CHAPTER VI.

CONNECTION OF TWO LOWER PAIRS BY HIGHER PAIRING.

Section I.—Tension and Pressure Elements.

57. *Preliminary Remarks—Tension Elements.*—If one of the elements of a pair be not rigid, or if the contact be not of the simple kind considered in the preceding chapter, the pairing is said to be "higher," because the relative motion of the elements is more complex. Higher pairing is seldom employed alone; it is generally found in combination with lower pairs, the elements of which it serves to connect. The most important case is that where a chain of two lower pairs is completed by contact between their elements or by means of a link which is flexible or fluid. Motions may thus be produced in a simple way which are impossible or difficult to obtain by the use of lower pairing alone. The present chapter will be devoted to mechanisms derived from chains of this kind, the fixed link being generally a frame common to the two lower pairs. The velocities of each of the pairs are thus the same as those of their moving elements. We commence with the case of non-rigid elements.

A body which was incapable of resistance to any kind of change of form and size would of course be incapable of being used as part of a machine, for it could not furnish any constraining force whereby the motion of other pieces could be affected, but if it resists any particular kind of change it will supply a corresponding partial constraint which may be supplemented by other means. The first case we take is that of a flexible inextensible body, such as is furnished approximately by a rope, belt, or chain. This is called a Tension Element, being capable of resisting tension only, and it is plain that when any two points are connected by it, their distance apart, measured along the element itself, must be invariable so long as the rope remains tight. If the rope be straight, it may be replaced by a link, and we obtain the mechanisms already considered, but we now suppose it to pass over a surface of any form.

Fig. 59a.

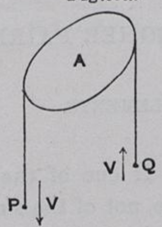


Fig. 59b.

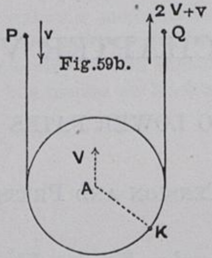
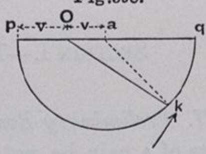


Fig. 59c.

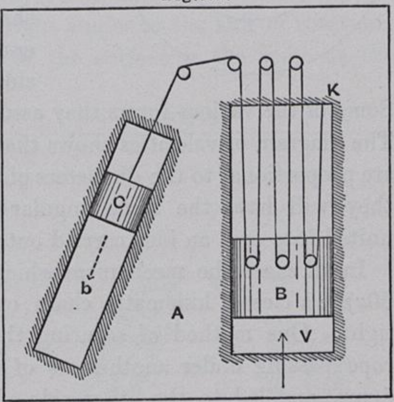


In Fig. 59*a*, let A be a fixed body of any shape, round which an inextensible rope PQ passes, the ends hanging down. If P moves downwards with velocity V, Q moves upwards with the same velocity, the rope slipping over A at all points with velocity V. In practice A is generally circular, and is mounted on an axis, upon which it revolves. We have then a "pulley block," of which A is the "pulley" or "sheave," and the rope causes it to rotate instead of slipping over it, but this makes no difference in the motion, and the only object of the arrangement is to diminish friction and wear.

Next suppose the pulley moveable (Fig. 59*b*), and imagine P attached to a fixed point, while Q moves upwards with the same velocity V relatively to A as before. Then A must move upwards with velocity V, because its motion relatively to the fixed point P is unaltered, and hence Q moves with velocity $2V$. More generally, if P, instead of being fixed, moves downwards with velocity v, Q must move upwards with a velocity $2V + v$, or to express the same thing otherwise — *the difference of velocities of the two sides of the rope is twice the velocity of lifting* — a principle applicable to all questions relating to pullies. The velocity of rotation of the pulley is $V + v$, its radius being the "radius of reference" (Art. 46). The motion of rope and pulley may be represented by a diagram of velocity. Thus, in Fig. 59*c*, describe a semi-circle with radius equal to $V+v$, then the radius of that circle represents the velocity of rotation or the velocity of any point in the rope relatively to the centre of the pulley. The actual velocity of any point K in the rope is found by compounding this with V, the velocity of the centre of the pulley. The pole of the diagram is therefore a point O, distant V from the centre of the circle, so that if k be the point in the diagram corresponding to the point K of the rope, Ok represents the velocity of K.

58. *Simple Pulley Chain—Blocks and Tackle.*—We have now a simple means of solving one of the most important problems in mechanism—namely, to connect two sliding pieces with a constant velocity ratio.

Fig.60a.



In Fig. 60*a*, B, C are pieces sliding in guides attached to a frame-piece A, thus forming two sliding pairs with one link common. In B a number of pins are fixed, and in A an equal number placed as in the figure, so that a rope passing round them as shown may form a number of plies parallel to B's motion.*

The rope is attached at one end to C, and led to the nearest fixed pin, over a guide pin placed so that this part of the rope may be parallel to C's motion, while the other end is attached to a fixed point K. The effect of this arrangement is that when C moves in the direction of the arrow, B also must move with a velocity which is readily found by the principle just explained, for the difference of velocities of the two parts of each ply must be the same, being twice the velocity of B. Thus reckoning from the fixed end, if B's velocity

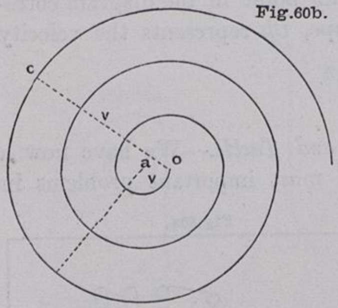
* This Figure is taken, with some modifications, from the 2nd edition (1870) of Willis's Mechanism.

be V, the velocities of the several parts of the rope must be

$$0, 2V, 2V, 4V, 4V, 6V, 6V\ldots\ldots,$$

so that if there are n pins in B, the velocity of the other end of the rope must be $2nV$, and the velocity ratio $2n:1$. The diagram of velocities consists of a number of semi-circles (Fig. 60b), the lower set struck with centre a and the upper with centre O, where O is the pole and Oa the velocity of lifting.

Fig. 60b.

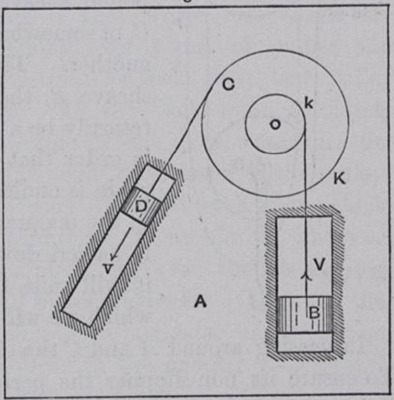


The simple kinematic chain here described may be inverted, by fixing B or C instead of A. In the blocks and tackle so common in practice, the pins are replaced by moveable sheaves, usually, but not always, of equal diameters, and placed side by side so as to rotate on the same axis. Some of the various forms they assume will be illustrated hereafter. The diagram of velocities shows that, if the diameters of the sheaves are proportional to the diameters of the circles shown in the diagram, they will have the same angular velocity, and may therefore be united into one, an idea carried out in White's Pulleys.

In all cases the mechanism which we have been considering (Fig. 60a) is a closed kinematic chain only so long as the rope remains tight. One method of securing this would be to supply a second rope passing under another set of pins below B (not shown in the figure) and led to the other side of C by a suitably placed guiding pulley; we should then, by tightening up the ropes, have a self-closed chain similar to those considered in the preceding chapter. In practice, however, forces are applied to B and C which produce tension in the rope; thus, for example, when employed for hoisting purposes the weight which is being lifted keeps the rope tight. This is the simplest example of what is called force closure, where a kinematic chain, which is not in all respects closed, is made so by external forces applied during the action of the mechanism. In practical applications the principle of force-closure is carried still further, for the guides which compel the pieces B and C to move in straight lines are usually omitted. In the case of B the weight and inertia of the load which is being raised or lowered supply sufficiently the necessary closure, while in the case of C the end of the rope may be guided by the hand.

59. *Wheel and Axle.*—When mechanical power is employed for hoisting purposes, the end of a rope is frequently wound round an axle, the rotation of which raises or lowers the weight, and this leads us at once to a different and equally important method of employing tension elements — namely, by attaching one end to a fixed point in the cylindrical surface of an element of a turning pair. The rope in this case passes over the surface and is guided by it, but does not slip over it as it does over the pins of the previous arrangement. The most useful case is that where the transverse section of the surface is a circle, and the direction of the rope always at right angles to the axis of rotation; then it is clear that the motion of the surface is the same as the motion of the rope.

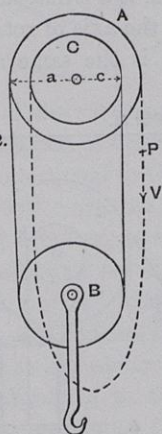
Fig.61.



The well known Wheel and Axle is a combination of two chains of this kind. In its complete ideal form it consists of two sliding pairs AB, AD, with planes parallel and one link A (Fig. 61) common. A rope is attached to D and, passing partly round a wheel, is attached to it at a fixed point K in its circumference; a second rope is attached to B, and passing partly round an axle, is attached to a fixed point k in its circumference, the two ropes lying in parallel planes. The wheel and axle are fixed together, and form with A the turning pair AC. We have thus a second means of connecting two sliding pieces so that their velocity-ratio may be uniform, for the velocities of B and D must be inversely as the radii of the wheel and the axle. As before, the ropes must be kept tight, also the guides of the pieces B and D may be omitted and replaced by force-closure, and this will be necessary if the wheel is to make more than one revolution, for then a lateral movement is required to enable the rope to coil itself on the surfaces.

In practical applications the second rope is generally omitted and the wheel turned by other means; the lateral movement is sometimes provided for by permitting the axle to move endways in its bearings, but more often, in cases where the load is not free to move laterally, the effect of a moderate inclination of the rope to the axis is disregarded. We may, however, escape this difficulty by the use of force-closure of a different kind. Instead of attaching the rope to a fixed point in the surface, let it be stretched over it by a force at each end, there will then be friction between the rope and the surface, which will be sufficient to prevent slipping if the tendency to slip be not too great.

Fig.62.



The Differential Pulley is a good example of the application of these principles. As is shown in Fig. 62, there are two blocks, of which the upper, which is fixed, carries a compound sheave, consisting of two pullies A and C, of somewhat different diameters, fixed to one another. The lower block carries a single sheave B, the diameter of which should theoretically be a mean between those of A and C, in order that the chain may be vertical. The chain is endless, and passes round the pullies in the manner shown, so that when the side P is hauled downwards with a given velocity V, it will raise the lower block B with a velocity which we will now determine.

In passing around A and C the chain is not capable of slipping. To ensure its non-slipping the periphery may be recessed to fit the links of the chain. In passing around B the slipping is immaterial; the raising of B would take place with the same velocity, whether there were an actual slipping of the chain round the circumference, or whether B were a rotating pulley.

When the point P is hauled downwards with velocity V, it necessitates the rotation of A, and with it of C. Thus the left hand portion of the chain passing round B will be hauled upwards with the same velocity as the point P downwards, and the right hand will descend with a velocity which is less in the ratio of the radii, c, a, of the united pullies, and thus on the whole there will be an ascending motion given to B. Now, since the upward velocity of B is $\frac{1}{2}$ the difference between the velocities of the two portions of the chain,

$$v = \tfrac{1}{2}\left(V - \frac{c}{a}V\right) = \frac{V}{2}\left(1 - \frac{c}{a}\right) = \frac{a-c}{2a}V.$$

Thus, by making the difference between a and c small, the relative velocity of B to P may be made as small as we please.

This apparatus, in a somewhat modified form, is much employed. It is called Weston's Differential Pulley Block, and possesses the valuable property that the weight will not descend when the hauling force is removed, for reasons which will be explained hereafter (Ch. X.).

60. *Pulley Chains with Friction-Closure. Belts.*—A tension element may also be employed to connect the elements of two turning pairs. The most important case is that where two shafts are connected by an endless belt passing over a pair of pullies and stretched so tightly that the friction between belt and pulley is sufficient to prevent slipping. If the belt were absolutely inextensible the speed of centre line of the belt would be the same at all points, and therefore the angular velocities of the pulleys would be inversely as their radii each increased by half the thickness of the belt. This mode of connection is unsuitable where an exact angular velocity-ratio is required, for even though the belt may not slip as a whole, yet it will be seen hereafter (Chap. X.) that its extensibility causes a virtual slipping to a greater or less extent. In the case of leather belts, the error in the angular velocity-ratio due to this cause is said to be about 2 per cent.

There are two ways in which the belt may be wrapped around the pullies, being either *crossed or open.* If the belt is crossed, the pullies will run in opposite directions of rotation. The crossed belt embraces a larger portion of the circumference of the pullies than the open belt, and there is thus less liability to slip.

There is a proposition of some importance connected with the length of a crossed belt, which it will be useful to give here.

AC and BD (Fig. 63) being radii, each drawn at right angles to the straight portion of the belt CD, will each make the same angle θ with the line of centres. Thus

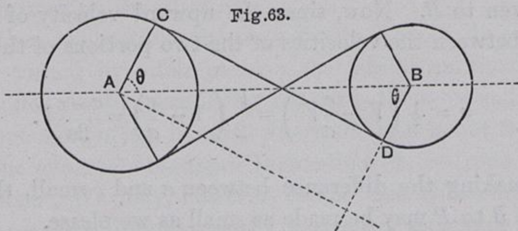


Fig. 63.

the portion of the belt in contact with the pulley $A = (2\pi - 2\theta)\, r_A$ and that in contact with the pulley $B = (2\pi - 2\theta) r_B$.

The length not in contact $= 2\,.\, CD = 2(r_A + r_B) \tan\theta$.

Thus whole length of belt $= 2(\pi - \theta + \tan\theta)(r_A + r_B)$.

Now $\cos\theta = \dfrac{r_A + r_B}{AC}$.

Thus, if the distance AB between the centres is a constant quantity, and if, further, the sum of the radii $r_A + r_B$ is constant, then the angle θ will be constant. That being so, the total length of the belt will be a constant quantity.

This property is made use of when it is desired to connect two parallel shafts with an angular velocity-ratio, which may be altered at pleasure. A set of stepped pullies, such as are shown in Fig. 1, Plate III., are keyed to each shaft, and the belt being shifted from one pair to another of the pullies, the angular velocity-ratio is altered at will. If the belt is crossed, then the same belt will be tight on any pair of pullies, if the sum of the radii is the same for each pair. This does not hold good for open belts. The actual length of belt required in any given example is best found by construction.

The tightness of the belt necessary to effect closure by friction of this kinematic chain may be produced simply by stretching the belt over the pullies so as to call into play its elasticity, but the axis of rotation of one pulley is sometimes made moveable, so that the belt may be tightened by increasing the distance apart of the shafts, while in other cases an additional straining pulley is provided. The belt may then be tightened and slackened at pleasure, a method frequently used in starting and stopping machines.

In order that the belt may remain on the pulleys they must be provided with flanges, or, as is more common in practice, they must be slightly swelled in the middle, for when the shafts are properly in line, a belt always tends to shift towards the greater diameter. Great care, however, is necessary in lining the shafts that each side of the belt lies exactly in the plane of the pulley on to which it is advancing. Thus, for example, if the shafts be in the same plane, they must be exactly parallel, otherwise the belt will shift towards the point of intersection. This remark, however, does not apply to the receding side of the belt, and the shafts may make a considerable angle with each other, subject to the above restriction.

Friction-closure is always imperfect, because the magnitude of the friction is limited, but this is often a great advantage, since it permits the chain to open when the machine encounters some unusual resistance, which would otherwise produce fracture. By the use of grooved pullies provided with clips the friction may be increased to any extent, so that great forces may be transmitted, but these devices are only suitable for low speeds, as in steam-ploughing machinery. Slipping may be avoided altogether by the employment of gearing chains, the links of which fit on to projections on the pullies; force-closure is here replaced by chain-closure, and the action is in other respects analogous to toothed gearing. The speed is limited, as will be seen hereafter.

61. *Shifting of Belts. Fusee Chain.* — By the use of drums of considerable length as pullies, the belt may be shifted laterally at pleasure. This principle is much employed in practice, as for example—

(1.) To stop and set in motion a machine.—The drum on one of the shafts is divided into two pullies, one fast and the other loose on the shaft.

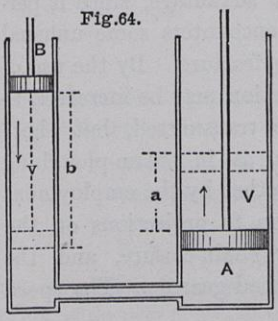
(2.) To reverse the direction of motion.—The drum is divided into three pullies, the centre one fast, the two end ones loose on the shaft. Two belts, one crossed and the other open, are placed side by side. By shifting the belt either is made to work on the fast pulley at pleasure.

(3.) To produce a varying angular velocity-ratio.—The drums are made conical instead of cylindrical. The fusee employed in watches to equalize the force of the main spring is a common example.

The kinematic character of these devices will be considered in the next chapter.

62. *Simple Hydraulic Chain. Employment of Springs.* — Incompressible fluids may be employed to connect together two or more rigid pieces forming a class of elements which may be called "pressure-elements," since they are capable of resisting pressure only. The pressure must be applied in all directions, and the fluid must therefore be enclosed in a chamber which pairs with the different pieces to be connected. For constructive reasons lower pairing must generally be adopted, and almost all cases are included in the following investigation.

Fig. 64.



Suppose two cylinders, each fitted with a piston (A and B in Fig. 64), to be connected by a pipe, the space intervening between the pistons being filled with fluid. Then when the piston B moves downwards with velocity v, the piston A will rise with velocity V, which is easily found by considering the spaces traversed by the two pistons in a given time. Let A, B be the areas of the pistons, a, b the spaces traversed, then, since the volume of the fluid remains the same, we must have $Aa = Bb$, and therefore

$$\frac{V}{v} = \frac{a}{b} = \frac{B}{A}.$$

The chain here considered, in which the elements of two sliding pairs are connected by a fluid, is kinematically identical with the arrangement of Fig. 60, p. 137, the replacement of a tension-element by a pressure-element constituting merely a constructive difference between the mechanisms. In the hydraulic press, in pumps, in water-pressure engines driven from an accumulator, and in other cases this kinematic chain is of constant occurrence, and will be frequently referred to hereafter. Combinations of an hydraulic chain with blocks and tackle are common in hydraulic machinery. (See Part V.)

Springs, compressible fluids, and even living agents, are employed in mechanism, not only in a manner to be explained hereafter as a source of energy, by means of which the machine does work, but also in force-closure, and especially for the purpose of supplying the force necessary to shift pieces which open and close, or lock and unlock kinematic chains, and so produce changes in the laws of motion of the mechanism. The force of gravity, which, as has already been shown, frequently produces closure, should be regarded as the tension of a link of indefinite length connecting the frame-link of the mechanism with the link we are considering. The inertia of moving parts likewise gives rise to forces which are not unfrequently applied to similar purposes. Examples will be given in a later section.

EXAMPLES.

1. A shaft making 90 revolutions per minute carries a driving pulley 3 feet in diameter, communicating motion by means of a belt to a parallel shaft, 6 feet off, carrying a pulley 13 inches diameter. Find the speed of belt and its length—1st, when crossed, and 2nd, when open. Find also the revolutions of the driven shaft, allowing a slip of two per cent.

Speed of belt	=	847·8 feet per minute.
Length when crossed	=	19 feet 2 inches.
,, open	=	18 ,, 8 ,,
Revolutions of the follower	=	$244\frac{1}{4}$

2. Construct a pair of speed pullies to give two extreme velocity-ratios of 7 to 1 and 3 to 1, and two intermediate values. The belt is to be crossed and the least admissible diameter is 5 inches.

$$\text{Velocity-ratios} \quad - \quad - \quad \frac{21}{3} \quad \frac{17}{3} \quad \frac{13}{3} \quad \frac{9}{3},$$

$$\text{Diameter of pullies} \quad \begin{cases} 5 & 6 & 7\frac{1}{2} & 10 \\ 35 & 34 & 32\frac{1}{2} & 30. \end{cases}$$

3. The diameters of the compound sheave of a differential pulley block are 8 inches and 7 inches respectively; compare the velocities of hauling and lifting.

Velocity-ratio = 16 to 1.

4. In a pair of ordinary three-sheaved blocks compare the velocity of each part of the rope with the velocity of lifting.

5. In a hydraulic press the diameter of the pump plunger is 2 inches and that of the ram 12 inches, determine the velocity-ratio.

SECTION II.—WHEELS IN GENERAL.

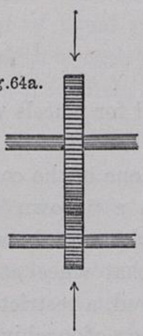
63. *Higher Pairing of Rigid Elements.*—We next consider pairs of rigid elements in which the relative motion is not consistent with continuous contact over an area. The elements then touch each other at a point or along a line which is not fixed in either surface, but continually shifts its position. The form of the surfaces is not then limited as in lower pairing, but may be infinitely varied, with a corresponding variety in the motion produced.

This kind of pairing occurs when a chain of two lower pairs is completed by simple contact between their elements. In the double slider-crank chain shown in Fig. 4, Plate II., of the last chapter, let us omit the block C and enlarge the crank pin so as just to fill the slot. By so doing the relative motions of the remaining parts will be unaltered, but we shall have three pairs instead of four, the turning pair BC and sliding pair CD being replaced by a single higher pair BD. This process is called *Reduction* of the chain, and when higher pairing is admissible the reduced chain serves the same purpose as the original, but with fewer pieces. The crank pin and slot are in contact along a line only which during the motion continually shifts its position. In practice, the elements not being perfectly rigid, the contact extends over an area, but this area is of very small breadth, and consequently if heavy pressures are to be transmitted at high velocities the wear is excessive. If we trace the development of pieces of mechanism we observe that in the earlier stages higher pairing is much employed for the sake of simplicity of construction, but is gradually replaced by lower pairing. Nevertheless, where the object of the machine is mainly to transmit and convert motion rather than to do work, or where the velocity of rubbing is low, higher pairing may be employed. In many cases it is necessary, because the required motion cannot be produced by any simple combination of lower pairs.

Higher pairing of rigid elements may be divided into two classes according as the surfaces in contact do or do not slip over one another, just as in the case of tension elements considered in the last section. In the first case the contact is spoken of as Sliding Contact and in the second as Rolling Contact. In rolling contact the difficulty of wear does not occur, and hence it is always used when possible. The relative motion of the two elements is determined by considering that as the surfaces do not slip, the space moved through by the line of contact along each surface must be the same. Or as we may otherwise express it, if A, B be a pair of elements in rolling contact, the velocity of the surface of A must be the same as that of the surface of B.

64. *Rolling Contact.*—Rolling contact may be employed for the communication of motion between two shafts, the centre lines of which are either parallel or intersect, by means of surfaces rigidly attached to the shafts. In the first case the surfaces are cylindrical and in the second conical, the apex of the cone being the intersection of the shafts. By far the most important case, and the only one we shall here consider, is that in which the transverse sections of the surfaces are circular. Portions of the surfaces are used, as in Figs. 64*a*, 64*b*,

Fig.64a.



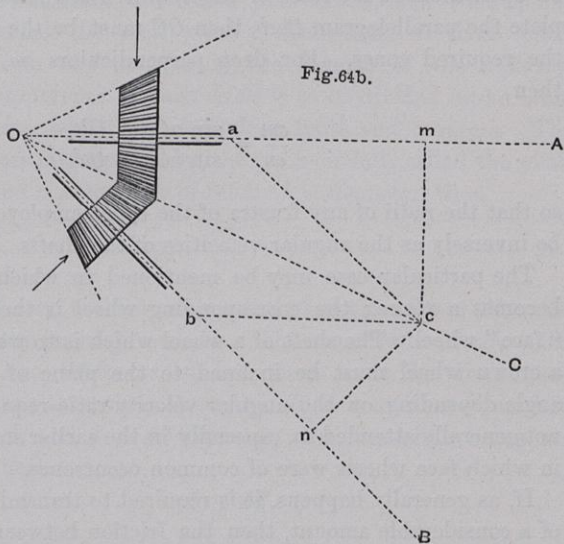


Fig.64b.

and are pressed together by external forces, so that sufficient friction is produced to prevent the slipping of the surfaces. In other words, force-closure is necessary, as in the case of connection by a belt. This being supposed, it will immediately follow that the velocity of the two surfaces at the points of contact is the same, and hence, as before, the angular velocity-ratio of the shafts is inversely proportional to the radii of the wheels. In the case of intersecting shafts, the surfaces are frustra of cones called "bevel," or, if the semi-angle of the cone be 45°, "mitre wheels," and their radii may be reckoned as the mean of that at the inner and outer periphery. The shafts revolve in opposite directions, unless one of the surfaces be hollow so that the other may be inside it, in which case the corresponding wheel is said to be "annular." When it is inconvenient to use an annular wheel, the same result may be obtained by transmitting the motion through an intermediate or "idle" wheel. If the radius of a wheel be infinite, it becomes a "rack," and the surface a plane.

In the case of bevel wheels the corresponding cones may be found, when the centre lines of the shafts and the angular velocity-ratio are given, by a simple construction. In Fig. 64*b*, let OA, OB be the centre lines of the shafts, and let distances Oa, Ob be marked off upon them in the ratio of the required angular velocities. Complete the parallelogram $Oacb$, then OC must be the line of contact of the required cones. For drop perpendiculars cm, cn, on OA, OB, then

$$\frac{cm}{cn} = \frac{\sin aOc}{\sin bOc} = \frac{Ob}{Oa},$$

so that the radii of any frustra of the cones employed for wheels will be inversely as the angular velocities of the shafts.

The particular case may be mentioned in which one of the cones becomes a plane; the corresponding wheel is then a "crown" or "face" wheel. The shaft of a wheel which is to work correctly with a crown wheel must be inclined to the plane of that wheel at an angle depending on the angular velocity-ratio required, a restriction not generally attended to, especially in the earlier stages of machinery in which face wheels were of common occurrence.

If, as generally happens, it is required to transmit a working force of a considerable amount, then the friction between the two circumferences will be found not to be sufficient to prevent slipping taking place, unless a considerable pressure to force the shafts together is employed, which involves an excessive friction on the bearings. In what is known as "frictional gearing," this is partially avoided by the use of wheels with triangular grooves fitting each other as the thread of a screw fits into its nut; but, in general, to prevent slipping, teeth are cut on the two peripheries, and the motion is transmitted by the gearing together of the teeth. Since this is a substitution for the rolling contact of two surfaces, it is required to so design the number and form of the teeth that the wheels on which they are cut shall turn one another with the same constant angular velocity-ratio as that due to the two original surfaces. If recesses are cut in each wheel, and projections be added between the recesses so as to fit into the corresponding recesses of the other wheel, then the two wheels may be placed to gear together at such a distance that the two original surfaces would have been in contact and would have rolled together. In the case of a pair of toothed wheels, such a pair of imaginary surfaces which will roll together with the same angular velocity-ratio as that obtained from the toothed wheels, are called *pitch surfaces.* Considering first the case of parallel shafts, the transverse sections of these surfaces are called *pitch circles,* and their point of contact is called the *pitch point.* The radii of these pitch circles must be to one another in the inverse of the velocity-ratio. The circumference of each circle is to be divided into a number of equal parts, which will include a tooth and a recess. The length of each part measured along the pitch circle is called the *pitch.* Let p = pitch, and n = number of teeth, d = diameter, then

$$p = \frac{\pi d}{n}.$$

The thickness of each tooth is made a little less than $\frac{1}{2}p$ to allow the clearance necessary for easy working. The magnitude of the pitch which governs the thickness of the teeth must be determined from considerations as to their strength. If n' = number of teeth in the second wheel, and d' = its diameter, then the pitch being the same for each wheel

$$p = \frac{\pi d}{n} = \frac{\pi d'}{n'}.$$

The distance apart of the shafts is generally adjusted to allow the pitch to be some exact number of inches, half, or quarter inches. The pitch is to be measured along the pitch circle, and is not the chord of the arc, as is sometimes stated.

In some small wheels used for spinning machinery, another kind of *pitch* is referred to. The diameter of the pitch circle is divided by the number of teeth, and the result is called the *diametral pitch.* In the smallest class of wheelwork used in clocks, the dimensions of the teeth are stated as so many to the inch. The proper form of teeth will be considered farther on.

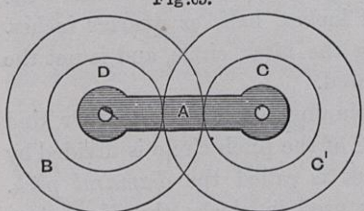
65. *Augmentation of a Kinematic Chain. Trains of Wheels.*—Another important application of rolling contact is to diminish friction by the intervention of rollers, hence called Friction Rollers. Thus

the friction between the elements of a sliding pair, subject to heavy pressure, will be so great as to require a great force to overcome it, but if rollers be placed between the elements the friction is greatly reduced, as will be seen hereafter. In this case sliding friction is wholly replaced by rolling friction; in carriage wheels the sliding velocity which, without the wheel, would be the actual velocity of the carriage, is reduced to that at the periphery of the axle, that is to say, in the ratio of the diameters of the axle and the wheel. The sheaves of an ordinary pulley block are examples of the same principle. In all these cases where additional pieces are added to a kinematic chain, in order to reduce friction or to serve some other non-kinematical purpose, the chain is said to be "augmented."

Chains are frequently augmented for purely constructive reasons; thus, if the velocity-ratio of a pair of shafts is great, the diameters of a single pair of wheels necessary in order to obtain it will be inconveniently large or small. A train of wheels is then resorted to. This is also the case where the shafts to be connected are too near or too far apart; in the latter case bevel wheels and an intermediate transverse shaft may be employed.

When, however, the shafts to be connected are in the same straight line, a train of wheels is kinematically necessary, and forms virtually a new mechanism. This is a common case in practice when a pulley or wheel is loose on a shaft, and it is required to connect the wheel and the shaft so as to revolve with different velocities. Such a train is shown in Fig. 65 in a simple ideal form. B and D are two wheels turning on the same centre but disconnected. C, C' are two wheels gearing with B and D and turning about another centre but united. The two centres are connected by the frame-link A. When B revolves it drives C, and C' drives D. If the numbers of teeth in these wheels be denoted by the letters which distinguish them, and the velocity of B be unity, the velocity of C or C' will be B/C, and that of D will be BC'/DC. Let it now be observed that the wheels B and D form a pair, the velocity of which will be the difference between the velocities of these wheels.

Fig. 65.



We have then altogether four turning pairs in this train of wheels, the relative velocities of which are—

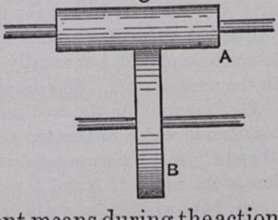
Pair, -	BA	CA	DA	DB
Velocity,	Unity	$-\frac{B}{C}$	$\frac{BC'}{CD}$	$1-\frac{BC'}{CD}$

One of the wheels in this train may be annular, and all may be bevel; in either case the wheels C, C' may be equal, and the train reduced to three wheels, though the number of simple pairs remains as before four. Examples are given in the figures of Plate III.

Either this or any other train of wheels may be inverted by fixing one of the wheels instead of the frame-link, the resulting mechanism is then called an Epicyclic Train; the velocity-ratios of the various pairs are unaltered, and are therefore shown by a table similar to that given above. Should the angular velocity of any wheel be required relatively to the fixed wheel, we have only to add to the velocity of the corresponding pair the velocity of the frame-link. Some examples of epicyclic trains are shown in the figures, but for detailed descriptions we must refer to a work on mechanism. Their use in compound chains will be further referred to in the next chapter.

66. *Wheel Chains involving Screw Pairs.*—In a simple wheel chain (Fig. 66) consisting of a wheel B, a pinion C, and a frame-link A, not shown on the figure, suppose C to be of considerable length, then there will be nothing to prevent the endways movement of B in its bearings if they be supposed cylindrical. This circumstance is often taken advantage of in machinery in shifting wheels in and out of gear, but the case to be examined here is that in which

Fig. 66.



the endways movement is given by independent means during the action of the mechanism. The simplest example is a three-link chain derived from the train of wheels just considered by changing the turning pair

BA into a screw pair; B then travels endways through the pitch of the screw in each revolution. The pinion C sometimes slides on the shaft which carries it, but quite as often it is made long enough to permit the necessary traverse of B. A well known example of this mechanism is that of the feed motion common in drilling and boring machines, in which the train of wheels of the last article is used with B and D nearly equal, so that the velocity of the pair BD is very small. B is attached to the nut and D to the screw, so that BD is a screw pair. D then traverses through B by a space each revolution which may be made very small.

To illustrate and explain preceding articles Plate III. has been drawn, giving examples of trains of wheels, especially of the differential trains of Fig. 65.

Fig. 1 shows the slow motion of a lathe. D is a wheel keyed on the mandrel and connected with B, the driving pulley, when the motion is not in use. B rides loose on the mandrel, and by means of a pinion gears with C, a wheel on the same shaft with C', which gears with D. CD being large compared with BC', the speed of the mandrel is much less than that of the pulley. For lighter work CC' are thrown out of gear by an endways movement of the shaft.

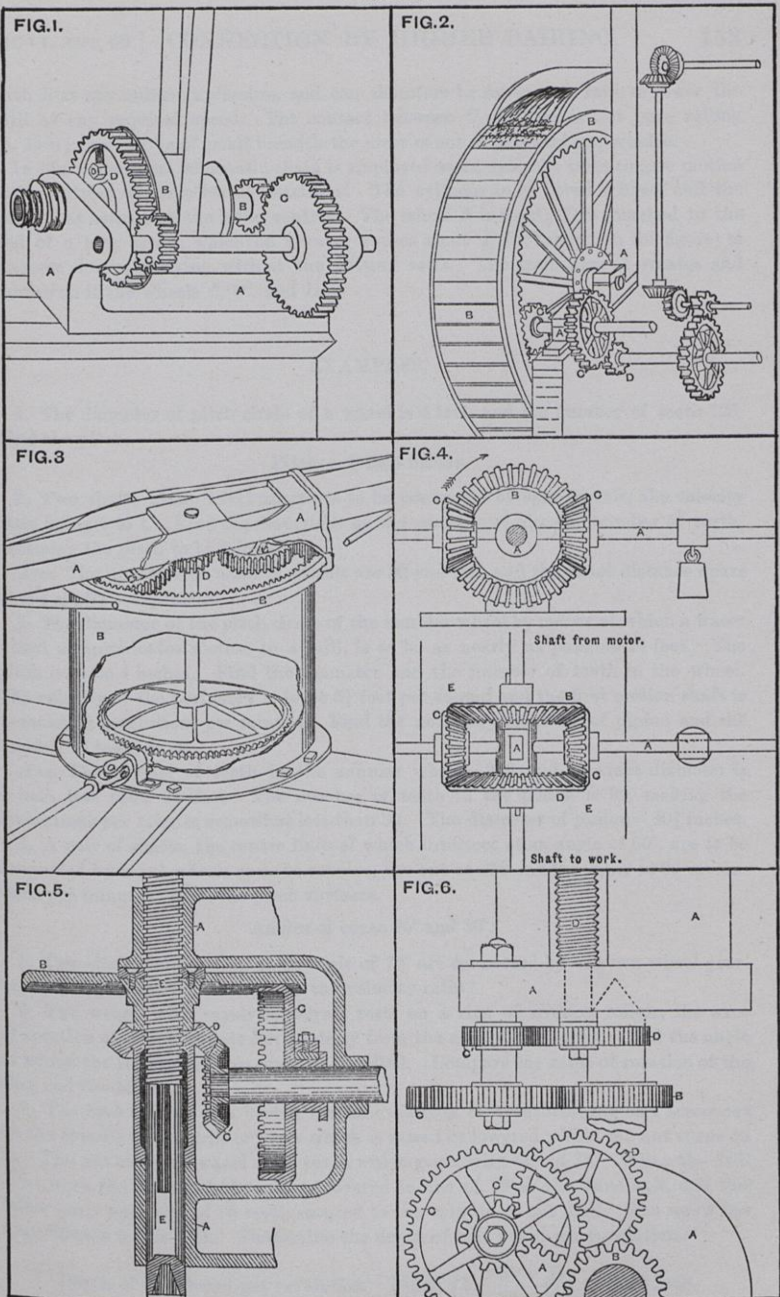
Fig. 2 represents the train of wheels by which the slow movement of a water-wheel is multiplied and transmitted to all parts of a factory. B is now an annular wheel attached to the water-wheel gearing with C, C' with D, and so on. A vertical shaft F with bevel wheels transmits the motion to the upper floors. The bearings of the secondary shafting are omitted for clearness, but they all form part of a frame-link A, which is fixed.

In Fig. 3 the kinematic chain is inverted. B is a fixed annular wheel, CC' are of equal diameter and reduce to one wheel, which, however, is in duplicate, in order to balance the driving forces. This epicyclic train is applied to many purposes. In the example shown the frame-link is a long arm, at the end of which a horse is attached, and a rapid motion thus given to the central pinion D. The motion is further multiplied by the bevel gear shown below, and applied to drive a thrashing machine or some similar purpose. The same mechanism is employed as a purchase in capstans and tricycles.

In Fig. 4 the train consists of three bevel wheels, BCD, C and C' reducing to one, as in the preceding case. The simple chain consists of these wheels and the train arm A. When A is fixed the wheels B and D turn in opposite directions with equal velocities; when B is fixed A revolves with half the velocity of D. The mechanism is much employed, but usually as a compound chain, and as such will be considered in the next chapter. The example shown is a dynamometer.

Fig. 5 represents the feed motion of a drilling machine. A is the frame of the machine in which rotates the vertical drill spindle E driven by a pair of mitre wheels D and C' from a horizontal shaft. A screw thread is cut on the spindle, of which B forms the nut. If B and D rotate at the same speed the drill moves neither up nor down, but any difference will result in a motion of the screw pair BE, and will thus give the necessary feed or raise the drill out of the hole. In the example chosen B is driven by a flat disc gearing by friction with a wheel C' turning with D (Naish's patent). This wheel, by means of a lever, can be moved along the shaft so as to gear

Plate.III.



To face page 152.

with B at any radius at pleasure, and can therefore be set so as to raise or lower the drill at any required speed. The contact between C, D here is not pure rolling (p. 148); but as C is of small breadth the error is not of practical importance.

In Fig. 6 the same kinematic chain is employed as an epicyclic train to give motion to the cutters of large boring machines. The cylinder to be bored is fixed, and the boring bar rotates on the lathe centres. The wheel B is fixed; D is attached to the end of a long screw, which on turning causes a nut E (not shown in the figure) to traverse slowly, carrying with it the cutting tools. The train arm A rotates and carries on it the wheels C, C', and D.

EXAMPLES.

1. The diameter of pitch circle of a wheel is 4 feet, and the number of teeth 120. Find the pitch.

Pitch = 1·2566 inches.

2. Two shafts about 4 feet apart are to be connected by spur wheels, the velocity ratio being 4 to 1. Find the diameters of the wheels and also the number of teeth, assuming the pitch to be 2 inches.

Ans. The numbers of teeth in wheels are 30 and 120, and the exact distance apart of the shafts = $47\frac{3}{4}$ inches.

3. The diameter of the pitch circle of the annular wheel by means of which a water wheel communicates motion to a mill, is to be as nearly as possible 24 feet. The pitch is to be 4 inches. Find the diameter and the number of teeth in the wheel. The velocity of the periphery is to be $5\frac{1}{4}$ feet per second and the first motion shaft is to make 30 revolutions per minute. Find the necessary diameter of pinion and the number of teeth in it.

Ans. The number of teeth in the annular wheel = 226, and its exact diameter is $\frac{1}{4}$ inch less than 24 feet. The number of teeth in the pinion is 32, making the revolutions per minute somewhat less than 30. The diameter of pinion = $40\frac{3}{4}$ inches.

4. A pair of shafts, the centre lines of which intersect at an angle of 60°, are to be connected by bevel wheels so as to revolve, the one at 250 and the other at 90 revolutions per minute. Find the pitch surfaces.

Angles of cones 90° and 30°.

5. Two shafts intersecting at an angle of 75° are connected by a crown wheel gearing with a pinion. What must be the velocity-ratio?

6. The weight of a revolving turret rests on a ring of friction rollers, the axes of rotation of which radiate horizontally from the axis of the turret: find the angle at which the rolling surfaces must be bevelled. Compare the rates of rotation of the ring and the turret.

7. The feed motion of a boring machine consists of a nut working on a screw cut on the spindle of the drill or borer which is raised or lowered whilst the nut turns on it. The nut carries a wheel of 96 teeth which gears with one of 35. When the drill is at work the wheel of 35 teeth is secured to one of 36 on the same axis, and this latter gears with one of 95 teeth secured to the spindle of the drill. The screw has four threads to the inch. Determine the depth of hole bored per revolution.

$$\text{Depth of hole bored per revolution} = \tfrac{1}{4}\text{ inch}\left(1 - \frac{35 \times 95}{36 \times 96}\right) = 0095 \text{ inch.}$$

8. The train of wheels in the preceding question is used as an epicyclic train by fixing the wheel of 96 teeth. Find the direction and number of revolutions of the train arm for each revolution of the spindle.

Ans. For each revolution of 95 wheel forwards, the arm turns backwards through

$$\frac{95 \times 35}{96 \times 36 - 95 \times 35} = 25{\cdot}4 \text{ revolutions.}$$

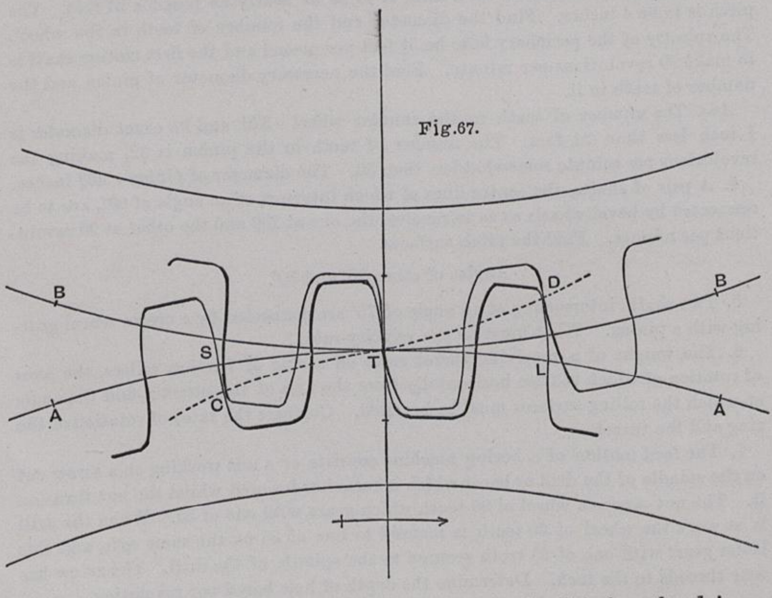
Section III.—Teeth of Wheels.

67. *Preliminary Explanations.*—Even though the number of teeth in a pair of wheels be such as to give the correct mean angular velocity-ratio due to the rolling together of the pitch circles, yet if they be of improper form they will jam or work roughly.

Theoretically the form of the teeth of one of a pair of wheels may be chosen at pleasure if a proper corresponding form be given to the teeth of the other; the problem of rightly determining the form is therefore one which admits of many solutions. We commence with some general explanations applicable to all forms of teeth.

The diagram (Fig. 67) shows a section of a pair of spur wheels in

Fig. 67.



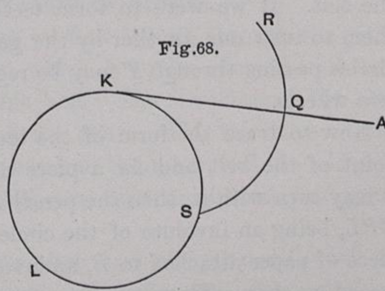
gear, with three teeth in action, the lower wheel being the driver. *BTB*, *ATA* are the pitch circles in contact at the pitch point *T*.

$ST = TL$ is the pitch, being the distance of a point in one tooth from the corresponding point in the next consecutive measured along the pitch circle. The teeth as shown in the figure partly project beyond the pitch circle and fit into corresponding recesses in the other wheel, so that each tooth is divided into two parts, a part within and a part without the pitch circle. The corresponding acting surfaces are called the Flank and the Face of the tooth respectively. In annular wheels the flank is outside and the face inside the pitch circle. The teeth commence action before reaching the line of centres by the flank of a tooth of the driver A coming into contact with the face of a tooth of the follower B, as shown at C in the diagram, and gradually approach that line till after the wheels have turned through a certain arc, which measured on the pitch circle is called the Arc of Approach; they are then in contact at T the pitch point. After passing the line of centres they remain in contact till the wheels have turned through a second arc called the Arc of Recess and then cease contact as shown at D, the face of a tooth of the driver being always in contact with the flank of a tooth of the follower. The sum of these arcs is called the Arc of Action, and must be great enough to permit at least two teeth to be in contact at once. Their magnitudes depend on the projection of the teeth beyond the pitch circle, a quantity which is called the Addendum of the corresponding wheel, the arc of approach depending on the addendum of the follower, and the arc of recess on the addendum of the driver.

68. *Involute Teeth.*—The question of the form of the teeth requires much explanation to render it completely intelligible; we shall only give a brief sketch, referring for full details to the works cited on page 100. Some points will be further considered at a later period. We commence with what are known as Involute Teeth.

Imagine a string AKL wound on a cylinder (Fig. 68). If the string be gradually unwound, the string being kept tight all the time, a point Q of the string will trace out a curve SQR called the *Involute*

Fig. 68.



of the Circle. Instead of causing the string to be unwound around the fixed circle we may if we please move A in a fixed straight line and cause the unwinding to take place by the revolution of the circle. If now a piece of paper be fixed to and revolve with the circle, the same involute curve will be traced on it as before.

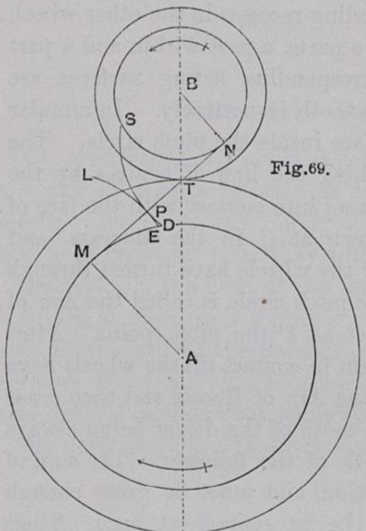


Fig. 69.

Now let A and B (Fig. 69) be two circles not in contact which are each capable of revolution about its centre. If we connect them by a crossed belt, of which one half is shown in the diagram by the line MTN, each will be capable of driving the other with a constant angular velocity ratio, namely, the inverse ratio of the radii. If, therefore, T be the point where the belt crosses the line of centres,

$$\frac{A_A}{A_B} = \frac{r_B}{r_A} = \frac{BT}{AT}.$$

Now, with centres A and B and radii AT and BT, describe circles which touch one another. These two circles would turn one another by rolling contact with the same angular velocity-ratio as that due to the belt. If we were to form teeth on the two wheels and cause them to turn one another by the gearing of the teeth, then the two circles passing through T may be regarded as the pitch circles of the two wheels.

Now to trace the form of the teeth. Attach a pencil (P) to any point of the belt and fix a piece of paper to the wheel A so that it may turn with it, then the pencil will trace on the paper the curve EPL, being an involute of the circle A. Similarly, if we imagine a piece of paper attached to B, an involute DPS of the circle B will be traced on that. These two curves will be in contact at the tracing point P, and will always remain in contact as the circles turn. If, therefore, we construct teeth of this form with any given pitch, and then remove the belt, the two toothed wheels will drive one another with the constant angular velocity required. In this form of tooth the face and flank are one continuous curve, which is a property practically confined to involute teeth. From this fact a practical advantage follows. By the continual action of the teeth together they wear and cause a looseness of fit, which may be remedied by bringing the centres of the wheels more nearly together, and this without altering the smooth action of the teeth or the exact uniformity of the angular velocity-ratio. In no other form of tooth occurring in practice is this possible.

The line of action of the mutual pressure between the teeth is always along the tangent line to the two base circles, from which the teeth are generated, thus tending always to force the axles apart. If the angle between this line and the common tangent to the two pitch circles, or as it is called, the "obliquity," be large, much friction in the bearings would result. On this account the obliquity is made as small as possible, not being allowed to exceed $14\frac{1}{2}°$ or $15°$. With this a limit is introduced to the smallness of the number of teeth which may be used. The action of the teeth must always be along the line MTN, and hence cannot extend beyond the point N. If it is essential that when two teeth are in contact at the pitch point another pair of teeth should just be coming into action whilst a third pair are just ceasing action, then the length of the arc of the pitch circle which corresponds to an arc on the base circle equal to TN will be the greatest length that can be given to the pitch of the teeth, and when the obliquity is $14\frac{1}{2}°$ there will be about twenty-five such pitches on the pitch circle, and hence the number of teeth cannot be less than twenty-five.

Having given the pitch circles we first lay off, through the pitch point, the line of oblique action which is to be allowed, and then draw the base circles touching this line. The involutes of the base circles will give us the form of the teeth. The thickness of the tooth is to be taken a little less than half the pitch, and the addenda of the teeth such as to give a sufficient number of teeth in contact at the same time. (Art. 71.)

All involute teeth of the same pitch and obliquity will work together; they have never been much used in practice, although there appears to be no reason why they should not be in cases where it is not necessary to have less than twenty-five teeth. Their wear is said to be greater than that of teeth of other kinds.

69. *Path of Contact the Pitch Circle.*—In involute teeth the tracing point is attached to a belt stretched over pulleys, and therefore describes a straight line on paper, which is fixed to the line of centres so as not to revolve with either wheel. Now, the tracing point is also the point of contact of the two teeth, and therefore the path of this point, or, as it is conveniently called, the "path of contact," is a straight line. Teeth of any shape may be traced by this method if, instead of simply stretching the belt over the pulleys, we pass it over a fixed curve between the pulleys, so that the tracing point describes the curve in question instead of a straight line, provided the fixed curve be such that the curves traced on the rotating circles touch one another. In other words, we may assume various "paths of contact" at pleasure and obtain teeth which will work together correctly. We shall next suppose the tracing point attached to the circumference of a rotating wheel, in which case the path of contact is a circle.

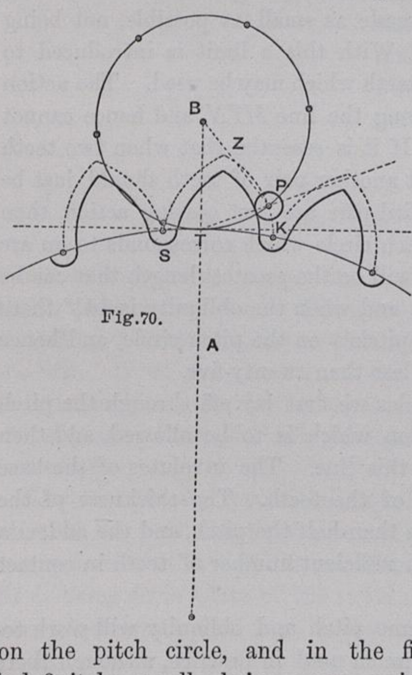


Fig.70.

In the use of toothed wheels the earliest idea was, for simplicity of construction, to form the smallest wheel of a number of cylindrical pins projecting from a disc. Supposing one of a pair of wheels to be so constructed, it is required to determine the proper form of the teeth for the other wheel.

On the wheel *B* (Fig. 70) let pins be placed at equal distances, with their centres on the pitch circle, and in the first place suppose the pins indefinitely small, being mere points. Now, if at one of the points P a pencil be attached, then if B be caused to roll without slipping over the surface of A kept fixed, the pencil P will trace a curve on a piece of paper attached to the wheel A. The same curve will be drawn if we cause one wheel to drive the other without slipping, the centres A and B being fixed, while the paper is attached to A and turns with it. If the tracing point started from the pitch point T, then the curve KP will have been drawn on the paper, which, by the further rotation of the circles, will be produced to Z. This curve is called an Epicycloid, and will be the proper form of teeth for the wheel A to drive the pinion B. For the pin P will be always in contact with the tooth KZ as the wheels revolve with uniform angular velocity-ratio. We complete the form of the teeth by drawing a similar curve ZS for the other face, SK being the pitch, in order to enable the wheels to be turned in the opposite direction if necessary. Placing a number of such teeth on the pitch circle A, we see they all touch one another at the roots on the pitch circle. The reason is because we have imagined the pins of B to have no definite dimensions, but to be mere mathematical points. In practice some definite dimensions must be given to the pins of B. In such a case the proper form for the teeth of A is derived from the previous construction by drawing a curve which at all points shall be at a distance from the epicycloid, when measured along the normal, equal to the radius of the pin. Below the pitch circle A a semi-circular recess must be formed, as shown by the full curve in figure.

These teeth possess the peculiar property of having faces but no flanks. The consequence is that, the toothed wheel A being the driver, the action of the teeth is wholly after the line of centres; there is no arc of approach, but only an arc of recess. On this account the pin-wheel must always be the follower, for if it be the driver the action of the teeth would be wholly before the line of centres, in consequence of which the friction is said to be more injurious.

The angle which PT makes with the common tangent is, as in the case of involute teeth, called the "obliquity"; it is now no longer constant, but varies from zero, when P passes the line of centres at T, to a maximum value when P escapes. It is easily seen that this angle is always one-half the angle PBT, which PT subtends at the centre of the pin-wheel, and hence the obliquity increases uniformly as the wheels turn; its mean value may be taken at half the maximum, and is limited in the same way as in involute teeth to about 15°, so that the greatest value of the angle PBT may be taken as 60°.

If the two sides of the teeth are alike, as in the figure, the pin then comes to the point of the tooth at Z. This circumstance determines the smallest number of pins which can be used, for one pin must not escape before the next comes to the line of centres; that is to say, PT cannot be greater than the pitch, the pitch then must not be greater than one-sixth the circumference of the pin-wheel, whence it appears that the least number of pins is six.

Pins are now rarely employed unless in clock and watch work; they have the great practical disadvantage that the toothed wheel to work with them must be specially designed, as it will work with only one diameter of pinion.

If we imagine a pin-wheel to work with an annular wheel, the teeth may be traced in the same manner as shown in Fig. 71 (below), to which the same letters are attached. The point P now traces out a curve called a Hypocycloid, the general character of which may be seen by joining P to F, the other extremity of the diameter TF of the circle B; for since the angle FPT must be a right angle, the angle APT will be greater than a right angle if, as in the figure, F lies between A and T, and less than a right angle if F lies beyond A. Thus the hypocycloid must reduce to the radius AK if F coincides with A, that is, if the diameter of the pin-wheel be half the diameter of the annular wheel; while, for smaller diameters, it forms a curve always concave towards T. Hence it appears that to work with a pin-wheel of half its diameter the teeth of the annular wheel should be constructed simply by drawing radii of the pitch circle. With a larger diameter of pin-wheel the teeth would be undercut, and therefore weak; the annular wheel must be the driver as before.

In all epicycloids and hypocycloids the normal to the curve at the tracing point P passes through the point of contact T of the circles considered—an important geometrical property, which we shall presently make use of, and hereafter prove.

70. *Path of Contact any Circle.*—Teeth traced in the way just described are wholly within the pitch circle, and this circumstance

suggests that by a combination with the preceding case, where they were wholly without, a form may be found which may be more suitable for practical use.

In Fig. 71 a third circle C is shown, touching the two others

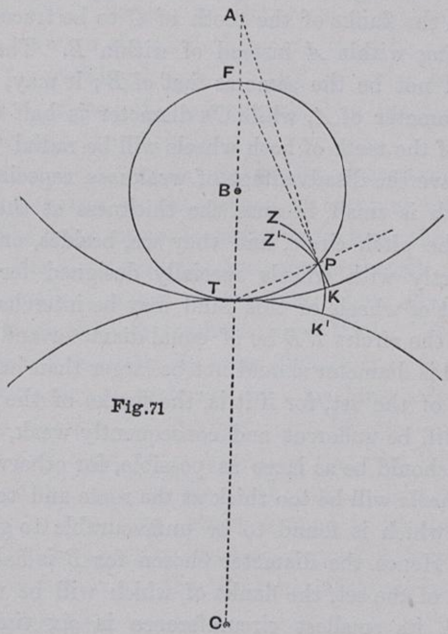


Fig. 71

at the same pitch point T. The three circles ABC turn each about its own centre without slipping. Imagine paper attached to A and C and rotating with them, while a pencil P is attached to B as before; then P will trace out two curves as in the case of involute teeth, one outside the circle C, the other inside the circle A. A's curve will be an hypocycloid KPZ, starting from K in the circle A, while C's curve is an epicycloid $K'PZ'$, starting from K' in the circle C'. Now these curves will, as in involute teeth, touch one another, having a common normal PT, and hence it follows that, while the circles turn with uniform angular velocity-ratio, the curves will always be in contact, and may be taken as face and flank of a pair of teeth. Thus it appears that we can obtain the faces of the teeth of C, and the flanks of the teeth of A, by causing a third circle B of any diameter to rotate within the circle C. If the diameter of B be half the diameter of C, the flanks for C will be simply radial lines, but if it be less the teeth will be concave towards T, the effect of which is that the teeth will spread out at the root, which is desirable on the score of strength. We can now imagine the faces of the teeth of A and the flanks of the teeth of C to be traced by another circle B' rotating within A instead of within B. The diameter of this circle need not be the same as that of B; it may, for example, be half the diameter of A, while C's diameter is half that of B; if so, the flanks of the teeth of both wheels will be radial. Teeth with radial flanks have the disadvantage of weakness, especially when the number of teeth is small, because the thickness at the root is less than that at the pitch circle, and they are, besides, only capable of working correctly with wheels specially designed for them. In order that a set of wheels of this kind may be interchangeable, it is necessary that the circles $B'B$ be of equal diameter and the same for all the set. This diameter should not be larger than half that of the smallest wheel of the set, for if it is, the flanks of the teeth of the small wheels will be undercut and consequently weak, while, on the other hand, it should be as large as possible, for otherwise the teeth of the large wheels will be too thick at the roots and too thin at the points, a form which is found to be unfavourable to good wearing. (See p. 184.) Hence the diameter chosen for B is half that of the smallest wheel of the set, the flanks of which will be radial. As B is a pin-wheel, its smallest circumference is six times the pitch (Art. 69), and the smallest wheel of the set has consequently 12 teeth; but if no wheel is required with so small a number of teeth as this, it will be better, for the reason stated above, to take a larger describing circle.

71. *Addendum and Clearance of Teeth.*—In any form of teeth it is clear from what has been said that the point of contact travels along the path of contact DT (Fig. 67, page 154) from the pitch point T to the end of the tooth at D, where the contact ceases. The length of the path of contact thus traversed is equal to the arc of recess in all kinds of cycloidal teeth, and less than that arc in a given ratio in involute teeth. By stepping off a suitable length on the path of contact then, we can find the end of the tooth for any given arc of recess, and the distance of this point from the pitch circle A of the driver is what we have already defined as the "addendum" of that wheel. The position of this point on the flank of the tooth of the follower B gives the working length of flank necessary. Similarly the length of face in the follower and flank in the driver depend on the arc of approach. The depth of the recesses between the teeth, however, must be made greater than is necessary for working length of flank, in order to allow the ends of the teeth to clear; the amount usual in practice appears to be about one-fifteenth the pitch.

The allowance necessary in practice for clearance in the thickness of the teeth depends on the degree of accuracy attainable in construction. The value formerly employed for teeth shaped by hand was one-eleventh the pitch, but the best modern teeth are machine cut, and a much smaller amount is sufficient. Less clearance is required for involute teeth than in teeth of other kinds. The setting out of bevel teeth is not theoretically more difficult than in the case of spur gear, but their accurate execution by a machine is far from easy. If the machine operate by straight cuts like an ordinary shaping machine, the tool must be mounted so that the line of cut always passes through the apex of the pitch cone. Gear cutting machines generally employ revolving cutters formed to fit the space between two teeth. Much ingenuity has been expended on giving the cutter a lateral movement to suit the bevel, but an exact bevel tooth cannot be formed in this way.

72. *Endless Screw and Worm Wheel.*—When two shafts are to be connected which are not parallel, and the centre lines of which do not intersect, it is necessary to resort to skew bevel, or screw, teeth. Only one case of this kind need be mentioned here as being of common occurrence, namely, the endless screw and worm wheel employed when the shafts are at right angles, and a slow motion of one of them is desired. In a common screw let the thread be so formed that the longitudinal section of the screw thread shows a range of teeth like those of a rack which would gear with a given spur wheel. Let the teeth of the wheel be set obliquely at an angle equal to the pitch angle of the screw; strictly speaking they also are screw threads, the pitch angle of which is the complement of the pitch angle of the screw. Then the screw and wheel will gear together, and the wheel moves through one tooth for each revolution of the screw. Like screws in general, this combination is non-reversible unless the pitch of the screw be coarse (Ch. X.), and for this reason, and on account of its simplicity, is much employed in practice. The method of constructing the teeth of a worm wheel is explained in a work by Prof. Unwin, cited on page 134.

1. A pair of wheels have 25 and 120 involute teeth respectively, and the addendum of each is $\frac{3}{10}$ths the pitch. Find the arcs of approach and recess in terms of the pitch, assuming the obliquity $14\frac{1}{2}°$, the large wheel being the driver. (See Art. 71.)

Ans.—Arc of approach = 1·07 × pitch.
Arc of recess = ·88 × pitch.

2. If the arcs of approach and recess in involute teeth are each to be equal to the pitch, show that the addenda of the wheels should be calculated by the approximate formula

$$\text{Addendum} = \left(\frac{1}{4} + \frac{3}{n}\right) \times \text{pitch},$$

where n is the number of teeth.

3. A pair of wheels have 25 and 120 teeth respectively, the flanks being in each case radial. Find the addendum of each wheel that the arcs of approach and recess may each be equal to the pitch.

Ans.—Addendum of driver = ·283 × pitch.
Addendum of follower = ·178 × pitch.

4. Connect two shafts which are not parallel and which do not intersect by bevel wheels, an intermediate idle wheel being admissible.

SECTION IV.—CAMS AND RATCHETS.

73. *Reduction of a Crank Chain by Omission of the Coupling Link.*—A pair of spur wheels in gear form a particular case of a three-link kinematic chain consisting of two lower pairs with parallel axes, two elements of which are united and generally form the frame-link, while the other two pair by contact.

Such a chain may be derived from the four-link crank chain of Art. 52, page 122, by omission of the coupling link, a process of reduction which has already been employed on page 146.

In Figure 72*a*, *ab*, *dc* are levers turning about fixed centres and connected by a coupling link *bc*, all three links being in one plane as in the article referred to. Imagine now the crank pins at *b* and *c* enlarged until they touch one another as shown by the dotted circles and then remove the coupling link. Suitable forces being applied to close the chain by keeping the surfaces in contact, the link bc may be removed without in any way altering the motion, and therefore the angular velocity-ratio will still be as before $aT : dT$, where T is now

Fig. 72b.

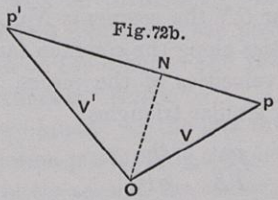
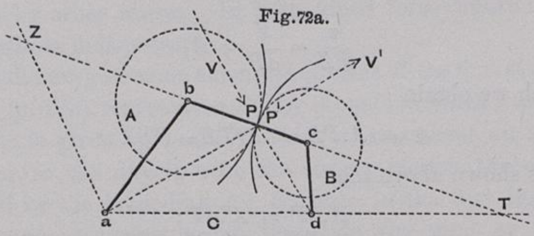


Fig. 72a.



the intersection of the common normal at the point of contact with the line of centres. Now the instantaneous motion of the levers cannot be affected by the shape of the pins except at the point of contact, and it therefore follows that if we replace the pins by any surfaces such as those indicated by the full lines in the figure, which have the same common normal at the point of contact, the result will be the same.

We may reach this conclusion directly by constructing a diagram of velocities for the two pieces in question. For let P, P' be points in the profiles which at the instant considered coincide by becoming the point of contact. Then P's velocity in the direction of the normal must be the same as that of P', for otherwise the surfaces would interpenetrate or move out of contact. If then from a given point O (Fig. 72*b*) we draw Op, Op' parallel to the lines aP, dP', to meet a parallel to the normal in pp', it follows by the same reasoning as in the case of linkwork that Opp' is a triangle of velocities of which the sides Op, Op' represent the velocities of P, P'. Hence drawing aZ parallel to dP' it appears as before that the angular velocity-ratio of the lines aP, dP' is dT / aT, and these lines are fixed in the rotating pieces so as to have the same velocity-ratio.

The third side pp' of the triangle of velocities represents in this case the velocity with which the surfaces rub against one another, for dropping the perpendicular ON the segments Np, Np' represent the resolved part of the velocities along the common tangent. Suppose A, A' to be the angular velocities of the pieces, V, V' the actual velocities of P, P', then by similar triangles

$$\frac{pp'}{PZ} = \frac{Op}{aP},$$

that is, if v be the velocity of rubbing,

$$\frac{v}{PZ} = \frac{V}{aP} = A,$$

from which we obtain

$$v = A.PZ = A(TZ - PT).$$

But it was shown above that

$$A.aT = A'.dT;$$
$$\therefore A.TZ = A'.PT;$$

hence

$$v = (A' - A)PT.$$

This formula suppose the pieces to turn in the same direction, as in the figure. If they turn in opposite directions, as in a pair of toothed wheels,

$$v = (A + A')PT,$$

a simple and important result which we shall hereafter verify.

It follows at once that for rolling contact the point of contact must lie on the line of centres, and that for a constant angular velocity ratio T must be a fixed point. Thus in all forms of teeth for wheels the common normal at the points of contact of the teeth must always pass through a fixed point on the line of centres, as is easily seen to be the case in the examples already considered. The velocity with which the teeth slide over one another is given by the above formula.

The diagram of velocities may when necessary be completed by laying down on it the velocities of all points rigidly connected with either rotating piece as explained before in the case of linkwork.

74. *Cams with Continuous Action.*—In toothed wheels the revolution of one wheel is always accompanied by that of the other in the same or in opposite directions, according as the gearing is inside or outside, or, in other words, the directional relation is always the same. We now pass on to cases in which the directional relation varies, the continuous rotation of one piece being accompanied by an oscillating motion on the other. The rotating piece is then called a "Cam," or sometimes a "Wiper."

Cams are of two kinds. In the first the contact is continuous, and the oscillating motion produced is completely defined by the form of the cam; while, in the second, the contact is only during the forward vibration of the oscillating piece, while the backward vibration is produced by other causes. In both kinds force-closure is common, and sometimes indispensable.

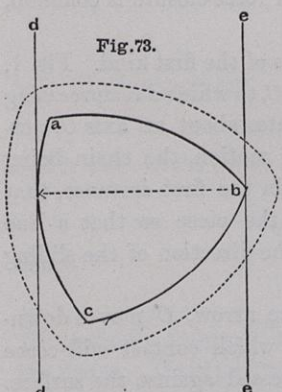
We shall now give some examples of cams of the first kind. Fig. 1, Plate IV. (p. 173), represents a sliding piece C, to which a reciprocating movement is given by a cam B, which rotates about an axis O, perpendicular to the direction of the sliding motion, the chain being completed by the frame-link A. Suppose, in the first instance, that the cam presses against a pin placed in the piece so that a line joining it to the centre of rotation gives the direction of the sliding motion.

As the cam turns in the direction of the arrow, C moves downwards to a certain limiting position, after which contact will cease unless some force be applied to keep it pressed against the surface. With suitable force-closure, however, supplied by the spring shown in the figure, C will return upwards to a second limiting position, and so on, continuously oscillating to and fro.

By properly taking the shape of the cam, any required relation may be obtained between the motions of the cam and slider; we have, in fact, only to draw a curve of position such as that constructed in Fig. 46, page 105, showing the position of the sliding piece for each position of the rotating piece. This curve will be the proper profile for the cam. In practice the chain is usually augmented by the addition of a friction roller, and the shape of the cam is modified by cutting away its surface to a depth equal to the radius of the friction roller, as was done in the case of the teeth of a wheel which drives a pin-wheel.

Force-closure, though common, is not necessary for the action of a cam chain of this kind; it may be avoided in two ways, both of which occur frequently in practice, though the mechanism would not always be described as a cam. First, the pin of the last example may be made to work in a slot cut in the face of a cam-plate, the centre line of the slot being formed to the profile of the original cam. Secondly, a slot may be cut in the sliding piece at right angles to the direction of sliding, and the cam may fit into the slot. Thus, for example, the cam may be a pin or an eccentric of any size; the chain is then merely a reduced double-slider crank motion, as explained on page 146. With other forms of cam other kinds of motion may be obtained; a common example is the Triangular Eccentric formed by three circular arcs (Fig. 73), each struck from one of the corners of an equilateral triangle *abc*. Such a curved triangle will fit between the sides *dd*, *ee* of a rectangular slot, and may therefore be used as an eccentric by fixing it to an axis passing through any point in it. In practice a figure would be used with rounded off corners, derived by striking small circular arcs with centres *a*, *b*, *c*, and uniting them by larger arcs having the same centres, thus obtaining a profile shown by dotted lines in the diagram, possessing the same essential property of uniform breadth, so that it will fit a rectangular slot of somewhat larger size. The mechanism is shown in Fig. 3, Plate IV.; it is sometimes used for a valve motion, the opening and closing of the valve taking place more rapidly than with a common eccentric. It has also been used in the "man engine" employed in mines to enable the miners to reach the surface without the fatigue of ascending ladders.

Fig. 73.



In these, as well as all other cam motions, a triangle of velocities can be constructed by the general method explained in Art. 73, and hence curves can be drawn showing the comparative velocities of the cam, the slider, and the rubbing between the two.

75. *Mechanisms with Intermittent Action.*—In all cases of higher pairing by contact between rigid elements, the closure of the chain is imperfect in the absence of external forces, for an exact fit between the surfaces, even if it exist originally, is soon destroyed by wear during the action of the mechanism. Thus, for example, when a pin works in a groove, as in the last article, the smallest looseness of fit will prevent the grooved piece from exactly following the movement of the pin when the contact passes from one side to the other of the groove. The same effect is produced by the clearance necessary for the safe action of the teeth of a wheel. In cam mechanisms, where the contact is continually changing from one side to the other, the chain opens for a short interval at every change unless force-closure be employed as described above. The pair, of which the oscillating piece forms an element, is locked by friction during the interval.

Suppose now that the groove is purposely made of much greater dimensions than the pin, the oscillating piece will remain at rest for a considerable interval, and will thus have an intermittent motion. The same thing occurs in wheels which work by the successive action of a number of teeth when some of the teeth in one of the wheels are removed. The pair which moves intermittently may be locked during the interval of rest either by friction or by the special means described in the next article.

Intermittent motions of both the cam and wheel class occur frequently in mechanism. Two common examples may be mentioned.

(1.) A wheel with one tooth may be employed to turn another wheel with any number of teeth through a small space at each revolution.

(2.) A wheel with one or more teeth may move a sliding piece alternately backwards and forwards.

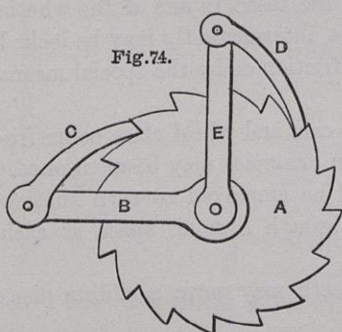
In all cases, during the interval of motion, we have a chain of the kind already described which closes at the commencement of the interval. The closure is accompanied by a shock which renders such mechanisms unfit for the transmission of considerable forces, and limits the speed at which they can be run. (See Ch. XI.)

76. *Ratchets.*—The oscillating motion of the piece C may be a turning instead of a sliding motion, as is often the case in shearing machines for example, but no new principle is here involved, and we now proceed to the second class of cam motions in which the forward vibration alone is subject to the action of the cam, while the backward vibration is effected by independent causes, generally by means of springs or of gravity. In such cases the forward vibration follows the same laws as in cams of the first kind, but during the backward vibration the oscillating piece forms a distinct machine by itself, working by means of energy supplied by the cam during the forward movement. In tilt hammers and stampers the work of the machine is done in this way and we need not here further consider them; but the object may be merely to shift the position of the piece and so to lock or unlock a pair, to open or close a kinematic chain. The piece is then called in general a Ratchet, though it may receive other names according to circumstances, and a chain in which it occurs is thus known as a Ratchet Chain.

(1.) The shifting piece may lock a turning or a sliding pair in one or both directions. A common latch for example rises to permit a gate to close and then drops into its place and fastens the gate until again raised by external means.

The piece C (Fig. 74) forming a turning pair with a fixed piece B, fits in the hollows of the teeth of a wheel A which also pairs with B. The teeth are formed as in the figure so as to permit A to move in one direction by raising C till it drops by the action of a spring or by gravity into the next hollow. In the other direction the pair AB is locked. C is then called a pawl, and the arrangement is the ordinary one employed in windlasses, capstans, and lifts to prevent the machine reversing when the hauling power is removed.

Fig.74.



(2.) Two shifting pieces may be employed to lock alternately two pairs which have a common element. This is the ratchet mechanism proper from which the name of the class is derived.

Returning to Fig. 74, A, B, C are the same as in the previous case, E is an additional piece which pairs with B: in the figure the axis of the pair has been supposed concentric with A, but this is not necessary: D is the ratchet pairing with E and at the same time fitting like C into the teeth or the ratchet wheel. If now an oscillating movement be communicated to E, the ratchet wheel A will be locked alternately with B and E according to the direction of motion of E. Accordingly A has an intermittent movement moving with E in its forward oscillation and resting in the backward. Instead of a pawl C, friction may be relied on to lock AB in the backward movement as in the common ratchet brace, but the nature of the mechanism is the same always. It sometimes happens that the pairs AB, BE are not concentric; the chain $ABED$ is then an ordinary four-link chain which opens when moved in one direction and closes when moved in the other, while the pair CA unlocks and locks as before, so as to permit A to move intermittently. In both cases the movement is single acting, but two such chains may be employed which move in opposite directions and open and close alternately; the movement may then be described as double acting. The well known "Levers of Lagourousse" (Fig. 6, Plate IV.) is a double acting ratchet mechanism in which the two chains have all the links common except the ratchets. The ratchet wheel then moves continuously in one direction, and the locking pawl C may be omitted. The ratchet wheel employed in the case of a turning pair may of course be replaced by a rack when a sliding pair is required, but no new principle is here involved.

(3.) The shifting piece may be connected with a pendulum or balance wheel which vibrates in equal times. Time may be thus measured by unlocking a kinematic chain at intervals. In clocks and watches a tooth of the ratchet wheel escapes from the action of the ratchet at each vibration or semi-vibration; the mechanism is therefore called an escapement.

(4.) In pumps various kinds of ratchet mechanisms are universal. The common reciprocating pump is a true ratchet mechanism, the column of water being locked and pairing with the plunger alternately; it may be single or double acting. It is needless to say that the ratchet is here called a "valve."

77. *Other Forms of Ratchet Mechanism.*—In all the examples of the preceding article the shifting piece is not subject to the action of the rest of the mechanism during its return oscillation, but it may also be worked by a cam movement of the first kind, or by linkwork mechanism: the slide valves of a steam engine are a familiar instance. Also it may be worked by external agency instead of by the machine itself, as in all kinds of starting and reversing gear. The ratchet chains form a large and interesting class of mechanical combinations, but their discussion would be out of place here.

78. *Screw Cams.*—The three-linked chain of Art. 72 may have the axes of its lower pairs inclined at an angle instead of parallel, and a number of mechanisms of the cam class may thus be derived which are analogous to those already considered. Some of these may also be derived from a screw chain, and may here be briefly mentioned.

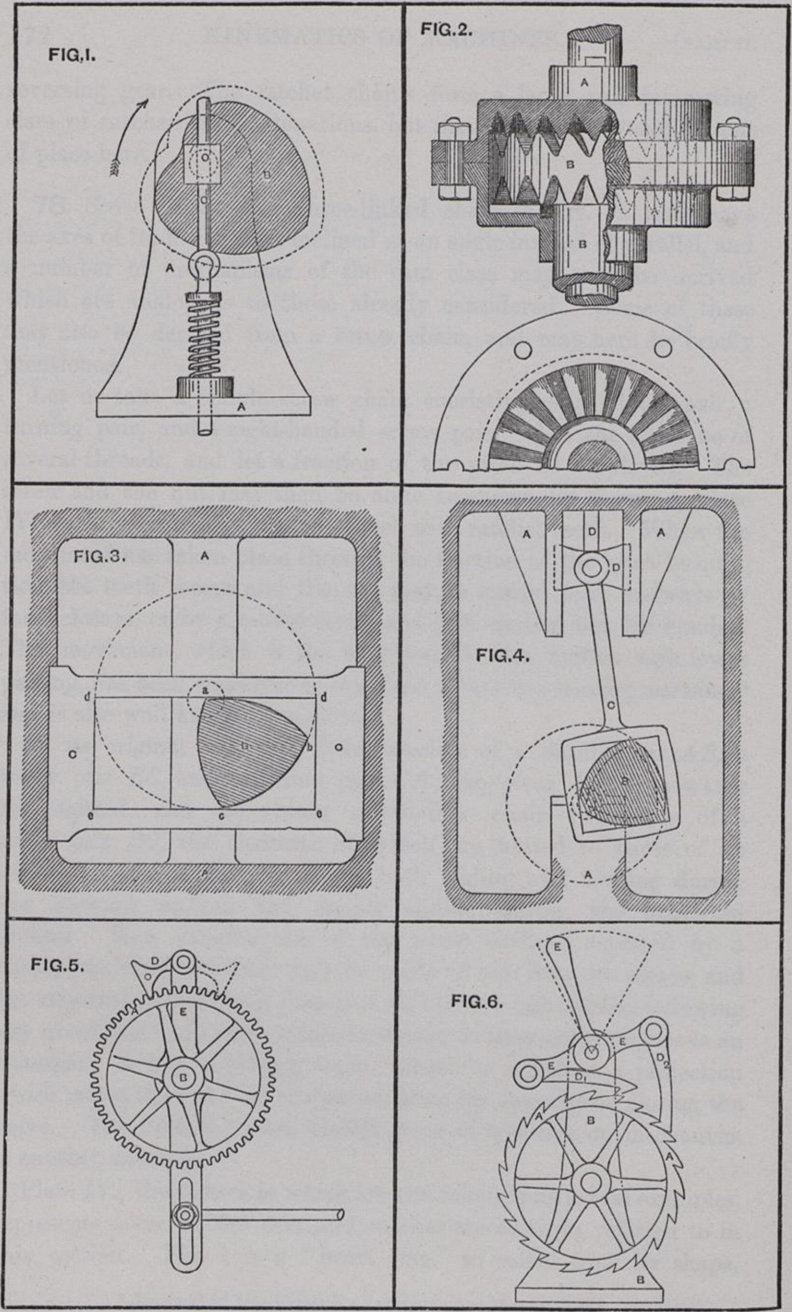
Let us take a simple screw chain consisting of a sliding pair, a turning pair, and a right-handed screw pair. Let the screw be of several threads, and let a fraction of the pitch be employed. The screw and the nut may then be alike as shown in Figure 2, Plate IV., each resembling a crown wheel with ratchet teeth. When the movement has taken place through the fraction of the pitch in question, the teeth escape and the nut may be moved back endways by force closure, or by a second screw and nut similar, but left-handed. This movement, which is the only possible cam motion with lower pairing, has been employed to work the shears in a reaping machine,* and is also well known as a clutch.

In its original form the chain consists of a sliding pair AB, a screw pair BC, and a turning pair CA; the piece A may however be omitted, and we obtain a two-link chain consisting of a screw pair BC, the elements of which are united to those of an incomplete lower pair, B and C both sliding and turning during the forward motion and simply sliding during the backward motion. Now imagine one of the screw surfaces replaced by a simple pin, then the other may be made of any form we please, and the elements of the incomplete pair will have a cam motion following any given law. A valve motion common in stationary engines is an example. B is a revolving crown wheel on which is a projection which raises the rod C at the proper time for opening or closing the valve. The "swash" plate usually given in treatises on mechanism is another example.

Plate IV., the figures in which are not taken from actual examples, represents some of the cam and ratchet mechanisms referred to in this section. Fig. 1 is a "heart cam," so called from its shape,

* Journal of the Franklin Institute for March, 1880.

Plate.IV.



in which the sliding and rotating pieces are connected with uniform velocity-ratio. Fig. 2 is the screw cam just described. Figs. 3 and 4 are two forms of the triangular eccentric motion (p. 181). Fig. 5 shows a ratchet motion (p. 171) in a form common in the feed motions of machine tools: the direction of movement of the ratchet wheel *A* is here reversible by putting over the ratchet *D* into the dotted position. Fig. 6 is referred to on page 171.

EXAMPLES.

1. A reciprocating piece moves in guides under the action of a cam attached to a shaft which rotates uniformly, and the centre of which lies in the line of motion. Trace the form of the cam that the piece may slide uniformly and make one complete movement in each revolution. Suppose a friction roller used of diameter equal to $\frac{1}{6}$ stroke, and suppose also that the least radius of the cam is $\frac{1}{4}$ the stroke.

2. A stamper is raised by a cam such that the rise takes place uniformly during a part of the revolution of a shaft which is distant from the stamper half the rise. Trace the proper form of cam, and find the fraction of the revolution in which the rise takes place.

The best solution is that in which the profile of the cam has the form of the involute of the dotted circle, whose radius is half the lift of the stamper; for then the pressure of the cam on the pin is always in the vertical direction. The rise takes place whilst the cam turns through an angle, the arc of which is equal to twice the radius, or $\frac{1}{\pi}$ of a revolution.

3. Draw a curve of velocity for a reciprocating piece moved by a uniformly rotating triangular eccentric.