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CHAPTER X.

FRICTIONAL RESISTANCES.

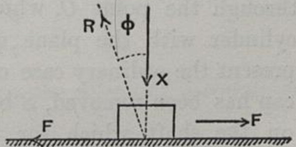
112. *Preliminary Remarks.*—The action of a machine consists, as we have seen, in a transmission of energy from a driving pair to a working pair, through a number of intermediate pairs, which change in a given way the motions proper to the source of energy. In the absence of friction, the energy transmitted from piece to piece in a complete period would be the same for all the pairs, but, in consequence of frictional resistances, a certain part of the energy is lost at each transmission. These frictional resistances are of two kinds, one due to the relative motion of the elements of the pairs one upon another, the other to the changes of form which the flexible parts of the machine undergo, for example to the bending of ropes and belts. It is to the first kind that the word "friction" is specially appropriated, although it is not essentially different from the second kind which in some cases is also called "stiffness."

We commence with the case of linkwork mechanisms in which the friction is due simply to the sliding of one surface upon another. The pairing is in this case of the lower class.

SECTION I.—EFFICIENCY OF LOWER PAIRING.

113. *Ordinary Laws of Sliding Friction.*—If one body rests on another (Fig. 102) and is pressed against it with a force X, a mutual action takes place between the two which resists sliding. The magnitude of this mutual action or tangential stress (Ch. XII.) is measured by the force F which is necessary to produce sliding, and the ratio F/X is called the co-efficient

Fig.102.



of friction and will be denoted by f. The value of f depends on the nature and condition of the surfaces in contact, whether rough or smooth, dry or lubricated. Under certain circumstances and within certain limits it is independent of the area of the surfaces in contact and of the velocity of sliding. These statements may be called the "ordinary" laws of friction. The evidence on which they rest and the limitations to their truth will be considered hereafter; for the present we assume them as applicable to all the cases we consider.

The work done in overcoming friction may be estimated just as in the case of any other resistance. If the body move through a space x the work done is Fx or $f.Xx$ if X be uniform, and if it be not, a curve is constructed giving X at every point, then the area under that curve multiplied by the co-efficient f is the work done (see Ex. 2). If R be the re-action of the surface upon which the body we are considering rests, ϕ the angle its direction makes with the normal to the plane,

$$R \,.\, \cos\phi = X : \; R \,.\, \sin\phi = F;$$
$$\therefore \tan\phi = f,$$

an equation which shows, that the total mutual action between two plane surfaces, which slide over one another, makes an angle with the normal to the plane, the tangent of which is the co-efficient of friction. The magnitude of this angle then is fixed, but its direction varies according to the direction of the sliding. It may therefore be called the "friction angle," but it is also often called the "angle of repose," because it is the greatest inclination of a plane on which the body can rest under the action of gravity without slipping. In the solution of questions respecting friction, graphically or otherwise, it is often convenient to suppose it known.

114. *Friction of Bearings.*—Next suppose the surfaces in contact cylindrical. In Fig. 103 ABA represents a cylinder pressed down into a semicircular bearing by a force S, the direction of which passes through the point O, which is the intersection of the axis of the cylinder with the plane of the paper. We may take this to represent the ordinary case of a shaft and its bearing from which the cap has been removed, S being the resultant of all the forces acting on the shaft which for the moment are supposed to have no tendency to turn the shaft. The force S is balanced by the reaction of the bearing which, when the bearing is in good condition, consists of a pressure distributed over the whole semicylindrical surface. Let DE be a small element of the surface,

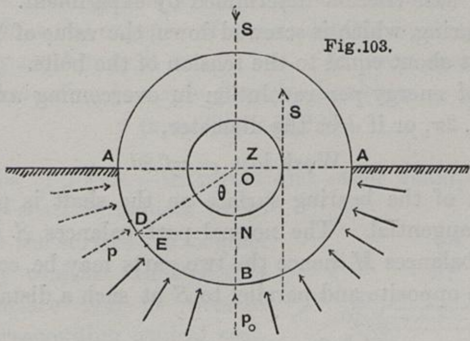


Fig. 103.

p the pressure, θ the angle the radius of DE makes with the direction of S, then we must have

$$\Sigma p\, DE \cos\theta = S.$$

If now we knew the law according to which p varies from point to point, we could by use of this equation find the actual value of p and also find the total amount of the distributed pressure, that is to say, $\Sigma p.\, DE$ which we will call X. Evidently then we shall have

$$X = k\,.\,S,$$

where k is a co-efficient depending on the law of distribution and therefore to some extent uncertain. When a bearing is well worn it is probable that (see Art. 115) if p_0 be the pressure at B

$$p = p_0\,.\,\cos\theta,$$

that is, that the intensity of the pressure at any point varies as ON the distance of the point below the centre. This is the same law as that which the pressure of a heavy fluid follows, supposed occupying the semicylinder ABA, and it is shown in books on hydrostatics that

$$\frac{\text{Total pressure}}{\text{Resultant pressure}} = \frac{4}{\pi} = k.$$

Next suppose the shaft to be turned by the action of a couple M applied to it, then if a be the radius

$$M = \Sigma f\,.\,p\,.\,DE\,.\,a = f\,.\,Xa = fk\,.\,Sa.$$

In this formula we have some doubt as to the value of k, and we are not sure that the co-efficient f would be the same for a curved as for a plane surface; we therefore replace fk by f', where f' is a special co-efficient of axle friction determined by experiment. If there is a cap on the bearing, which is screwed down, the value of S is increased by an amount about equal to the tension of the bolts.

The loss of energy per revolution in overcoming axle friction is evidently $M \cdot 2\pi$, or if d be the diameter,

$$\text{Work lost} = \pi f' S d.$$

The reaction of the bearing surface on the shaft is partly normal and partly tangential. The normal part balances S and the tangential part balances M, hence the two parts may be combined into a single force opposite and parallel to S at such a distance z from O that

$$Sz = M, \text{ or } 2z = f'd,$$

that is to say, the line of action of the mutual action between the shaft and its bearing always touches a circle, the diameter of which is f' times the diameter of the shaft. This circle is called the Friction Circle of the shaft or pin considered. When the bearing has a cap on, the force S must be increased by the tension of the bolts in calculating M, but not for any other purpose, and the diameter of the friction circle is consequently increased, it may be very considerably. The utility of this rule will be seen presently.

The real pressure between a shaft and its bearing varies from point to point, as we have seen. What is conventionally called the "pressure on the bearing" is something different. Let l be the length of the bearing, then ld is the area of the diametral section, and

$$p = \frac{S}{ld}$$

is the quantity in question. It is a sort of mean value of the actual pressure, and will bear some definite relation to it depending on the law of pressure. For the particular law of pressure given above

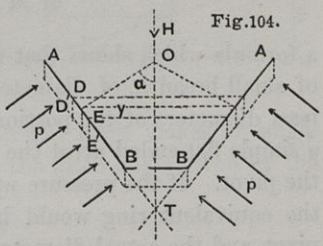
$$p = p_0 \cdot \frac{\pi}{4}.$$

The work lost by friction per square inch of bearing surface per 1′ is evidently proportional to pv, where v is the rubbing velocity in feet per minute. An equivalent amount of heat is generated as we shall see hereafter, and it is upon the rate at which this heat can be abstracted by the cooling influences to which the bearing is exposed that the amount of bearing surface required depends. In marine engine bearings the value of pv is sometimes as much as 60,000, though at the expense of a considerable liability to heating, and in railway machinery it is not less. At lower speeds the value is smaller. According to a rule given by Rankine,

$$p\,(v + 20) = 44{,}800.$$

115. *Friction of Pivots.*—In pivots and other examples in which the revolving shaft is subject to an endways force the surfaces in contact are frequently conical. In Fig. 104 a conical surface AB is pressed against a corresponding conical seating by a force H, and revolves at a given rate. If the surface be divided into rings, one of which is seen in section at DE, the pressure on those rings may be resolved vertically upwards, and must then balance H. Hence if p be the pressure on DE a ring the radius of which is y,

Fig.104.



$$\Sigma p \,.\, DE \,.\, 2\pi y \cos \alpha = H,$$

where α is the angle a normal to the conical surface makes with the axis.

When the bearing is somewhat worn the conical surface will have descended through a certain space, and it may be assumed that all points such as DE will descend through an equal space, so that the wear of the surface measured normal to itself is proportional to $\cos \alpha$. But if v be the velocity of rubbing of the ring DE, the wear will be proportional to pv, that is to py: hence

$$py \propto \cos \alpha.$$

This principle determines the most probable distribution of the pressure on worn surfaces in any case, and has already been used above for the case of a journal. In the present case α is constant, and we have

$$py = \text{constant} = p_1 y_1 = p_2 y_2,$$

where the suffixes 1 and 2 refer to the upper and lower edge; hence, by substitution, if l be the length AB of the conical surface,

$$py \,.\, 2\pi l \,.\, \cos \alpha = H,$$

a formula which determines the pressure at every point. The moment of friction is evidently

$$M = f\Sigma pDE2\pi y^2$$
$$= f \,.\, py \,.\, 2\pi \,.\, \Sigma y \,.\, DE = \frac{fpy2\pi\Sigma y\Delta y}{\cos\alpha},$$

where Δy is written for the projection of DE on the transverse plane. By use of the integral calculus this is readily seen to be

$$M = fpy2\pi\frac{y_1^2 - y_2^2}{2\cos\alpha} = fpy2\pi l \,.\, \frac{y_1 + y_2}{2},$$
$$\text{or } M = f \,.\, H \,.\, \frac{y_1 + y_2}{2\cos\alpha},$$

a formula which shows that the friction is the same as that of a ring of small breadth, of diameter equal to the mean of the greatest and least diameters of the portion of a cone considered. In the case of a simple flat-ended pivot the equivalent ring is half the diameter of the pivot. If the pressure were uniform throughout, the diameter of the equivalent ring would be $\frac{2}{3}$ instead of $\frac{1}{2}$ the diameter of the pivot, and the actual diameter in practice will probably vary between these limits.

Pivots are sometimes used in which the surfaces in contact are not cones, but are curved, so that in wearing the pressure and wear are the same throughout (Schiele's pivots). That this may be the case we must have, since p is constant,

$$y \propto \cos\alpha,$$

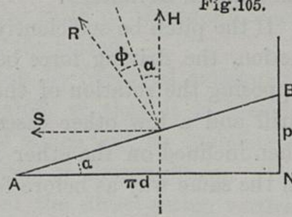
that is to say, if we draw a tangent DET to meet the axis in T, ET must be constant. The curve which possesses this geometric property is called the "tractrix." It is traced readily by stepping from point to point, keeping the tangent always of the same length. Pivots of this kind are very suitable for high speeds, as the wear is very smooth.

116. *Friction and Efficiency of Screws.*—In any case of a machine in steady motion the principle of work takes the form (Art. 96)

$$\left.\begin{array}{c}\text{Energy exerted}\\ \text{in a period}\end{array}\right\} = \left\{\begin{array}{l}\text{Useful work done + Work wasted}\\ \text{in overcoming frictional resistance.}\end{array}\right.$$

The simplest case is that of a screw which we will suppose to be square threaded and applied to a press, or to some similar purpose. The pressure between the nut and the thread is distributed uniformly along the thread, if the screw be accurately constructed and slightly worn. As shown in the last article in the similar case of a pivot, the friction may be regarded as concentrated on a spiral traced on a cylinder the diameter of which may be expected to be about the mean of the external and internal diameter of the screw. Fig. 105 shows one convolution of this spiral unrolled. AB is the thread, BN, parallel to the axis of the screw, is the pitch p, and AN is the circumference πd. H is the thrust of the screw, being the force which the screw is overcoming by means of a couple applied to turn it about its axis. R is the action of the screw thread which (Art. 113) makes an angle ϕ with the normal, where ϕ is the angle of repose. The normal itself makes an angle α with the axis of the screw, where α is the pitch angle given by the formula

Fig.105.



$$\tan \alpha = \frac{p}{\pi d}.$$

This force R arises from the turning forces applied to the screw, and must have the same moment M about the axis of the screw; its vertical component therefore must be H and its transverse component a force S such that

$$S \cdot \frac{d}{2} = M.$$

Hence the equations

$$M = \frac{Rd}{2} \cdot \sin(\alpha + \phi),$$

$$H = R \cdot \cos(\alpha + \phi).$$

Also considering a complete revolution of the screw,

$$\text{Energy exerted} = M \cdot 2\pi = R\pi d \cdot \sin(\alpha + \phi),$$

$$\text{Useful work done} = H \cdot p = Rp \cdot \cos(\alpha + \phi),$$

from which it follows that the efficiency of the screw is

$$\text{Efficiency} = \frac{\tan \alpha}{\tan(\alpha + \phi)}$$

It is not difficult to show that this fraction is greatest when $\alpha = 45^\circ - \frac{1}{2}\phi$, and its value is then

$$\text{Maximum efficiency} = \left(\frac{1 - \frac{1}{2}f}{1 + \frac{1}{2}f}\right)^2 \text{ approximately.}$$

For ordinary values of f then, the best pitch angle is approximately 45° and the efficiency is considerable.

In practice, however, the pitch angle is much smaller, its value in bolts and the screws used in presses ranging from ·035 in large screws to ·07 in smaller ones; the efficiency is then less, often much less, than one third, the object aimed at being not efficiency but a great mechanical advantage.

If the pitch be sufficiently coarse, it will be possible to reverse the action, the driving force being then H and the resistance a moment opposing the rotation of the screw. In a well known kind of hand drill and a few other cases this occurs in practice; the force R is then inclined on the other side of the normal, and the efficiency is in the same way as before found to be

$$\text{Efficiency} = \frac{\tan\,(\alpha - \phi)}{\tan\,\alpha}.$$

In most cases, however, α is less than ϕ, and the screw is then incapable of being reversed. Non-reversibility is often a most valuable property in practical applications, the friction then serving to hold together parts which require to be united or to lock a machine in any given position.

In estimating the efficiency of screw mechanisms the friction of the end of the screw acting like a pivot or of the nut upon its seat must be included; in screw bolts this item is generally as great as the friction of the threads. The friction due to lateral pressure of the screw on its nut may usually be neglected, but when necessary it may be estimated by the same formula as is used for shafts. The above investigation, strictly speaking, applies only to square-threaded screws; it has, however, been shown that the efficiency is only slightly diminished by the triangular or other form of thread usually adopted for the sake of strength.* The formulæ here given for screws may be applied to any case of a sliding pair in which the driving effort is at right angles to the useful resistance. A simpler case is that in which the driving effort is parallel to the direction of sliding. This is given in Example 1, page 271. In all cases observe that the efficiency diminishes rapidly when the velocity-ratio is increased. This, which is common to most mechanisms, limits the

* *Cours de Mécanique Appliquée aux Machines*, par J. V. Poncelet, p. 386. Paris, 1874.

mechanical advantage practically attainable. The hydraulic press is an exception, as will be seen hereafter.

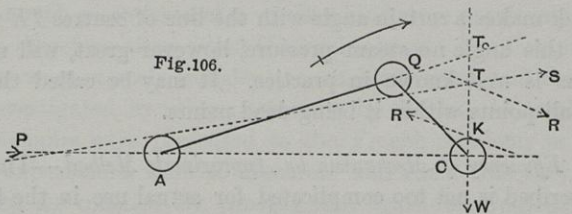
117. *Efficiency of Mechanism by Exact Method.*—In the preceding cases the efficiency is the same for any motion of the mechanism whether large or small. Generally, however, it will be different in each position of the mechanism, and by the "efficiency of the mechanism" is then to be understood the ratio of the useful work done in a period to the energy exerted in the period.

The exact calculation of the loss of work by frictional resistances in mechanism is generally very complicated, so that it is best to proceed by approximations the nature of which will be understood on considering an example with some degree of thoroughness. The case we select is that of the mechanism of the direct-acting vertical steam engine such as is represented in Plate I., p. 119.

The losses by friction are (1) the loss by piston friction, (2) friction of guide bars, (3) friction of crosshead pin, (4) friction of crank pin, (5) friction of crank-shaft bearings. Of these, the first two are considered separately (Ex. 2, p. 271), and for the present neglected, while the last three are treated by a graphical method as follows.

In Fig. 106 CQA are the friction circles of the three parts in

Fig.106.



question, which for the sake of clearness are drawn on a very exaggerated scale while the bearings themselves are omitted. We will neglect the weight of the connecting rod and its inertia; of these the first is generally relatively inconsiderable, but in high-speed engines the last is often very large and makes the friction very different at high speeds and low speeds (see Ch. XI.) The weight of the crank shaft and all the parts connected with it is supposed to act through the centre of the shaft; for simplicity we will call it W. The pressure on the piston after correction for piston and guide-bar friction is denoted by P. Then, in the absence of friction, the line of action of the thrust on the connecting rod is the line joining the centres of the friction circles, and the moment of crank effort is $P.CT_0$, where T_0 is the intersection of that line with the vertical through C. But the line of action in question must now touch the friction circles (Art. 114), and the true moment of crank effort on the same principle must be $P.CT$, where T is the intersection of this common tangent with the vertical CT. Thus $P.TT_0$ is the correction for friction of the crosshead and crank pins. Next observe that the forces acting on the crank shaft are W the weight and S the thrust of the connecting rod; these may be compounded into one force R passing through T as shown in the diagram. The reaction of the crank-shaft bearing is an equal and opposite force R which must touch the friction circle and cut CT in a certain point K. Now the horizontal component of R is the same as that of S, namely P; therefore the true moment of crank effort after allowing for friction is $P.TK$.

By performing this construction for a number of positions, as in the last chapter, we obtain a diagram of crank effort corrected for friction. The area of this curve will give us the useful work done in a revolution, the ratio of which to the energy exerted is the efficiency of the mechanism: and its intersections with the line of mean resistance will give the points of maximum and minimum energy and the fluctuation of energy as corrected for friction. When the crank makes a certain angle with the line of centres TK vanishes. Within this angle no steam pressure, however great, will move the crank, as is well known in practice. It may be called the "dead angle," all points within it being dead points.

118. *Efficiency of Mechanism by Approximate Method.*—The process just described is not too complicated for actual use in the foregoing example, but in many cases it would be otherwise, and it may therefore be frequently replaced with advantage by a calculation of the efficiency of each of the several pairs of which the mechanism is made up taken by itself.

Each pair consists of two elements, one of which transmits energy to the other, with a certain deduction caused by the friction between the elements. The ratio of the energy transmitted to the energy received may be called the efficiency of the pair. If e_1, e_2, e_3 ... be the efficiencies of all the pairs in the mechanism it is evident from the definition that the efficiency of the whole mechanism must be

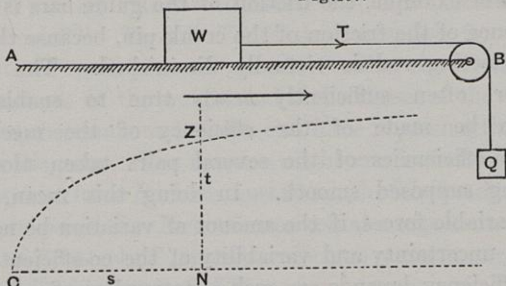
$$e = e_1 . e_2 . e_3 \ldots$$

In some cases the efficiency of each pair will be independent of the frictional resistances of all the other pairs, and may be found separately. In general this is approximately, but not exactly, true, a point which will be best understood by a consideration of the foregoing diagram. For example, the friction of the guide bars is diminished in consequence of the friction of the crank pin, because the obliquity of the connecting rod is virtually diminished. The supposition is, however, often sufficiently nearly true to enable a rough estimate to be made of the efficiency of the mechanism by finding the efficiencies of the several pairs taken alone, all the others being supposed smooth. In doing this mean values are taken for variable forces, if the amount of variation be not considerable. The uncertainty and variability of the co-efficients on which frictional efficiency depends are such as to render refined calculations of little practical value.

119. *Experiments on Sliding Friction* (*Morin*).—The ordinary laws of friction, which may be comprised in the single statement that the co-efficient of friction depends on the nature of the surfaces alone, and not on the intensity of the pressure or on the velocity of rubbing, were originally given by Coulomb in a memoir, published in 1785, although some facts of a similar kind were previously known. They are therefore often called Coulomb's laws. Yet Coulomb's experiments were scarcely sufficient to establish them, and the subject was reinvestigated by others, especially by the late General Morin, whose memoirs were presented to the French Academy in 1831-4. Morin's experiments were so elaborate and exact that they may be considered as conclusively proving the truth of Coulomb's laws within certain limits of pressure and velocity, and under the circumstances in which they were made: it will therefore be advisable to explain them briefly.

A sledge loaded with a given weight was caused to slide along a horizontal bed AB more than 12 feet long (Fig. 107), the rubbing surfaces being formed of the materials to be experimented on. The necessary force was supplied by a cord passing over a pulley at B to a descending weight Q. The tension of the cord T was measured by a spring dynamometer, and could likewise be inferred from the magnitude of the weight after correction for the stiffness of the cord and the friction of the pulley. In one form of experiment the weights were so arranged that the sledge moved nearly uniformly: the corresponding friction was measured and found to be constant. In a second form, the times occupied by the sledge in reaching given

Fig.107.



points were automatically measured and compared with the spaces traversed, by setting them up as ordinates of the curve *CZ* shown below. The curve proved to be a parabola, showing that the space varied as the square of the time, from which it was inferred that the acceleration of the sledge was constant.

From both methods it appeared that the co-efficient of friction was exactly the same, whatever the pressure and whatever the velocity, provided the nature and condition of the surfaces were the same. A few important results are given in the annexed table; they are taken from Morin's latest memoir,* containing, besides many new experiments, tables of the results of the whole series. The limits to their application will be considered presently.

Nature of Surfaces.	Condition of Surfaces.	Co-efficient of Friction.
Wood on Wood,	Perfectly dry and clean, - -	·25 to ·5
Metal or Wood on Metal or Wood,	Slightly oily, -	·15
Do. do.,	Well lubricated,	·07 to ·08
Do. do.,	Lubricant constantly renewed,	·05

* Nouvelles Experiences . . . . faites à Metz en 1834. Page 99.

Full tables of Morin's results will be found in Moseley's work cited on page 267. The friction between surfaces at rest is often greater than when they are in motion, especially when the surfaces have been some time in contact: the excess, however, cannot be relied on, as it is liable to be overcome by any slight vibration.

120. *Exceptions to the Ordinary Laws in Plane Surfaces.*—From the exactitude with which Coulomb's laws were verified by Morin's experiments the inference was naturally drawn that they were universally true, but this is probably erroneous. Although no complete and thorough investigation has been made, it can hardly now be a matter of doubt that there are cases in which the laws of friction are widely different. The known cases of exception for plane surfaces may be grouped as follows:—

(1) At low pressures the co-efficient of friction increases when the pressure diminishes. This has been shown by various experimentalists, as, for example, by Dr. Ball.* The lowest pressure employed by Morin was about three fourths of a lb. per square inch, and this is about the pressure at which the deviation noticed by Ball becomes insensible. This effect may be due to a slight adhesion between the surfaces independent of friction proper.

(2) At high pressures, according to certain experiments by Rennie,† the co-efficient increases greatly with the pressure. The upper limit of pressure in Morin's experiments was from 114 to 128 lbs. per square inch. At 32·5 lbs. per square inch Rennie found for metallic surfaces at rest ·14 to ·17, nearly agreeing with Morin; but on increasing the pressure the co-efficient became gradually greater, ranging from ·35 to ·4 at pressures exceeding 500 lbs. per square inch. The metals tried were wrought iron on wrought and cast iron, and steel on cast iron. Tin on cast iron showed only a slight increase in the co-efficient. In fully lubricated surfaces in motion we shall see presently the results are exactly opposite. This increased friction at high pressures may be due to abrasion of the surfaces.

(3) At high velocities the co-efficient of friction, instead of being independent of the velocity, diminishes greatly as the velocity increases. This was shown by M. Bochet in 1858. Similar results

* *Experimental Mechanics*, by R. S. Ball, page 78. Macmillan, 1871.

† *Phil. Trans.* for 1829.

have been obtained by others, especially by Capt. Galton in some important experiments on railway brakes.* The limit of velocity in Morin's experiments was 10 feet per 1″, and at somewhat greater velocities than this the diminution becomes perceptible. Morin's results have been shown to be applicable at the very lowest velocities by Professor F. Jenkin and Mr. Ewing.†

It appears difficult to explain the diminution at high speeds merely by a change in the condition of the surfaces; it should, probably, be regarded as part of the law of friction. Professor Franke in the *Civil Ingenieur* for May, 1882, has proposed the formula

$$f = f_0 \cdot \epsilon^{-av},$$

where f_0 is about ·29, and a (for velocities in metres per 1″) ranges from ·02 to ·04, according to the nature and state of the surfaces.

121. *Axle Friction.*—It has already been pointed out that the co-efficient of axle friction is not necessarily the same as that for plane surfaces sliding on one another, and, besides, the continuous contact of a shaft and its bearing is very different from the brief contact occurring in sledge experiments. Morin however made special experiments on the friction of axles and showed that the co-efficients were constant and nearly the same in the two cases. The diameters employed however were 4 inches and under, while the revolutions did not exceed 30 per minute, so that the rubbing velocity was not more than 30 feet per minute. The pressures were not great, the value of pv not exceeding 5,000.

Much greater values of pv than this occur in modern machinery, and then it is tolerably certain that the value of the co-efficient is much less and diminishes with the pressure. Already in 1855 M. Hirn had made a long series of experiments on friction, especially of lubricated surfaces. The following summary of his results is given by M. Kretz, editor of the third edition of the *Mécanique Industrielle.*‡

(*a*) That a lubricant may give a regular and minimum value to the friction it must be "triturated" for some time between the rubbing surfaces.

* See *Engineering*, vol. 25, pages 469–472.

† *Phil. Transactions*, vol 167, part II.

‡ *Introduction à la Mécanique Industrielle*, par J. V. Poncelet. Troisième édition. Paris, 1870. Page 516.

(*b*) The friction of lubricated surfaces diminishes when the temperature is raised, other things being equal.

(*c*) With abundant lubrication and uniform temperature friction varies directly as the velocity. When the temperature is not maintained uniform, the relation between friction and velocity depends on the law of cooling of the special machine considered. In ordinary machinery friction varies as the square root of the velocity.

(*d*) The friction of lubricated surfaces is nearly proportional to the square root of the area and the pressure.

The last result is equivalent to saying that the co-efficient of friction varies inversely as the square root of the pressure per unit of area. It is remarkable that this law has also been deduced by Professor Thurston from experiments made apparently without any knowledge of what Hirn had done* with pressures from 100 to 750 lbs. per square inch and a velocity of 150 per 1′.

It may be open to question whether Hirn's experiments are sufficient to establish all the above statements, but it cannot be doubted that for values of pv exceeding 5000 the co-efficient of friction of well lubricated bearings of good construction diminishes with the pressure, and may be much less than the value at low speeds as determined by Morin. How far the diminution can be regarded as due to a change of condition consequent on continuous wear is uncertain.

We now proceed to consider higher pairing, commencing with the case of rolling contact. The friction is then described as "rolling friction."

SECTION II.—EFFICIENCY OF HIGHER PAIRING.

122. *Rolling Friction.*—When a wheel rolls on soft ground the resistance to rolling is due to the fact that the wheel makes a rut and depresses the ground as it advances over it. Thus the resistance to motion is proportioned to the product of the weight moved into the depth of the depression. The depth of the rut depends on the radius as well as the breadth of the wheel. It is found that the resistance may be expressed by

$$R=\frac{bW}{r},$$

where W = weight, r = radius of wheel, and b is approximately a constant length. This might have been anticipated, since the depth

* *Friction and Lubrication*, by R. H. Thurston. New York, 1879.

of the rut is the versed sine of the arc of contact, and therefore for a given small arc is inversely as the radius. If the wheel roll on hard ground over a succession of obstacles of small height the law of resistance will be expressed by the same formula.

When the surface rolled over is elastic and the pressure on it is not sufficient to produce a permanent rut, the resistance to rolling is not so easily explained. If we consider an extreme case, as for instance a heavy roller rolling on india-rubber, we shall be able to see to what action the resistance is due. The wheel will sink into the rubber, which will close up around it both in advance and behind as shown in Fig. 108. At C the rubber will be most compressed.

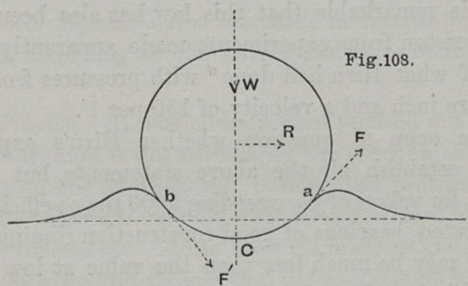


Fig. 108.

As the wheel advances and commences to crush the rubber in advance of it the rubber moves away to avoid the compression, heaping itself up continually in advance of the wheel. In this movement it rubs itself over the surface Ca of the wheel, exerting on it a frictional force in the direction shown by the arrow F, which opposes the onward motion of the wheel. Again, the rubber in the rear is continually tending to recover its normal position and form of flatness, and in doing so rubs itself over the surface bC of the wheel in the direction shown by the arrow F', which also tends to oppose the onward motion of the wheel. The effect of this creeping action of the rubber over the surface of the wheel is to cause the onward advance of the centre of the wheel to be different from that due to the circumference rolled out.* Moreover the vertical component of the reaction of the surface no longer passes through the centre of the wheel as it must do in the absence of friction, but is in advance by a small quantity b such that Wb is the moment of resistance to rolling.

* See a paper by Prof. Osborne Reynolds, *Phil. Trans.*, vol. 166, to whom the true explanation of resistance to rolling in perfectly elastic bodies is due.

Experiments on rolling resistance present considerable discrepancies, but within the limits of dimension of rollers which have been tried it appears that b is independent of the radius; this leads to a formula of the same form as before for the force necessary to draw the roller, namely

$$R = \frac{Wb}{r},$$

where b is a constant which for dimensions in inches is from ·02 to ·09 according to the nature of the surfaces. With very hard and smooth surfaces of wood or metal, the lower value ·02 may be employed. Rolling friction is not sensibly diminished by lubricants, but depends mainly on smoothness and hardness of the surfaces. It is probably influenced by the speed of rolling, but this does not appear to have been proved by experiment unless in cases where the resistance of the atmosphere and other causes make the question more complicated.

In many cases of rolling the surfaces are partly elastic and partly soft, so that the resistance to rolling is partly due to surface friction and partly to permanent deformation. The value of the constant b is then much increased. For wagon wheels on macadamized roads in good condition the value of b is about ·5″, and on soft ground four to six times greater. The draught of carts is said to be increased by the absence of springs.

123. *Friction of Ropes and Belts.*—Frictional resistances are also produced by the changes of form and dimension of the parts of a machine occasioned either by the stresses necessarily accompanying transmission of energy or by shocks. In the present chapter we consider tension elements only, that is to say, chiefly ropes and belts.

In Fig. 109 AB is a pulley, the centre of which is O, over which a rope passes embracing the arc AKB and acted on by forces $T_1 T_2$ at its ends. If there be sufficient difference between T_1 and T_2 the rope will slip over the pulley notwithstanding the friction which tends to prevent it. Let the rope be just on the point of slipping, then its tension will gradually diminish from T_1 at A to T_2 at B. Let T, T' be the tensions at the intermediate points K, L, then the portion KL of the rope is kept in equilibrium by the forces T, T' at its ends, and a third force S due to the reaction of the pulley, the three forces meeting in a point E.

On OL set off to Ol to represent T, and draw lk perpendicular to S to meet OK in k, then the sides and the triangle Okl will be proportioned by the three forces, so that Ok represents T' and ak S. The angle S makes with the radius will be the same for all arcs of the same length, and if KL be taken small enough will be the angle of friction (Art. 113).

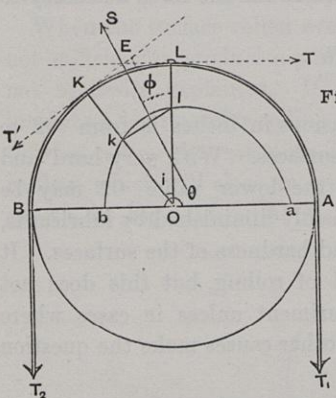


Fig.109.

This construction can, if we please, be commenced at A and repeated for a number of small portions of the rope till we arrive at B; we shall obtain a spiral curve $alkb$, the last radius Ob of which represents T_2 on the same scale as the first Oa represents T_1. It is convenient however to have an algebraical formula to calculate T_2. Let the angle KOL be i and the angle S makes with the radius ϕ, then

$$\frac{T}{T'}=\frac{Ol}{Ok}=\frac{\sin\ Okl}{\sin\ Olk}=\frac{\cos\ (i+\phi)}{\cos\ \phi}=\cos\ i+\sin i \tan\ \phi.$$

If now the angle i be diminished indefinitely we may write $\cos\ i=1$ and $\sin\ i=i$, so that

$$\frac{T-T'}{T'}=i\,.\tan\ \phi.$$

Replacing i by $\Delta\theta$, $T-T'$ by ΔT, and proceeding to the limit

$$\frac{1}{T}\frac{dT}{d\theta}=\tan\ \phi=f,$$

which being integrated gives

$$\frac{T_1}{T_2}=\epsilon^{f\theta},$$

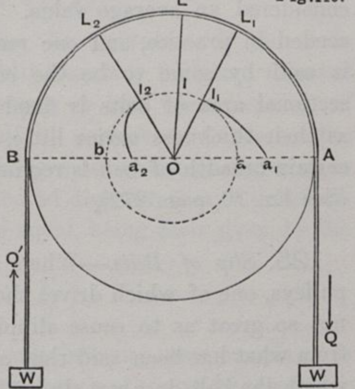
where f is the co-efficient of friction, θ the angle subtended by the part of the pulley embraced by the rope, and ϵ the number 2·7288 being the base of the Napierian system of logarithms. The formula is applicable even if the pulley be not circular. For a circular pulley the spiral curve, representing graphically the tension at every point, is the equiangular or logarithmic spiral of which the formula may be regarded as the equation. In constructing it graphically, the value of ϕ, for a small yet finite angle i, is found by replacing T/T' by ϵ^{fi} and expanding the exponential: we thus get approximately

$$1 + fi = \cos i + \sin i \,.\, \tan \phi = 1 - \tfrac{1}{2} i^2 + i \,.\, \tan \phi,$$
$$\therefore \tan \phi = f + \tfrac{1}{2} i.$$

With small values of the co-efficient $2f$ may be a sufficiently small angular interval, but in general it will be advisable to take the angular interval equal to the angle of friction, then the value of ϕ is $1\frac{1}{2}$ times that angle. The construction being one in which errors accumulate, the formula is preferable when great accuracy is desired.

124. *Driving Belts.*—When a belt is stretched over a pulley by equal weights, the tension of the belt is not necessarily the same everywhere in the first instance; but if the pulley move steadily and the stiffness of the belt be disregarded, it must be so. Assuming this, let one of the weights be increased by a certain quantity Q and the pulley be held fast, then the tension of that side of the belt will be increased by an amount equal to Q at A, but diminishing to zero at L, a point determined by the intersection of the friction spiral $a_1 l_1$ (Fig. 110) with the circle alb, the radius of which represents the weight W.

Fig.110.



Similarly, if the other weight be diminished by Q', the tension will be diminished by an amount equal to Q' at B, but diminishing to zero at L_2. The portion L_1L_2 will remain at the original tension W. If QQ' be increased sufficiently, L_1, L_2 will coincide in one point L, the position of which will depend on the proportion between Q and Q'. While these changes take place in tension, corresponding changes of length must occur in the parts of the belt exposed to them, AL increases and BL_2 diminishes in length. Hence both these parts slip over the pulley and work is lost by friction, while L_1L_2 remains fixed. If now, instead of altering the weights W, we imagine these weights held fast and the pulley forcibly rotated so as to increase A's tension by Q, and diminish B's tension by Q', L_1L_2 will rotate with the pulley, and the total increase of length of the one side must be equal to the total diminution on the other, from which consideration it is possible to calculate the ratio Q bears to Q'. In practical cases, however, the difference between Q and Q' is so small that it may be neglected without sensible error, and therefore, in all questions relating to the working of belts, it may be assumed that the mean tension of the two sides of the belt is independent of the power which is being transmitted. The difference of tensions, however, is directly proportional to the power, and may at once be calculated if the speed be known, while the ratio of tensions may be determined, so that the belt shall just not slip, by means of the formula above obtained. The value of the co-efficient of friction of leather on iron ranges from ·15 to ·46 according to the degree of lubrication: under ordinary circumstances ·25 may be considered an average value. This, however, is often greatly exceeded in practice, and one reason why large values are admissible is said by some to be the effect of atmospheric pressure. The sectional area of belts is fixed by considerations of strength, and as their thickness varies little, this is equivalent to saying that a certain breadth of belt is required for each horse-power transmitted. (See Ex. 10, page 272).

125. *Slip of Belts.*—When a belt is stretched over a pair of pulleys, one of which drives the other, notwithstanding a resistance not so great as to cause slipping of the belt as a whole, it appears from what has been said that a certain arc exists on each pulley on which the belt does not slip. The length of these arcs has already been found, but in the present cases the movement of the pulleys causes them to place themselves where the belt winds on to the pulleys, so that the driving pulley has the speed of the tight side of the belt and the driven pulley that of the slack side. The two sides have different speeds, because the same weight of belt must pass a given point in a unit of time, wherever that point be situated, and therefore the speed must be greater the greater the elongation, that is to say the greater the tension. Hence the driving pulley moves quicker than the driven pulley by an amount which can be calculated when the tensions and the elasticity of the leather are known, and this "slip" measures the loss of work due to the creeping of the belt over the pulleys described above. In ordinary belting

this loss is small, not exceeding 2 per cent. The length of belts, however, must not be too great, or its extensibility will be inconvenient, especially if the motion of the machine be not sufficiently uniform.* Within moderate limits extensibility is favourable to smooth working.

126. *Stiffness of Ropes.*—When a rope is bent it is found that a certain moment is required to do it depending on the dimensions of the rope and, besides, on its tension. The reason of this is best understood by referring to the corresponding case in a chain with flat links united by pin joints. If d be the diameter of the pin, T the tension of the chain, there will be a certain moment of friction resisting bending which, if the pin be an easy fit, will be simply $\frac{1}{2} fTd$, but if it be tight will be

$$M = \tfrac{1}{2}fTd + \tfrac{1}{2}fT_0d,$$

where T_0 is a constant depending on the tightness. If the chain pass over a rotating pulley without slipping, this frictional moment has to be overcome both when bending on and when bending off the pulley. The effect shows itself by a shift outwards on the advancing and inwards on the retiring side of the chain, so as to increase the leverage of the resistance and diminish that of the effort. In the present case the two shifts are equal, being each given by the formula

$$x = \tfrac{1}{2}fd\left\{1 + \frac{T_0}{T}\right\}.$$

The case of a rope differs from this only in being more complex: in the act of bending, the fibres move over each other, and the relative motion is resisted by friction due to pressures which are partly constant and partly proportional to the tension. The shift of the centre line of the rope is visible on the side of the resistance, but hardly perceptible on the side of the hauling force, showing that most of the loss of work is due to the bending on the pulley. The magnitude of the shift varies so much according to the mode of manufacture and the condition of the rope that it is useless to attempt more than a very rough estimate. According to a formula given by Eytelwein, if d be the diameter of the rope,

$$x = c \,.\, d^2,$$

* See a footnote by M. Kretz, *Cours de Mécanique Appliquée aux Machines par Poncelet*, page 264.

where c is a constant, which for dimensions in inches is taken as ·47 for hemp ropes; but this value is too large, except for light loads, and small diameters of pulley. The loss of work per revolution is $T \cdot 2\pi x$, and if D be the effective diameter of the pulley,

$$\text{Efficiency} = \frac{D}{D+2x}.$$

There is a loss of work by the stiffness of belts of a similar kind, but of uncertain amount. By most authorities it is considered so small as to be negligible.

The shift of the line of action of the tension of a rope due to its stiffness has the effect of diminishing its strength.

127. *Friction of Toothed Wheels and Cams.*—The friction of toothed wheels is partly rolling and partly sliding, but the first is relatively small and may be neglected. To determine the sliding friction, let $PT = z$ (see Fig. 71, page 161), then (page 166) the velocity of rubbing is given by the formula

$$v = (A + A')z,$$

which may be written, if V be the speed of periphery of the pitch circles, R, R' the radii,

$$v = z\left\{\frac{1}{R} + \frac{1}{R'}\right\}V.$$

If, therefore, the wheels be supposed to turn through a small space δx measured on the pitch circles, the pair of teeth will slide on one another through the small space δy, given by the formula

$$\delta y = \left(\frac{1}{R} + \frac{1}{R'}\right)z\delta x.$$

This enables us to find the work done in overcoming friction, for if P be the pressure between the pairs of teeth,

$$\text{Work done} = f\int Pdy = f \cdot \left(\frac{1}{R} + \frac{1}{R'}\right)\int Pzdx.$$

The pressure between the teeth will vary as the wheels turn according to some unknown law, depending on the way the teeth wear, but the variation is probably not great. Assuming it constant, and further, supposing that the chord PT (Fig. 71) is equal to the arc PT, and therefore to x the arc turned through by the wheels after the teeth pass the line of centres,

$$\text{Work done} = f \cdot P \cdot \left(\frac{1}{R} + \frac{1}{R'}\right)\frac{x^2}{2}.$$

The same formula applies before the line of centres, and if we assume the arcs of approach and recess each equal to the pitch p, we shall have for the whole work lost by the friction of a pair of teeth,

$$\text{Whole Work lost} = fP\left(\frac{1}{R} + \frac{1}{R'}\right)p^2.$$

The energy transmitted during the action of a pair of teeth is $2Pp$, therefore the counter efficiency is

$$1 + e = 1 + f\left(\frac{1}{R} + \frac{1}{R'}\right)\frac{p}{2} = 1 + f\pi\left(\frac{1}{n} + \frac{1}{n'}\right),$$

where n, n' are the numbers of teeth in the wheels. A smaller arc of action is sometimes employed in practice, and the friction will then be less. This is also the case in bevel gear. The formula shows that the friction is diminished by increasing the number of teeth.

A more exact solution of this question * can be obtained on the assumption that P varies as it would do if there were only one pair of teeth ; but as this is uncertain it is not practically useful.

In all cam and wheel mechanisms the efficiency for a small movement in any position can be determined exactly by a graphical or other process. For the velocity ratio can be found, as shown in Part II., and the force-ratio is determinate by the principles of statics, therefore the quotient which gives the efficiency can also be found. In the case of toothed wheels this method shows at once † that the friction of the teeth before the line of centres is greater than the friction after the line of centres. The difference appears insufficient to account for the injurious effects generally ascribed to friction before the line of centres, which however may be due to other causes. In cam mechanisms the efficiency in one position is little guide to the efficiency in a complete period, which can only be found by a process too intricate to be useful, or by making some supposition as the mean value of the pressure between the rubbing surfaces.

The counter efficiency of a train of m equal pairs of wheels is

$$1 + e = 1 + mf\pi\left(\frac{1}{n} + \frac{1}{n'}\right).$$

Assume now that a given velocity-ratio is to be provided by the train, and that the number of teeth in one wheel is given, then it is

* See Moseley's *Mechanical Principles of Engineering.*

† Ibid., page 286.

possible to find the value of m that the friction may be least. The solution of this problem is the same as that of finding the least possible number of teeth, and it was shown by Young that, for this, we ought to take m, so that the velocity-ratio for each pair of wheels is, as nearly as possible, 3·59. For example, if the train is to give a total velocity-ratio of 46, there should be three pair of wheels. The gain over a single pair in this case is one third, but will be much greater for higher velocity-ratios. The solution (first given by Mr. Gilbert) takes no account of axle friction, a circumstance which would greatly modify the result.

SECTION III.—FRICTIONAL RESISTANCES IN GENERAL.

128. *Efficiency of Mechanism in general.*—It appears from what has been said that an exact calculation of the frictional resistances is impracticable, partly because the process is too complex to be useful, but chiefly because the co-efficients to be employed are variable according to circumstances, and within limits, which are not precisely known. Hence when possible the efficiency of a machine is estimated, not by considering each particular element, but by direct experiment on the machine as a whole, and we conclude this chapter with some general principles which bear on this question.

The effort employed to drive a machine may be greater or less, according to the resistance which is being overcome, and therefore the stress between each element will also vary according to this effort. As, however, these stresses depend also on other forces, such as weight and elasticity, which have no connection with the effort, but are always the same, they will not increase so fast, and the frictional resistances will accordingly be proportionally less the greater the effort. Some resistances are absolutely constant, for example, the friction of bearings, the load on which is simply the weight of a fly-wheel or other moving part: or the friction of a piston rod in its stuffing box. Others are sensibly proportional to the driving effort or the useful resistance, in which case, when the ordinary laws of friction apply, the loss of work increases in direct proportion to these quantities. The greater number depend on both variable and constant forces, but these may be in great measure separated into two parts, one of which is approximately constant and the other approximately proportional either to the driving effort or to the useful resistance. Hence, if U be the useful work done and E the energy exerted in a period of the machine,

$$E = U + kU + k'.E + B,$$

where k, k' are numerical co-efficients and B the work done in overcoming the constant resistances. In hydraulic and other machines, where fluid resistances occur, terms depending on the speed of the machine must be added, indeed this is so in all machines when driven at a high speed; because forces due to inertia increase the friction, and besides shocks and the resistance of the atmosphere have to be considered. Such cases, however, are not considered here.

If we transfer the term $k'E$ to the other side of the equation and divide by $1-k'$, we get

$$E=(1+e)U+E_0,$$

where e, E_0 are two new constants derived from the former ones, of which E_0 is the work done in driving the machine when unloaded, and $1+e$ the counter-efficiency when the load is very great.

The same formula may also be written in a way which is sometimes more convenient. Let P be the mean value of the driving effort and R that of the useful resistance during a complete period, r the mean value of the velocity-ratio of the working and driving pairs, then

$$P=(1+e)Rr+P_0,$$

where P_0 is now the effort required to drive the machine when unloaded. In hoisting machines R is the weight lifted and P the hauling force usually called the power, R/P is the mechanical advantage or purchase.

In the steam engine, if p_m be the actual mean effective pressure, p'_m the part of that pressure employed in overcoming the useful resistance, p_0 the pressure necessary to drive the engine when unloaded,

$$p_m=(1+e)\,p'_m+p_0.$$

The value of e may be taken as ·15 or in large engines somewhat less. The constant p_0, often called the "friction pressure," is from 1 to $1\frac{1}{2}$ lbs. or in marine engines 2 lbs. or more per square inch. At high speeds and pressures the ordinary laws of friction fail and e is diminished, the constant friction is then relatively of more importance.

If the direction of motion of the machine be reversed so that the original resistance becomes the driving effort and the effort the resistance, the same general formula is approximately true, but the constants k, k' are interchanged. Unless under special conditions the efficiency is not the same in the two cases, and in fact is generally very different. Let us suppose that in a machine working against a known reversible resistance, the driving effort is gradually diminished until the machine reverses, and let E' be the work done when reversing, we have the equations

$$E = U + kU + k'E + B,$$
$$U = E' + k'E' + kU + B,$$

from which by subtraction and dividing by U we find

$$\frac{E'}{U} = \frac{2}{1+k'} - \frac{1-k'}{1+k'} \cdot \frac{E}{U},$$

a formula which gives the efficiency when reversing. If the original efficiency be less than $\frac{1}{2}(1-k')$, the machine will not reverse even when the driving force is entirely removed. In most forms of hoisting machines k' is small enough to be neglected, and we have the important principle that a machine will not reverse if its efficiency is less than ·5. It will not reverse under any circumstances if $k > 1$. As previously explained in the case of a screw, non-reversibility is a property so valuable in practical applications as to be worth obtaining at the sacrifice of efficiency. The differential pulley block is a common example.

129. *Friction Brakes.*—Frictional resistances are not only a source of loss, they are also usefully employed in machines for various purposes. In screws and driving belts we have already found them employed for the purpose of locking a pair or closing a kinematic chain, and many instances of the same kind might be referred to. Another application of equal importance is for the purpose of absorbing surplus energy, which might otherwise produce dangerous effects, or which requires to be disposed of in order to stop a machine. An apparatus for this purpose is called a "brake."

The most powerful brakes are those in which fluid resistances are used, but when the amount of energy is small as compared with the surfaces available, the friction of solids may be employed. The energy thus absorbed is converted into heat, and is dissipated by radiation and conduction. Sufficient surface must be provided to prevent the temperature rising too high.

A brake is generally applied to a rotating wheel or drum, and consists either of a solid block of wood or metal pressed against the wheel by some suitable mechanism; or else of a strap of metal, often lined with small blocks of wood, embracing the drum and tightened by a lever or otherwise. Three common forms are shown in Plate VII.; two of these (Figs. 1 and 2) are used as dynamometers, and will be referred to as such in the next chapter.

EXAMPLES.

1. A weight is moved up a plane inclined at 1 vertical to n horizontal by an effort parallel to the plane: show that the counter-efficiency is $1+nf$, where f is the co-efficient of friction. Find the value of n for a mechanical advantage of 10 : 1 and a co-efficient ·05. *Ans.* $n=20$.

2. Show that the pressure on the guide bars of a direct-acting engine is approximately proportional to the ordinates of an ellipse, and deduce the work lost per stroke.

Referring to Fig. 91 let X be that pressure, then

$$X=S\,.\,\sin\phi=P\,.\,\tan\phi=\frac{P}{n}\sin\theta \text{ approximately.}$$

If the radius of the crank circle represent P, and an ellipse be drawn with the same major axis, and minor axis $=P/n$, X will be the ordinate of the ellipse at a point representing position of piston.

$$\text{Loss of work per stroke}=f\times\text{Area of semi-ellipse}$$

$$=\tfrac{1}{2}f\,.\,\pi\,.\,a\,\frac{P}{n}=\frac{\pi fsP}{4n},$$

where s is the stroke and f the co-efficient of friction.

3. A bearing 16″ diameter is acted on by a horizontal force of 50 tons and a vertical force of 10 tons. Find the work lost by friction per revolution, using a co-efficient of one-eighteenth. Find also the horse power lost by friction at 70 revolutions per minute. *Ans.* Loss of work = 11·87 foot-tons. H.P. = 56·4.

4. The thrust of a screw propeller is 20 tons, the pitch 20 feet. The thrust block is 18″ diameter at the centre of the rings. Find the efficiency with a co-efficient of friction of ·06. *Ans.* Efficiency = ·986.

5. Find the efficiency of a common screw and nut with pitch angle 45° and co-efficient ·16. *Ans.* Efficiency = ·72.

6. A screw bolt is ½″ diameter outside and ·393″ at the base of the thread. The effective diameter of the nut is ¾″, and the co-efficient of friction ·16; supposing it screwed up by a spanner two feet long, find the mechanical advantage.

Tension of bolt = 234 × pull on spanner.

7. Find the efficiency of a pair of wheels, the numbers of teeth being 10 and 75, and the co-efficient of friction ·15. *Ans.* ·954.

8. The stroke of a direct-acting engine is 4 feet, piston load 50 tons, load on crank-shaft bearings 10 tons, connecting rod 4 cranks : trace the curve of crank effort when friction is taken into account, assuming all bearings 16″ diameter and co-efficient one eighteenth. Find the "dead angle."

9. In the last question, if the engine drive the screw propeller of question 4, find the efficiency of the mechanism, including thrust block, by the approximate method. The connecting rod may be supposed indefinitely long except for the purpose of estimating the efficiency of the guide bars.

$$\text{Efficiency} = \cdot989 \times (\cdot97)^2 \times \cdot986 = \cdot92.$$

10. A rope is wound thrice round a post, and one end is held tight by a force not exceeding 10 lbs. What pull at the other end would be necessary to make the rope slip, the co-efficient of friction being supposed ·366? *Ans.* 1,000 lbs.

11. Find the necessary width of belt three sixteenths inch thick to transmit 1 h.p., the belt embracing 40 per cent. of the circumference of the smaller pulley and running at 300 feet per 1′. Co-efficient = ·25. *Ans.* Breadth = $4\frac{1}{2}$″.

12. In question 10 construct the friction spiral showing the tension of the rope at every point.

13. The axles of a tramway car are $2\frac{1}{2}$″ diameter, and the wheels 2′ 6″: find, the resistance being given, that the co-efficient of axle friction is ·08 and that for rolling ·09. *Ans.* Resistance = $28\frac{1}{2}$ lbs. per ton.

14. Find the efficiency of a pulley 6″ diameter, over which a rope $\frac{1}{2}$″ diameter passes, the axis of the pulley being $\frac{1}{2}$″ diameter, and the load on it twice the tension of the rope. Co-efficient of axle friction ·08. Co-efficient for stiffness of rope ·47. *Ans.* Efficiency = 94 per cent.

15. From the result of the preceding question deduce the efficiency of a pair of three-sheaved blocks. *Ans.* Efficiency = 71 per cent.

16. A wheel weighing 20 lbs., radius of gyration 1′, is revolving at 1 revolution per second on axles 1″ diameter. It is observed to make 40 revolutions before stopping : find the co-efficient of axle friction. *Ans.* Co-efficient = ·059.

17. In a pair of three-sheaved blocks it is found by experiment that a weight of 40 lbs. can be raised by a force of 10 lbs., and a weight of 200 lbs. by a force of 40 lbs. Find the general relation between P and W, and the efficiency when raising 100 lbs.

$$P = \tfrac{3}{16}W + \tfrac{5}{2}. \quad \text{Efficiency} = \cdot784 \text{ when raising 100 lbs.} \quad e = \tfrac{1}{8}.$$

18. Find the distance to which power can be transmitted by shafting of uniform diameter, with a loss by friction due to its weight of n per cent, assuming that the angle of torsion is immaterial, and co-efficient for strength 9,000 lbs. per square inch.

If f be co-efficient of friction, then the length of shafting is $13\frac{1}{2} \cdot \dfrac{n}{f}$.

References.

On the graphical determination of the efficiency of mechanism the reader is referred to two papers by Prof. F. Jenkin in the *Transactions* of the Royal Society of Edinburgh. On the stiffness of ropes, see Weisbach, *Ingenieur-Mechanik*, vol. I., 3rd German edition, p. 300.