CHAPTER VII.

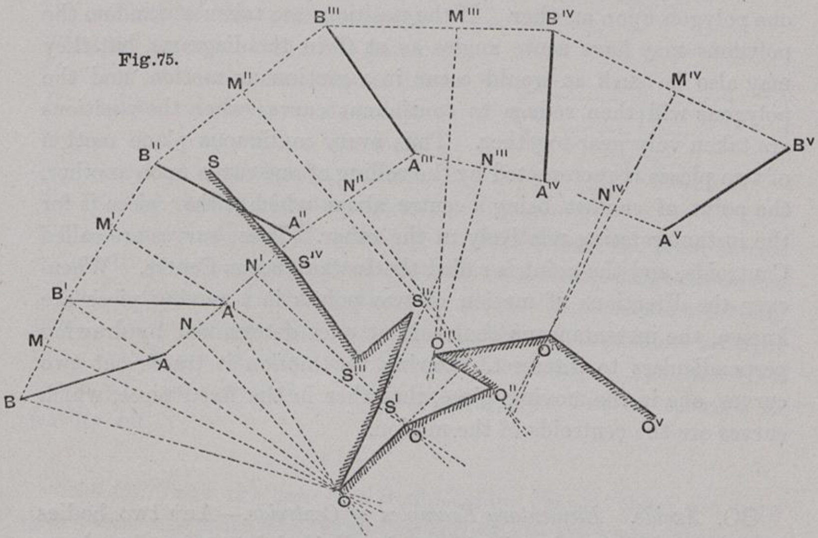
MECHANISM IN GENERAL.

79. *Plane Motion in General. Centroids.*—In the two preceding chapters the mechanisms considered have been composed either wholly of lower pairs or else of two lower pairs connected by higher pairing. The velocity-ratios of the various lower pairs have been considered, and diagrams of velocity have been drawn for the complete mechanism, but without attempting to form any idea of the comparative motion of pieces which do not pair with each other, or which form the elements of a higher pair. It will now be necessary to consider the comparative motion of two pieces more generally.

First, suppose the two pieces to move in such a way that any point in one moves parallel to a plane fixed in the other. The motion is then the same as that of a plane area which slides on a fixed plane, and may be called for brevity "plane motion." If the position of any two points in the moving area be given, all the rest can be found, and the motion is therefore completely defined by the movement of the straight line joining these points.

Let AB, $A'B'$, $A''B''$ ... (Fig. 75) be successive positions of such a line. Join AA', BB', and from the middle points of these lines draw perpendiculars NO, MO to meet in O, then $OA = OA'$ and $OB = OB'$, from which it can be proved that $AOB = A'OB'$, so that AB might be moved to $A'B'$ by attaching it to a plane area, and rotating that area about O as a centre. Obtain similar centres O', O'', O''' ... for the succeeding changes of position, then it is clear that the motion of AB, and therefore of the plane area to which it is attached, may be completely represented by the rotation of the area about the centres O, O', O''... in succession through certain angles which are given,

Fig. 75.



being the inclinations to each other of the successive positions of AB.

Next, through O draw OS', making it equal to OO' and inclined to OO' at the first angle of rotation, $S'S''$ equal to $O'O''$ and inclined to it at an angle equal to the sum of the first and the second angle of rotation, and so on; we thus obtain a second polygon $OS'S''$..., the sides of which are equal to those of the original polygon $OO'O''$... Imagine this polygon rigidly attached to AB so as to move with it, then during the motion the polygon will rotate about O till S' reaches O', then about O' till S'' reaches O'', and so on in succession; that is to say, the changes of position of AB may be produced by the rolling of one polygon upon the other. Thus, by properly determining the polygons, any given set of changes of position of a plane area may be produced at pleasure by rolling the moveable polygon on the fixed one.

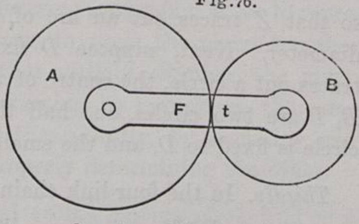
Now imagine the moving area to become fixed in its original position, and let the originally fixed area move by rolling the polygon $OO'O''$... which is attached to it upon the polygon $OS'S''$ which is now fixed. Evidently the two areas take up the same relative positions, and we obtain the very important proposition that any set of changes of relative position of two areas may be obtained by the rolling of one polygon upon another. If the positions are taken at random the polygons may have acute angles as at O'' in the diagrams, but they may also be such as would occur in a continuous motion, and the polygons will then reduce to continuous curves when the positions are taken very near together. Thus every continuous plane motion of two pieces is represented by the rolling of one curve upon another, the point of contact being a centre about which either piece is for the instant rotating relatively to the other. These curves are called Centroids, and the point is called the Instantaneous Centre. Whenever the directions of motion of two points in a moving piece are known, the instantaneous centre is at once determined by drawing perpendiculars to intersect. During the motion it traces out two curves, one in the moving piece, the other in the fixed piece, which curves are the centroids of the motion.

80. *Axoids. Elementary Examples of Centroids.*—Any two bodies moving in the way described may be divided into slices by planes parallel to the plane of motion, the centroids of which will of course be all similar and equal, so that we may regard them as the transverse sections of cylindrical surfaces in contact with each other along a generating line. The surfaces are called Axoids, and the line the Instantaneous Axis. The relative motion of the bodies is represented by the rolling of the axoids upon one another, endways motion being supposed prevented.

Any two parts of a mechanism have a relative motion which is completely defined by the nature of the mechanism, as has been sufficiently explained already; and it follows, therefore, that they must have given axoids, the nature of which completely defines the motion of the pieces. In every kinematic chain there are as many sets of axoids as there are sets of two pieces, and these surfaces are the same for all the mechanisms derived from that chain by inversion. These remarks apply even when the motion is not plane, as will be seen further on.

First. Take the case of a pair of spur wheels AB in gear, F being the frame-link (Fig. 76), forming the three-link chain considered in the last chapter. Let the pitch circles touch at the pitch point t, then, as before explained, those circles roll together without slipping, and therefore must themselves be the centroids, the pitch surfaces being the axoids. Hence the point t is the instantaneous centre of B's motion relatively to A, or A's motion relatively to B. We shall return to this immediately, but for the present merely remark that if the centres of A and B move up to each other, the pitch circles reduce to points, and the axoids become coincident straight lines, the point t is fixed in A and B, the two pieces then become a turning pair. In lower pairing, then, the axoids are coincident straight lines, which are at infinity if the pair be sliding. The case of a screw pair in which the motion is not plane will be referred to farther on.

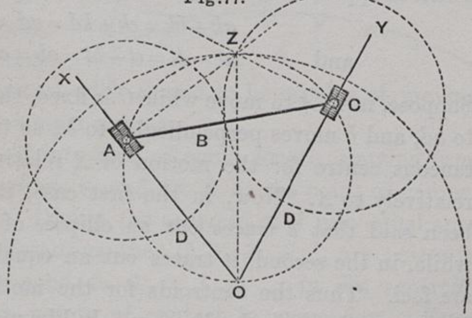
Fig.76.



Secondly. Take the case of a double-slider chain; there are here four pieces which may be taken two and two in six ways; there are, therefore, six sets of axoids. Four of these, however, are only the four axes of the four lower pairs, and it remains to determine the other two.

In Fig. 77 the blocks A, C are connected by a link B and slide on a piece D along lines OX, OY, forming the chain described fully in a former chapter. The blocks A, C form two turning pairs with the link B, and the velocities of these pairs are equal because B makes angles with OX, OY, the difference of which is constant. The centroids for A, C are therefore equal circles, the centres of which are the centres of A, C (see note, p. 191). Next, to find the centroids of B, D, through those centres draw perpendiculars to OX, OY to

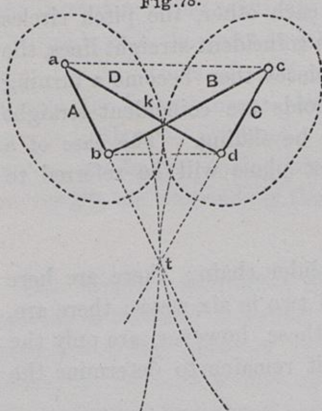
Fig.77.



meet in Z, then Z is the instantaneous centre for B when D is fixed, and for D when B is fixed. First, suppose B fixed, then the angle at Z is the supplement of the angle at O, and is therefore constant, so that Z traces out an arc of a fixed circle, of which OZ is the diameter. Next, suppose D fixed, then, since OZ is constant, Z traces out a circle, the centre of which is O. Thus the centroids of B, D are two circles, one half the diameter of the other; the large circle is fixed to D, and the small circle to B.

Thirdly. In the four-link chain A, B, C, D, consisting of four turning pairs with parallel axes, the sections of which are represented by the points a, b, c, d (Fig. 78); suppose opposite links equal, but set so as not to be parallel. This is the case referred to already (page 122) as "anti-parallel" cranks.

Fig. 78.



Joining ac, bd by the dotted lines in the figure, the quadrilateral $abdc$ has two sides and two diagonals equal, hence the triangles bac, cda must be equal in every respect, so that bd is parallel to ac. Hence if k be the intersection of the diagonals, and t the intersection of the sides, $ak = ck : bk = dk : bt = dt$, from which it appears that

$$ak + bk = ck + kd = ad = bc$$

and $$at - dt = ct - bt = ab = cd.$$

Suppose, now, A to move while C is fixed, then a moves perpendicular to ad, and b moves perpendicular to bc, so that k must be the instantaneous centre for the motion of A relatively to C, or for that of C relatively to A. Now, in the first case, it appears from what has been said that k traces out an ellipse, of which c and d are foci, while, in the second, it traces out an equal ellipse, of which a and b are foci. Thus the centroids for the motion of A and C are equal ellipses, as shown in the diagram. In like manner the centroids for the motion of B and D are the equal hyperbolæ traced out by the point t.

The four other pairs of centroids are the points ab cd, which are the centres of motion of the four turning pairs.

81. *Profiles for given Centroids.*—Any given motion of one piece relatively to another may be produced in an infinite number of ways. One way of doing this is by rolling contact, for if the motion is given the centroids will be given, and by forming the profiles so as to represent the centroids, and applying forces to press the pieces together and cause them to roll on one another without slipping, the given motion may be produced. But if slipping be permitted, the same motion may be produced, at least theoretically, by assuming any form whatever for one profile and properly determining the other.

(1.) Let a given profile be attached to the moving piece, and as it rolls into different positions let that profile be traced on paper attached to the fixed piece. If the positions be taken near enough together, a curve may be drawn through their ultimate intersections which will envelope them all, and if a profile formed to that envelope be attached to the fixed piece, the two pieces will fit one another and yet be capable of relative motion of the prescribed kind.

(2.) To apply the foregoing process a model would be necessary, but by a simple modification, a geometrical construction may be obtained. In Fig. 79, A and B are the pieces, which move so that the centroids are 0, 1, 2, 3 ..., 0′, 1′, 2′, 3′ ..., curves which are shown touching at the point t. P is a profile of given form attached to B; it is required to find a profile attached to A, which will always remain in contact with P, and so be capable of moving it in the required way by simple contact.

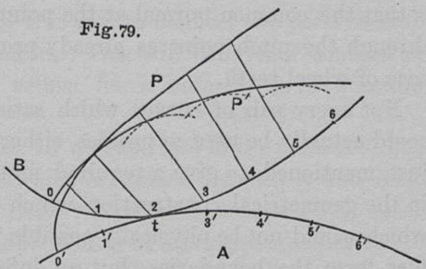


Fig. 79.

Divide the centroid of B into arcs of equal length, starting from O, the point where P intersects it, and let 2 be the point of contact at the instant considered. Divide the centroid of A into equal arcs, stepping from 2 in both directions, then 0′, 1′, 3′, 4′ ... are points in A centroid, which correspond to 0, 1, 3, 4 ... in B's centroid, being during the motion points of contact in succession. From 1, 2, 3 ... drop normals on to the curve P, and with these normals as radii trace circular arcs with centres 1′, 2′, 3′ ...; the envelope of these arcs must be the required profile P'.

(3.) Instead of assuming one profile and determining the other to suit it, it is generally more convenient to employ some method of determining pairs of profiles which satisfy the required conditions.

In Fig. 80 A, B are the centroids as before, C is a third curve, theoretically of any form, which rolls on A and B, always touching these curves at their point of contact t. P is a tracing point which is attached to C and traces out two curves during the motion, one on A, the other on B.

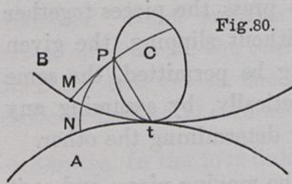


Fig.80.

First, suppose A fixed, then, since t is the instantaneous centre of the motion of C, Pt must be normal to the curve NP traced out on A. Similarly supposing B fixed, Pt is normal to the curve MP traced out on B. Thus the two curves touch one another at the point P and therefore may be taken as profiles which will give the required motion. If A, B, C be circles, this construction becomes that already considered when discussing the form of teeth for a wheel. This and the preceding method show clearly that the condition which the two profiles must always satisfy is that the common normal at the point of contact must always pass through the pitch point as already proved otherwise for the special case of wheel teeth.

Not every pair of curves which satisfy the geometrical conditions could actually be used as profiles, either for centroids, or, in the cases just mentioned, to give a required motion, because there is nothing in the geometrical construction which excludes an interpenetration which would not be physically possible in the areas of which the profiles form the boundaries, but an infinite variety of forms can be found, for given centroids, which might be so used.

In all cases in which the centroids are known for the relative motion of two pieces, one of which is fixed, the velocity-ratio of any two points (a, b) in the moving piece is known for each position of the pieces. For, joining the two points to the instantaneous centre O, the ratio of the distances Oa, Ob must be the velocity-ratio in question, since the moving piece is for the moment turning about O. It is easily seen that the triangle Oab is similar to the triangle of velocities constructed as in Art. 49, p. 108.

82. *Centroids for a Higher Pair connecting Lower Pairs.*—Among the

infinite variety of profiles which correspond to given centroids it is frequently possible to find some which are closed curves, one completely surrounding the other. If these curves be used as the external and internal boundaries of two areas, the two pieces thus formed will fit one another and be capable of no motion except that of the prescribed kind without requiring any additional constraint. In Figure 4, Plate IV., a form of the triangular eccentric motion is shown, which has been occasionally used and which furnishes an example. On reference to Art. 74 it will be seen that such an eccentric will exactly fit a square within which it is enclosed, and therefore forms with it a higher pair which is "complete" in itself.

Complete higher pairs are very unusual in mechanism, higher pairing being employed almost exclusively to complete a chain of lower pairs as in the preceding chapter. It is then generally "incomplete," the necessary constraint being furnished by the rest of the kinematic chain to which it belongs, as for example in the triangular eccentric motion shown in Figure 3, Plate IV. The general problem in mechanism is not to connect two pieces in a given way, but to convert the motion of a given pair into the motion of a different pair—that is to say, to connect two pairs so as to have a prescribed relative motion. This will be further considered presently, but we must first return for a moment to a question considered in the last chapter.

In the three-link chain of Art. 73 we have two lower pairs AC, BC, with axes parallel, connected by simple contact between A and B at the point P (Fig. 72, p. 165). Draw the common normal PT to meet ad in T, then when B is fixed the motion of a is perpendicular to ad, and the motion of P perpendicular to PT, therefore T must be the instantaneous centre for the motion of A relatively to B. Let v be the velocity of rubbing at P; A, A' the angular velocities of the pairs AC, BC: further let $ad = l$ and $PT = z$; then, since B is fixed and A is rotating round T,

$$\frac{v}{z} = \frac{\text{velocity of } a}{aT} = A' \cdot \frac{l}{aT}.$$

Similarly supposing A fixed,

$$\frac{v}{z} = \frac{\text{velocity of } d}{dT} = A \cdot \frac{l}{dT}.$$

from which it appears that

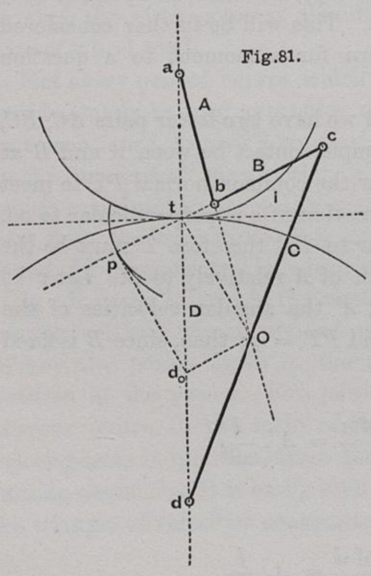
$$\frac{A}{A'} = \frac{dT}{aT}; v = z(A' - A),$$

results which agree with those obtained in the article cited by a different method.

The centroids in this case, as well as in that of the four-link chain from which it was derived by reduction, may be traced graphically by plotting the position of T for a number of positions of the pieces, but they are known curves only in exceptional cases such as those of Art. 80, and generally have infinite branches which render their use inconvenient.

When the point P lies on the line of centres it coincides with T, and the velocity of rubbing is zero; the centroids are then no other than the profiles themselves of A and B. The curves are then said to roll together: a particular example is that of the equal ellipses of Art. 80 which are not unfrequently used to connect two revolving shafts with variable angular velocity-ratio. In this case the velocity-ratio is the ratio of the focal distances of the point of contact, but by properly determining the profiles it is theoretically possible to give any velocity-ratio to the shafts at pleasure. The question, however, is not one of much practical interest.

Fig.81.



83. *Construction of Centres of Curvature of Profiles — Willis's Method.*—In the four-link chain $ABCD$ shown in Fig. 81, D is the fixed link and B the coupling link: a, b, c, d are sections of the axes of the pairs which are supposed parallel.

If the coupling link bc be prolonged to meet the line of centres ad in the point t, and ab to meet cd in O, it appears as in previous cases that O must be the instantaneous centre of B, and that the angular velocity-ratio of A and C is $dt : at$. Join Ot, and imagine bt an actual prolongation of the bar bc, so that t is rigidly connected with it, then t's motion will be perpendicular to Ot. Suppose now that the proportions of the links are taken so that Ot is perpendicular to bt, then t moves in the direction of the length of the rod, and the rod therefore may be imagined to slide through a fixed swivel at t.

This reasoning shows that the levers A and C, when in this position, will move for a short interval with uniform angular velocity-ratio, and the movement of a pair of wheels in gear is thus imitated by a link-work mechanism.

Let us now form a reduced chain by omission of the coupling-link, and we shall be able to solve the important problem of finding a pair of circular arcs which will serve for the profiles of a pair of teeth in contact. For this purpose, with centres b and c, strike arcs through any point p on cbt produced, and let these arcs be rigidly connected with A and C respectively; the coupling-link may now be removed, and A imagined to drive C by direct contact of the arcs. Evidently wherever p is, the pieces will move for the moment with uniform angular velocity-ratio and pitch point t. The uniformity, however, is only momentary, because the position of O changes, and to trace the profiles with accuracy it would be necessary to perform the construction for a succession of positions of cbt, hence the face and flank of a pair of teeth in contact cannot be exactly represented by a pair of circular arcs. When it is sufficiently approximate to do so, the arcs are found by assuming a mean position for the point p, and a mean value for the obliquity i, found by experience to give good results. The method here described was invented by the late Professor Willis, and the value of i recommended by him was $\sin^{-1}\cdot 25$, or about $14\frac{1}{2}°$, being about the actual mean value of the obliquity in cycloidal teeth of good proportions. Also the value of pt was taken by him as half the pitch, p being then about midway between the pitch point t and the point of the tooth.

Having made these assumptions, it still remains to fix the position of the point O, which may be taken anywhere on a line through t inclined at $14\frac{1}{2}°$ to the line of centres. This is done by observing that O must be the same for all wheels D intended to work with a given wheel A, and that teeth never should be undercut (Art. 70); that is, c and b must lie on the same side of t. Hence in the smallest wheel intended to work with A, c is at infinity, so that if d_0 is its centre, d_0O is parallel to pt, and therefore perpendicular to Ot. The flank of the tooth in this case becomes a radius d_0p. The position of O is thus completely determined for all the wheels of a set when the pitch is given.

Willis's method is of great theoretical interest, and has consequently been given here, but the form of teeth obtained is not always sufficiently approximate. It may, therefore, with advantage be replaced by other methods, as to which the reader is referred to a work by Professor W. C. Unwin on Machine Design.

84. *Sphere Motion.*—When a body moves about a fixed point its motion is completely represented by that of a portion of a spherical shell of any radius which fits on to a corresponding sphere, and moves on it just as in the case of plane motion. Everything which has been said respecting plane motion also applies to sphere motion, but the axoids are conical instead of cylindrical surfaces, the centroids spherical instead of plane curves, and all straight lines are replaced by great circles of the sphere on which the motion is imagined to take place. The corresponding crank chains are called "conic" crank chains, the axes of the pairs lying on a cone instead of a cylinder.

85. *Screw Motion.*—In the plane motion of two pieces, endways motion of the cylindrical axoids is supposed to be prevented by some suitable means. Let us now remove this restriction and imagine the axoids to slide endways, while continuing to roll together, the relative movement will now not be completely defined, but additional constraint will be required. In the first place take the case of a lower pair in which the axoids are coincident straight lines; if endways sliding be permitted we obtain an incomplete pair, unless the nature of the surfaces in contact define the relation between the endways motion and the rolling motion. In the simple screw pair the two are in a fixed ratio, in the screw cams of Art. 76 they have a varying ratio. In every case of non-plane motion with cylindrical axoids, not only must the axoids be given, but also a connection between the endways sliding and the motion of rotation.

In the most general case possible the instantaneous axis changes its direction as in spherical motion, its position as in plane motion, and in addition there may be an endways sliding. This is expressed by the rolling and sliding of certain surfaces on one another, which are now neither cylindrical nor conical. These surfaces are in all cases of the kind known as "ruled" surfaces, being generated by the motion of a straight line, along which they touch each other. The surfaces are still called Axoids, and the line is the Instantaneous Axis. The hyperboloidal pitch surfaces for wheels connecting two shafts which do not intersect are examples of this kind; but for the discussion of this question, which is not of very common occurrence, the reader is referred to the works already cited.

86. *Classification of Simple Kinematic Chains.*—On observing the action of any mechanism, several of the pieces of which it is constructed may be readily distinguished as having functions different from the rest. These pieces, like the rest, occur in pairs, and may be described as such, though the pairing is not necessarily kinematic. First, one or more perform the operations which are the object of the mechanism, these may be called the Working Pairs, as, for example, the tool and the work in machine tools, the weight raised and the earth in the hoisting machines. Second, one or more form the source from which the motion is transmitted, as, for example, the crank handle and frame of a windlass, the piston and cylinder of a steam engine. These may be called the Driving Pairs. Thirdly, various subsidiary working pairs carry out various operations incidental to the working of the machine. The object of the mechanism is always to convert the motion of the driving pairs into that of the working pairs.

The simplest case is that in which the motion has only to be transmitted without alteration; a single pair will then suffice. Thus, by means of a long rod sliding in guides or turning in bearings, a motion of translation or rotation may be transmitted to a distance only limited by non-kinematical considerations. By use of flexible elements—among which should be included the flexible shafts recently introduced—the direction may be altered at pleasure and any desired position reached.

If, however, the magnitude of the motion is tobe altered, a mechanism must be employed in which at least one element of the driving and working pairs is different. The driving pairs are usually kinematic lower pairs and the working pairs are so very frequently, and this is why so many of the simplest and most important mechanisms are examples of the connection of lower pairs. The peculiar motions of lower pairs being translation and rotation, a number of mechanisms may be classed as examples of the conversion of rotation into translation or rotation and conversely, with uniform or varying directional relation or velocity-ratio. This is especially the case when, as so frequently happens, the driving and working pairs have a common link which is fixed.

It has been shown, however, that many apparently different mechanisms are in reality closely connected, being derived from the same kinematic chain. Mechanisms are therefore to be classed according to the kinematic chains to which they belong. The number of simple chains actually employed in mechanism is limited by the preceding considerations to those already described, which are ranged by Reuleaux in the following classes, the names of which are derived from the most important piece in some example of common occurrence :—

(1) Crank chains.
(2) Screw chains.
(3) Pulley chains.
(4) Wheel chains.
(5) Cam chains.
(6) Ratchet chains.

In the first two are included all combinations of sliding, turning, and screw pairs; in the third, all cases where tension or pressure elements are employed; in the fourth, all cases of connection by contact where the directional relation remains the same; in the fifth, all cases where it varies; while in the last all combinations are included where, by action of a shifting piece, the law of motion is periodically varied.

87. *Compound Kinematic Chains.*—In a complete machine, the motions required are generally too complex to be carried out by a single kinematic chain of this simple kind; it is necessary to combine together a number of such chains, and we conclude this part of the subject with some general remarks on such combinations which may all be regarded as compound chains derived from two or more simple chains by union of their links.

(1.) From any two closed chains a third may be derived by uniting two links. The links must have the same relative motion, for otherwise the chains would lock each other, and they generally form a pair.

This is one of the commonest of all combinations. When two machines are driven from the same shaft, or when the same shaft is driven by two separate engines, we have examples in which the driving pairs or the working pairs are common, but the mechanisms are otherwise independent. Further, in every complete machine we find, in addition to the principal chain which does the work, a number of auxiliary chains which carry out various operations necessary to the working of the machine. Thus, in the steam engine, besides the slider-crank or other mechanism which turns the crank, we have the valve motion which governs the distribution of steam, the air pump motion which produces the vacuum in the condenser, and frequently others as well. Each of these auxiliary mechanisms has a pair in common with the principal chain which serves as a driving pair, but the chains are otherwise independent. Again, in trains of mechanism which, as previously remarked (page 150), are frequently simple chains augmented for non-kinematical reasons, a number of such chains are arranged so that the working pair of one chain is the driving pair of the next in succession. A train of wheels or the mechanism of a beam engine are examples already referred to, in which one link is common to all the separate chains, but cases occur in which this is not so, as, for example, the well-known Lazy Tongs.

The case here considered is that where the movements of various driving pairs have to be transmitted to various working pairs, but no new motion is required in a working pair other than could be produced by a simple chain. All such combinations may be called Trains, and may be divided into "converging," "diverging," and "transmitting" trains.

(2.) If two closed chains have only one link common they are completely independent, like two machines standing on the same floor, but disconnected. It might, therefore, be supposed that nothing was obtained that was new. In fact, however, this is a combination which is as common as the preceding, being employed to give a motion to a working pair which is too complex to be produced by simpler means, or which requires to be varied at pleasure. The working pair consists of two elements, one of which is supplied by one chain, the other by the other, and the motion of the pair is thus a combination of the motions of the two independent chains. Completely new motions are obtained in this way, and they may be varied at pleasure by alteration of either or both of the primary motions.

Take, for example, the common planing machine. The working pair consists of the table, upon which the work is mounted, and the tool. To the first a reciprocating movement is communicated by means of a suitable kinematic chain connecting it with the driving shaft. The other is mounted on a slide rest, forming an element of a screw chain, which gives it a horizontal movement. This chain has one link in common with the principal chain, but is otherwise independent. In the ordinary working of the machine this chain is locked by friction, except at the end of each reciprocating movement of the table when it moves to take the next cut. The tool thus traces out a complete plane surface.

In this example the common link is fixed, but this need not be the case, and in fact in the planing machine a third independent chain is added to adjust the tool vertically, the tool being mounted on a vertical slide having an independent movement. Also, one element of the working pair may be fixed and both movements given to the other, which is common to both chains. Double and treble chains of this kind occur whenever it is necessary to move the elements of the working pair into all possible positions. In cranes of all kinds we find a treble movement, one to raise and lower the jib, a second to swing the jib round, and a third to raise or lower the load. In traversing cranes the three movements are rectangular, as in the planing machine. In either case we find the methods employed by mathematicians to define the position of a point in space by rectangular or polar co-ordinates exactly imitated by the mechanism.

The elements of the working pair need not be wholly disconnected as we have hitherto supposed, they may form an incomplete kinematic pair. Thus if the axoids be cylindrical, endways motion may still be possible and may be given by an independent chain. A common example is a drilling machine, the working pair in which consists of a table on which the work is mounted, and a spindle carrying the drill which rotates and at the same time descends as the hole is drilled; the two movements may be quite independent, the one proceeding from a driving shaft, the other operated by the workman.

A similar combination is employed when a train is varied by shifting one of its links. Fig. 5, Plate III. (p. 152), represents a case of this kind. The wheel C' is mounted on a shaft which can be shifted endways by an independent mechanism. The shifting of belts (Art. 61, p. 143) is another example.

Again, the movements of the working pieces may be connected by a transmitting train connecting the chains which produce them. In the self-acting feeds of planing and shaping machines the connection is intermittent, but it may also be continuous, and we then have a fertile means of producing complex movements variable at pleasure. In a screw-cutting lathe the tool is mounted on a slide-rest moved by a screw, and the work is attached to a rotating mandrel. Connecting these independent chains by a train of "change" wheels, the tool cuts a screw of any pitch.

The principle of all combinations of this kind is the closure of an incomplete or disconnected pair by independent chains. We may describe them as Multiple Chains.

(3.) If two closed chains have two or more pairs common, they must be of the same kind, for otherwise the pairs would not have the same relative motion, and the chains would lock each other. The differential mechanisms, examples of which have been already given, are cases of this kind. Thus in the differential pulley (Fig. 62, p. 140), if A and C be disconnected we have two simple pulley chains with common moveable pulley B and separate axles. Either of these might be operated independently. In the actual mechanism A and C are united, and the movement of B is the difference of the movements due to each separate chain.

Complex examples of similar combinations occur in the epicyclic mechanisms. Fig. 82 (p. 191) shows a combination of two of the differential trains described on p. 150. C, C' are wheels turning about the same axis in the frame-link A and united; E, E' are also united, but have a different frame-link A'. Both gear with the wheels B, D, which are disconnected, but turn on an axis common to A and A'. On comparing this with Fig. 65 it will be seen that two trains have been compounded by uniting the wheels B, D, which are common to both. If now one of the frame links, say A', is fixed, and EE' be rotated, the other frame-link A will rotate with a velocity which can be found on the principles of the article cited. For simplicity, EE' have been supposed to gear directly with B, D, but they may also gear with wheels of other diameters fixed to B, D, or the wheels may be replaced by a different train of mechanism, all that is necessary being that the motions of the pairs BA', DA' should be connected.

Many examples of this mechanism may be found—especially in the case where C, C' are equal and the train reduces to three bevel wheels (pp. 151-2). In traction engines and tricycles, for instance, a mechanism of this kind is sometimes employed to facilitate turning. A' is then the frame of the machine, B and D are equal bevel wheels attached to the axle, which is divided into halves, each connected with one of the driving wheels. If now the motive power be applied to A, B and D will rotate, but not necessarily with the same velocity, and the machine may therefore be guided in a curve by the front wheel without the slipping which would occur if the driving wheels were fixed to an undivided axle.

Combinations of this class are not essentially different from multiple chains in which the elementary chains are connected by a train, as described above. They may be called Compound Trains; all consisting of simple trains compounded in various ways, either for non-kinematical reasons or to enable the train to be varied at pleasure.

(4.) All the preceding combinations are formed of simple closed chains united together in various ways; no new chain is obtained, but merely an aggregation of forms already known. Certain mechanisms, however, occur which if taken to pieces by separation of united links are found to contain one or more chains which are not closed.

Take for simplicity a common slider-crank mechanism, and imagine the crank pin, instead of being fixed to the crank, to be mounted on a slide so as to be free to move to and from the centre. The chain is now augmented by an additional sliding pair, and is no longer closed, so that it cannot be used as a mechanism. If, however, we introduce a screw, which moves the slide, we may lock the sliding pair in any position and thus obtain a closed chain, one link of which can be varied at pleasure. This mechanism is used in practice to obtain a varying stroke in a sliding piece. It is often required to make the stroke increase or diminish at each revolution of the crank. A wheel attached to the screw then comes in contact with a projecting piece and moves through a small space, the screw chain being locked by friction during the rest of the revolution. The mechanism thus varies at intervals its own law of motion.

By a suitable transmitting train however a continuous variation may be produced, and the combination then furnishes us with an entirely new mechanism. An important example is the wheel crank chain (Fig. 83), formed by combining a simple wheel chain with an open crank chain of five links. A number of mechanisms may

Fig. 82.

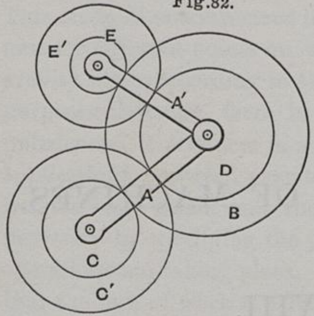
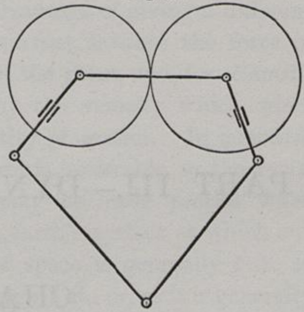


Fig. 83.



be derived from this chain by inversion, but for particulars the reader is referred to Reuleaux's work already cited.

Another example is shown in Fig. 6, Plate II. (p. 121), which represents a mechanism employed in sewing machines to give two strokes to a sliding piece for one revolution of a shaft. We have here a closed double slider chain combined with a single slider rendered incomplete by omission of the crank pin. Combinations of this class are called by Reuleaux "true" compound chains to distinguish them from the preceding classes, in which no new mechanism results from the combination. Perhaps the words "higher" and "lower" would more clearly express the meaning.

NOTE.—In Fig. 77, page 177, the blocks A, C revolve in the same direction, and the centroids are circles of infinite size. To represent them equal circles of finite size are employed, which give the same motion but in opposite directions.