CHAPTER V.

LOWER PAIRING.

46. *Definition of Lower Pairs.*—Each piece of a machine is in direct connection with at least one other, and constitutes with it what is called a PAIR, of which the two pieces are said to be the Elements. The whole machine may be regarded as made up of pairs, and the nature of the mechanism depends on the nature of the pairs of which it is constructed.

In the present chapter we consider exclusively mechanism composed of pairs of rigid elements which are in contact with each other, not merely at certain points or along certain lines, but throughout the whole or part of the area of certain surfaces. Such pairs are of peculiar importance from the simplicity of the relative movement of their elements, from their resistance to wear when transmitting heavy pressures, and from their tightness under steam and water pressure. They are called Lower Pairs, and in many cases this kind of pairing is alone admissible.

In order that two rigid surfaces may be capable of moving over each other while continuing to fit, they must either be cylindrical, including under that head all surfaces generated by the motion of a straight line parallel to itself, or surfaces of revolution, or screw surfaces. In the first case the relative motion of the elements is one of translation along the line, in the second of rotation about the axis of revolution, in the third the motion of translation and rotation are combined in a fixed proportion. Hence there are three kinds of lower pairs, known as Sliding Pairs, Turning Pairs, and Screw Pairs. In each case one of the surfaces is hollow, and wholly or partly encloses the other which is solid, and the motion depends on the surfaces only, and not on the other parts of the elements which assume very various forms, according to the purpose of the mechanism. Either element may be fixed and the other move, or both elements may move in any way whatever, the relative motion is still of the same kind.

As an example of a sliding pair may be taken a piston and cylinder, in which either the cylinder may be fixed and the piston move, or the piston be fixed and the cylinder move, as in some steam hammers, or both cylinder and piston move, as in the oscillating engine. The relative motion is always a simple translation. Velocities of translation are most conveniently measured in feet per 1″ or feet per 1′, but miles per hour and knots per hour are also used, as to which it is convenient to remember that one mile per hour is 88 feet per 1′, and one knot per hour approximately 101 feet per 1′.

As examples of turning pairs may be taken a cart and its wheel, a shaft and its bearing, or a connecting rod and crank pin. The relative motion here is one of simple rotation, which may be measured by the number of revolutions (n) per unit of time, or by the speed of periphery (V) of a circle of given radius (r), or by the angle (A) turned through per unit of time. The first two modes of measurement are common in practice, the third is used for scientific purposes only. When employed the angle is always expressed in circular measure, and the three methods are therefore connected by the equations

$$V = Ar = 2\pi nr.$$

When angular velocity is used as a measure of speed of rotation, the unit of time is always 1″, but the minute and hour are common in other cases.

A screw pair consists of a screw and its nut, and the relative motion consists of a motion of translation along the axis of the screw combined with a rotation about that axis. The motion of translation is often called the "speed of the screw," and is equal to np, where p is the pitch, that is to say, the space traversed in one revolution, and n the revolutions in the unit of time. Strictly speaking, the two first lower pairs are limiting cases of the screw pair: in the turning pair the pitch is zero, and in the sliding pair infinite.

In all three cases the motion of either element relatively to the other is identically the same, and the rate of that motion may properly be called the Velocity of the Pair, whether the movement considered be translation or rotation. When the velocity of a sliding pair and a turning pair are compared, rotation may be measured by the speed of periphery of a circle of given diameter; it is the velocity with which bearing surfaces of that diameter would rub each other. The radius of this circle may be called the "radius of reference." The velocity of a screw pair may be measured by the rate either of its translation or its rotation.

In these three simple pairs the motion of one element relatively to the other is completely defined, each point describing a definite curve. Such a pair is called a "complete" or "closed" pair, but we may have pairs in which the motion is not defined unless further constraint be applied, and the pair is then said to be "incomplete." An incomplete pair cannot be used in mechanism without employing such constraint, and this process is called "closing" the pair. A pair may be incomplete, because there is nothing to prevent the disunion of its elements, as, for example, a shaft and its bearing when the cap is removed, but it also may be incomplete in itself. Lower pairing is sometimes, though not very frequently, incomplete in this latter sense; there are three possible cases, first, when the surfaces are spherical, as in a ball and socket joint; second, where a rod fits into a hole, and is free to move endways as well as rotate; third, where a block fits in between parallel plane surfaces. The methods of producing closure will be considered hereafter.

It may be here remarked, in anticipation of what will be said hereafter, that cases of lower pairing may be imagined in which the elements are not in contact over an area but along a line. For example, a rod may fit into a square hole. It is the simplicity of the relative motion which is the essential characteristic.

The motion of the elements of a pair may be prevented by a pin key or other fastening removable at pleasure: the pair is then said to be "locked." In capstans and windlasses, provided with ratchet wheel and pawls, we have examples in which a pair is locked in one direction only.

47. *Definition of a Kinematic Chain.*—It has been already said that a machine consists of a number of parts so connected together as to be capable of moving relatively to one another in a way completely defined by the nature of the machine. Each part forms an element of two consecutive pairs, and serves to connect the pairs so that the whole mechanism may be described as a chain, of which the parts form the links. Such a series of connected pieces is called a Kinematic Chain.

The motion of any piece may be considered either relatively to one of the pieces with which it pairs, or with reference to any other piece which we may choose to regard as fixed. In the first case the rate of movement has already been defined as the Velocity of the Pair. In the second, the fixed piece is usually the frame of the machine, which unites the rest of the pieces, and is commonly attached to the earth or some structure of large size, such as a vessel. For pieces which pair with the frame the velocity of the pair is the same as the velocity of the moving element, and this element alone need be mentioned. In some common practical cases the speed of an element means the speed of one of the pairs of which it forms part. For example, the speed of piston of an oscillating engine would be understood to mean its velocity relatively to the cylinder, in other words, the speed of the "cylinder-piston pair." In the present chapter we consider exclusively chains of closed lower pairs, so that the motion of the pairs is a simple translation, rotation, or screw motion. The motion of some of the pieces relatively to the frame may be much more complex, but this is a subject for subsequent investigation; it is the motion of the pairs alone we now consider. We shall first direct our attention to the very common and important piece of mechanism employed in direct-acting steam engines. An example is shown in Fig. 1, Plate I., p. 118, which represents a direct-acting engine of the vertical inverted cylinder type which is common in marine engines and often occurs in other cases.

Let us consider the pairs of which this mechanism is constructed. We have, *first*, a cylinder, to which are rigidly attached guides for the crosshead, and bearings for carrying the crank shaft. The cylinder-guide bars and crank-shaft bearings all form one part rigidly connected together, and must be considered as being one piece or link of the kinematic chain. It may conveniently be called the frame. *Secondly*, there is a piston, which fits and slides in the cylinder. To the piston a rod and crosshead are rigidly attached, forming practically one piece. Not only is the piston guided in the cylinder, but the crosshead also between the guide bars, and the piston rod in the stuffing box; but yet, since there are practically two pieces only which move relatively to one another, we must look on the cylinder, stuffing box, and guide bars as altogether forming the hollow element of a sliding pair, and the piston, rod, and crosshead as together forming the solid element of the pair. *Thirdly*, there is a connecting rod which is attached by a gudgeon or crosshead pin to the piston-rod head. These two parts will together compose a turning pair. At the other end the connecting rod embraces the crank pin, forming a second turning pair with it. The crank pin is one of the elements contained in the *fourth* piece of the mechanism. This piece consists of the crank pin, crank arms, and shaft with its journals. The journals turn in the bearings of the fixed frame of the machine, the first link mentioned, and so form a third turning pair. Thus the chain is complete. It consists of four links forming one sliding pair and three successive turning pairs.

The same mechanism, in a different form, is shown in Fig. 2 of the same plate which represents the air-pump of a marine engine worked, as is not unusual, by a large eccentric keyed on the crank shaft. The crank pin is here enlarged so as to become an eccentric; and to save room, the piston rod is replaced by a trunk within which the eccentric rod vibrates. We have, however, exactly the same pairs arranged in the same way, and the difference between the mechanisms is therefore merely constructive, the motions of the parts being identical.

48. *Mechanism of Direct-Acting Engine—Position of Piston.*—This is such an important piece of mechanism that we will examine its motion somewhat fully.

First as to the relative positions of the crank in its revolution and the piston in its stroke. The position of the piston in its stroke will compare exactly with the position of the crosshead, so instead of

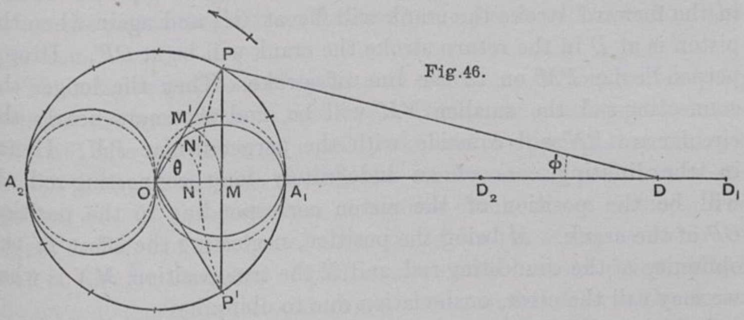


Fig. 46.

introducing the length of the piston rod into the diagram, we may just as well determine the relative positions of the point D (Fig. 46) in its straight-line path, and P in its circular path.

Suppose the line of stroke to pass through the centre of the crank-pin circle. Let OP = length of the crank arm, and PD the length of the connecting rod. When the crank arm is in the line of stroke, away from the piston, the piston will be in one extreme position, and when the crank is in the line of stroke towards D, the piston will be in its other extreme position. The points A_1 A_2 on the crank-pin circle are called the dead points. If we take distances A_1D_1 A_2D_2 $= PD$, the length of the connecting rod, the points D_1 D_2 represent the ends of the stroke of the piston. If now we place the crank in any position OP we obtain the corresponding position of the piston by cutting the line of stroke with a circular arc of radius $= PD$ and with centre P. DD_1 DD_2 will be the distances of the piston from the ends of its stroke. Since $A_1A_2 = D_1D_2$, the length of the stroke, it will be convenient to find the point in A_1A_2 which corresponds to the position of the piston in its stroke. This may be readily done by striking a circular arc PN with centre D. N will be the point for $A_1D_1 = PD = ND$, therefore $A_1N = D_1D$, and the point N is the same distance from A_1 and A_2 as the piston is from the ends of its stroke.

We may just as easily solve the converse problem of finding the position of crank corresponding to any given position of the piston in its stroke. Let D be any position, cut the crank-pin circle by a circular arc of which D is the centre and DP the radius, then OP or OP' will be the corresponding position of the crank. Let the direction A_1 PA_2 be the ahead direction of the crank, and let us call the motion D_1D_2 towards the crank the forward stroke, and D_2D_1 the back or return stroke of the piston, then when the piston is at D in the forward stroke the crank will be at OP, and again when the piston is at D in the return stroke the crank will be at OP'. Drop a perpendicular PM on to the line of stroke. Then the longer the connecting-rod the smaller NM will be, and the more nearly the circular arc PN will coincide with the perpendicular PM. Hence in the limiting case of an indefinitely long connecting-rod, M will be the position of the piston corresponding to the position OP of the crank. M being the position, neglecting the effect of the obliquity of the connecting-rod, and N the true position, MN is what we may call the error, or deviation due to obliquity.

In general the slide valve is worked by an eccentric, the radius of which is set at a particular angle on the shaft, so that the cut-off takes place when the crank occupies a certain angular position in its revolution, and it consequently follows that the fraction of stroke completed before cut-off takes place will be affected by the obliquity of the connecting-rod, so that in the ordinary setting of the slide valve the rates of cut-off will be different in the two strokes. This is well illustrated by Ex. 4, page 112.

We may obtain a convenient approximate expression for MN, the error due to obliquity. Referring to Fig. 46.

$$NM = DN - DM = DN(1 - \cos\phi).$$

Now the length of the connecting-rod may be conveniently expressed as a multiple of the length of the crank radius a or stroke s.

$$DN = na \text{ suppose} = \tfrac{1}{2}ns.$$

$$\therefore \quad NM = n\frac{s}{2}(1 - \cos\phi) = ns\sin^2\frac{\phi}{2}.$$

In the triangle POD, the sides being proportional to the sines of the opposite angles,

$$\text{Sin}\,\phi = \frac{OP}{DP}\sin\theta = \frac{1}{n}\sin\theta.$$

Now, the angle ϕ is in all practical cases a small angle, so we may write approximately

$$2\sin\frac{\phi}{2} = \sin\phi,$$

$$\therefore \quad NM = n \,.\, s \,.\, \frac{\sin^2\theta}{4n^2} = \frac{s}{4n}\sin^2\theta.$$

This is greatest when $\theta = 90$. $NM_{\text{max.}} = \dfrac{s}{4n}$.

If the connecting-rod is four times the crank, the greatest error due to obliquity $= \frac{1}{16}$ stroke.

We see that, in the forward stroke, the effect of the obliquity of the connecting-rod is to put the piston in advance of the position due to an indefinitely long connecting-rod, and, in the return stroke when the piston moves from the crank, the piston will be behind that position.

The relative positions of piston and crank may be very conveniently represented by a curve in this way. Divide the crank-pin circle (see Fig. 46) into a number of equal parts, and supposing the crank-pin at the points of division P, find the corresponding positions of the piston N. If then we take along the crank arm a distance ON' equal to ON, the distance of the piston from the centre of its stroke, and do this for a number of positions, we shall find the points N' will lie on a double-looped closed curve, shown in full lines in the figure. This may be called a curve of position of the piston. If we had supposed the connecting-rod to be indefinitely long, and had taken a distance OM' along $OP = OM$, the curve of position in such a case would have been a pair of circles, dotted in the figure, on OA_2 and OA_1 as diameters. The true curves of position will deviate from these circles more the shorter the connecting-rod. For the half stroke nearer the crank the curve will lie outside the dotted circle, and for the further half stroke inside. In Zeuner's valve diagram the obliquity of the eccentric rod is neglected, and the circles employed to show the position of the slide valve.

49. *Velocity of Piston.*—We will now pass on to the question of the relative velocity of the piston and crank-pin.

We will suppose the crank to turn uniformly at so many revolutions in the unit of time. If n = number of revolutions and a = length of crank-arm, s = stroke.

$$\text{Velocity of crank-pin } V_0 = 2\pi an = n\pi s.$$

Now, as the crank-pin moves with uniform velocity, the piston undergoes continual changes of velocity, from being zero at the ends to a maximum at about the centre of the stroke. What is commonly spoken of as the speed of piston is the mean speed. If in the unit of time a complete number of revolutions are performed at a uniform rate, the mean speed will be the actual distance traversed by the piston in the unit of time. In each revolution the piston will complete a double stroke, so that speed of piston $= \overline{V} = 2ns$. This may be compared with the speed of crank-pin V_0,

$$\frac{V_0}{\overline{V}} = \frac{\pi ns}{2ns} = \frac{\pi}{2}.$$

Next, as to the actual velocity of the piston at any point of its stroke.

Fig.47a.

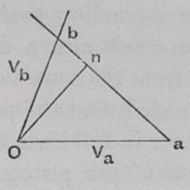
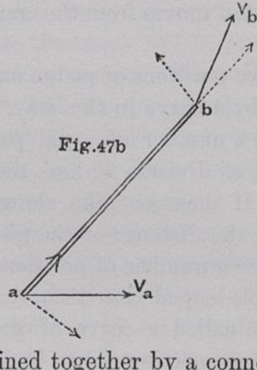
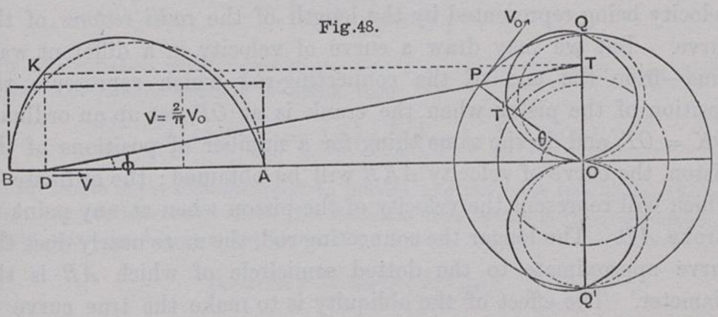


Fig.47b



The piston and crank-pin are joined together by a connecting-rod of invariable length; one end of this rod has the velocity of the piston and the other that of the crank-pin. In Fig. 47*b* let *ab* be a rod, the ends of which move with velocities V_a, V_b in given directions. If one of these velocities be given, the other can be determined. For in Fig. 47*a* draw *Oa* parallel and equal to V_a and *Ob* parallel to V_b to meet a line *ab* which is perpendicular to the line *ab* of the first figure; then, if we drop a perpendicular *On* on *ab*, this will be parallel to *ab* of the first figure, and must represent the resolved part of the velocity V_a along the rod. But the velocities of *a* and *b* resolved along the rod must be equal, because the length *ab* of the rod is invariable; hence *On* also represents the resolved part of V_b along the rod, and consequently *Ob* must represent that velocity in magnitude as well as in direction. The figure *Oab* is called the Diagram of Velocities of the rod, and from it we can find the velocity of any point we please either in, or rigidly connected with, the rod. We shall return to the properties of this diagram frequently hereafter: it will be sufficient now to remark that the triangle *Oab* determines the velocity-ratio of the two ends. In drawing the triangle it is generally convenient to turn it through 90°, so that the lines *ab* in the two figures become parallel, while the sides *Oa*, *Ob* become perpendicular to the velocities they represent.

Fig. 48.



In Fig. 48 *OP* is the crank arm, *PD* the connecting-rod; through *O* draw *OT* at right angles to the line of stroke to meet the connecting rod produced in *T*, then *P* moves perpendicular to *OP*, and *D* to *OT*, therefore *OPT* is a triangle of velocities, so that if *V* be the velocity of the piston, V_0 that of the crank pin,

$$\frac{V}{V_0} = \frac{OT}{OP}.$$

This simple construction enables us very conveniently to draw a curve of piston velocity. In the first place, set off along OP a length $OT' = OT$, and do this for a number of positions of the crank. The points T' will be found to lie on a pair of closed curves, shown in full lines in the figure, passing through O and also through Q, Q', the upper and lower ends of the vertical diameter of the crank circle. Had the connecting-rod been indefinitely long, the points T' would have been found to lie on a pair of circles, of which the diameters are OQ and OQ', shown in dotted lines. On account of the obliquity of the connecting-rod, the curve of actual velocity lies outside the circle on the cylinder side of the crank, and inside the circle when the crank lies away from the cylinder.

When the crank is at right angles to the line of dead centres, the velocity of the piston is the same as that of the crank-pin, and neglecting the obliquity of the connecting-rod this will be the maximum velocity of the piston. If the obliquity is taken into account, the greatest velocity of piston occurs when the crank is inclined a little towards the cylinder, it is very approximately when the crank is at right angles to the connecting-rod, and the maximum velocity will a little exceed the velocity of the crank-pin.

The curve just described is a polar curve, the magnitude of the velocity being represented by the length of the *radii vectores* of the curve. But we may draw a curve of velocity in a different way, thus—from the end of the connecting-rod which represents the position of the piston when the crank is at OP, set up an ordinate $DK = OT'$, and do the same thing for a number of positions of the piston, the curve of velocity AKB will be obtained; the ordinate of which will represent the velocity of the piston when at any point of stroke AB. The longer the connecting-rod, the more nearly does the curve approximate to the dotted semicircle of which AB is the diameter. The effect of the obliquity is to make the true curve of velocity lie outside the semicircle in the first half of the stroke of the piston towards the crank, and inside for the second half of the stroke.

The mean velocity of the piston may be conveniently represented by an addition to the diagram, thus:—On the same scale that OP, the length of the arm, represents the velocity V_0 of the crank-pin, take a length to represent

$$\overline{V} = \frac{2}{\pi} V_0.$$

In the polar diagram draw a circle with O as centre and radius of this length. Where this circle cuts the polar curve of velocity the positions of the crank are given at which the actual speed of the piston is equal to its mean speed. In the second diagram of velocity, set up an ordinate to represent $\overline{V}$, and draw a line parallel to the line of stroke. It will cut the curve of piston velocity in two points.

An approximate expression for the velocity of the piston may be determined thus:

$$V = V_0 \frac{\sin OPT}{\sin OTP} = V_0 \frac{\sin(\theta + \phi)}{\cos \phi};$$

or expanding the numerator,

$$V = V_0 \{\sin\theta + \cos\theta \tan\phi\}.$$

Since ϕ is in all practical cases a small angle, $\tan\phi$ may be written $= \sin\phi$ without sensible error.

$$\therefore V = V_0\{\sin\theta + \cos\theta\sin\phi\}.$$

$$\text{Now} \quad \frac{\sin\phi}{\sin\theta} = \frac{OP}{PD} = \frac{1}{n}.$$

$$\therefore V = V_0\left\{\sin\theta + \frac{1}{n}\sin\theta\cos\theta\right\}$$

$$= V_0\left\{\sin\theta + \frac{1}{2n}\sin 2\theta\right\}.$$

By differentiation with respect to the time t we obtain the acceleration of the piston. Let a be the length of the crank, then

$$\frac{dV}{dt} = \frac{dV_0}{dt}\left\{\sin\theta + \frac{1}{2n}\sin 2\theta\right\} + \frac{V_0^2}{a}\left\{\cos\theta + \frac{1}{n}\cos 2\theta\right\}.$$

If the length of the connecting-rod be infinite, and the crank turn uniformly, we obtain a simple harmonic motion, the deviation from which is therefore, approximately, assuming n large and dV_0/dt small,

$$Deviation = \frac{V_0^2}{na}\cos 2\theta + \frac{dV_0}{dt}\sin\theta.$$

The graphical construction for the acceleration when the crank turns uniformly will be found in Ch. IX.

EXAMPLES.

1. The driving wheels of a locomotive are 6 feet in diameter, find the number of revolutions per minute and the angular velocity, when running at 50 miles per hour. If the stroke is 2 feet, find also the speed of piston.

Revolutions per minute,	=	$233\frac{1}{2}$.
Angular velocity,	=	$24\frac{4}{9}$ per second.
Speed of piston,	=	933·6 feet per minute.

2. The pitch of a screw is 24 feet, and revolutions 70 per minute. Find the speed in knots per hour. If the stroke is 4 feet, find also the speed of piston in feet per minute.

Speed of screw = 16·58 knots per hour.
,, piston = 560 feet per minute.

3. The stroke of a piston is 4 feet, and the connecting-rod is 9 feet long. Find the position of the crank, when the piston has completed the first quarter of the forward and backward strokes respectively. Also find the position of the piston when the crank is upright.

Ans. The crank will make, with the line of dead centres, the angles 55° and 66°. When the crank is upright the piston will be $2\frac{3}{4}$ inches from the middle of its stroke.

4. The valve gear is so arranged in the last question as to cut off theste am when the crank is 45° from the dead points both in the forward and backward strokes. Find the point at which steam will be cut off in the two strokes. Also when the obliquity of the connecting rod is neglected.

Ans. Fraction of stroke at which steam is cut off is —

·175 in forward stroke,
·118 in backward ,,
·146 neglecting obliquity.

5. Obtain the results of the two last questions for the case of an oscillating engine, 6 feet stroke, the distance from the centre of the trunnions to the centre of the shaft being 9 feet.

Ans. Angles 68° and 51° : Cut off ·2 and ·115.

6. In Ex. 3 construct both curves of piston velocity. If the revolutions be 70 per minute, find the absolute velocity of the piston in the positions given. Find also the maximum velocity of the piston.

Ans.	$\frac{1}{4}$ stroke forward, velocity	=	810 feet per 1′.
	$\frac{1}{4}$,, back, ,,	=	730 ,,
	Maximum, ,,	=	900 ,,

Find also the points in the stroke at which the actual speed of piston is equal to the mean speed.

Ans. $4\frac{3}{4}$ in. from commencement of forward stroke.
$6\frac{3}{4}$ in. ,, end ,, ,,

7. The travel of a slide valve is 6 in., outside lap 1 in. Find, in feet per second, the velocity with which the port commences to open when the revolutions are 70 per minute.

Ans. Port commences to open when the valve is 1 in. from the centre of its stroke. Neglecting the obliquity of the eccentric rod, velocity of valve is then 1·72 feet per second.

8. Show that the maximum velocity of the piston occurs when the crank is nearly at right angles to the connecting rod, the difference being a small angle, the sine of which is $\frac{1}{n(n^2+2)}$ nearly, where n is the ratio of connecting rod to crank.

50. *Mechanisms Derived from the Slider-Crank Chain.*—In the investigation just given it has been supposed, for simplicity, that the crank turns uniformly, but if this be not the case the curve constructed will show the ratio of the velocities of the piston and crank pin. In all cases it is the velocity-ratio of two parts, not the velocities themselves, which are determined by the nature of the mechanism. The velocities are of course reckoned relatively to the frame, but as both piston and crank pair with the frame, they are also the velocities of the piston-frame pair and the crank-frame pair (see p. 104), the crank being the radius of reference. The velocities of the other pairs will be determined presently, but in this mechanism are of less importance. We will now direct our attention to other examples of the simple chain of lower pairs, of which the direct-acting engine is only a particular case. In Fig. 49, *D* is a

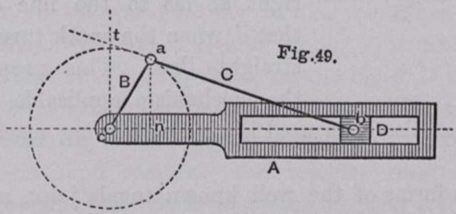


Fig.49.

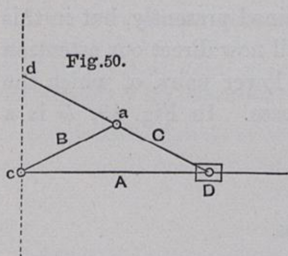
block capable of sliding in the slot of the piece *A*. By means of a pin this block is connected with one end of the link *C*. *B* is a crank capable of rotating about a pin attached to the piece *A*, and united to *C* by another pin. Each of the four pieces of which this mechanism is composed, together with either of the adjacent pieces, constitutes a "pair," of which there are *four*, viz., three turning pairs, *AB*, *BC*, *CD*, and a sliding pair *DA*. This simple combination of pairs is known, in the modern theory of machines, as a Slider-Crank Chain.

Since the relative motions of the parts depend solely on the form of the bearing surfaces of the pairs and the position of their centres, not on the size and shape of the pieces in other respects, we may vary these at pleasure, and thus adapt the same chain to a variety of purposes. Especially we may interchange the hollow and solid elements of the pairs, a process which occurs constantly in kinematic analysis, and is called "inversion of the pair."

Again, any one of the four pieces may be fixed and the others move, so that we can obtain four distinct mechanisms from the same chain, simply by altering the link which we regard as fixed, a process called "inversion of the chain."

(1.) Let A be fixed, then we obtain the mechanism of the direct-acting engine already fully considered. In this, however, the connecting link C is much longer than the crank B; by supposing them equal we obtain a mechanism well known in various forms. In Fig. 50 C is prolonged beyond the crank pin a to a point d, such that $ad = ac$, a circle struck with centre a then passes through c, d, and the centre of the block, thus cd is at right angles to the line of stroke, so that d, when the crank turns, describes a straight line. This property renders the mechanism applicable as a parallel motion. It has also been used in air-compressing machinery. (See page 129.)

Fig.50.



The various forms of the well known toggle joint, some of which will be given hereafter, are examples of the same mechanism with different proportions of C to B.

(2.) Instead of A, let us suppose C to be the fixed link, so that A and the other pieces have to take a corresponding motion. With this, by a change in the shape of the pieces, we are able to derive a mechanism well known in two forms. C being fixed, and B caused to rotate, A will have given to it an oscillating motion about the block D, and, at the same time, will slide to and fro on the block, the block itself having a vibrating motion about the other end of the piece C. Now, the relative movement of the parts of this mechanism is identical with that of the oscillating steam engine, and by a suitable alteration in the shape of the pieces, that mechanism may be derived. Thus, suppose, in the first place, the hollow element of A to become the solid one, in the shape of a piston-rod and piston, whilst the block D is enlarged into a cylinder to surround the piston, and so becomes the hollow element of the pair. The cylinder D will oscillate on trunnions, in bearings in the fixed piece C, which must be so constructed as to be a suitable frame for carrying the engine, and have bearings in which the crank shaft and crank B can turn.

The oscillating cylinder is in general mounted on bearings, the centre line of which coincides with the centre of the stroke of the piston, so that the distance apart of the shaft and trunnion bearings is equal to the length of the piston-rod. An example is shown in Fig. 4, Plate I.

Next let us consider the relative motions of the parts. Returning to Fig. 49 above, suppose a, b, c, to be the centres of the turning pairs, and draw ct, an perpendicular to the line of centres bc, to meet C and A in t and n, then it was shown above (page 109) that the velocity-ratio of the pairs DA, BA in the direct-acting mechanism was ct/ac, and as fixing a link makes no difference in the relative motions, this must also be the ratio of the speed of the piston of the oscillator in its cylinder, to the speed of the turning movement of the crank *relatively* to the piston-rod. Again, when C is fixed, as in the oscillator, the link A (Fig. 49) slides on the block D with a velocity the direction of which is perpendicular to an, while the point c in it moves perpendicular to ac. Hence it follows that the triangle of velocities is acn, and therefore the velocity ratio of piston and crank pin is an/ac. The curve of piston velocity can be drawn as before; it differs little in form from that of the direct actor, but the maximum velocity of the piston is equal to that of the crank pin, instead of being somewhat greater. Once more, remembering that fixing a link does not alter the relative motions, it appears that, in all cases, the velocity-ratio of the pairs BC, DA must be an/ac, so that we have determined the ratio of the speed of piston in the direct actor to the speed of the turning movement of the crank *relatively* to the connecting rod.

Comparing our results, we see that the velocity-ratio of the turning pairs BC, BA must be $ct : an$, or what is the same thing $bt : ab$. Since the three angles of the triangle abc are always together equal to 180°, it is clear that the sum of the speeds of the three turning pairs must be zero, due regard being taken of the direction of rotation, and it follows, therefore, that in any slider-crank chain the speeds of the three turning pairs are as $at : ab : bt$. By the introduction of a suitable radius of reference, we may compare these velocities with that of the sliding pair. The most convenient radius to take is that of the crank, then assuming, as before, $ab = n \, . \, ac$, the velocities of the pairs are shown by the annexed table :—

Velocity Ratios in a Slider-Crank Chain.				
Pair,	DA	BA	BC	DC
Velocity,	ct	ac	$\frac{bt}{n}$	$\frac{at}{n}$

In the oscillator the angular velocity-ratio of the cylinder and crank is the velocity-ratio of the pairs CD, CB and is therefore $at : bt$ or $cn : bc$. This can readily be constructed by drawing nz parallel to ac, then, since ab is constant, the angular velocity of the cylinder is proportional to az, as may be verified by an independent investigation. The result may be exhibited by a polar curve similar to the curve of piston velocity already drawn. In Fig. 51, c is the crank pin, $TST'S'$ the crank circle, az' is set off along the crank radius equal to az, then a curve with two unequal loops is obtained, which shows the law of vibration of the cylinder.

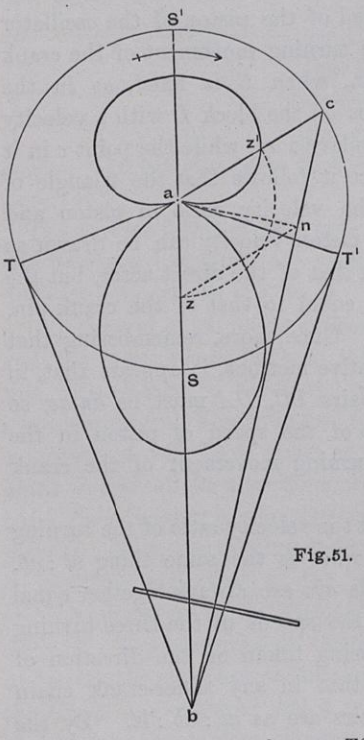
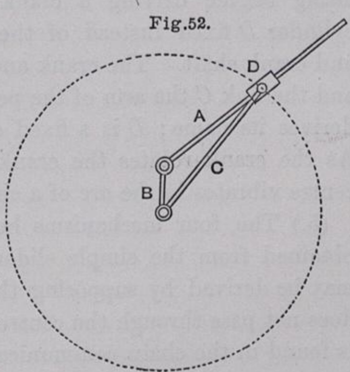


Fig.51.

The motion of the cylinder is such that, in the swing to the left, whilst the crank pin moves along the arc $T'ST$, the angular velocity is much greater than in the return swing to the right, whilst the crank pin moves along the arc $TS'T'$. Supposing the crank to revolve uniformly, the times occupied by the forward and return swings are as the arcs $T'ST$ and $TS'T'$, which are proportional to the angles subtended by them. By measuring or otherwise estimating these angles, the mean angular velocities in the forward and backward oscillation may be determined. This peculiar vibration, rapid one way and comparatively slow the other, has been made use of to obtain a quick return motion of a cutting tool in a shaping machine. The velocity with which a tool will make a smooth cut in metal is limited, and since in general the tool is made to cut in one direction only, time is saved by causing the return stroke to be made more quickly. One construction of such a quick return motion may be thus described. A slotted lever *D* vibrates on a fixed centre in the frame piece *C*, its motion being derived from the revolution of a crank *B* on another fixed centre in the same frame piece *C*. The crank pin of *B* turns in the block *A*, which slides in the slotted lever *D*. There is in addition a connecting rod, by means of which a to-and-fro motion of a headstock carrying the cutting tool is communicated from the oscillating lever, the headstock sliding in a guide.

Omitting the connecting rod, we have the same kinematic chain, with the same fixed link *C*, as in the oscillating engine. There has been a change made only in the form of some of the pieces. What was the oscillating cylinder is now the slotted lever, and instead of a piston and rod, we have here the simple block *A* sliding in the slot. The crank *B* and frame-link *C* remain practically unaltered. The slotted lever will vibrate according to the same law which we have investigated for the oscillating cylinder, and thus with a uniform rotation of the crank, a quick return motion of the tool will be obtained. This mechanism is shown in Fig. 5, Plate I., in a form employed for giving motion to the table of small planing machines.

Fig. 52.



(3.) Let us next take an example in which *B* is the fixed link, and becomes the frame, its form being of course modified to suit the new conditions.

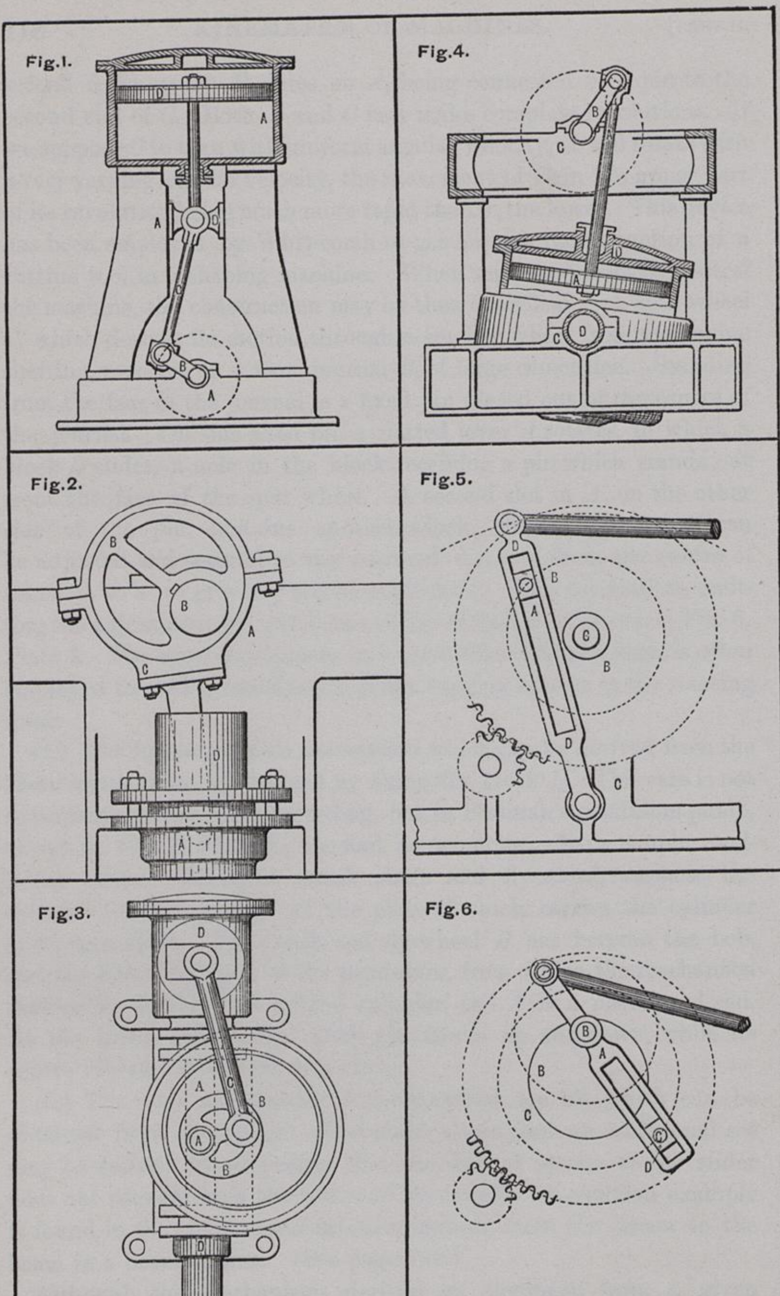
A crank arm *C* (Fig. 52) turns on a fixed centre in the frame piece *B*; so also does another arm *A* on a second fixed centre, D slides on A, being connected by a pin to the second end of C. Both A and C may make complete revolutions. If we suppose C to turn with uniform angular velocity, A will rotate with a very varying angular velocity, the movement of A in the upper part of its revolution being much more rapid than in the lower. This device has been employed by Whitworth to get a quick return motion of a cutting tool in a shaping machine. When separated from the rest of the machine, the construction may be thus described :—A spur wheel C, which derives its motion through a smaller wheel from the engine shafting, revolves on a fixed journal B, of large dimension. Standing from the face of the journal is a fixed pin placed out of the centre of the journal. On this fixed pin a slotted lever A rotates, in which a block D slides, a hole in the block receiving a pin which stands out from the face of the spur wheel. A second slot in A, on the other side of the pin, contains another block, which, by a screw, can be adjusted and secured at any required distance from the centre of rotation, so as to give any stroke at pleasure. This mechanism, omitting the adjustment by which the stroke is varied, is shown in Fig. 6, Plate I. The same mechanism in a somewhat different form is often employed in sewing machines to give a varying motion to the rotating hook.

(4.) The fourth possible mechanism which can be derived from the slider-crank chain is obtained by fixing the block D. This case is not so common as the three preceding, but in Stannah's pendulum pump, shown in Fig. 3, Plate I., we find an example. In a simple oscillating engine driving a crank shaft and fly-wheel, suppose the cylinder D fixed instead of the piece C which carries the cylinder and crank shaft. The crank and fly-wheel B has become the bob, and the link C the arm of the pendulum, from which the mechanism derives its name; D is a fixed cylinder, and A is a piston and rod. As the crank rotates the crank pin moves up and down, while its centre vibrates in the arc of a circle.

(5.) The four mechanisms here described are all which can be obtained from the simple slider-crank chain, but an additional set may be derived by supposing that the line of stroke of the slider does not pass through the centre of the crank. A common example is found in the chain communicating motion from the piston to the beam in a beam engine. (See page 129.)

Although the mechanisms derived by inversion from a given

Plate. I.



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kinematic chain may be described as distinct, it must be carefully observed that there is in reality no kinematic difference between them, the distinction consisting merely in a different link being chosen to reckon velocities from. If we consider the velocities of the pairs which constitute the chain, those velocities are always related to each other in the same way, and the same machine may be regarded sometimes as one mechanism and sometimes another. For example, suppose a direct-acting engine working on board ship; the ship may be imagined to roll so that the connecting rod of the engine is at rest relatively to the earth, and the engine becomes an oscillator to an observer outside the ship. Dynamically and constructively, however, there is a great difference, for the fixed link is the frame, and is attached to the earth or other large body, the predominating mass of which controls the movements of all bodies connected with it. To illustrate and explain the inversion of a slider crank-chain, Plate I. has been drawn. The six examples which have just been described are here placed side by side with the same letters $ABCD$ attached to corresponding links, so that they may readily be recognized. It will be seen that each link assumes very various forms; thus, for example, the link A is the frame and cylinder in Figs. 1 and 2, a piston and rod in Figs. 3 and 4, a block in Fig. 5, and a rotating arm in Fig. 6. The relative motions of corresponding parts are, however, always the same.

51. *Double Slider-Crank Chains.*—We now pass on to the consideration of a kinematic chain consisting of two turning pairs and two sliding pairs. We will commence by showing how this chain may be derived from that previously described. Suppose the piece D, instead of being simply a block, is a sector shaped as shown in Fig. 1, Plate II., having a slot curved to the arc of a circle of centre O, while the piece C, which was before the connecting rod, is compressed into a block sliding in the curved slot. The law of relative motion of the parts of this mechanism will be precisely the same as in the direct-acting engine, for the block C will move just as if it were attached by a link, shown by the dotted line, to a point O, a fixed point in the piece D. The piece D will slide in A, just as if there were a connecting link from C to O and no sector—that is, it will slide just as the piston does in the cylinder of a direct-acting engine. Moreover, there are, in reality, exactly the same pairs in this as in the mechanism of the direct-acting engine, for C and D together make a turning pair, although only portions of the surfaces of the cylindric elements are employed.

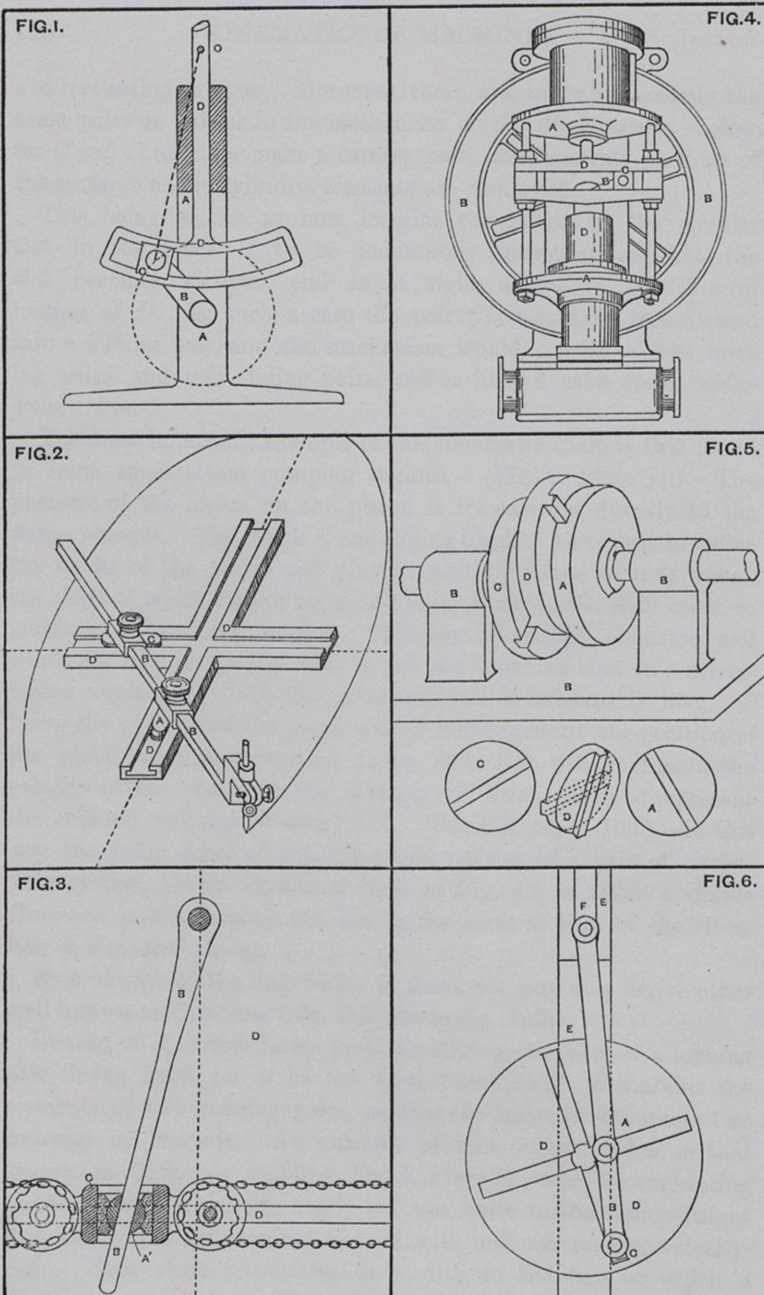
This being so, let us now imagine the radius of the circular slot in the piece D to be indefinitely increased, so that the slot becomes straight, and is at right angles to the line of motion of D. In such a case the pair CD would be transformed into a sliding pair, and the mechanism would consist of two turning pairs, and two sliding pairs, and is known as a *double slider-crank chain.*

The most important example of this kinematic chain is that found in some small steam pumping engines. (Fig. 4, Plate II.) The pressure of the steam on the piston is transmitted directly to the pump plunger. The crank B and sliding block C serve only to define the stroke of the piston and plunger, and, by means of a fly-wheel, the shaft of which carries an eccentric for working the slide valve, to maintain a continual motion. The law of motion of piston and crank pin may be readily seen to be the same as that in a direct-acting engine, in which the connecting rod is indefinitely long. P being the position of the crank pin, M will represent the position of the piston and reciprocating piece, and PM will represent the velocity of the piston at the instant, OP being taken to represent the uniform velocity of crank pin. (See Fig. 48, p. 109.) In this case the polar curve of velocity would consist of a pair of circles. This motion, shown in dotted lines in Fig. 48, is called a simple *Harmonic motion*, because the law is the same as that of the vibration of a musical string.

By a change of the link which is fixed, we may now derive other well known mechanisms from this kinematic chain.

Instead of A, which forms part of a sliding and part of a turning pair, being fixed, let B be the fixed frame link. B contains the elements of two turning pairs, so that the frame must contain two bearings or journals. An example of such a mechanism is that known as Oldham's coupling, Fig. 5, Plate II., used for connecting parallel shafts, which are nearly but not quite in the same straight line, and which are required to turn with uniform angular velocity-ratio. Each shaft terminates in a disc, in the face of which a straight groove is cut. The two discs, A and C in the figure, with

Plate.II.



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the grooves face each other, and are placed a little distance apart, with the grooves at right angles to each other. Filling up the space between them is placed a disc D, on the two faces of which are straight projections at right angles to one another, which fit into the grooves in the shaft discs. In the revolution of the shafts each of these projections slides in the groove in which it lies, and rotates with it. The two grooves are, therefore, maintained always at right angles to one another, and the two shafts rotate one exactly with the other.

Next, let the fixed link of the chain contain the elements of two sliding pairs, which would be obtained if we made D the frame piece. An interesting example of this is the instrument sometimes employed in drawing ellipses. (Fig. 2, Plate II.) Two blocks slide in a pair of right-angled grooves. By means of clamp screws a rod unites them at a constant distance from one another. Pins fitting in holes in the blocks allow the rod to rotate relatively to the blocks. Any point in the rod will describe an ellipse, as indicated in the figure.

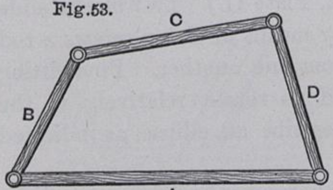
If the link C be fixed, the resulting mechanism does not differ from that derived by fixing A, and the three mechanisms just described are therefore all which can be obtained by inversion of a double-slider chain. In Figs. 2, 4, 5 of the plate referred to, they are shown side by side with the same letters attached to corresponding links, as in Plate I.

The directions of motion of the two sliding pairs have been supposed at right angles, but any other angle may be assumed, and mechanisms obtained which we need not stop to examine. A more important change is to suppose that the sliding pairs and turning pairs alternate, so that each link forms an element of one sliding and one turning pair. A mechanism known as "Rapson's Slide," employed as a steering gear in large ships, will furnish an example. Fig. 3, Plate II., shows one way in which it is applied. A' is an enlarged pin made in two pieces between which the tiller B slides while turning about an axis fixed in the ship D. A' is carried by the piece C, which slides in a groove fixed transversely to the ship being drawn to port or starboard by the tiller chains passing round pullies mounted on C, as shown in the figure. The further the tiller is put over the slower it moves (Ex. 8, p. 133), and therefore the greater the turning moment (Ch. VIII.), a property of considerable practical value. In this kinematic chain the same mechanism is obtained whichever link is fixed.

The mechanism shown in Fig. 6 of this Plate is a compound chain, to be referred to hereafter.

52. *Crank Chains in General.*—Instead of having a chain of turning pairs connected by one or two sliding pairs, we may have turning pairs alone. The number will be four, and their axes must meet in a point or be parallel. Taking the second case, the chain in its most elementary form consists of 4 bars united by pin joints at their extremities, as in Fig. 53. It is called a crank or four-bar chain, and from it may be derived the slider-crank chain already considered, in the same way as from that chain we derived the double slider chain. All the mechanisms hitherto considered may therefore be regarded as particular cases of it. In its present form, however, many new mechanisms are included, some of which will be briefly indicated, referring for descriptions and figures to works specially devoted to mechanism.

Fig. 53.



Assuming A the fixed link, B and D which pair with it are called for distinction cranks or levers, according as they are or are not capable of continuous rotation, while C the connecting link is called for shortness the *coupler*.

(1.) Let B be a crank and D a lever, then the mechanism is a "lever-crank," an example of which occurs in the common beam engine, D being the beam, B the crank, C the connecting rod, and A the entablature, foundation, and all other parts connected therewith.

(2.) The links B and D may be equal, and C may be equal to A. This may be called "parallel cranks" when B and D are set parallel, as in the coupled outside cranks found in locomotives, or "anti-parallel cranks" when they are set crosswise, a case to be hereafter referred to. (Page 178.)

(3.) The links D and B may still both be cranks if C be greater than A, provided that the difference between B and D be not too great. The mechanism is called "double cranks," and occurs in the common drag-link coupling, and also in the mechanism of feathering paddles.

(4.) If the coupling link be too short, neither B nor D will be capable of a complete rotation. The mechanism is then a "double lever," and an example occurs in the common parallel motion to be considered hereafter.

(5.) A number of additional mechanisms may be derived by supposing the axes of the four turning pairs to meet in a point, instead of being parallel; we thus obtain a "conic crank chain." Hooke's joint is a particular case of this, but in general these mechanisms are of less importance.

53. *Diagram of Velocities in Linkwork.*—A simple construction has already been given, by means of which the velocity-ratios of the parts of a slider-crank chain are determined, and we will now consider this question for any case of linkwork in which the axes of the pairs are parallel.

Figure 54*a* represents a chain of links *zOabcd*... united by pins so as to form a succession of turning pairs. The first link *Oz* is fixed,

Fig. 54a.

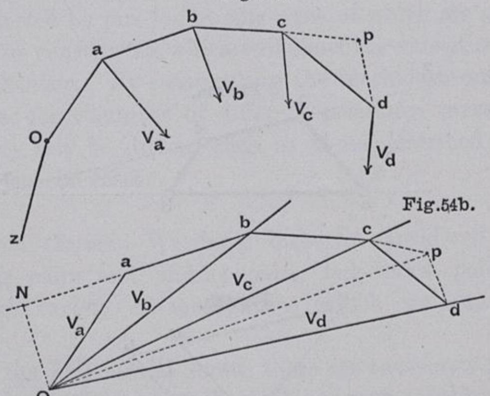


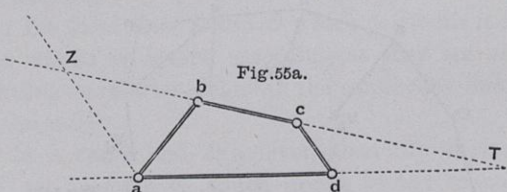
Fig. 54b.

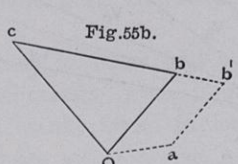
so that the second turns about a fixed O point as centre, and therefore a moves perpendicularly to Oa, with a velocity V_a, which we may suppose known. The other points b, c, d... move in directions which we suppose given, and with these data it is required to find the magnitudes of the velocities. In Figure 54*b*, from a pole *O* draw radiating lines perpendicular to the given directions, and set off on the first *Oa* to represent V_a, then draw *ab*, *bc*, *cd*... parallel to the links of the chain to meet the corresponding rays, then the lengths of those rays represent the velocities.

For drop a perpendicular *ON* from *O* on to *ab*, or *ab* produced, then *ON* represents the component of V_a in the direction of the second link, but this must also be the component of V_b in that direction, since *ab* is of invariable length; that is, *Ob* must represent V_a. Similarly all the other rays must represent the velocities of the corresponding points.

The figure thus drawn may be called the Diagram of Velocities of the chain. It may be constructed equally well, if the magnitudes of the velocities be given instead of their directions, also any of the turning pairs may be changed into sliding pairs. If both ends of the chain be attached to fixed points, the diagram will evidently be a closed polygon. Its sides, when divided by the lengths of the corresponding links of the chain, represent their angular velocities, for each side is the algebraical difference of the velocities of the ends of the link perpendicular to the link.

In the four-link chain (Fig. 55*a*), consisting of two links turning





about fixed centres *a*, *d*, coupled by a link *bc*, the diagram of velocities is a simple triangle *Obc* (Fig. 55*b*), the sides of which, when divided by the lengths of the links to which they are parallel, represent the angular velocities of the links. Through a draw aZ parallel to cd, and prolong bc to meet it in Z, and the line of centres in T, then, since the triangle Zab is similar to the triangle of velocities, the angular velocities of the levers cd, ab will be proportional to Za/cd and ab/ab. The last fraction is unity, and therefore we have

$$\text{Angular Velocity-Ratio} = \frac{Za}{cd} = \frac{aT}{dT},$$

showing that the ratio in question is the inverse of the ratio of the distance of T from the centres.

If, instead of the link ad being fixed, the chain of four bars be imagined to turn about one joint such as d, the diagram of velocities would be a quadrilateral $Oab'c$, with sides parallel to $abcd$.

Returning to the general case, let p be any point rigidly connected with one of the links of the chain, say cd, in the figure; then if we lay down on the diagram of velocities a point p, similarly situated with respect to the corresponding line cd of that diagram, it follows at once, by the same reasoning, that the ray Op, drawn from the pole O, must represent the velocity of p in the same way that the other rays represent the velocities of the points a, b... Thus it appears that for any linkwork mechanism, consisting of pieces of any size and shape connected by pin joints, the axes of which are parallel, a diagram may be constructed which will show the velocities of all points of the mechanism. By constructing the mechanism and its diagram of velocities for a number of different positions, curves of position and velocity may be drawn, such as those described in preceding articles for special cases.

54. *Screw Chains.*—We have hitherto considered only chains of turning pairs and sliding pairs, but screw pairs also occur in a great variety of mechanisms which we can only briefly indicate.

(1.) In the Differential Screw, there are two screw pairs with the same axes but a different pitch, combined with a sliding pair, forming a three-link chain. The connection between the common velocity of rotation of the screws and the velocity of translation of the sliding pair is the same as that between the rotation and translation of a screw, the pitch of which is the difference between the pitches of the actual screws. The arrangement has often been proposed for screw presses, a mechanical advantage being obtained, at least theoretically, with screws of coarse pitch, which would otherwise require a thread so fine as to be of insufficient strength. The right and left-handed screw is an example in common use.

(2.) In the Slide Rests of lathes and other machine tools, the traversing motion of planing machines, and many other cases we find a three-link chain, consisting of a screw pair, a turning pair, and a sliding pair. This may be regarded as a particular case of the preceding, the pitch of one of the screws being zero.

(3.) In presses, steering gear, and many other kinds of machinery we find a simple screw chain employed to work a slider-crank chain. Some examples will be given hereafter.

55. *Parallel Motions Derived from Crank Chains.*—In beam engines the connecting rod by which the reciprocating motion of the piston is communicated to the vibrating beam is necessarily short, in order to diminish the height of the machine, and therefore, if guides are employed to retain the end of the piston-rod in a straight line, there will be considerable lateral

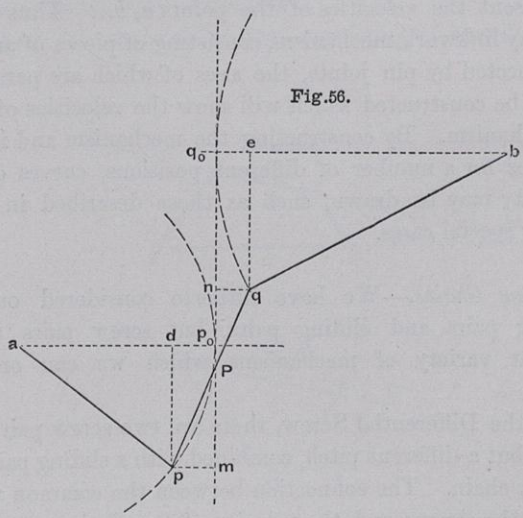


Fig.56.

pressure which is difficult to provide against, and which involves a large amount of friction. The guides may then be replaced with advantage by some linkwork or other mechanism. Such a mechanism is called a Parallel Motion, and in the early days of engineering was employed more extensively than at the present time. In its most simple form it consists of two levers capable of turning about the fixed centre a and b. (Fig. 56.) The ends of the levers are connected by a coupling link pq, then, so long as the angular movement of the levers is not too great, there is a point in the link pq which will describe very approximately a straight line. In the first instance let us suppose the links so set that when ap_0 and bq_0 are parallel, p_0q_0 is at right angles to them. Let $apqb$ be the extreme downward movement of the levers, then p lying to the left and q to the right, there will be some point P in pq which in this extreme position lies in the straight line p_0q_0. In the upward extreme position the same point of pq will, approximately, also lie in this line. If, then, p_0q_0 be the line of stroke, and the point P be selected for the point of attachment of the piston-rod head, then this point will be exactly in the line at the middle and bottom of the stroke, and at other points will deviate but little from it.

To find the point where pq intersects p_0q_0, we must first obtain expressions for the amount that the point p deviates to the left of p_0 and q to the right of q_0; these amounts being the versines of the arcs in which the points move, and shown by dp_0 and eq_0, where pd and qe are drawn perpendicular to ap_0 and bq_0. By supposing the circle of which a is the centre to be completed, it is easy to see that

$$(ad + ap_0)dp_0 = pd^2,$$

$$\therefore \quad dp_0 = \frac{pd^2}{ad + ap_0}.$$

If the angle p_0ap is not greater than 20°, we may write

$$dp_0 = \frac{pd^2}{2.ap_0},$$

the error not being greater than 1 per cent. Now, neglecting the small effect due to the obliquity of the connecting link when in the extreme positions, $pd = \frac{1}{2}$ stroke; therefore, supposing $ap = r_a$ and $bq = r_b$,

$$pm = dp_0 = \frac{(\text{stroke})^2}{r_a},$$

and

$$qn = eq_0 = \frac{(\text{stroke})^2}{r_b}.$$

Now P being the point where pq intersects p_0q_0, we have similar triangles in which

$$\frac{pP}{qP} = \frac{pm}{qn} \text{ and } \therefore \; = \frac{r_b}{r_a}.$$

Thus the point P, which has most correctly the straight-line motion, is such that it divides the coupling link into segments which are inversely proportional to the lengths of the levers. If the levers be placed into all possible positions, then in the motion the connecting link will be inverted and the point P will trace a closed curve resembling a figure of 8. There are two limited portions of this curve which deviate very little from a straight line.

We may approximate still more nearly to a straight line by a little alteration in the setting of the levers. Suppose the centres of vibration a, b, are brought a little nearer together so that the line of stroke bisects the two versines dp_0 and eg_0. Then when the levers are parallel, the link slopes to the left upwards, whereas at the ends of the stroke the link will slope to the right upwards. At two intermediate positions about quarter stroke from the ends, the link will be vertical. If we choose the point P as previously described, the maximum deviation will be only about one fifth of its former amount. In practice, the final adjustment of the centres of motion is performed by trial.

The use of parallel motions is almost exclusively confined to beam engines. In that case bq will be the half length of the beam of the engine, and in order that the angle through which the beam vibrates should not exceed 20° above and below the horizontal, the length of the beam should not be less than three times the stroke. The radius rod may be somewhat shorter than the half beam, but should not be less than the stroke, or the error in the motion of P will be too great. This mechanism will, therefore, occupy a considerable space. To economise space, and also to provide a second straight-line path to guide the air-pump rod, a modification of the mechanism is made use of.

In Fig. 57, be being the half length of beam, a point q is chosen so that

$$\frac{bq}{be} = \frac{\text{stroke of air pump}}{\text{stroke of piston}},$$

and a parallelogram of bars $qeQp$ provided, united by pins. The point p is jointed to the end of the radius rod ap, vibrating on the fixed centre a. Consequently there will be some point P in

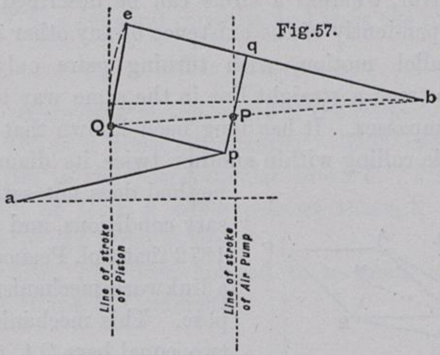


Fig.57.

the back link qp which will describe very nearly a straight line. This point is such that

$$\frac{Pp}{Pq} = \frac{bq}{ap}.$$

Now, if the proportions of the links are such that bPQ is a straight line, bQ/bP will be constant, and therefore the path described by Q will be an enlarged copy of the path described by P. That is to say if P moves approximately in a straight line, then Q will do so also. If then the radius rod is of suitable length we provide a point Q for the attachment of the piston rod, and also a point P for the attachment of the air-pump rod. To find this length we have

$$\frac{bq}{pQ} = \frac{qP}{pP},$$

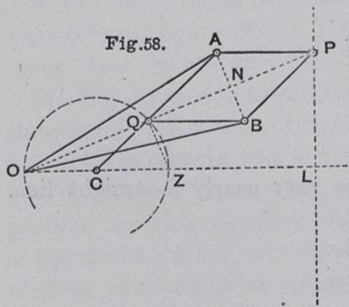
whence multiplying by the preceding equation

$$bq^2 = pQ \times ap,$$

$$\text{or Length of radius rod} = \frac{(bq)^2}{eq}.$$

The parallel motion just described which was introduced by Watt is the only one much used in practice, but there is another form which possesses great theoretical interest because it is exact and yet involves only turning pairs. Scott Russell's parallel motion (Fig. 50, page 114) is exact, but as it involves a sliding pair its accuracy depends on the exactness with which the guides of the slides are constructed. Now, a straight edge or a plane surface can only be constructed by a process of copying from some given plane surface or by trial and error, whereas a circle can be described by a pair of compasses independently of the existence of any other circle. Hence an exact parallel motion, with turning pairs only, enables us theoretically to trace a straight line in the same way that a circle is traced with compasses. It has long been known that this could be done by a circle rolling within another twice its diameter, but this method does not satisfy the necessary conditions, and it was not till 1872 that Col. Peaucellier invented a linkwork mechanism for the purpose. This mechanism consists of two equal bars OA, OB, jointed to each other at O, and at A, B to a parallelogram of equal bars $AOBQ$, so that OQP are in a straight line (Fig. 58). This being so then, however the bars are placed there will always be some fixed relation existing between OQ and OP. Thus drop a perpendicular AN on OP, then $OQ = ON - QN$ and $OP = ON + NP$. Also since $AQ = AP$, $QN = NP$,

Fig. 58.



$$\therefore OQ \, . \, OP = ON^2 - QN^2.$$

But $ON^2 = OA^2 - AN^2$ and $QN^2 = QA^2 - AN^2$, therefore $OQ \, . \, OP = OA^2 - QA^2$, and is a constant quantity for all positions; that is to say, if we cause Q to move over any curve, then P will describe its reciprocal.

We can now show how this mechanism may be employed to draw a straight line. Let O be a fixed centre and PL be the straight line which it is required to describe. Draw the perpendicular OL on PL. Then the mechanism being placed in any position with P at any point on the line to be drawn, draw QZ at right angles to OQ. Bisect OZ in C and attach Q to C by means of a jointed rod which can turn on the fixed centre C. The circle which Q describes during the motion of the bars will have OZ as a diameter, for OQZ is a right angle, and therefore the angle in a semicircle. We observe now that we have similar triangles OQZ and OLP.

$$\therefore OL = \frac{OP.OQ}{OZ};$$

but $OZ = 2.OC$ is a constant quantity and so is the product OP, OQ.

$$\therefore OL \text{ is constant.}$$

That is to say, wherever P is, the length of the projection of OP on the perpendicular OL is a constant quantity. This can be true only so long as P lies in the perpendicular line PL. Thus, by the constrained motion of Q in a circle passing through O, P is caused to move perfectly in a straight line.

This mechanism has been applied to a small engine used for ventilating the House of Commons.

56. *Closure of Kinematic Chains. Dead Points in Linkwork.*—A kinematic chain, like a pair (p. 103), may be "incomplete," that is, the relative movements of its links may not be completely defined. It then cannot be used as a mechanism without employing some additional constraint, a process called "closing" the chain. In order that a chain may be closed it must be endless, and the number of links must not be too great; for example, in a simple chain of turning pairs with parallel axes we cannot have more than 4 links. If there be 5 the motion of any one link relatively to the rest will not be definite, but may be varied at pleasure.

So also a chain may be "locked" either by locking one of the pairs of which it is constructed; or by rigidly connecting two links not forming a pair; it then becomes a frame, such as was considered in a previous part of this book.

A chain is often incomplete or locked for special positions of its links, though closed and free to move in all other positions; this, for example, is the case at the dead points which occur in most linkwork mechanisms. A well known instance is that of the mechanism of the steam engine, in which the chain is locked and the direction of motion of the crank indeterminate when the connecting rod and crank are in the same straight line. This instance further shows that it is necessary to distinguish between the two directions in which motion may be transmitted through the mechanism, for the dead points in question would not occur if the crank moved the piston instead of the piston the crank. A piece, then, which transmits motion is called a "driver," in relation to the piece to which motion is transmitted, which is called a "follower," terms which will be frequently used hereafter in cases where both pair with the fixed link. (Compare Arts. 47, 86.) The dead points in a mechanism may be passed either by the union of two similar mechanisms, with dead points in different positions, as in a steam engine with a pair of cranks at right angles, or by aid of the inertia of the moving parts. This last method involves what is called "force-closure," a term which will be explained presently.

EXAMPLES.

1. The stroke of an oscillating engine is 6 feet, and the distance between the centre line of the trunnions and the centre of shaft is 9 feet. Find the maximum and mean angular velocity of the cylinder in each of its two oscillations as compared with that of the crank. Find also the velocity of the piston at half stroke as compared with the speed of piston.

Ans. Maximum angular velocity-ratios of cylinder and crank, $\frac{1}{2}$ and $\frac{1}{4}$.
Mean " " ·277 and ·178.

$$\frac{\text{Velocity of piston at } \frac{1}{2} \text{ stroke}}{\text{Mean speed of piston}} = \frac{1{\cdot}54}{1}.$$

2. The travel of the tool of a shaping machine is to be 9 inches, and the maximum return, three times the maximum cutting, velocity. Neglecting the obliquity of the connecting link, find the proper proportions of the quick return motion (Fig. 5, Plate I.). Find also the revolutions per minute for a maximum cutting velocity of 6 inches per second, and compare the times of cutting and return.

Ans. The length of slotted lever = 9 inches.

Distance apart of centres = twice length of crank.
Revs. per minute of crank = 19·1.

$$\frac{\text{Time of cutting}}{\text{Time of return}} = \frac{2}{1}.$$

Remark.—The lever vibrates through 60°, an inconveniently large angle. Whitworth's mechanism would be preferable, as it readily gives an even greater inequality.

3. In Whitworth's quick return motion find the proportions that the maximum return may be three times the maximum cutting velocity, and compare the times of cutting and return.

4. In example 1, draw curves showing the angular velocity and position of the piston for any position of the crank.

5. In example 2, draw a diagram of velocities for any position of the mechanism, and hence trace curves showing the velocity of the tool at any point of the cutting and return stroke.

6. In question 1, p. 111, supposing two pairs of driving wheels coupled, the lengths of cranks 1 foot, find the velocity of the coupling-rod in any position. First, relatively to the locomotive; second, relatively to the earth.

7. In Ex. 5, p. 112, find in feet per second the maximum and minimum velocity of rubbing of the crank pin, assuming its diameter 12 in. Draw a curve showing this velocity in any position of the crank.

8. In Rapson's Slide (p. 121), if the tiller be put over through an angle θ, show that the velocity-ratio of tiller and slide varies as $\cos^2\theta$, and draw a curve of velocity.

9. In a drag-link-coupling the shafts are 6 in. apart, the drag-link 1 foot long, and the cranks each 3 feet long. By construction, determine the four positions of the following crank when the leading crank is on the line of centres, and at right angles to the line of centres.

10. The length of the beam of an engine is three times the stroke. Supposing the end of the beam when horizontal is vertically over the centre of the crank shaft at a height equal twice the stroke, and the crank also is then horizontal, find the length of connecting rod and the extreme angles through which the beam will sway. Adjust the crank centre so that the beam may sway through 20° above and below the horizontal.

Length of rod = 2·06 stroke. The beam sways $22\frac{1}{2}°$ above the horizontal, and 17° below.

11. The depth of the floats of a feathering paddle wheel is $\frac{1}{6}$th the diameter of the wheel, and the immersion of the upper edge in the lowest position $\frac{1}{4}$th the depth of the float. Assuming the stem levers $\frac{3}{5}$ths the depth of the floats, find the position of the centre of the collar to which the guide rods are attached. Determine the length of the rods, and draw the float in its highest position.

If O be centre of wheel, K centre of collar, OK = ·054 of diameter of wheel, and is horizontal (very approximately).

Length of guide rods = 1·01 radius of wheel.

12. In Ex. 9, find the angular velocity-ratio of the shafts when the cranks are in the positions mentioned. Find also the maximum and minimum angular velocity ratio.

13. In Oldham's coupling, show that the centre of the coupling disk revolves twice as fast as the shafts, and hence show how to give two strokes of a sliding piece for one revolution of a shaft.

14. In a simple parallel motion the lengths of the levers are 3 feet and 4 feet respectively, and the length of the connecting link is $2\frac{1}{2}$ feet. Find the point in the link which most nearly moves in a straight line, and trace the complete curve described by this point as the levers move into all possible positions, the motion being set so that, when the levers are horizontal, the link is vertical.

Ans. The required point in link is $17\frac{1}{7}$ in. from the 3-feet lever,
and $12\frac{6}{7}$ in. ,, 4-feet ,,

15. In a beam engine the stroke of piston is 8 feet, of air pump $4\frac{1}{2}$ feet, length of beam 24 feet, the front and back links of the parallel motion being 4 feet. Find the proper length of radius rod, and the point in the back link where the air-pump rod should be attached.

Ans. Length of radius rod = 8 feet $8\frac{1}{8}$ inches.
Point of attachment of air-pump rod = 3 ,, 3 ,, below beam.

16. Suppose in last question the parallel motion set for least deviation from a straight line, find the correct positions of the centre lines of air pump and piston, and the position of the centre of motion of the radius rod.

Ans. Horizontal distances from centre of beam—

Line of stroke of piston, -	11 feet	8 inches.
,, air pump,	6 ,,	$6\frac{3}{4}$,,
Centre of motion of radius rods,	15 ,,	$1\frac{1}{8}$,,

Reference.

A good collection of linkwork and other mechanisms, some of which do not occur in the larger works cited on page 100, will be found in the 4th edition (1880) of Professor Goodeve's *Elements of Mechanism.* Much valuable information on the details of machine design is contained in a treatise on Machine Design by Professor W. C. Unwin, M.I.C.E. (Longman.)