 1′, 2′, 3′ ... ; the envelope of these arcs must be the required profile P'.

(3.) Instead of assuming one profile and determining the other to suit it, it is generally more convenient to employ some method of determining pairs of profiles which satisfy the required conditions.

In Fig. 80 A, B are the centroids as before, C is a third curve, theoretically of any form, which rolls on A and B, always touching these curves at their point of contact t. P is a tracing point which is attached to C and traces out two curves during the motion, one on A, the other on B. First, suppose A fixed, then, since t is the instantaneous centre of the motion of C, Pt must be normal to the curve NP traced out on A. Similarly supposing B fixed, Pt is normal to the curve MP traced out on B. Thus the two curves touch one another at the point P and therefore may be taken as profiles which will give the required motion. If A, B, C be circles, this construction becomes that already considered when discussing the form of teeth for a wheel. This and the preceding method show clearly that the condition which the two profiles must always satisfy is that the common normal at the point of contact must always pass through the pitch point as already proved otherwise for the special case of wheel teeth.

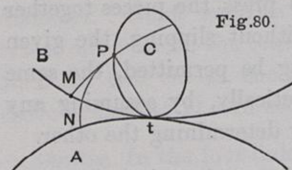


Fig. 80.

Not every pair of curves which satisfy the geometrical conditions could actually be used as profiles, either for centroids, or, in the cases just mentioned, to give a required motion, because there is nothing in the geometrical construction which excludes an interpenetration which would not be physically possible in the areas of which the profiles form the boundaries, but an infinite variety of forms can be found, for given centroids, which might be so used.

In all cases in which the centroids are known for the relative motion of two pieces, one of which is fixed, the velocity-ratio of any two points (a, b) in the moving piece is known for each position of the pieces. For, joining the two points to the instantaneous centre O, the ratio of the distances Oa, Ob must be the velocity-ratio in question, since the moving piece is for the moment turning about O. It is easily seen that the triangle Oab is similar to the triangle of velocities constructed as in Art. 49, p. 108.

82. *Centroids for a Higher Pair connecting Lower Pairs.*—Among the

infinite variety of profiles which correspond to given centroids it is frequently possible to find some which are closed curves, one completely surrounding the other. If these curves be used as the external and internal boundaries of two areas, the two pieces thus formed will fit one another and be capable of no motion except that of the prescribed kind without requiring any additional constraint. In Figure 4, Plate IV., a form of the triangular eccentric motion is shown, which has been occasionally used and which furnishes an example. On reference to Art. 74 it will be seen that such an eccentric will exactly fit a square within which it is enclosed, and therefore forms with it a higher pair which is "complete" in itself.

Complete higher pairs are very unusual in mechanism, higher pairing being employed almost exclusively to complete a chain of lower pairs as in the preceding chapter. It is then generally "incomplete," the necessary constraint being furnished by the rest of the kinematic chain to which it belongs, as for example in the triangular eccentric motion shown in Figure 3, Plate IV. The general problem in mechanism is not to connect two pieces in a given way, but to convert the motion of a given pair into the motion of a different pair—that is to say, to connect two pairs so as to have a prescribed relative motion. This will be further considered presently, but we must first return for a moment to a question considered in the last chapter.

In the three-link chain of Art. 73 we have two lower pairs AC, BC, with axes parallel, connected by simple contact between A and B at the point P (Fig. 72, p. 165). Draw the common normal PT to meet ad in T, then when B is fixed the motion of a is perpendicular to ad, and the motion of P perpendicular to PT, therefore T must be the instantaneous centre for the motion of A relatively to B. Let v be the velocity of rubbing at P; A, A' the angular velocities of the pairs AC, BC: further let $ad = l$ and $PT = z$; then, since B is fixed and A is rotating round T,

$$\frac{v}{z} = \frac{\text{velocity of } a}{aT} = A' . \frac{l}{aT}.$$

Similarly supposing A fixed,

$$\frac{v}{z} = \frac{\text{velocity of } d}{dT} = A . \frac{l}{dT},$$

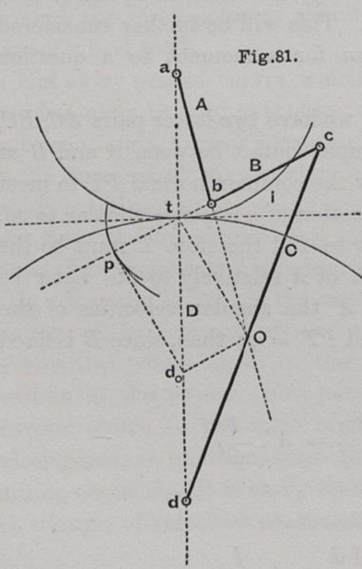
from which it appears that

$$\frac{A}{A'} = \frac{dT}{aT}; v = z(A' - A),$$

results which agree with those obtained in the article cited by a different method.

The centroids in this case, as well as in that of the four-link chain from which it was derived by reduction, may be traced graphically by plotting the position of T for a number of positions of the pieces, but they are known curves only in exceptional cases such as those of Art. 80, and generally have infinite branches which render their use inconvenient.

When the point P lies on the line of centres it coincides with T, and the velocity of rubbing is zero; the centroids are then no other than the profiles themselves of A and B. The curves are then said to roll together: a particular example is that of the equal ellipses of Art. 80 which are not unfrequently used to connect two revolving shafts with variable angular velocity-ratio. In this case the velocity-ratio is the ratio of the focal distances of the point of contact, but by properly determining the profiles it is theoretically possible to give any velocity-ratio to the shafts at pleasure. The question, however, is not one of much practical interest.



83. *Construction of Centres of Curvature of Profiles — Willis's Method.*—In the four-link chain $ABCD$ shown in Fig. 81, D is the fixed link and B the coupling link: a, b, c, d are sections of the axes of the pairs which are supposed parallel.

If the coupling link bc be prolonged to meet the line of centres ad in the point t, and ab to meet cd in O, it appears as in previous cases that O must be the instantaneous centre of B, and that the angular velocity-ratio of A and C is $dt : at$. Join Ot, and imagine bt an actual prolongation of the bar bc, so that t is rigidly connected with it, then t's motion will be perpendicular to Ot. Suppose now that the proportions of the links are taken so that Ot is perpendicular to bt, then t moves in the direction of the length of the rod, and the rod therefore may be imagined to slide through a fixed swivel at t.

This reasoning shows that the levers A and C, when in this position, will move for a short interval with uniform angular velocity-ratio, and the movement of a pair of wheels in gear is thus imitated by a link-work mechanism.

Let us now form a reduced chain by omission of the coupling-link, and we shall be able to solve the important problem of finding a pair of circular arcs which will serve for the profiles of a pair of teeth in contact. For this purpose, with centres b and c, strike arcs through any point p on cbt produced, and let these arcs be rigidly connected with A and C respectively; the coupling-link may now be removed, and A imagined to drive C by direct contact of the arcs. Evidently wherever p is, the pieces will move for the moment with uniform angular velocity-ratio and pitch point t. The uniformity, however, is only momentary, because the position of O changes, and to trace the profiles with accuracy it would be necessary to perform the construction for a succession of positions of cbt, hence the face and flank of a pair of teeth in contact cannot be exactly represented by a pair of circular arcs. When it is sufficiently approximate to do so, the arcs are found by assuming a mean position for the point p, and a mean value for the obliquity i, found by experience to give good results. The method here described was invented by the late Professor Willis, and the value of i recommended by him was $\sin^{-1}\cdot 25$, or about $14\frac{1}{2}°$, being about the actual mean value of the obliquity in cycloidal teeth of good proportions. Also the value of pt was taken by him as half the pitch, p being then about midway between the pitch point t and the point of the tooth.

Having made these assumptions, it still remains to fix the position of the point O, which may be taken anywhere on a line through t inclined at $14\frac{1}{2}°$ to the line of centres. This is done by observing that O must be the same for all wheels D intended to work with a given wheel A, and that teeth never should be undercut (Art. 70); that is, c and b must lie on the same side of t. Hence in the smallest wheel intended to work with A, c is at infinity, so that if d_0 is its centre, d_0O is parallel to pt, and therefore perpendicular to Ot. The flank of the tooth in this case becomes a radius d_0p. The position of O is thus completely determined for all the wheels of a set when the pitch is given.

Willis's method is of great theoretical interest, and has consequently been given here, but the form of teeth obtained is not always sufficiently approximate. It may, therefore, with advantage be replaced by other methods, as to which the reader is referred to a work by Professor W. C. Unwin on Machine Design.

84. *Sphere Motion.*—When a body moves about a fixed point its motion is completely represented by that of a portion of a spherical shell of any radius which fits on to a corresponding sphere, and moves on it just as in the case of plane motion. Everything which has been said respecting plane motion also applies to sphere motion, but the axoids are conical instead of cylindrical surfaces, the centroids spherical instead of plane curves, and all straight lines are replaced by great circles of the sphere on which the motion is imagined to take place. The corresponding crank chains are called "conic" crank chains, the axes of the pairs lying on a cone instead of a cylinder.

85. *Screw Motion.*—In the plane motion of two pieces, endways motion of the cylindrical axoids is supposed to be prevented by some suitable means. Let us now remove this restriction and imagine the axoids to slide endways, while continuing to roll together, the relative movement will now not be completely defined, but additional constraint will be required. In the first place take the case of a lower pair in which the axoids are coincident straight lines; if endways sliding be permitted we obtain an incomplete pair, unless the nature of the surfaces in contact define the relation between the endways motion and the rolling motion. In the simple screw pair the two are in a fixed ratio, in the screw cams of Art. 76 they have a varying ratio. In every case of non-plane motion with cylindrical axoids, not only must the axoids be given, but also a connection between the endways sliding and the motion of rotation.

In the most general case possible the instantaneous axis changes its direction as in spherical motion, its position as in plane motion, and in addition there may be an endways sliding. This is expressed by the rolling and sliding of certain surfaces on one another, which are now neither cylindrical nor conical. These surfaces are in all cases of the kind known as "ruled" surfaces, being generated by the motion of a straight line, along which they touch each other. The surfaces are still called Axoids, and the line is the Instantaneous Axis. The hyperboloidal pitch surfaces for wheels connecting two shafts which do not intersect are examples of this kind; but for the discussion of this question, which is not of very common occurrence, the reader is referred to the works already cited.

86. *Classification of Simple Kinematic Chains.*—On observing the action of any mechanism, several of the pieces of which it is constructed may be readily distinguished as having functions different from the rest. These pieces, like the rest, occur in pairs, and may be described as such, though the pairing is not necessarily kinematic. First, one or more perform the operations which are the object of the mechanism, these may be called the Working Pairs, as, for example, the tool and the work in machine tools, the weight raised and the earth in the hoisting machines. Second, one or more form the source from which the motion is transmitted, as, for example, the crank handle and frame of a windlass, the piston and cylinder of a steam engine. These may be called the Driving Pairs. Thirdly, various subsidiary working pairs carry out various operations incidental to the working of the machine. The object of the mechanism is always to convert the motion of the driving pairs into that of the working pairs.

The simplest case is that in which the motion has only to be transmitted without alteration; a single pair will then suffice. Thus, by means of a long rod sliding in guides or turning in bearings, a motion of translation or rotation may be transmitted to a distance only limited by non-kinematical considerations. By use of flexible elements—among which should be included the flexible shafts recently introduced—the direction may be altered at pleasure and any desired position reached.

If, however, the magnitude of the motion is tobe altered, a mechanism must be employed in which at least one element of the driving and working pairs is different. The driving pairs are usually kinematic lower pairs and the working pairs are so very frequently, and this is why so many of the simplest and most important mechanisms are examples of the connection of lower pairs. The peculiar motions of lower pairs being translation and rotation, a number of mechanisms may be classed as examples of the conversion of rotation into translation or rotation and conversely, with uniform or varying directional relation or velocity-ratio. This is especially the case when, as so frequently happens, the driving and working pairs have a common link which is fixed.

It has been shown, however, that many apparently different mechanisms are in reality closely connected, being derived from the same kinematic chain. Mechanisms are therefore to be classed according to the kinematic chains to which they belong. The number of simple chains actually employed in mechanism is limited by the preceding considerations to those already described, which are ranged by Reuleaux in the following classes, the names of which are derived from the most important piece in some example of common occurrence :—

(1) Crank chains.
(2) Screw chains.
(3) Pulley chains.
(4) Wheel chains.
(5) Cam chains.
(6) Ratchet chains.

In the first two are included all combinations of sliding, turning, and screw pairs; in the third, all cases where tension or pressure elements are employed; in the fourth, all cases of connection by contact where the directional relation remains the same; in the fifth, all cases where it varies; while in the last all combinations are included where, by action of a shifting piece, the law of motion is periodically varied.

87. *Compound Kinematic Chains.*—In a complete machine, the motions required are generally too complex to be carried out by a single kinematic chain of this simple kind; it is necessary to combine together a number of such chains, and we conclude this part of the subject with some general remarks on such combinations which may all be regarded as compound chains derived from two or more simple chains by union of their links.

(1.) From any two closed chains a third may be derived by uniting two links. The links must have the same relative motion, for otherwise the chains would lock each other, and they generally form a pair.

This is one of the commonest of all combinations. When two machines are driven from the same shaft, or when the same shaft is driven by two separate engines, we have examples in which the driving pairs or the working pairs are common, but the mechanisms are otherwise independent. Further, in every complete machine we find, in addition to the principal chain which does the work, a number of auxiliary chains which carry out various operations necessary to the working of the machine. Thus, in the steam engine, besides the slider-crank or other mechanism which turns the crank, we have the valve motion which governs the distribution of steam, the air pump motion which produces the vacuum in the condenser, and frequently others as well. Each of these auxiliary mechanisms has a pair in common with the principal chain which serves as a driving pair, but the chains are otherwise independent. Again, in trains of mechanism which, as previously remarked (page 150), are frequently simple chains augmented for non-kinematical reasons, a number of such chains are arranged so that the working pair of one chain is the driving pair of the next in succession. A train of wheels or the mechanism of a beam engine are examples already referred to, in which one link is common to all the separate chains, but cases occur in which this is not so, as, for example, the well-known Lazy Tongs.

The case here considered is that where the movements of various driving pairs have to be transmitted to various working pairs, but no new motion is required in a working pair other than could be produced by a simple chain. All such combinations may be called Trains, and may be divided into "converging," "diverging," and "transmitting" trains.

(2.) If two closed chains have only one link common they are completely independent, like two machines standing on the same floor, but disconnected. It might, therefore, be supposed that nothing was obtained that was new. In fact, however, this is a combination which is as common as the preceding, being employed to give a motion to a working pair which is too complex to be produced by simpler means, or which requires to be varied at pleasure. The working pair consists of two elements, one of which is supplied by one chain, the other by the other, and the motion of the pair is thus a combination of the motions of the two independent chains. Completely new motions are obtained in this way, and they may be varied at pleasure by alteration of either or both of the primary motions.

Take, for example, the common planing machine. The working pair consists of the table, upon which the work is mounted, and the tool. To the first a reciprocating movement is communicated by means of a suitable kinematic chain connecting it with the driving shaft. The other is mounted on a slide rest, forming an element of a screw chain, which gives it a horizontal movement. This chain has one link in common with the principal chain, but is otherwise independent. In the ordinary working of the machine this chain is locked by friction, except at the end of each reciprocating movement of the table when it moves to take the next cut. The tool thus traces out a complete plane surface.

In this example the common link is fixed, but this need not be the case, and in fact in the planing machine a third independent chain is added to adjust the tool vertically, the tool being mounted on a vertical slide having an independent movement. Also, one element of the working pair may be fixed and both movements given to the other, which is common to both chains. Double and treble chains of this kind occur whenever it is necessary to move the elements of the working pair into all possible positions. In cranes of all kinds we find a treble movement, one to raise and lower the jib, a second to swing the jib round, and a third to raise or lower the load. In traversing cranes the three movements are rectangular, as in the planing machine. In either case we find the methods employed by mathematicians to define the position of a point in space by rectangular or polar co-ordinates exactly imitated by the mechanism.

The elements of the working pair need not be wholly disconnected as we have hitherto supposed, they may form an incomplete kinematic pair. Thus if the axoids be cylindrical, endways motion may still be possible and may be given by an independent chain. A common example is a drilling machine, the working pair in which consists of a table on which the work is mounted, and a spindle carrying the drill which rotates and at the same time descends as the hole is drilled; the two movements may be quite independent, the one proceeding from a driving shaft, the other operated by the workman.

A similar combination is employed when a train is varied by shifting one of its links. Fig. 5, Plate III. (p. 152), represents a case of this kind. The wheel C' is mounted on a shaft which can be shifted endways by an independent mechanism. The shifting of belts (Art. 61, p. 143) is another example.

Again, the movements of the working pieces may be connected by a transmitting train connecting the chains which produce them. In the self-acting feeds of planing and shaping machines the connection is intermittent, but it may also be continuous, and we then have a fertile means of producing complex movements variable at pleasure. In a screw-cutting lathe the tool is mounted on a slide-rest moved by a screw, and the work is attached to a rotating mandrel. Connecting these independent chains by a train of "change" wheels, the tool cuts a screw of any pitch.

The principle of all combinations of this kind is the closure of an incomplete or disconnected pair by independent chains. We may describe them as Multiple Chains.

(3.) If two closed chains have two or more pairs common, they must be of the same kind, for otherwise the pairs would not have the same relative motion, and the chains would lock each other. The differential mechanisms, examples of which have been already given, are cases of this kind. Thus in the differential pulley (Fig. 62, p. 140), if A and C be disconnected we have two simple pulley chains with common moveable pulley B and separate axles. Either of these might be operated independently. In the actual mechanism A and C are united, and the movement of B is the difference of the movements due to each separate chain.

Complex examples of similar combinations occur in the epicyclic mechanisms. Fig. 82 (p. 191) shows a combination of two of the differential trains described on p. 150. C, C' are wheels turning about the same axis in the frame-link A and united; E, E' are also united, but have a different frame-link A'. Both gear with the wheels B, D, which are disconnected, but turn on an axis common to A and A'. On comparing this with Fig. 65 it will be seen that two trains have been compounded by uniting the wheels B, D, which are common to both. If now one of the frame links, say A', is fixed, and EE' be rotated, the other frame-link A will rotate with a velocity which can be found on the principles of the article cited. For simplicity, EE' have been supposed to gear directly with B, D, but they may also gear with wheels of other diameters fixed to B, D, or the wheels may be replaced by a different train of mechanism, all that is necessary being that the motions of the pairs BA', DA' should be connected.

Many examples of this mechanism may be found—especially in the case where C, C' are equal and the train reduces to three bevel wheels (pp. 151-2). In traction engines and tricycles, for instance, a mechanism of this kind is sometimes employed to facilitate turning. A' is then the frame of the machine, B and D are equal bevel wheels attached to the axle, which is divided into halves, each connected with one of the driving wheels. If now the motive power be applied to A, B and D will rotate, but not necessarily with the same velocity, and the machine may therefore be guided in a curve by the front wheel without the slipping which would occur if the driving wheels were fixed to an undivided axle.

Combinations of this class are not essentially different from multiple chains in which the elementary chains are connected by a train, as described above. They may be called Compound Trains; all consisting of simple trains compounded in various ways, either for non-kinematical reasons or to enable the train to be varied at pleasure.

(4.) All the preceding combinations are formed of simple closed chains united together in various ways; no new chain is obtained, but merely an aggregation of forms already known. Certain mechanisms, however, occur which if taken to pieces by separation of united links are found to contain one or more chains which are not closed.

Take for simplicity a common slider-crank mechanism, and imagine the crank pin, instead of being fixed to the crank, to be mounted on a slide so as to be free to move to and from the centre. The chain is now augmented by an additional sliding pair, and is no longer closed, so that it cannot be used as a mechanism. If, however, we introduce a screw, which moves the slide, we may lock the sliding pair in any position and thus obtain a closed chain, one link of which can be varied at pleasure. This mechanism is used in practice to obtain a varying stroke in a sliding piece. It is often required to make the stroke increase or diminish at each revolution of the crank. A wheel attached to the screw then comes in contact with a projecting piece and moves through a small space, the screw chain being locked by friction during the rest of the revolution. The mechanism thus varies at intervals its own law of motion.

By a suitable transmitting train however a continuous variation may be produced, and the combination then furnishes us with an entirely new mechanism. An important example is the wheel crank chain (Fig. 83), formed by combining a simple wheel chain with an open crank chain of five links. A number of mechanisms may

Fig. 82.

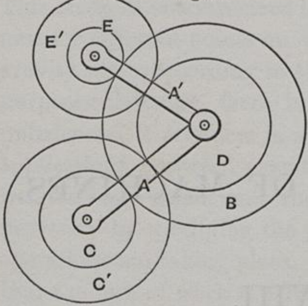
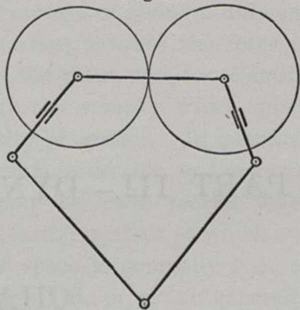


Fig. 83.



be derived from this chain by inversion, but for particulars the reader is referred to Reuleaux's work already cited.

Another example is shown in Fig. 6, Plate II. (p. 121), which represents a mechanism employed in sewing machines to give two strokes to a sliding piece for one revolution of a shaft. We have here a closed double slider chain combined with a single slider rendered incomplete by omission of the crank pin. Combinations of this class are called by Reuleaux "true" compound chains to distinguish them from the preceding classes, in which no new mechanism results from the combination. Perhaps the words "higher" and "lower" would more clearly express the meaning.

NOTE.—In Fig. 77, page 177, the blocks A, C revolve in the same direction, and the centroids are circles of infinite size. To represent them equal circles of finite size are employed, which give the same motion but in opposite directions.