CHAPTER IX.

DYNAMICS OF THE STEAM ENGINE.

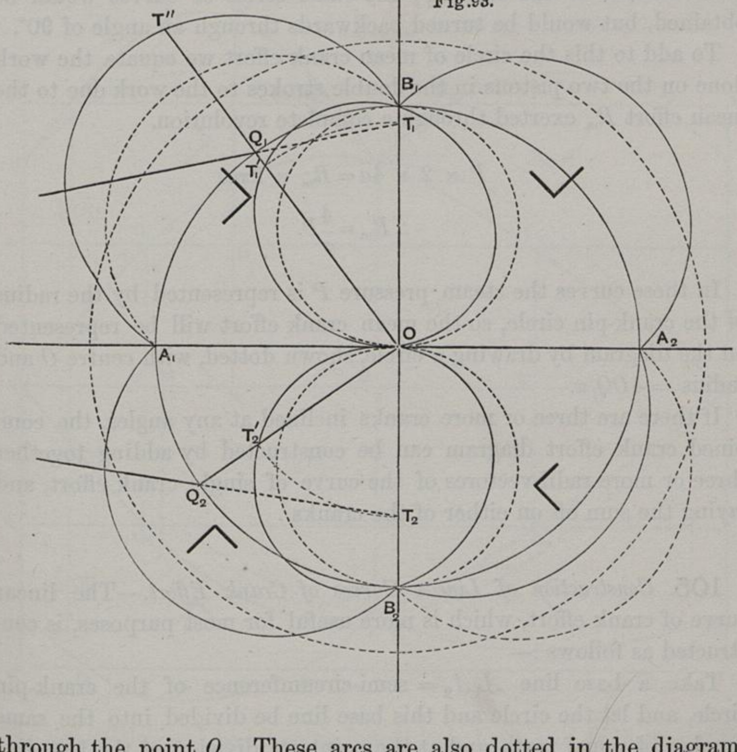
104. *Construction of Polar Curves of Crank Effort.*—One of the most common and important applications of the principles of the preceding chapter is to the working of steam engines, and we shall investigate this question, chiefly with reference to fluctuations of stress, energy and speed. Throughout, frictional resistances are neglected.

In Ch. V. a curve was constructed which shows the velocity ratio of piston and crank pin, and it has been proved (p. 204) that this curve must also give the ratio of the effort tending to turn the crank to the pressure of the steam on the piston, so that it may also be called a Curve of Crank Effort. If there are two or more cranks, the crank effort can be obtained by suitably combining the results for each taken separately, and a curve may then be drawn representing the combination. There are two kinds of such curves, the Polar and the Linear. First suppose two cranks at right angles, steam pressure uniform, and the same on both pistons. Let us commence with the polar curve.

Suppose $OT_1'B_1$, $OT_2'B_2$ (Fig. 93) to represent the polar curve of crank effort for an engine constructed as in Art. 49, and let the two cranks be in the positions OQ_1, OQ_2, each pointing towards the cylinder. Add together the corresponding crank efforts OT_1', OT_2', which are given by the curve, and set off their sum along OQ, we thus obtain a radius OT'', which represents the total crank effort for the two engines taken together. It may also be considered as the leverage at which the pressure on one piston must act to produce the same turning moment. Performing this construction for a number of positions of the cranks, we obtain a polar curve showing the crank effort in every position.

If the connecting rod is indefinitely long the single curve of crank effort consists of the pair of circles on OB_1, OB_2, shown dotted in the diagram. If we add together radii of these circles, the combined curve of crank effort will consist of four portions of circles passing the points $A_1B_1A_2B_2$; each of the circular arcs if produced would pass

Fig. 93.



through the point O. These arcs are also dotted in the diagram. When the crank is in a quadrant lying towards the engine, the actual crank effort is in excess of that due to a long connecting rod. So for the positions OQ_1,OQ_2, shown, for each the crank effort is in excess, and thus the curve of combined effort will for the quadrant A_1B_1 lie outside the circular arc. When the cranks are in the two upper quadrants the effort for the leading crank is less than when the connecting rod is long, whereas for the following crank it is greater; and the diminution of one is very approximately equal to the excess of the other; and the sum is the same as that, neglecting the shortness of the rod. The true combined effort is then for the quadrant B_1A_2 represented by the circle. In the next quadrant both are in diminution; and the true curve will lie inside the circle A_2B_2, while for the fourth quadrant it will again coincide with the circular arc.

We may, if we please, lay off the sum of the radii on the following crank instead of the leading; the same series of curves would be obtained, but would be turned backwards through an angle of 90°.

To add to this the circle of mean crank effort we equate the work done on the two pistons in the double strokes to the work due to the mean effort R_m exerted through a complete revolution.

$$P \times 2 \times 4a = R_m \times 2\pi a.$$

$$\therefore R_m = \frac{4}{\pi}P.$$

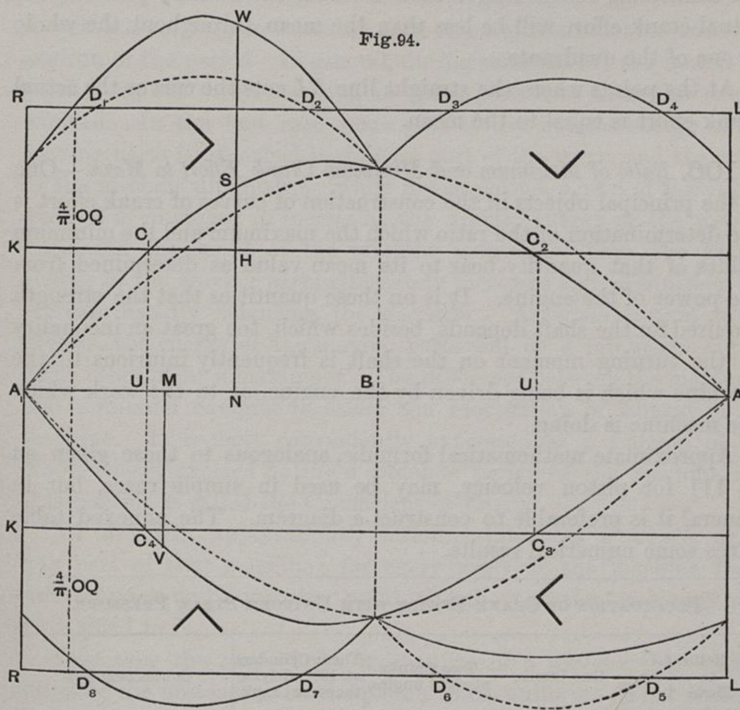
In these curves the steam pressure P is represented by the radius of the crank-pin circle, so the mean crank effort will be represented on the diagram by drawing a circle, shown dotted, with centre O and radius $= 4OQ/\pi$.

If there are three or more cranks inclined at any angles, the combined crank effort diagram can be constructed by adding together three or more radii vectores of the curve of single crank effort, and laying the sum off on either of the cranks.

105. *Construction of Linear Curves of Crank Effort.*—The linear curve of crank effort, which is more useful for most purposes, is constructed as follows:—

Take a base line A_1A_2 = semi-circumference of the crank-pin circle, and let the circle and this base line be divided into the same number of equal parts, and at the points of division of the base line set off ordinates such as SN, VM both above and below the base equal to lengths of the common ordinates of the single crank effort diagram such as OT'_1, OT'_2, and so we construct the linear crank effort diagram for a single crank. Neglecting the obliquity of the connecting rod, the diagram will consist of two curves of sines shown dotted, one above, the other below (Fig. 94). To get the combined crank effort diagram we have only to add together proper ordinates according to the angle between the cranks, just as we did in drawing the polar diagram. When the cranks are at right angles it will be seen that when the leading crank is, for example, at Q_1 or N the following crank is at Q_2 or M; and if the ordinate MV is laid off on the top of ordinate NS we obtain a point W on the curve of combined crank effort. If the same process be followed throughout we obtain the diagram shown in Fig. 94, consisting of four curves. If

Fig.94.



the connecting rod be taken as indefinitely long, and ordinates of the dotted curve be added together the combined diagram will consist of four curves, also curves of sines shown dotted in the diagram, all alike and all of the same height. But taking proper account of the shortness of the rod, we observe that for one quadrant of the revolution when both cranks lie towards the cylinder, each ordinate added is in excess of that, neglecting obliquity, and then we obtain the highest curve. In the next quadrant the height of the curve is less and is the same as if we neglected the shortness of the rod. In the next quadrant when both cranks are away from the cylinder the shortness of the rod makes the crank effort for each engine less, and we get a very low curve for the combination. This is followed in the last quadrant by a curve like the second.

The mean crank effort will be represented by a horizontal line at a height $4OQ/\pi$, as before. Setting off this line we observe that unless the connecting rod is longer than is usual in ordinary practice, the actual crank effort will be less than the mean throughout the whole of one of the quadrants.

At the points where the straight line RL cuts the curves the actual crank effort is equal to the mean.

106. *Ratio of Maximum and Minimum Crank Effort to Mean.*—One of the principal objects in the construction of curves of crank effort is the determination of the ratio which the maximum and the minimum values of that quantity bear to its mean value as determined from the power of the engine. It is on these quantities that the strength required for the shaft depends, besides which, too great an inequality in the turning moment on the shaft is frequently injurious to the machine which is being driven by the engine, or to the work which the machine is doing.

Approximate mathematical formulæ, analogous to those given on p. 111 for piston velocity, may be used in simple cases, but in general it is preferable to construct a diagram. The annexed table gives some numerical results.

Fluctuation of Crank Effort with Uniform Steam Pressure.				
Ratio to Mean for	One Crank.	Two Cranks at right angles.	Three Cylinders at 120°, driving the same Crank.	Connecting Rod.
Maximum.	1·57	1·112	1·047	Indefinitely long.
Minimum.	0	·785	·907	
Maximum.	1·62	1·31	1·077	Four Cranks.
Minimum.	0	·785	·794	

The great influence which the length of the connecting rod has on the results should be especially noticed; we shall return to this hereafter, but now go on to consider the motion of the engine under the action of the varying crank effort.

107. *Fluctuation of Energy.*—We have already referred to the periodic character of the motion of a machine, and explained that when the mean motion is uniform we have for a complete period

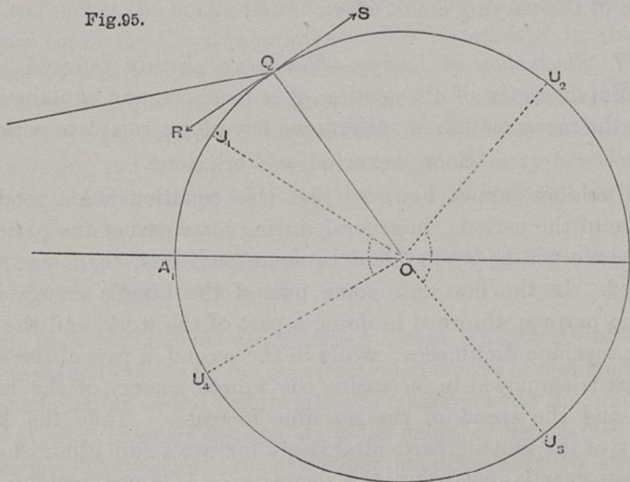
$$\text{Energy exerted} = \text{Work done.}$$

It will seldom happen however that this equation holds good for a portion of the period. In general, during some part of the period the work done will be greater, and in some part less, than the energy exerted. In the first case some part of the kinetic energy of the moving parts is absorbed in doing a part of the work, and the speed of the machine diminishes; while in the second, a part of the energy exerted is employed in increasing the kinetic energy of the moving parts and the speed of the machine increases. Thus the kinetic energy of the moving parts alternately increases and diminishes, the increase exactly balancing the decrease. At some instant in its motion, the energy of the moving parts will be a minimum, and at some other point a maximum. The difference between the maximum and minimum energies is called the Fluctuation of Energy of the machine. It is most conveniently expressed as a fraction of the whole energy exerted during a complete period of the machine, and this fraction is called the Co-efficient of Energy.

All this will apply to any machine taken as a whole, or to any part of that machine; for every piece of the machine has a driving point and a working point, and the equation of energy may be applied to it.

Take now the case of the mechanism of a direct-acting engine. Suppose the pressure P on the piston to be uniform. This through the connecting rod will produce a crank effort S, the magnitude of which for each position of the crank may be found as just now shown. To the crank and shaft S is the driving force and furnishes the energy exerted. At every point of the revolution of the shaft a certain resistance will be overcome, which resistance will tend to prevent the shaft from turning; it will not depend on the steam pressure, but on the sort of work that is being done. As the most simple ordinary case we will suppose the resistance overcome to be uniform, and we will neglect the inertia of the reciprocating parts (Art. 110). We may represent this constant resistance by a constant force R applied to the crank pin Q (Fig. 95), at right angles to the

Fig. 95.



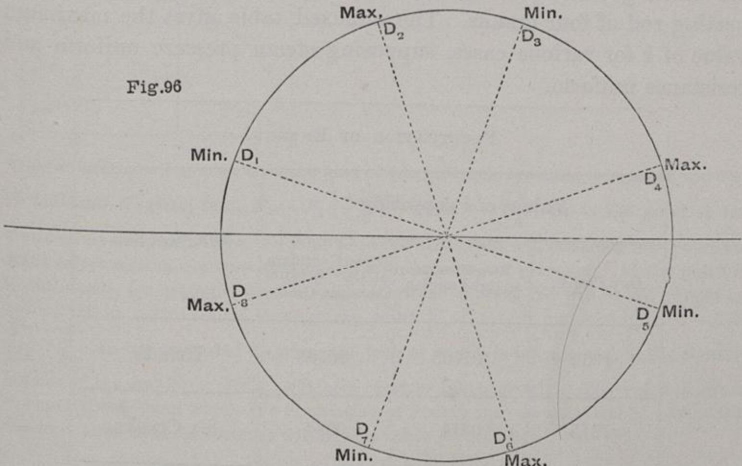
crank arm, resisting its motion. The magnitude of R is immediately determined by the application of the principle of work to a complete period, say one revolution. We have

$$P4a = R \times 2\pi a. \quad \therefore R = \frac{2}{\pi} P.$$

This constant resisting force is the same as the mean crank effort. Then, so long as $S > R$ the speed of the crank shaft will increase, and when $S < R$ it will diminish.

Referring to the linear curve of crank effort (Fig. 94, p. 223) let $A_1N =$ the arc A_1Q (Fig. 95), then $NS =$ crank effort S for this position of the crank. If an ordinate A_1K be set up to represent the constant resistance or mean crank effort, and a horizontal line parallel to base line be drawn, then NH being the representation of R the resistance overcome, the effort S will be greater for this position of the crank, and the difference HS will be employed in accelerating the motion of the machine. From the commencement of the revolution up to this position, the energy exerted is represented by the area A_1NS, whereas the work done is represented by the area A_1KHN. As the crank revolves from the position A_1 the crank effort increases until when at U_1 it is equal to the resistance. Up to this point the speed of rotation will have been diminishing. After passing the point U_1 the effort will be greater than the resistance and the speed of the engine will increase. Thus U_1 is a point of minimum speed at which the kinetic energy is a minimum. When the crank reaches the position U_2 the effort will again be equal to the resistance; and, since from U_1 to U_2 the effort has been greater than the resistance, during the whole of which time the engine has been increasing its speed, it follows that at the point U_2 the speed and the kinetic energy will have reached a maximum. The energy stored during this interval will be equal to the area C_1SC_2, and this will be the fluctuation of energy. During all the movement from U_2 to U_3 the speed of the engine will diminish, so that U_3 is another point of minimum kinetic energy. The kinetic energy stored from U_2 to U_3 is negative and represented by $C_2A_2C_3$, which quantity also is the fluctuation of energy. Again at U_4 the kinetic energy is a maximum. If the resistance had not been uniform, but its varying magnitude represented by the ordinates of some curve of resistance, then where the curve of resistance intersected the curve of crank effort would be the points where the kinetic energies would be maximum and minimum, as just explained. By the graphical construction of such a curve of resistance the fluctuation of energy may be estimated by measuring

Fig.96



the area of the crank-effort curve cut off above or below the curve of resistance, which area will lie between consecutive points of maximum and minimum energies. If the energy be E, the fluctuation of energy may properly be denoted by ΔE. It is convenient to express this as a fraction of the total energy $4Pa$ exerted in a revolution. We have then for the co-efficient of fluctuation of energy $\dfrac{\Delta E}{4Pa} = k$.

The value of k does not depend on the size of the engine, but only on the length of the connecting rod and the way in which the steam pressure and resistance vary. If the connecting rod is indefinitely long, steam pressure and resistance uniform, $k = \cdot 1052$. The shorter the connecting rod the greater will be the value of k.

An equally important case is that of two cranks at right angles also shown in Fig. 94. Neglecting the shortness of the connecting rod, then the line of resistance cuts each of the four curves in two points, the first of which is a point of minimum energy as shown in Fig. 96, on the preceding page. For this case $k = \cdot 01055$ or one-tenth of its value for a single crank: eight fluctuations of equal magnitude occur in each revolution. When the connecting rod is short the curves of crank effort are not the same in each quadrant (see Fig. 94), and one of them lies wholly below the line of resistance. There are then six fluctuations in each revolution: four of these are nearly the same as before, but the other two are much greater, the values of k being ·037 and ·042, with a connecting rod of four cranks. The annexed table gives the maximum value of k for various cases, supposing steam pressure uniform and resistance uniform.

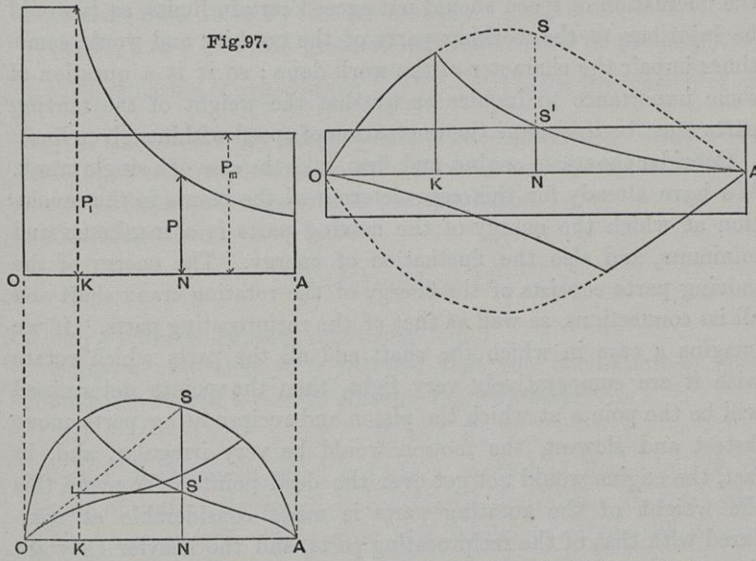
Fluctuation of Energy.

Values of k supposing			Length of Rod.
One Crank.	Two Cranks, at right angles.	Three Cylinders, at 120°, driving the same Crank.	
·1052	·01055	·00325	Infinite.
·1245	·0314	·0084	Six Cranks.
·1358	·0418	·0115	Four Cranks.

As before, the great influence of length of connecting rod on the results should be noticed. Frictional resistances, which are here neglected, generally increase the value of k.

In general the pressure of the steam in the cylinder of an engine varies throughout the stroke, and the construction of the curve of crank effort previously described must be modified on account of this. Suppose, instead of the steam being admitted throughout the stroke, it is cut off at a certain point and expanded so that the expansion curve is hyperbolic. For simplicity neglect the back pressure. At the point N in the stroke (Fig. 97) the pressure will have fallen to P, such that $\frac{P}{P_1}=\frac{OK}{ON}$. If we

Fig.97.



draw an ordinate P_m such that the area of the rectangle enclosed is equal to the area of indicator diagram, then $P_m=P_1\frac{1+\log_\epsilon r}{r}$ where $r=\frac{OA}{OK}$. Up to the point K the crank-effort diagram will be the same as previously described, but after that point the crank effort will be less than that due to a uniform steam pressure. At the point N in the stroke, for example, the crank effort instead of being NS will be NS', found by joining OS to cut a vertical through the point K of cut-off and making $NS'=KL$, $\frac{NS'}{NS}=\frac{P}{P_1}$. In the expanded diagram, the base of which is taken equal to the circumference of the crank-pin circle, ordinates must be taken equal to NS', and a diagram so constructed, from which the fluctuation of energy may be calculated. Assuming the resistance to be uniform, it will have a value R such that

$$R\pi a=P_m 2a=2aP_1\frac{1+\log_\epsilon r}{r},$$

$$R=\frac{2}{\pi}P_1\frac{1+\log_\epsilon r}{r};$$

and drawing a horizontal line above the base at a height to represent R, it will cut off an area above it which will be the fluctuation of energy. The diagram for the return stroke is shown below. It is not exactly the same as that for the forward stroke, because the effect of obliquity is different. A general method of procedure applicable with any given indicator diagram is explained at the end of this chapter.

108. *Fluctuation of Speed. Fly-Wheels.*—Fluctuation of energy in an engine or any other machine is necessarily always accompanied by a fluctuation of speed; but the heavier the moving parts the less will be the fluctuation of speed. In most cases it is necessary that the fluctuation of speed should not exceed certain limits, as it would be injurious to the working parts of the machine and would sometimes impair the character of the work done; so it is a question of some importance to inquire as to what the weight of the moving parts must be to confine the fluctuation of speed within a given limit.

Consider the steam engine, and, first, take the case of a single crank. We have already for this case determined the points in the revolution at which the energy of the moving parts is a maximum and minimum, and also the fluctuation of energy. The energy of the moving parts consists of the energy of the rotating crank shaft and all its connections, as well as that of the reciprocating parts. If we imagine a case in which the shaft and all the parts which rotate with it are comparatively very light, then the points determined will be the points at which the piston and reciprocating parts move fastest and slowest, the motion would be very irregular, and, in fact, the engine would not get over the dead points. To avoid this the weight of the rotating parts is made considerable as compared with that of the reciprocating parts, and the heavier they are the more uniform the motion of the engine will be. To increase the uniformity, the weight must generally be artificially increased by the addition of a heavy fly-wheel to the shaft, and the inertia of this is predominant over that of the other moving parts of the engine. For the present we may neglect the inertia of the reciprocating parts and consider the fly-wheel alone.

On this supposition the energy and speed of the fly-wheel will be greatest and least at the points previously described, viz., where the curve of crank effort is cut by the line of uniform resistance. Let W be the weight, V the velocity of rim of fly-wheel; then

$$\frac{WV^2}{2g} = \text{Energy of Rim.}$$

The energy of the arms and boss may be estimated by the addition of a percentage to the weight of the rim, or be considered as furnishing a margin in favour of uniformity. On account of the danger of fracture the speed of periphery V should not exceed 80 feet per second. This is the limit of speed commonly stated, but the liability to fracture depends very much on the straining action on the arms of the wheel due to inequality between the crank effort and the resistance, and not merely on centrifugal forces. (See Ch. XI.). In large wheels the rim is in segments, and the speed is not more than from 40 to 50 feet per second.

Let V_1 and V_2 be the greatest and least speed of periphery due to the fluctuation of speed, then $\frac{W}{2g}(V_1^2 - V_2^2)$ is the fluctuation of energy of the wheel. By the graphical process previously described we have been able to determine the fluctuation of energy in terms of the total energy E_0 expended in one revolution.

Equating these two we have

$$\frac{W}{2g}(V_1^2 - V_2^2) = kE_0,$$

where k is the co-efficient previously found.

Suppose now that it is required that the fluctuation of speed should not exceed a certain amount, then we may write

$$V_1 - V_2 = q \, . \, V_0,$$

where V_0 is the mean speed and q is a co-efficient depending on the degree of uniformity which is considered desirable. In some cases q must not exceed ·02 or even less, whilst in others ·05 or even more may be sufficient.

We may generally assume with sufficient accuracy that

$$V_0 = \frac{V_1 + V_2}{2}$$

(see next Article), then we find by substitution that, at the mean speed,

$$\text{Energy of Wheel} = \frac{k}{2q} \, . \, E_0.$$

In a single crank non-expansive engine the value of k ranges, as we have seen, from ·1 to ·14 when the resistance is uniform. In expansive engines k may be ·25 even with a uniform resistance, and when an engine is doing very irregular work k may be unity.

If we have a pair of cranks at right angles, the kinetic energy of the reciprocating parts is the same, at the same speed, for all positions of the cranks. (Ex. 4, p. 218.) Consequently these parts may be considered as so much added to the weight of the fly-wheel. Besides this the value of k is much less, seldom reaching ·1 if the resistance is approximately uniform. Hence a lighter fly-wheel may be used. The difference however is not so great as it might appear, for in estimating the weight of wheel required, it is important to consider not merely the change of speed, but also the time in which the change takes place. A small change taking place rapidly may be as injurious as a much greater change taking place slowly. The values of the acceleration and retardation at any instant are proportional to the difference between the crank effort and resistance at that instant, which can be found from tables such as that on page 224, and some regard should be paid to these numbers in considering what value of q should be employed.

In any case then we may write

$$\text{Energy of Wheel} = K \,.\, E_0,$$

where K is a co-efficient, which will vary within much narrower limits than the two co-efficients of speed and energy on which it depends. In general, in the very cases in which the resistance is most irregular a greater variation in speed is admissible.

The old rule for fly-wheels, dating from the time of Watt, was that the energy of the wheel should be 3·75 times the energy exerted per stroke. This corresponds to $K = 1{\cdot}875$, and would be satisfied by $k = 1$, $q = {\cdot}267$, or by $k = {\cdot}125$, $q = \frac{1}{30}{}^{\text{th}}$. The first of these cases would be a very irregular resistance with a great variation in speed, and the second a moderately uniform resistance with a uniformity of speed which would be sufficient for most purposes. Heavier wheels are not unusual in modern practice, and it may be here remarked that the minimum weight necessary may depend partly on the rigidity of the shafting.

There is another method of obtaining the fluctuation of energy which, though not practically so convenient, is for some purposes advantageous. A curve representing the energy exerted may be constructed in this way: Suppose the steam pressure P constant, then in the movement of the crank pin from A to Q the piston moves from A to N and the energy exerted $= P \times AN$, which will be proportional to AN. Now in Fig. 98 take a base line AA' equal to the semi-circumference, and at the various points, such as Q, set up ordinates $QK = AN$,

$A'A'' = AA'$, and so on; a curve $AKLA''$ will be obtained, which will represent by its ordinates the energy which has been exerted from the commencement up to the various points in the stroke. At the same time, the resistance being uniform,

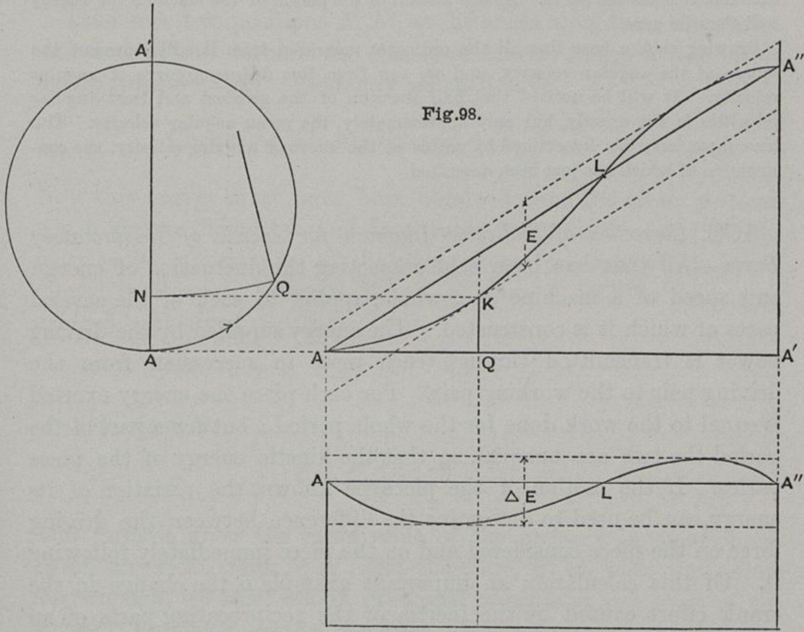


Fig.98.

the work done will be proportional to the length of the arc AQ, since work done $= R \times AQ$. If from the base line AA' we set up ordinates to represent the work done, a straight sloping line will be obtained. If the work done = energy exerted in the complete stroke, they will both be represented by the same ordinate $A'A''$, and so the sloping line will meet the curve at the point A''. The intercept between the curve and line AA'' measured on the vertical ordinate will at any point be the difference between the energy exerted and the work done reckoned from the commencement of the stroke up to that point, and what we have called the fluctuation of energy will be the vertical intercept between two tangents to the curve $AKLA''$ drawn parallel to AA''.

From this we can derive a curve which will represent the varying angular velocity of the crank; but, in order to simplify the measurement and description, let the vertical intercepts of the curve just described be laid off from a horizontal base line, as shown below.

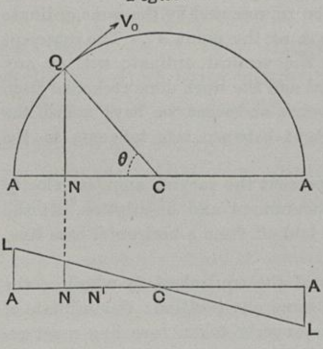
For suppose we know the moment of inertia of the equivalent fly-wheel of the engine and the angular velocity of the crank in some one position: the ordinate of the curve ALA'' at this point measured from a properly taken base line must represent the energy of the moving parts. Thus, if the base line be drawn in proper position, all ordinates measured from it will represent the square of the velocity of revolution of the crank shaft. If the speed of the machine is great, the base line will be some distance below the curve. On the other hand, if the speed is small, the base line will be close to the curve. There is manifestly a minimum speed at which the machine can be kept revolving ; it is that which corresponds to the case in which the base line touches the curve. At one instant of the period of the machine the energy will then be zero.

Drawing such a base line all the ordinates measured from it will represent the square of the angular velocity, and we can from this deduce a curve of angular velocity. It will be noticed that half the sum of the greatest and least angular velocities is not exactly, but only approximately, the mean angular velocity. The true mean may be determined by means of the curve of angular velocity, the construction of which has just been described.

109. *Correction of Indicator Diagram for Inertia of Reciprocating Parts.*—All that has been said respecting the fluctuation of energy and speed of a machine as a whole, applies to each of the several parts of which it is constructed. The energy supplied by the driving power is transmitted through each piece in succession from the driving pair to the working pair. For each piece the energy exerted is equal to the work done for the whole period ; but for a part of the period the two are unequal, so that the kinetic energy of the piece varies. If the motion of the piece be known, the variation of its energy can be used to determine the difference between the driving force on the piece considered and on the piece immediately following it. Of this calculation an important example is the change in the crank effort caused by the inertia of the reciprocating parts of an engine.

In this calculation we neglect, in the first instance, the obliquity of the connnecting rod, and suppose the crank to rotate uniformly. Let Q (Fig. 99) be the centre of the crank pin describing a circle AQA with velocity V_0, then the position of the piston is represented by N, and its velocity is

Fig.99.



$$V = V_0 . \sin\theta,$$

from which it follows that the kinetic energy of the reciprocating parts must be given by

$$\text{Kinetic Energy} = \frac{WV_0^2 \sin^2\theta}{2g} = \frac{WV_0^2}{2g}\left(1 - \frac{x^2}{a^2}\right),$$

where W is the weight of the piston, piston rod, and other reciprocating parts, and x is the distance of the piston from the centre of its stroke.

Take now two positions N, N', at distances x_1, x_2 from the centre, and find by this formula the change of kinetic energy as the piston moves from N to N'. Evidently we shall have

$$\text{Change of Kinetic Energy} = \frac{WV_0^2}{2g} \cdot \frac{x_1^2 - x_2^2}{a^2}.$$

Now this energy must have been obtained from the steam pressure which drives the piston and accelerates its motion. Let P be the mean value of that part of the whole steam pressure which is employed in this way between N and N', then $P \,.\, NN'$ is the energy exerted in this way, so that

$$P(x_1 - x_2) = \frac{WV_0^2}{2g} \cdot \frac{x_1^2 - x_2^2}{a^2},$$

or dividing by $x_1 - x_2$,

$$P = \frac{WV_0^2}{2g} \cdot \frac{x_1 + x_2}{a^2}.$$

This formula gives the mean value of the pressure in question between any two points N, N', and therefore, if we take the points near enough, we shall obtain the actual pressure at any point of the stroke. Putting $x_1 = x_2 = x$ we get

$$P = \frac{WV_0^2}{ga} \cdot \frac{x}{a}.$$

It is convenient to express our result as a pressure in lbs. per square inch by dividing by the area of the piston in square inches, then

$$p = p_0 \cdot \frac{V_0^2}{ga} \cdot \frac{x}{a},$$

where p_0 is the weight of the reciprocating parts divided by the area of the piston, or, as we may call it, the "pressure equivalent to the weight of the reciprocating parts."

When $x = a$ we get the pressure at the commencement of the stroke required to start the piston: here the pressure is greatest, and elsewhere varies as the distance from the centre. At the centre the pressure is zero: the piston then for the moment moves with uniform velocity and requires no force to change its motion. When past the centre the pressure is so much addition to the steam pressure because the piston is at every instant being stopped: this is shown by the formula, since x is then negative. All this is shown graphically by drawing a straight line LCL through C such that

$$AL = p_0 \cdot \frac{V_0^2}{ga}.$$

The ordinate of that straight line represents the pressure due to inertia for each position of the piston. After subtracting this from the actual steam pressure the effective pressure is found, which is transmitted to the crank pin, and furnishes the crank effort.

The value of p_0, the pressure equivalent to the weight of the reciprocating parts, varies considerably according to the size and type of engine, but in ordinary cases ranges from $1\frac{1}{2}$ to 3 lbs. per square inch. In return connecting rod engines, and in some other types where the reciprocating parts are exceptionally heavy, p_0 may reach $4\frac{1}{2}$ or 5 lbs. per square inch. This being given, the pressure due to inertia will vary inversely as the stroke and directly as the square of the speed; in the short-stroke high-speed marine engines common in the present day, the correction for inertia is sometimes very considerable. It is hardly necessary to say that it is only the value of the crank effort at particular points of the stroke which is affected. The mean value must remain unaltered, for any energy employed in overcoming inertia at one part of the stroke must be given out again at another part, so that the total energy exerted by the steam remains the same. Further, when there are a pair of cranks at right angles the total crank effort is little altered.

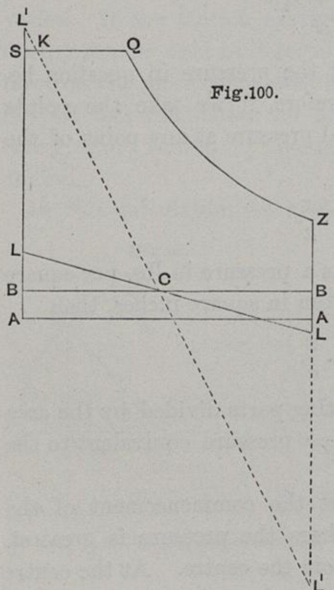


Fig. 100.

The effect is best seen by correcting an indicator diagram for the inertia of reciprocating parts in the following way. Consider, for simplicity, a theoretical indicator diagram (Fig. 100) $SQZA$, in which BB is the back-pressure line, QZ the expansion curve, then, but for inertia, the ordinates reckoned from BB of SQZ give the effective pressure of the steam. Set up BL equal to the pressure necessary to start the piston found above and draw the straight line LCL, then the actual effective pressure will be obtained by measuring the ordinates to the sloping base LCL instead of the original base BB. It will be seen that the general effect is to equalize the steam pressure throughout the stroke.

In engines running at a very high speed the pressure necessary to start the piston at the commencement of the stroke may be greater than the steam pressure (see Ex. 11, p. 243), which will be shown on the diagram by the point L rising above S, as shown by the dotted line $L'CL'$ of the figure. The direction of stress on piston rod and connecting rod is then reversed, which will produce a shock if the brasses are at all loose. This gives a limit to the speed with which the engine can safely be driven (see p. 244).

In obtaining the preceding results it has been supposed, first, that the crank rotates uniformly and, secondly, that the connecting rod is indefinitely long. To take account of the variation in the velocity of the crank, it would be necessary to draw a curve representing that velocity, and deduce from it a curve showing the kinetic energy of the piston in every position. In general, however, the inertia of the rotating parts will be sufficient to reduce the variation in speed within narrow limits, and the error caused by neglecting it may be disregarded. The effect of obliquity is of more importance: to obtain it we may either use the formula for piston velocity given on p. 111 instead of the simpler formula employed above (Ex. 13, p. 243), or we may derive a curve of kinetic energy from the known curve of piston velocity and take the differences of equidistant ordinates. For the sake of variety, however, we will employ a method depending on a different principle, which is perhaps more simple in practical application.

Divide the crank-pin circle into a number of equal parts, and supposing the connecting rods drawn, let them cut the vertical through 0 in the points $1'$, $2'$, $3'$ in Fig. 101. Also find and mark off the corresponding positions of the piston $1''$, $2''$, $3''$, &c. Now, since the lengths $01'$, $02'$, $03'$, &c., represent the velocities of the piston and reciprocating parts when in positions $1''$, $2''$, $3''$, &c., the difference between any two consecutive lengths, for example $1'$, $2'$, will represent the change of velocity that has taken place in the corresponding movement of the piston $1''$, $2''$. If we suppose the crank pin to revolve uniformly and divide the circle into equal parts, equal times will be occupied in the motions from point to point, and therefore equal times in the motions between consecutive positions $1''$, $2''$, $3''$, $4''$, &c., of the piston. Accordingly the differences $01'$, $1'2'$, $2'3'$, &c., will represent the force required to change the velocity of the reciprocating parts; and if we set them up as ordinates between the corresponding positions of the piston, we shall obtain the curve expressing the effect of inertia. The ordinate should be erected from the position of the piston when the crank-pin is at the middle of the intervals 1, 2, 3, &c.

It will be seen that the greater the number of parts into which we divide the crank-pin circle the less will be the ordinates representing the effect of inertia, though in all the curves the same character will be preserved. Accordingly it is possible to determine the number of parts into which the crank circle should be divided, or to determine the angle between consecutive radii, 01, 02, &c., such that the ordinates of the inertia curve be of such a length that they represent the pressure

Fig. 101.

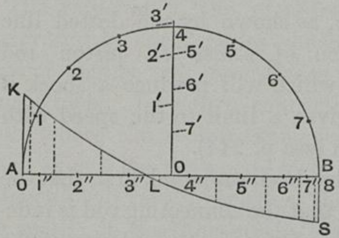
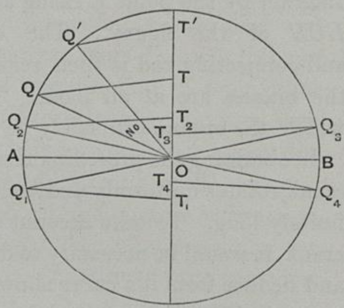


Fig. 102.



per square inch of piston area required for inertia on the same scale that the indicator diagram is drawn. The ordinates of the resulting inertia curve may then be directly employed to correct the indicator diagram.

Let N be the number of revolutions per minute; Q, Q' consecutive points on the crank-pin circle; and let $QOQ' = n°$. Further suppose that the crank-pin circle is drawn on a scale of x inches to the foot. Then

$$\frac{\text{change of velocity of piston}}{\text{velocity of crank pin}} = \frac{TT'}{OQ},$$

$$\therefore \text{ change of velocity of piston } \Delta v, \text{ in feet per second} = \frac{2\pi N}{60} TT',$$

where TT' is to be measured in feet on the scale x inches $= 1$ foot.

$$\therefore \Delta v = \frac{2\pi N}{60} \frac{TT'}{x}, \text{ where } TT' \text{ is to be measured in inches.}$$

Now this change of velocity takes place in the time occupied by the movement

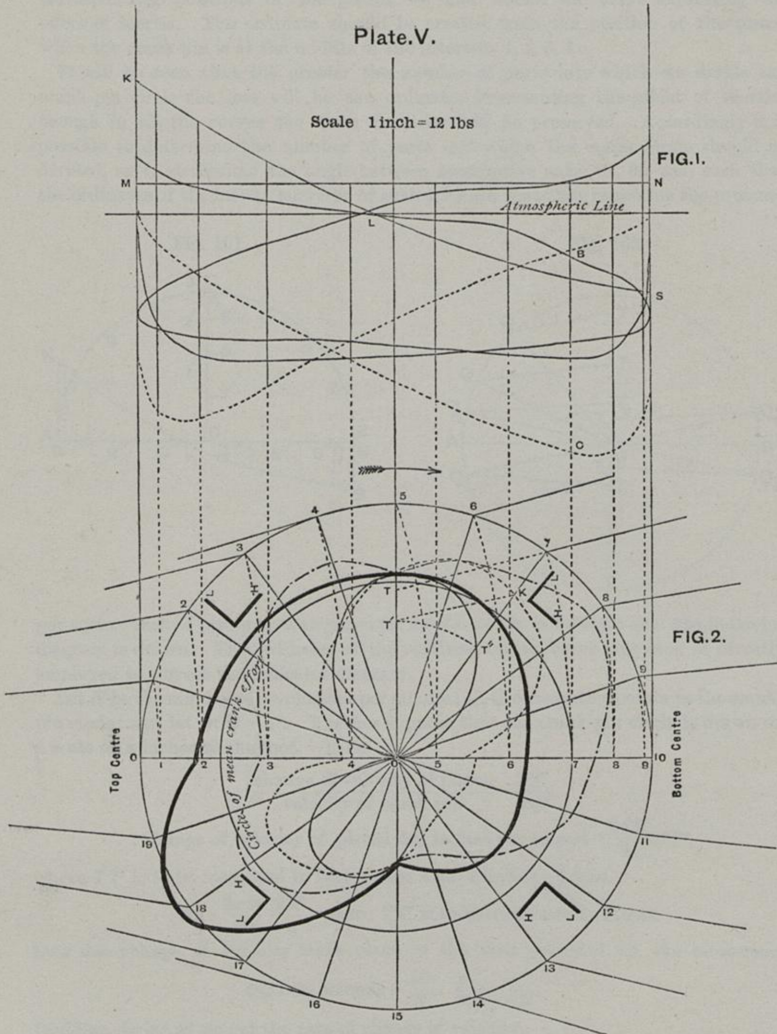
$$QQ' = \Delta t \text{ seconds} = \frac{60}{N} \cdot \frac{n°}{360} = \frac{n°}{6N}.$$

Dividing Δv by Δt we get the rate of change of velocity,

$$\frac{\Delta v}{\Delta t} = \frac{2\pi N}{60} \cdot \frac{(TT') \text{ inches}}{x} \cdot \frac{6N}{n°}.$$

Now the mass of the reciprocating parts $\times \dfrac{\Delta v}{\Delta t}$ will be the magnitude of the force due to inertia.

$$\therefore \text{ Force due to inertia} = \frac{W}{g} \frac{\Delta v}{\Delta t} = \frac{W}{g} \frac{2}{10} N^2 \frac{(TT' \text{ in inches}}{xn°}.$$



CO-EFFICIENTS—

Of Crank Effort, 1·72. Of Fluctuation of Energy, ·108.

To face page 239.

Divide both sides by the area of the piston, and let p = pressure per square inch due to inertia, which will be in lbs. if W is taken in lbs. Also let p_0 = pressure equivalent to weight of reciprocating parts in lbs. per square inch.

$$\therefore p = p_0 \frac{2\pi}{10g} N^2 \frac{(TT' \text{ inches})}{xn^\circ}.$$

If now the indicator diagram be drawn on a scale of y lbs. to the inch, the pressure p equivalent to inertia will be represented on the same scale by taking a length in inches—

$$\frac{p}{y} = p_0 \frac{2\pi}{10g} \frac{N^2}{xyn^\circ} (TT' \text{ inches}).$$

Now it is required that n° be so taken that p/y in inches shall be the same thing as TT' in inches. Consequently

$$p_0 \frac{2\pi}{10g} \frac{N^2}{xyn^\circ} = 1.$$

$$\therefore n^\circ = \frac{2\pi}{10g} p_0 \frac{N^2}{xy} = \cdot 0195\, p_0 \frac{N^2}{xy}.$$

If now we draw a number of cranks inclined to each other at an angle of n°, we may obtain as many points on the curve of inertia as we please.

In practice it will in general be sufficient if we determine the end ordinates AK, BS, and the point L (Fig. 101) and draw a fair curve through these points. The ordinates AK and BS will be determined if we take (Fig. 102) AOQ_1, AOQ_2, BOQ_3, BOQ_4 each equal $\frac{1}{2}n^\circ$, then Q_1OQ_2 and Q_3OQ_4 being each equal n°, T_1T_2 will equal AK and $T_3T_4 = BS$.

Further, the curve will cross the base line at the point L, at which the piston will have its maximum velocity, which will occur approximately when the crank is at right angles to the connecting rod.

$$\therefore OL = \sqrt{(\text{con. rod})^2 + (\text{crank})^2} - \text{connecting rod.}$$

110. *Construction of Curves of Crank Effort for any given Indicator Diagram.*—If the varying magnitude of the steam pressure is given by the actual indicator diagram of the engine we may deduce the true crank effort as follows :—Let Fig. 1, Plate V., be a pair of indicator diagrams. The examples chosen are from the low-pressure cylinder of H.M.S. "Nelson." * Before proceeding to make use of them they should be corrected for inertia, and, where the engines are vertical, for the weight of the reciprocating parts. The curve of pressure due to inertia is KLS in Fig. 1, which has been drawn, as just described, to the same scale as the indicator diagram. If we draw a line MN parallel to the base line of the inertia curve to represent p_0, the pressure due to the weight of the reciprocating

* I am indebted to Mr. T. Hearson for the example here given, and for the method of drawing the curve of inertia which has just been described.

parts, then the intercept between MN and KLS will be the necessary correction for inertia and weight combined. In applying the correction, the forward pressure in one of the pair of diagrams should be taken in conjunction with the back pressure of the other, for it is the difference between these which gives the true effective pressure on the piston. Let the dotted lower curves be the result of the correction, so that the virtual pressure which is transmitted to the crank pin is to be measured by the vertical intercept between the upper steam curve and the dotted curve, such as BC for example. Immediately below the diagram draw a crank-pin circle with diameter equal to the length of the indicator diagrams. Divide the crank-pin circle into, say 20, equal parts, and suppose the crank pin to be successively at these points of division; determine the corresponding positions of the piston in its stroke. Whilst doing this, mark the directions in which the connecting rod lies when the crank pin is in these several positions. Let the positions of the piston in the line of stroke be set off along the diameter 0, 10. Through these points draw verticals to intersect the indicator diagrams. The intercepts of these verticals will give us the virtual steam pressure at each of the points of the stroke and corresponding to each position of the crank in its revolution. Next, having in Fig. 2 drawn a number of radii through the points 1, 2, 3, &c., lay off from the centre O along each, the respective intercepts of the indicator diagram which represent the vertical pressures of the steam when the cranks are in those positions. We thus draw what we may call a polar curve of virtual steam pressure. We have for example taken OK equal to BC in the figure, and similarly for all other radii.

Now, referring to page 204, we observe that if the connecting rod in any position be drawn to cut the vertical through O, in a point T, as for example in Fig. 2 when the crank is at 7, then the length OT will represent the crank effort on the same scale that the length of the crank arm $O7$ represents the magnitude of the steam pressure. If now through K we draw KT' parallel to $7T$, then by similar triangles $\dfrac{OT'}{OK} = \dfrac{OT}{O7}$, and thus on the same scale that OK represents the steam pressure OT' will represent the crank effort. Now along the crank $O7$ set off a length $OT'' = OT'$, and perform a similar operation for each of the positions of the crank. If through the points so obtained we draw a continuous curve it will be the polar curve of crank effort which we require, for it will represent by its radii in any position the actual crank effort when the crank is in that position; and we see that, in the construction, account is taken not only of the angular position of the crank, but also of the steam pressure which is available for turning the crank. Taking both indicator diagrams we thus draw the curve for the complete revolution of the engine. By transfer of the radii of the polar curve to the crank circle unrolled we can construct a linear curve (Art. 105), and thus determine the fluctuation of energy.

In Fig. 2 the thick curve has been drawn to show the crank effort due to the high and low pressure cylinders combined, by adding to the radii of the original curve the corresponding radii of the high pressure curve (not shown in the figure). In this engine the high pressure crank is 90° in advance of the low: if it had been 90° behind the low the fluctuation of crank effort would have been less. This is shown by the large dotted curve in the figure. The circle of mean crank effort is added to facilitate comparison. The values of the co-efficients of crank effort and energy are given in the diagram.

111. *Periodic Motion of Machines in General.*—The motion of a steam engine, which we have been describing in detail in this chapter, may be taken as a typical example of the transmission of energy by any machine whatever. Neglecting frictional resistances the energy is transmitted without alteration from a driving pair to a working pair—when the complete period of the machine is considered; but the rate of transmission varies from instant to instant during the period. The alternate excess and deficiency of energy is provided for by the moving parts of the machine, which serve as a store of energy or "kinetic accumulator," which can be drawn upon at pleasure. For equable motion it is necessary that they should be sufficiently heavy, and that the rotating pieces should greatly predominate over the reciprocating pieces. If the speed be very great reciprocating pieces are to be avoided altogether, especially in cases of higher pairing with force closure (Ex. 17, p. 244).

It has been supposed that the mean resistance at the working pair is exactly equal to the mean effort at the driving pair. If this be not the case the machine will rapidly alter its mean speed, till the equality is restored by alteration of the effort or the resistance or both. The equality seldom exists for long, and some means of controlling the machine is therefore generally indispensable, but this is a matter for subsequent consideration.

EXAMPLES.

1. In the case of a pair of cranks at right angles, draw the polar diagram of crank effort when the connecting rod is indefinitely long, and find the ratio of maximum crank effort to mean. Find also the position of the cranks when the actual crank effort is equal to the mean.

Maximum crank effort = 1·11 mean.

2. Draw the diagram and obtain the results as in the last question, when the length of connecting rod is equal to 4 cranks.

Maximum crank effort = 1·307 mean.

3. Draw the linear diagram of crank effort, assuming two cranks at right angles and connecting rod = 4 cranks.

4. What is the maximum length of connecting rod for which the crank effort is less than the mean throughout one quadrant?

Connecting rod = 7·1 cranks.

5. From the diagram of crank effort constructed in question 3, determine the coefficient of fluctuation of energy, 1st. When the connecting rods are indefinitely long; 2nd. When the length equals 4 cranks.

Connecting rod indefinitely long. Co-efficient of fluctuation of energy = ·011.
Connecting rod = 4 cranks. Co-efficients are ·011, ·042, ·011, ·009, ·038, ·009.

6. A pair of engines of 500 h.p., working on cranks at right angles with connecting rods = 4 cranks, are running at 70 revolutions per minute. Find the maximum and minimum moments of crank effort, and the fluctuation of energy in ft.-lbs.; assuming the steam pressure and resistance uniform.

Maximum moment of crank effort = 49,125 ft.-lbs.
Minimum moment of crank effort = 29,465 ft.-lbs.
Mean moment of crank effort = 37,500 ft.-lbs.

Fluctuation of energy = 9,900 ft.-lbs. Co-efficient = ·042.

7. In the case of a single crank the steam is cut off at one-fourth of the stroke. Neglecting back pressure and inertia, find the ratio of maximum to mean crank effort, and also the ratio of the fluctuation of energy to the energy of one revolution.

Maximum = 2·9 mean crank effort.
Fluctuation of energy = $\frac{1}{4}$ energy of one revolution.

8. Construct a diagram of crank effort for three cranks at angles of 120°. The lines of stroke of the three pistons are parallel, the steam pressure constant, and the resistance uniform. Find the ratio of maximum to mean crank effort, and the coefficient of fluctuation of energy for a connecting rod of 4 cranks.

Maximum = 1·077 mean crank effort.
k = ·0115.

9. In a pair of cranks at right angles, connecting rod 4 cranks long, the reciprocating parts have a stroke of 4 feet and weigh 20 tons. The steam pressure is uniform, and equal to 50 tons on each piston, and the resistance moment is uniform. Find the least number of revolutions the engines can make without the aid of a fly-wheel, and draw a curve of angular velocity ratio for this case.

Ans. At the point of maximum speed the least number of revolutions will be 50 per 1′. To obtain the curve and the least number of *complete* revolutions, see p. 233.

10. The pressure equivalent to the weight of the reciprocating parts of an engine is 4 lbs. per square inch, the stroke is 4 feet. Find the pressure necessary to start the piston, when the engines are making 75 revolutions per minute. If the steam pressure be initially at 30 lbs. above the atmosphere, and the cut-off at $\frac{1}{4}$th the stroke, find the effective pressure at each eighth of the stroke, taking account of the inertia of the piston, and assuming a constant back pressure of 3 lbs.

Pressure equivalent to inertia at commencement of stroke = 15·3 lbs. per sq. in.

Effective pressure	at	commencement	= 26·4
,,	,,	1st eighth	= 30·3
,,	,,	2nd ,,	= 34·0
,,	,,	3rd ,,	= 23·
,,	,,	4th ,,	= 19·4
,,	,,	5th ,,	= 18·7
,,	,,	6th ,,	= 19·5
,,	,,	7th ,,	= 21·2
,,	,,	8th ,,	= 23·5

11. In the last question find the number of revolutions per minute necessary to produce a shock near the commencement of the stroke. If the steam be cut off at $\frac{1}{8}$th, or earlier, show that a shock occurs also at other points of the stroke. *Ans.* 124.

12. In question 10 construct a curve showing the kinetic energy of the piston at each point of the stroke, and deduce a curve showing the pressure due to inertia of the piston.

Take the curve of piston velocity previously constructed, and PN being any ordinate of it, the kinetic energy of the piston will be proportional to the square of PN, so we have only to draw a curve whose ordinates vary as $(PN)^2$.

Having drawn the curve of kinetic energy, take the difference between consecutive equi-distant ordinates of that curve and set them as an ordinate from a new base line AB as Cd, and so construct a curve whose ordinates will be proportional to the pressure equivalent to inertia.

13. By use of the formula

$$V = V_0\left(\sin\theta + \frac{1}{n}\,.\,\sin\theta\,.\,\cos\theta\right)$$

(page 111) for the velocity of the piston, prove that the pressure necessary to start and stop the piston at the ends of the stroke is given by

$$p' = p_0\frac{V_0^2}{ga}\left(1 + \frac{1}{n}\right).$$

14. Draw a curve of kinetic energy of an oscillating cylinder, assuming a mean radius of gyration for the cylinder and piston, and deduce the bending moment on the piston rod.

NOTE.—The force of inertia in this case is so great that the speed of oscillating engines is limited.

15. If n be the revolutions per minute of a fly-wheel and d its diameter : show that the weight of wheel necessary for a given regularity in an engine of given indicated power is

$$W = C \cdot \frac{IHP}{n^3 d^2},$$

where C is constant.

NOTE.—The diameter is generally about $3\frac{1}{2}$ times the stroke (S), and according to a well-known empirical rule for piston speed (V) employed in calculating nominal horse-power $V^3 \propto S$. If this be assumed $n^3 d^2$ is constant, and the weight of wheel is then proportional to the indicated horse-power, a rule sometimes employed, 100 lbs. being allowed for each horse-power.

16. The fluctuation of energy of an engine of 150 $I.H.P.$ is 13 per cent. of the energy exerted in one revolution. The revolutions are 35 per minute, find the weight of a fly-wheel 20 feet in diameter, that the fluctuation in speed may not exceed one-fortieth. *Ans.* 8 tons.

17. In the cam movement shown in Fig. 1, Plate IV., page 173, suppose the cam a circular disk of radius equal to the stroke of the sliding piece. Supposing the force of the spring twice the weight of sliding piece : find the greatest number of revolutions per 1′ the mechanism can make when the cam rotates uniformly.

Ans. If S be the stroke in inches, n the revolutions,

$$n = \frac{216}{\sqrt{S}}.$$

18. In a 3-cylinder Brotherhood engine, the stroke is S inches, the revolutions n per 1′, the total pressure on one piston P; show that, to avoid reversal of the stress on the piston rod, the weight of a piston and rod must not exceed

$$W = 70{,}500 \frac{P}{n^2 S}.$$

NOTE.—In a double-acting engine there is necessarily reversal at the ends of the stroke : in the Brotherhood this is avoided by the use of 3 cylinders at 120°, the inner ends of which communicate constantly with a central chamber containing steam at full pressure. These engines therefore may run at a very high speed if the cut off at the outer end be sufficient.