CHAPTER XI.

INCOMPLETE CONSTRAINT. STRAINING ACTIONS ON MACHINES.

130. *Preliminary Remarks.*—In the motion of a machine the relative movements of the several parts are completely defined by the nature of the machine, and the principal action consists in a transmission and conversion of energy. Hence it is that the principle of work is of such importance in all mechanical operations that it is desirable to consider it as an independent fundamental law verified by daily experience. Even in applied mechanics, however, we have sometimes to do with sets of bodies, the relative movements of which are not completely defined by the constraint to which they are subject, but partly depend on given mutual actions between them. When this is the case, the principle of work, though still of great importance, is not by itself sufficient to determine the motions.

Again, if we wish to study the forces which arise when the direction of a body's motion is changed, the principle of work does not help us, for no work is done by such forces. For example, the position of the arms of a governor, revolving at a given speed, cannot be found, except, perhaps, indirectly, by the methods hitherto employed. We then resort to the ordinary laws connecting matter and motion, which form the base of the science of mechanics, and of which the principle of work itself is often considered as a consequence.

The present chapter will be devoted in the first place to a brief summary of elementary dynamical principles, and afterwards to various questions relating to machines and the forces to which they are subject.

SECTION I.—ELEMENTARY PRINCIPLES OF DYNAMICS.*

131. *Impulse and Momentum.*—The effect of an unbalanced force P, acting during a certain time t, on a piece of matter, is to generate a velocity v, which is proportional to P and t directly and the quantity of matter inversely. When the force P is equal to the weight W, as in the case of a body falling freely, the velocity generated in $1''$ is known to be g, where g is a number which varies slightly for different positions on the earth's surface (Art. 99), but is precisely the same for all sorts of matter. We may express this by the equation

$$Pt = \frac{W}{g}v.$$

In this formula we may take W to mean the weight of the piece of matter as compared with that of a unit piece at a given point on the earth's surface. As formerly stated (Art. 88) this is called "gravitation measure," and has the defect of giving a varying unit of force, so that considerations of convenience alone induce us to employ it. If, instead of measuring W in units of weight, we compare it with the force P, which produces unit velocity in unit time, we have

$$W = Pg,$$

that is, the weight of the unit piece of matter is g units of force. Such units depend on nothing but the size of the unit piece of matter, and are hence called "absolute" units. For scientific purposes, and especially in electrical measurements, they are much employed.

Quantity ot matter is called MASS, and, when absolute measure is used, is simply measured by comparing it with that of a standard piece, for example, in Britain, with a certain piece of platinum called a pound. The unit of force is then that which is necessary to produce a velocity of 1 foot per $1''$ in this piece, a quantity for which the name "Poundal" was suggested by the late Professor Clerk Maxwell; the weight of a piece is then g poundals, so that what is called a pound-weight in the common gravitation measure is about 32·2 of these units. When absolute measure is used, however, the Continental system of units depending on the mètre and gramme is likely to be universally employed. No more need be

* The brief statement here made of principles assumed in subsequent articles of this treatise is not intended as a substitute for a treatise on elementary dynamics.

said on this point, as gravitation measure is exclusively used in this treatise.

When gravitation measure is used the unit of mass employed is that piece of matter in which a pound weight generates a velocity of 1 foot per second, that is the above mentioned piece of platinum divided by the numerical value of g, so that the unit of mass as well as the unit of force varies according to the place. If m be the mass, W the weight,

$$W = mg,$$

where g is taken equal to 32·2.

This explanation being premised we have

$$Pt = mv.$$

The products Pt, mv are called IMPULSE and MOMENTUM respectively, and the equation may be written

Impulse exerted = Momentum generated.

A unit of impulse is unit force exerted for unit time, usually 1 lb. for 1″, a quantity for which the expression "second-pound" may conveniently be used. If P be variable, then impulse is calculated in the same way as the energy exerted by a variable force (Art. 90), the abscissæ of the diagram now representing time instead of space.

The body we are considering may have a velocity at the commencement of the time t, and the force may be partially balanced; if so, v must be understood to be the *change* of velocity, and P the unbalanced part of the force.

132. *Centrifugal Force.*—So far the equation of momentum is analogous to the equation of work, impulse representing the time-effect of force as energy represents its space-effect. There are, however, two important differences.

Change of kinetic energy arises from a change in the magnitude of the velocity irrespectively of direction, whereas change of momentum must be estimated in the direction of the force producing it, and includes change of direction. Hence the equation is applicable when the direction of the force is perpendicular to the direction of motion, so that the only effect produced is change of direction. The rate of change of velocity, taken in the most general sense, is called Acceleration, and the equation of momentum may also be written

$$P = mf,$$

where f is the acceleration estimated in the direction of the force. By taking the force perpendicular to the direction of motion we get the equation which connects the curvature of the path of a moving body with the force R, which compels it to deviate from the straight line, namely,

$$R = \frac{mv^2}{r},$$

where v is the velocity and r the radius of the circle in which it is moving at the instant considered. Like other forces this arises from the mutual action between two bodies: one of these is the moving body; the other, the fixed body which furnishes the necessary constraint. If we are thinking of the fixed body instead of the moving body, we call the force R the Centrifugal Force, being the equal and opposite force with which the moving body acts on the body which constrains it. The two forces together constitute what we have already called a Stress (Art. 1). To determine a stress of this kind it is necessary to refer the direction of motion to some body which we know may be regarded as fixed, and we are not at liberty to choose any body we please for this purpose, as in kinematical questions. What constitutes a fixed body is a question of abstract dynamics, into which we need not enter. For practical purposes the Earth is taken as fixed.

If a body rotate about a fixed axis the centrifugal forces, arising from the motion of each particle, will not balance one another unless the axis be one of three lines, passing through the centre of gravity, which are called the "principal axes of inertia" at that point. In most cases occurring in practical applications the position of these lines can be at once foreseen as being axes of symmetry. This is the case, for example, in homogeneous ellipsoids and parallelopipeds. In the common case of a homogeneous solid of revolution, the axis of revolution, and any line at right angles to it through the centre of gravity, are principal axes. If the axis of rotation be parallel to one of these axes, but do not pass through the centre of gravity, the centrifugal forces reduce to a single force, which is the same as if the whole mass were concentrated at the centre of gravity. In all other cases there is a couple depending on the direction of the axis of rotation, as well as the force just mentioned. (Ex. 16, p. 297.)

133. *Principle of Momentum.*—Again, every force arises from the mutual action between two bodies, consisting in an action on one accompanied by an equal and opposite reaction on the other. Hence, if we understand by the total momentum of two bodies in any direction, the sum or the difference of the momenta of each, according as the bodies move in the same or in the opposite direction, it appears that the total momentum will not be affected by the mutual action between the two. And more generally, if there be any number of bodies we shall have

Total impulse exerted = Change of total momentum,

where, in reckoning the impulse, we are to take into account external forces alone, and not the internal forces arising from the mutual action of the parts of the set of bodies we are considering. This equation expresses one form of what we may call the Principle of Momentum; other forms will be explained hereafter in connection with questions relating to fluid motion (Part V.).

The total momentum of a number of bodies may be reckoned by direct summation, with due regard to sign, but it may also be expressed in terms of the velocity of the centre of gravity; for, let m be the mass of any particle of the system, the ordinate of which, reckoned from a given origin parallel to a given line, is x; also, let Σmx denote the sum of all the separate products mx, for all the particles of the system, and let M be the total mass, then we know that the ordinate of the centre of gravity * is given by the formula

$$\bar{x} = \frac{\Sigma mx}{M}.$$

Let the velocity of a particle parallel to the given line be u, then if x_1, x_2 be the ordinates at the beginning and end of 1″ we shall have

$$u = x_2 - x_1.$$

Hence, if $\bar{u}$ be the velocity of the centre of gravity parallel to the same line,

$$\bar{u} = \bar{x}_2 - \bar{x}_1 = \frac{\Sigma m\,(x_2 - x_1)}{M} = \frac{\Sigma mu}{M},$$

which equation may be written

$$M\bar{u} = \Sigma mu,$$

showing that the total momentum of the system is the same as if its total mass were concentrated in its centre of gravity. We conclude from this that the motion of the centre of gravity can only be

* Called more correctly by modern writers on mechanics the "centre of mass."

influenced by external forces and not by any action between the parts of the system.

134. *Internal and External Kinetic Energy.*—If we multiply the equation just obtained by $2\bar{u}$ and remember that $\bar{u}$ being constant may be placed within the sign of summation, we obtain

$$2M\bar{u}^2 = \Sigma m \,.\, 2u\bar{u},$$

which, adding Σmu^2 to each side and re-arranging the terms, may be written

$$M\bar{u}^2 + \Sigma m(\bar{u} - u)^2 = \Sigma mu^2.$$

This is true in whatever direction the velocities are estimated, and we can therefore write down two similar equations for the velocities in two directions at right angles to the first. Now the resultant of three velocities at right angles is the square root of the sum of the squares of the components, also $\bar{u} - u$ is the velocity parallel to x relatively to the centre of gravity; hence if U be the resultant velocity of the centre of gravity, v, $\bar{v}$ the velocities of any particle relatively to the body regarded as fixed and relatively to the centre of gravity respectively, we have, adding the three equations together, and dividing by 2,

$$\tfrac{1}{2}MU^2 + \Sigma\tfrac{1}{2}m\bar{v}^2 = \Sigma\tfrac{1}{2}mv^2.$$

The first term on the left-hand side of this equation is what the energy would be, if the whole mass were concentrated at its centre of gravity, a quantity which may be described as the External Energy, or otherwise as the Energy of Translation of the system. The second term is the energy relatively to the centre of gravity considered as fixed, which may be called the Internal Energy. The right-hand side is the total energy of motion, and we see therefore that this is the sum of the internal and external energies. In the case of a single rigid body the motion relatively to the centre of gravity is always a rotation about some axis, and therefore

Energy of Motion = Energy of Translation + Energy of Rotation,

a principle already employed in a preceding chapter (p. 214).

In the case of a set of rigid bodies the internal energy is the sum of the energies of rotation of each together with the internal energy of a set of particles of the same mass occupying the centres of gravity of the bodies and moving in the same way.

135. *Examples of Incomplete Constraint.*—In the cases which

occur in applications to machines and structures we usually have to consider two bodies moving in straight lines without rotation.

CASE I. *Recoil of a Gun.*—When a cannon is fired the shot is projected and the cannon recoils with velocities dependent on the relative weights of the shot, the cannon, and the charge of powder.

Here, the motion is due to the pressure of the gases generated by the combustion of the powder one way on the shot, the other way on the cannon. If the inertia of these gases could be neglected these pressures would be exactly equal at each instant and would cease as soon as the shot left the bore. The impulse exerted on shot and cannon would then be equal. In fact, the inertia of the powder gases causes the pressure to be greater and to last longer on the cannon than on the shot, so that the impulses on the two are not nearly equal. For the present we shall neglect this, and shall further suppose that the material of both shot and gun is sensibly rigid.

In general, recoil is checked by an apparatus called a "compressor," which supplies a gradually increasing resistance to the backward movement of the gun, while friction and the resistance to rotation of the shot resist the forward movement of the shot. In the first instance suppose there are no such resistances, let V be the velocity of recoil and M the mass of the gun, v the velocity and m the mass of the shot; then, since the impulse exerted is the same for both,

$$MV = mv.$$

Further, if the weight of the charge and the amount of work 1 lb. of it is capable of doing be known, the explosion will develop a definite amount of energy (E) which will be all spent in giving motion to the shot and the cannon.

$$\text{Energy of Explosion} = \tfrac{1}{2}MV^2 + \tfrac{1}{2}mv^2.$$

Here E is the sum of two parts—

$$\text{Energy of Shot} = \frac{M}{M+m}E,$$

$$\text{Energy of Recoil} = \frac{m}{M+m}E.$$

The energy of recoil has to be absorbed by the compressor, usually an hydraulic brake, which will be considered hereafter (see Part V.).

CASE II. *Collision of Vessels.*—When two vessels come into collision an amount of damage is done depending on the size and velocities of the vessels.

Here we may suppose the vessels moving in given directions with given velocities; let the velocities parallel to a given line be u_1, u_2, and the masses m_1, m_2, then, as in Art. 133, the velocity of the centre of gravity parallel to the same line is

$$\bar{u} = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2},$$

and therefore the velocities of the vessels relatively to their common centre of gravity must be

$$u_1 - \bar{u} = \frac{m_2(u_1 - u_2)}{m_1 + m_2}\ ;\ u_2 - \bar{u} = \frac{m_1(u_2 - u_1)}{m_1 + m_2}.$$

Two similar equations may be written down for the velocities in a direction at right angles to the first. Square and add corresponding equations, multiply by $\frac{1}{2}m_1$, $\frac{1}{2}m_2$, and add the pair of products, then (Art. 134)

$$\text{Internal Energy} = \tfrac{1}{2} \cdot \frac{m_1 m_2}{m_1 + m_2} V^2,$$

where V is the velocity of either vessel relatively to the other, a quantity found immediately from the given velocities of the vessels by means of a triangle of velocities.

The total kinetic energy of the vessels is found by adding the energy of translation. As, however, this quantity cannot be altered by the collision, it is clear that the amount of work done must depend on the internal energy alone: we may properly call it therefore the "energy of collision." If the displacements in tons of the vessels be W_1, W_2, we shall have, in foot-tons,

$$\text{Energy of Collision} = \frac{W_1 W_2}{W_1 + W_2} \cdot \frac{V^2}{2g}.$$

It is not, however, to be supposed that the whole of this is necessarily expended in damage to the vessels; if the circumstances of the collision be such that the vessels, even though completely devoid of elasticity, would have a motion of rotation or a velocity of separation of their centres of gravity, then the corresponding internal energy must be deducted. Also the influence of the water surrounding the vessels has been left out of account; this somewhat augments the effect by increasing the virtual mass of the vessels.

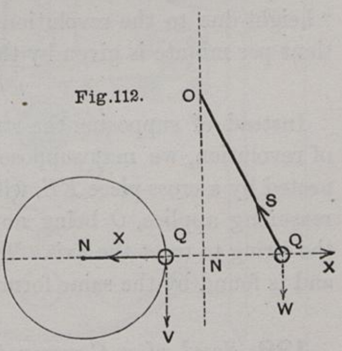
The same formula may be used for other cases of impact, but the effects of impact depend so much on the strength and stiffness of the colliding bodies that the subject must be postponed (Ch. XVI.).

Section II.—Centrifugal Regulators.

136. *Preliminary Remarks.*—Centrifugal forces may be employed in machines to do the work which is the object of the machine, as in certain drying machines where the substance to be dried is caused to rotate with great rapidity so that the fluid is expelled at the outer circumference: or, partially, in centrifugal pumps. More frequently they serve to move a kinematic chain connected with a shifting piece which regulates the speed of the machine. Such mechanisms are called Centrifugal Regulators or, more briefly, Governors.

137. *Simple Revolving Pendulum.*—In Fig. 112 Q is a heavy particle attached by a string to a fixed point O and revolving in a horizontal circle the centre of which is N vertically below O. This will be possible if the centrifugal force due to the motion of the particle just balances the horizontal component of the tension of the string. Let S be that tension, W the weight of the particle,

Fig. 112.



and let the string make an angle θ with the vertical, then the horizontal and vertical components of S are

$$X = S \cdot \sin\theta;\ W = S \cdot \cos\theta.$$

Let A be the angular velocity of the revolving particle, then it is shown in works on elementary dynamics that the centrifugal force is

$$X = \frac{W}{g} \cdot A^2 \cdot QN.$$

Equating these values of X and eliminating S,

$$W \cdot \tan\theta = \frac{W}{g} \cdot A^2 \cdot QN.$$

Since $QN = ON \cdot \tan\theta$, this reduces to the simple formula

$$ON = \frac{g}{A^2},$$

which shows that the vertical distance of Q below the point of suspension depends on the speed, not on the length of the string or the magnitude of the weight.

This distance is called the height of the revolving pendulum, and will be denoted by h. If t be the period, that is the time of a complete revolution, we find, since $At = 2\pi$,

$$t = 2\pi\sqrt{\frac{h}{g}},$$

showing that the period is the same as that of a double oscillation of a simple pendulum of length h (see Art. 103). The height of a simple revolving pendulum may often be conveniently adopted as a measure of a speed of revolution, and will then be spoken of as the "height due to the revolutions." Its value in inches for n revolutions per minute is given by the formula

$$n^2h = 35,232.$$

Instead of supposing the string attached to a point O in the axis of revolution, we may suppose it attached to a point K, rigidly connected by a cross-piece KE, with a revolving spindle ON. The same reasoning applies, O being now an ideal point, found by prolonging the string to meet the axis. The height of the pendulum is still ON, and is found by the same formula.

138. *Speed of a Governor to overcome given Frictional Resistances. Loaded Governors.*—In the simplest centrifugal governors two heavy balls are attached to arms, which are jointed either directly to a revolving spindle, or to the ends of a cross-piece attached to a spindle. Motion is communicated by links from the arms to a piece sliding on the spindle, the movement of which is communicated by a train of linkwork, either to a throttle valve directly controlling the supply of steam, or to an expansion valve which regulates the cut-off. In either case an upward movement of the arms has the effect of diminishing the mean effective pressure, and a downward movement of increasing it. Two forms of this mechanism are shown in the figures of Plate VI.: in one of these (Fig. 1) the weight of the sliding piece is increased by a large additional weight, the governor is then said to be loaded; while in the other (Fig. 2) the arms cross each other, the spindle being slotted, or the arms bent to permit this. The object of these arrangements we shall see presently.

Plate. VI.

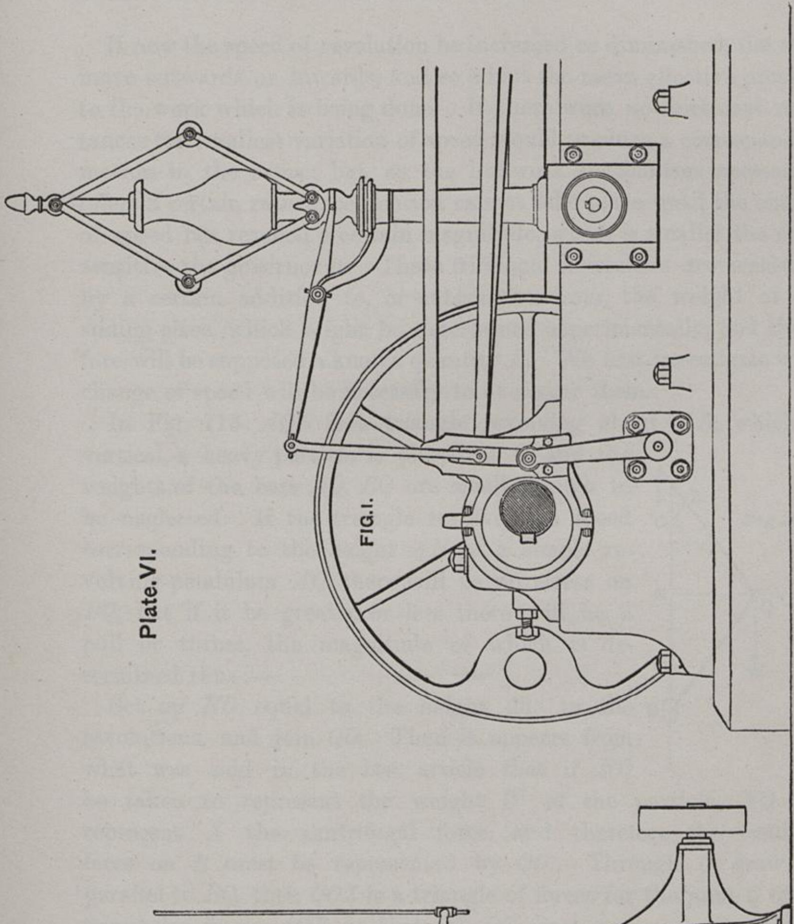


FIG. I.

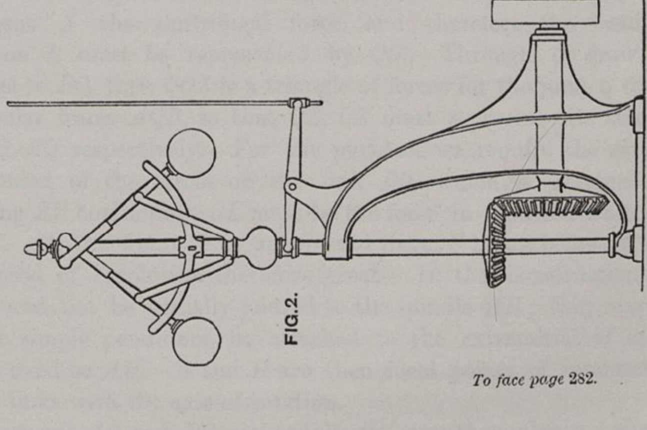


FIG. 2.

To face page 282.

If now the speed of revolution be increased or diminished, the arms move outwards or inwards, and so adapt the mean effective pressure to the work which is being done. If there were no frictional resistances the smallest variation of speed would produce a corresponding motion in the arms; but, as the linkwork mechanism necessarily offers a certain resistance, motion cannot take place until the change of speed has reached a certain magnitude, which is smaller the more sensitive the governor is. These frictional resistances are measured by a certain addition to, or subtraction from, the weight of the sliding-piece, which might be determined experimentally, and therefore will be supposed a known quantity F. We first investigate what change of speed will be necessary to overcome them.

In Fig. 113 AQB is a triangle revolving about AB, which is vertical, a heavy particle is placed at Q, and the weights of the bars AQ, BQ are small enough to be neglected. If the triangle revolve at a speed corresponding to the height AN of a simple revolving pendulum AQ, there will be no stress on BQ, but if it be greater or less there will be a pull or thrust, the magnitude of which is determined thus:—

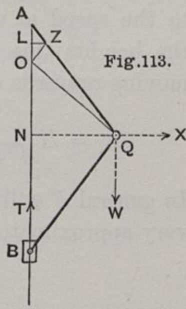


Fig.113.

Set up NO equal to the height due to the revolutions, and join QO. Then it appears from what was said in the last article that if NO be taken to represent the weight W of the particle, NQ will represent X the centrifugal force, and therefore the resultant force on B must be represented by QO. Through O draw OZ parallel to BQ, then QOZ is a triangle of forces for the joint Q of the triangular frame AQB, so that QZ, OZ must represent the stresses on AQ, BQ respectively. For our purposes we require the vertical component of the stress on the link BQ, which is obtained by drawing ZL horizontal: OL must be the force in question which we call T. In the figure T is an upward force, O being below A, and the speed of revolution therefore great. In this construction the links need not be actually jointed to the spindle AB; they may, as in the simple pendulum, be attached to the extremities of cross-pieces fixed to AB. A and B are then ideal points of intersection of the links with the axis of rotation.

In general AQ and BQ are equal; we may then obtain a simple formula for T. Let $NO = h$, a quantity given by the same formula as before for a given speed, and let AN, the actual height of the governor, be denoted by H, then $OA = H - h$; but in the case supposed, $OA = 2OL$, therefore

$$2T = W.\frac{H - h}{h}; \; h = H.\frac{W}{W + 2T}:$$

formulæ which give the pull for any speed, and conversely the speed for which the pull will have a given value. In practical applications there are always two balls, so that if W be the weight of one, $2T$ will be the pull due to both.

We can now find within what limits of speed the mechanism can be in equilibrium. Let w be the weight of the sliding-piece B, inclusive of any load which may be added to it, h the height due to the speed at which there is no tendency to move the arms, h_1, h_2 the heights due to the speeds at which they are on the point of moving upwards or downwards respectively, then

$$h_1 = H\frac{W}{W + w + F}; \; h = H\frac{W}{W + w}; \; h_2 = H\frac{W}{W + w - F}.$$

In general F will be small compared with $W + w$, and then we have very approximately,

$$h_2 - h = h - h_1 = h\frac{F}{W + w}.$$

These results show that loading a governor gives it a power of overcoming frictional resistances, which would otherwise require a weight of ball equal to the sum of the load and the actual weight. Light balls may therefore be used as in the figure (Plate VI.) without sacrificing power, as the load may be made great without inconvenience. The speed of a loaded governor is greater than that of a simple governor of the same actual height, as appears from the formula for h. It may be altered at pleasure by altering the load. This arrangement is known as Porter's governor, from the name of the inventor.

139. *Variation of Height of a Pendulum Governor by a Change of Position of the Arms.*—Next suppose the speed to alter so much that the arms actually change their position, then if H remained the same, the tendency to move would also be the same, and the movement must therefore continue until the speed is brought back within the limits for which rest is possible. In the ordinary pendulum governor, however, H alters in a way which depends on the mode of attachment and arrangement of the arms, as will appear from the annexed diagram (Fig. 114) which shows three cases.

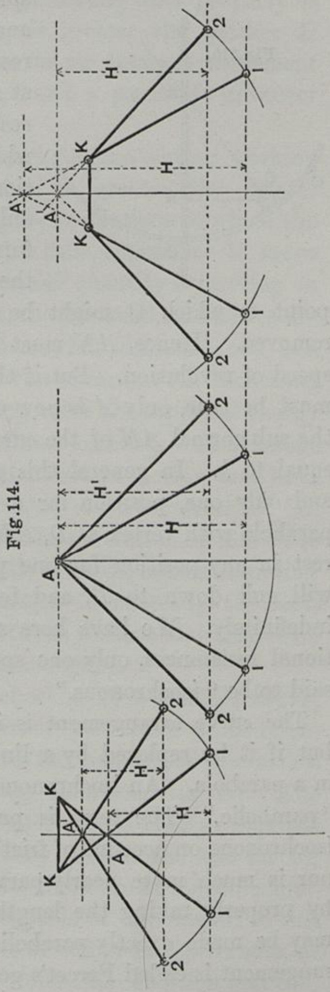


Fig. 114.

In the centre figure the arms are jointed to the spindle so that their centres of rotation are in the axis, in the two others they are jointed to a cross-piece KK, but differently arranged in the two cases. In all three, as explained in the preceding article, the height H is measured to A, the real or ideal intersection of the arms with the axis of rotation.

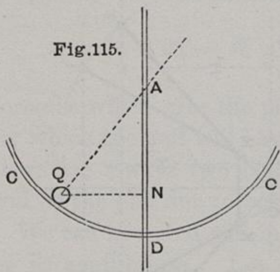
Suppose the arms to move from position 1 to position 2 in the figure; H diminishes to H', but the amount of diminution is different in the three cases: in the right-hand figure it is greatest, and in the left-hand least. Indeed in the latter case where the arms are crossed it is possible, by making KK long enough, to change the diminution into an increase. (Ex. 4, p. 295.)

If then, by an increase in the speed, the arms move into a new position, the speed required for equilibrium does not remain the same but increases, so that, when the adjustment has been effected between the energy and the work, the speed is increased, instead of being the same as before. Conversely, after adjustmen to suit a diminished speed, the speed actually attained is diminished. Thus the effect of the variation in H is to widen the limits within which the speed can vary.

140. *Parabolic Governors.*—A governor may be constructed in which H does not vary at all.

In Fig. 115 Q is a ball resting on a curve CC attached to a vertical spindle. The curve lies in a vertical plane, and D is the lowest point. When at rest the ball can only be in equilibrium at D, but, if the spindle revolve, it may rest at another point, the position of which depends on the speed of revolution. If the curve be a circle we have only the pendulum governor in a different form, for, drawing the normal QA and the perpendicular QN, A will be a point to which Q might be attached by a string and the curve removed. Hence, AN must be equal to h, the height due to the speed of revolution. But if the curve be not a circle the same thing must be true, only A is now not a fixed point; hence in every case the sub-normal AN of the curve at the point of equilibrium must be equal to h. In general this geometrical condition determines one, and only one, position for a given speed; but if the curve be a parabola with vertex at D, AN will be constant, and therefore Q will rest in any position for one particular speed, but for lower speeds will roll down to D, and for higher speeds will move upwards indefinitely. We have here a governor for which, neglecting frictional resistances, only one speed is possible. Such a governor is said to be "isochronous."

Fig.115.



The curve arrangement is inconvenient for constructive reasons, but if it be replaced by a linkwork mechanism the ball still moves in a parabola. An isochronous governor is therefore often said to be "parabolic." The term is preferable, for no governor is actually isochronous on account of frictional resistances. A pendulum governor is much more nearly parabolic when the arms are crossed, and by properly taking the length of the cross-piece (Ex. 4, p. 295) it may be made exactly parabolic for small displacements. This arrangement is called Farcot's governor from the name of the inventor.

141. *Stability of Governors.*—If the curve CC be not a parabola H will diminish or increase as the ball Q moves outwards. Take the first case and suppose Q in equilibrium at a certain point when the speed of revolution has a given value. Let Q now be moved up or down, then, if released, it will not remain at rest, but will return towards its original position and oscillate about it, or in other words the equilibrium of Q is stable. A governor possessing this property is described as "stable," and its stability is greater the quicker H diminishes. Similarly when H increases for an outward movement of the balls the governor is "unstable," and a parabolic governor may properly be described as "neutral."

A certain degree of stability is necessary for the proper working of a governor, and the amount required is greater the greater the frictional resistances. For assuming the revolutions at which the machine is intended to work to be n, the balls commence to move outward at the speed $n + x$, where x is a small quantity depending on the frictional resistance. After starting, the frictional resistances are not increased, but on the contrary will somewhat diminish; and, in a neutral governor, the balls therefore move outwards with increasing speed until by alteration of the regulating valve the supply of energy is diminished and the speed of the machine lessened. This change however requires time, and besides the balls are in motion and have to be stopped. The consequence is that they move outwards too far, and the supply of energy being too small the revolutions diminish to $n - x$, the speed necessary to move the balls inwards, notwithstanding the frictional resistance. Thus the motion is unsteady, the balls oscillating, and the speed fluctuating, between limits wider than $n \pm x$ without ever settling down to a permanent regime.

The oscillation of the balls may be checked by a suitable brake, but it is preferable to employ a governor possessing a moderate degree of stability; the tendency to move the balls then diminishes as soon as the balls move, and they stop before moving far. The greater the frictional resistances the greater is the change required to enable the balls to return at once if they have moved too far for equilibrium. An important characteristic therefore of a good centrifugal governor is that the stability be capable of adjustment to suit the frictional resistances. Certain forms of compound governors, as for example that known as the "cosine," fulfil this condition and can, probably, be made more perfect than the simple pendulum governor.

All such mechanisms are however imperfect in principle, for they cannot come into operation till a certain change of speed has actually existed for a perceptible length of time. Where the changes of resistance are sudden and violent the best governor will scarcely prevent violent fluctuations in speed. In screw vessels, where this difficulty is much felt, it has been proposed to employ an auxiliary engine rotating against a uniform resistance; any difference of speed of which and the screw shaft immediately shifts the regulating valve.

In the "cup governor," invented by Dr. Siemens,* a regulator and an hydraulic brake are combined. A cup containing water rotates within a cylindrical casing; at low speeds the water remains within the cup, but as soon as the speed exceeds a certain limit centrifugal action causes it to pour over the edge of the cup into the space between the cup and the casing. A set of vanes attached to the cup rotate between fixed vanes attached to the casing, and break up the descending water, which re-enters the cup by an orifice in the bottom. There is then a great resistance to the motion of the cup which absorbs surplus energy. Some other forms of governor will be considered hereafter.

SECTION III.—STRAINING ACTIONS ON THE PARTS OF A MACHINE.

142. *Transmission of Stress in Machines.*—We have seen (Art. 94, p. 202) that a mechanism becomes a machine if certain links are added capable of changing their form or size, and so producing forces which tend to move the mechanism combined with other forces which resist the motion. Each link so added exerts equal and opposite forces on the elements it connects, and for the pair of forces the general word "Stress" may be used, which has been already employed in Article 1 in the case of the bars of a framework structure.

When the machine is at rest the forces, being all in pairs, balance each other, and have no tendency to move the machine as a whole. For example in the direct-acting vertical engine represented in Fig. 1, Plate I., p. 119, the driving link is the steam, pressing with equal force, one way on the cylinder cover, and the other way on the piston; the working link is the resistance to turning of the crank shaft, which exerts equal and opposite forces, one way on the crank, the other way on the frame which carries the crank-shaft bearings. The steam pressure and the working resistance may each be de-

* *Phil. Trans.*, 1866.

scribed as a "Stress." The forces which make up the stress are transmitted from the piston through the connecting rod to the crank, and, in the opposite direction, from the cylinder cover through the frame to the crank shaft. The horizontal pressure of the cross-head on the guide bars is in like manner balanced by the equal horizontal thrust of the connecting rod on the crank pin, combined with the moment of the working resistance.

So in every machine, when at rest, or moving slowly and steadily, the stress is transmitted from the driving pair to the working pair, not only through the moveable parts of the machine, but in the opposite direction, through the framing; and this is a circumstance which must be always borne in mind in designing the framing. Thus, in our example, the steam cylinder and crank-shaft bearing must be rigidly connected by a frame strong enough to withstand the total steam pressure and, in addition, the bending due to the lateral pressure on the guide bars.

We have here one of the simplest examples of the transmission of stress; whether in a machine or in a structure it always takes place in a closed circuit.

If the driving pair and the working pair are the same, and acted on by the same stress, the whole state of stress is the same for all the mechanisms which are derived by inversion from the same kinematic chain. All such mechanisms are therefore statically as well as kinematically identical; it is only when we consider machines in motion, or the straining actions due to gravity, that it is necessary to consider which link (if any) is fixed to the earth. For example, the only difference between the direct-acting engine of Fig. 1, and the oscillating engine of Fig. 4, Plate I., is that the working pair is BA in the first and BC in the second. So again, in Plate III., the only difference between the water wheel of Fig. 2 and the horse gear of Fig. 3 is in the nature of the driving link, which in the first case is gravity acting on the falling water, and in the second a living agent.

A striking example of the balance of forces in a machine occurs in the hydraulic rivetting machines. Here the working pair is a small hydraulic cylinder and its ram, between which the rivet is compressed. This cylinder is suspended from a crane by chains, and can be moved into any position, as it communicates with the accumulator (Part V.) by a flexible pipe. Any portable machine, however, is an example of the same kind: machines which require foundations have no complete frame apart from the solid ground which connects their parts together.

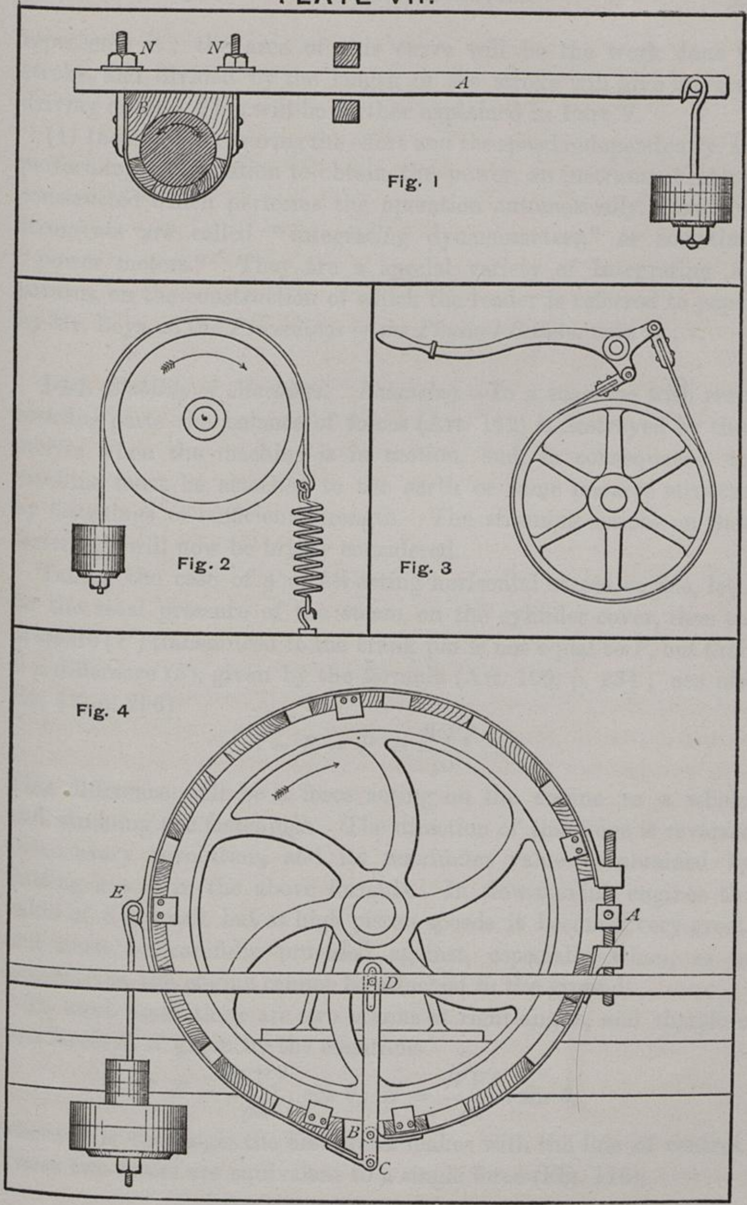
143. *Dynamometers.*—It is often a question of great practical importance to determine by direct experiment the power which is being expended in driving a machine. Instruments for effecting this are called Dynamometers; they are of very various construction, and only a few simple cases can be mentioned here. The most common are those in which the instrument measures the driving effort, while the speed is independently determined, and the power thence obtained, as in Art. 97, page 207.

(1) In Fig. 4, Plate III., a common "transmission dynamometer" is represented. A shaft transmitting the power is divided into parts, and bevel wheels *B*, *D* attached to each. A lever *A*, turning about an axis concentric with the shaft, has a weight suspended from it, which is found by trial just to balance the driving couple, transmitted through the bevel wheels *C* attached to *A* and gearing with *BD*. The whole forms the train described on page 152. Here the driving couple is measured in the act of transmission, and the revolutions of the shaft being known the power can be found.

(2) In Fig. 1, Plate VII., a "friction dynamometer" is represented in one of the various forms in which it is applied. *A* is a lever from which a weight is suspended; *B* is a block fixed to *A*, which rests on a revolving drum. A strap passes below the drum, and is tightened by the nuts *N*, *N*, till the friction just balances the weight, which by trial is made to balance the driving couple tending to turn the shaft. Here the driving couple and, consequently, the power are determined as in the preceding case, from which it only differs in the way in which the power is employed. Instead of being transmitted to the machine it is all absorbed by a friction brake which replaces it for the time being. A modification is shown in Fig. 2, in which the strap passes over a wheel and is tightened by a suspended weight, the difference between which and the tension of a spring balance measures the driving couple. (See Appendix.)

(3) In both the preceding cases the driving effort and the speed of the driving pair are constant, but in the indicator universally employed to measure the power of steam and other heat engines, we find an example in which both vary. The driving effort is now measured for each position of the piston and a curve drawn which

PLATE VII.



To face page 290.

represents it; the area of this curve will be the work done per stroke, and divided by the length of the stroke will give the mean driving effort. This will be further explained in Part V.

(4) Instead of measuring the effort and the speed independently, and performing a calculation to obtain the power, an instrument may be constructed which performs the operation automatically. Such instruments are called "integrating dynamometers," or sometimes "power meters." They are a special variety of Integrating Apparatus, on the construction of which the reader is referred to papers by Mr. Boys, in the *Proceedings of the Physical Society*, vols. iv., v.

144. *Stability of Machines. Balancing.*—In a machine with reciprocating parts the balance of forces (Art. 142) is destroyed by their inertia when the machine is in motion, and, in consequence, the machine must be attached to the earth or some massive structure by fastenings of sufficient strength. The straining actions on these fastenings will now be briefly considered.

Taking the case of a direct-acting horizontal steam engine, let P be the total pressure of the steam on the cylinder cover, then the pressure (P') transmitted to the crank pin is not equal to P, but there is a difference (S), given by the formula (Art. 109, p. 234; see also Ex. 13, p. 296)

$$S = P - P' = \frac{WV_0^2}{ga^2} . x.$$

This difference will be a force acting on the engine as a whole, and straining the fastenings. The direction of this force is reversed twice every revolution, and its maximum value is obtained by putting $x = a$ in the above formula. In slow-moving engines the value of S is small, but at high piston speeds it becomes very great, and must be carefully provided against, especially when, as in locomotives, the engine cannot be attached to the ground.

In most cases there are two cranks at right angles, and therefore two forces S, S' given by the equations

$$S = \frac{WV_0^2}{ga} . \cos\theta; \; S' = \frac{WV_0^2}{ga} . \sin\theta,$$

where θ is the angle the first crank makes with the line of centres. These two forces are equivalent to a single force (Fig. 116),

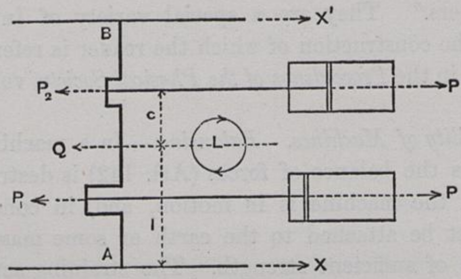
$$Q = S + S' = \frac{WV_0^2}{ga}(\sin\theta + \cos\theta),$$

acting midway between them, and a couple

$$L = (S - S')c = \frac{WV_0^2}{ga} \cdot c(\cos\theta - \sin\theta),$$

where c is the distance apart of the centre lines of the cylinders. The total effect therefore is the same as that of a single alternating

Fig. 116.



force combined with an alternating couple, which tends to turn the engine as a whole about a vertical axis. The maximum values are

$$Q_0 = \frac{WV_0^2}{ga}\sqrt{2}\,;\ L_0 = \frac{WV_0^2}{ga}\sqrt{2}c,$$

and they are each reversed twice in every revolution.

In locomotives this action produces dangerous oscillations at high speeds, and must therefore be counteracted by the introduction of suitably placed balance weights, so as to neutralize both the force and the couple.

Fig. 117.

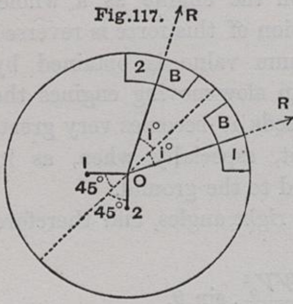


Fig. 117 shows a projection on a vertical plane of the two driving wheels and their cranks. On each wheel a balance weight is placed, occupying a segment between two or more spokes. The centre of gravity of each weight is in a radius nearly, but not exactly, opposite the nearer crank, the angle of inclination to the bisector being an angle i somewhat less than 45°. If B be the weight, r the radius of the circle in which its centre of gravity lies,

$$R = \frac{BV_0^2 r}{ga^2}$$

is its centrifugal force; and by rightly taking the values of B and i the horizontal components of these forces derived from the two balance weights, may be made to counteract both the force and the couple (Ex. 10, p. 296). In practice the weights are fixed approximately by a formula derived in this way, and the final adjustment is performed by trial. The engine is suspended by chains, and its oscillations, when perfectly adjusted, are very small even at very high speeds. (See Appendix.)

In high-speed marine engines similar forces arise, of great magnitude, which must add considerably to the strain on the fastenings, but no attempt is commonly made to balance them.

When the speed of a machine is excessive, reversal of stress must be avoided (see Exs. 17, 18, p. 244), and the greatest care is necessary that the axis of rotation of each rotating piece passes through its centre of gravity, and coincides with one of the axes of inertia of the piece (Art. 132). The magnitude of the forces which arise, in case of any error, may be judged of from the results of Exs. 13, 16, pp. 296, 297. The vibrations due to these forces will, however, in some cases be greatest at some particular speed—depending on the natural period of vibration of the frame of the machine—which could only be determined by trial. (Ch. XVI.)

In similar machines the forces due to inertia will be in a fixed proportion to the weight of the pieces, when the revolutions vary inversely as the square root of the linear dimensions of the machine.

145. *Straining Actions on the Parts of a Machine due to their Inertia.*—Another important effect of the inertia of a piece is to produce straining actions upon it. An important example is that of a ring rotating about its centre: the centrifugal force produces a tension on the ring which may be thus determined.

Suppose Fig. 121, p. 305, to represent the ring. Let the velocity of periphery be V, the weight W, and the radius r, then the centrifugal force on the small portion BB' of length z is

$$S = W \cdot \frac{z}{2\pi r} \cdot \frac{V^2}{gr}.$$

Resolve this in a given direction and sum the resolved parts, as in the article to which this figure refers, then the total is

$$P = W \cdot \frac{2r}{2\pi r} \cdot \frac{V^2}{gr} = \frac{W}{\pi r} \cdot \frac{V^2}{g}.$$

The stress to which this gives rise is evidently

$$q = \frac{W}{2\pi rA} \cdot \frac{V^2}{g} = w \cdot \frac{V^2}{g}.$$

where A is the sectional area of the ring and w is the weight of unit volume. The result here obtained is of great importance; it shows that the "centrifugal tension" of a revolving ring is independent of the radius for a given speed of periphery. Hence the result also applies to every point of a flexible element, such as a belt, whatever be the form of the surfaces over which it is stretched. In high-speed belts the tension is considerably increased by this cause, and additional strength has to be provided (Ex. 12, p. 293).

Another example of the straining actions due to inertia occurs in the motion of a rod, the ends of which describe given curves. Shearing and bending are produced, and at high speeds the magnitude of the stress thus arising is very great. Two common examples are given on page 296, but the limits of this work do not permit us to pursue the subject.

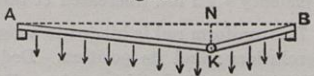
In similar machines the intensity of the stress occasioned by the straining actions we are here considering will be the same if the revolutions vary inversely as the linear dimensions of the machine.

146. *Virtual Machines.*—It has already been pointed out (Art. 94) that a machine may be regarded as a mechanism with two additional links applied as straining links, or, what is the same thing, a frame with one straining link (Art. 43). Further, as also remarked in the article cited, the external forces on any structure may be regarded as a set of straining links. It follows then that if in any framework or other structure one of its parts suffer a change of form or size of any kind, the rest remaining rigid, we shall have a machine in which the driving links exert a known stress and the working link is the bar in question. The principle of work then enables us to determine the stress on the bar, for the stress ratio must be the reciprocal of the velocity ratio. A machine thus formed may be called a "virtual machine," its movements being only supposed for the purpose of the calculation, not actually existing. It is especially in applying this method that we find in treatises on statics the principle of work employed under the title "principle of virtual velocities."

We must content ourselves with a single example of this method.

AB (Fig. 118) is a beam supported at the ends and loaded uniformly. Imagine the beam broken at K, and the pieces united by a stiff hinge, the friction of which is exactly equal to the bending moment M, then if the hinge be supposed gradually to yield under the weight, so that the joint K descends through the small space $KN(=y)$,

Fig.118.



$$\text{Energy exerted} = \tfrac{1}{2}yw(AK + BK).$$

$$\text{Work done} \quad = M(i_1 + i_2) = M\left(\frac{y}{AK} + \frac{y}{BK}\right),$$

where i_1, i_2 are the angles AK, BK make with the horizontal.

Equating the two

$$M\left(\frac{1}{AK} + \frac{1}{BK}\right) = \tfrac{1}{2}w(AK + BK),$$

which gives the known value (p. 44),

$$M = \tfrac{1}{2}w\,.\,AK\,.\,BK.$$

The advantage of this method is that it leads directly to the required result, without the introduction of unknown quantities which require to be afterwards eliminated.

EXAMPLES.

1. In Ex. 1, page 218, suppose the gun to weigh 35 tons, what additional powder will be required to provide for recoil? *Ans.* 1 lb. nearly.

2. Two vessels of displacements 8,000 and 5,000 tons are moving at 6 knots and 4 knots respectively. One is going north and the other south-west; find the energy of collision. *Ans.* 11,700 foot-tons.

3. Find the height of a governor revolving at 75 revolutions per 1′. *Ans.* 6·24″.

4. Find the dimensions of a Farcot governor to revolve at 40 revolutions per 1′, with the arms inclined at 30° to the vertical, and to be parabolic for small displacements. *Ans.* Height of governor = 22″. Length of arms = 34″. Length of cross piece to which arms are attached = $8\frac{1}{2}$″. More generally, if θ be the inclination, l the length of the arms, the length of the cross-piece is $2l\,.\,\sin^3\theta$.

5. In a simple governor revolving at 40 revolutions per 1′ find the rise of the balls in consequence of an increase of speed to 41 revolutions. Also find the weight of ball necessary to overcome a frictional resistance of 1 lb., the linkwork being arranged so that the slider rises at the same rate as the balls. *Ans.* Rise of balls = 1·1″. Weight of each ball = 5 lbs.

6. The balls of a governor weigh 5 lbs. each and it is loaded with 50 lbs. The linkwork is such that the slider rises and falls twice as fast as the balls. Find the height for a speed of 200 revolutions per 1′, and, if the speed be altered 2 per cent., find the tendency to move the regulating apparatus. How much is this tendency increased

by the loading? If the engine is required to work at three fourths its original speed, by how much should the load on the governor be diminished? *Ans.* Height = 9″·7. Tendency = 2·2 lbs. (increased 11 times).

7. A uniform rod is hinged to a vertical spindle and revolves at a given number of revolutions; find its position. Deduce the effect of the weight of the arms of a governor on its height. *Ans.* Height of rod $= \frac{3}{2} \cdot g/A^2$. Height of governor is increased in the ratio $1 + \frac{1}{2}n : 1 + \frac{1}{3}n$ where n is the ratio of the weight of the arm to the weight of the ball.

8. In Ex. 6, p. 133, find the ratio in which the bending moment at each point is affected by the inertia of the rod.

Every point of the rod describes relatively to the engine a circle and the centrifugal force of any portion of the rod = 18·6 times the weight. In the lowest position the centrifugal force acts with gravity, and so in this position the bending action is the same as if the weight of the material of the rod were 19·6 times its true weight.

9. In a horizontal marine engine with two cranks at right angles distant 8 feet from one another, weight of reciprocating parts attached to each crank 10 tons, revolutions 75 per minute, stroke 4 feet. Find the alternating force and couple due to inertia. *Ans.* Alternating force = 54·2 tons. Alternating couple = 216·8 foot-tons.

10. An inside cylinder locomotive is running at 50 miles per hour, find the alternating force and couple. Also find the magnitude and position of suitable balance weights, the diameter of driving wheels being 6 feet, the distance between centre lines of cylinders 2′ 6″, stroke 2′, weight of one piston and rods 300 lbs. Horizontal distance apart of balance weights 4′ 9″. Diameter of weight circle 4′ 6″. *Ans.* Alternating force = 7,871 lbs. Alternating couple = 9,839 foot-lbs. $B = 106{\cdot}5$ lbs. $i = 27\frac{3}{4}°$.

11. A fly-wheel 20 feet diameter revolves at 30 revolutions per 1′. Assuming weight of iron 450 lbs. per cubic foot, find the intensity of the stress on the transverse section of the rim, assuming it unaffected by the arms. *Ans.* 96 lbs. per sq. inch.

12. A leather belt runs at 2,400 feet per 1′, find how much its tension is increased by centrifugal action, the weight of leather being taken as 60 lbs. per cubic foot. *Ans.* 20·5 lbs. per square inch.

13. If r be the radius of the circle described by the centre of gravity of a rotating body, h the height due to the revolutions (page 282), show that the centrifugal force is

$$R = W\frac{a}{h}.$$

Obtain the numerical result (1) for a wheel weighing 100 lbs. with centre of gravity one sixteenth of an inch out of centre, revolving at 1000 revolutions per minute, (2) for a piece weighing 10 lbs. revolving at 300 revolutions per minute in a circle 1 foot diameter. *Ans.* (1) 178 lbs. (2) 154 lbs.

NOTE.—The formulæ of Art. 144 can all be expressed most simply in terms of h.

14. In Question 8 suppose the connecting rod of uniform transverse section, find how much the bending moment upon it due to its weight is increased by the effect of inertia.

Here the bending moment is greatest (very approximately) when the crank is at right angles to the connecting rod, and the forces due to inertia then consist (also very approximately) of a set of forces perpendicular to the rod, and varying as the distance from the crosshead pin. At the crank pin we have simply the centrifugal force due to the revolutions and length of crank. Thus the curve of loads is a straight line (p. 68) whence, proceeding by the methods of Chap. III., we find for the maximum moment

$$M=\frac{Wl}{9\sqrt{3}}\cdot\frac{a}{h},$$

where l is the length of rod, a the length of crank, h the height due to the revolutions. In the numerical example the effect of inertia is about $9\frac{1}{2}$ times that of the weight W.

15. A body rotates about an axis OE, lying in a principal plane through its centre of gravity G, and inclined to a principal axis OG at an angle θ. Show that the moment of the centrifugal forces about O is

$$L=W\frac{k^2-k'^2}{h}\cdot\sin\theta\cdot\cos\theta,$$

where h is the height due to the revolutions, and k', k are the radii of gyration about OG, and a line through O, perpendicular to OG in the plane GOE, respectively. Deduce the height of a compound revolving pendulum.

16. A disc rotates about an axis through its centre at 1000 revolutions per minute. The disc is intended to be perpendicular to the axis, but is out of truth by $\frac{1}{100}$th of the radius: find the centrifugal couple. *Ans.* If r be the radius in inches the couple in inch-lbs. is

$$L=\frac{Wr^2}{13\cdot 1}.$$

17. In question 10 find the alternate increase and diminution of the pressure of the driving wheel on the rail due to the inertia of the balance weight. *Ans.* 5,900 lbs.

NOTE.—This force of more than $2\frac{1}{2}$ tons produces great straining actions on both the wheel and the rails.

18. The power of a portable engine is tested by passing a strap over the fly-wheel, which is 4 feet 6 inches diameter, fixing one end and suspending a weight from the other. The weight is 300 lbs., and the tension of the fixed end is found by a spring balance to be 195 lbs: what is the power when running at 160 revolutions per minute. *Ans.* 7·8 h.p.