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CHAPTER XIV.

TENSION OR COMPRESSION COMPOUNDED WITH BENDING CRUSHING BY BENDING.

172. *General Formula for the Stress due to a Thrust or Pull in combination with a Bending Moment.*—The bars of a frame and the parts of other structures are often exposed, not only to a pull or thrust alone, or to a bending action alone, but to the two together; and the total stress at any point of a transverse section is then the sum of that due to each taken separately. That is to say, if H be the thrust, reckoned negative if a pull, M the bending moment, the stress at any point distant y from the neutral axis of the bending (see Art. 155), reckoned positive on the compressed side, must be given by

$$p=\frac{H}{A}+\frac{My}{I}=\frac{H}{A}\left\{1+\frac{q}{n}\cdot\frac{M}{Hh}\right\},$$

the notation being as in the article cited.

This formula shows how the effect of a thrust or pull is increased by a bending action: it has many important applications, some of which we shall now briefly indicate.

173. *Strut or Tie under the Action of a Force parallel to its Axis in cases where Lateral Flexure may be neglected.—Case I.* Bar under the action of a force in a principal plane parallel to its axis.

Let z be the distance from the axis of the line of action of the force, then

$$M=Hz\,;\ p=\frac{H}{A}\left(1+\frac{q}{n}\cdot\frac{z}{h}\right).$$

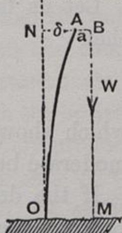
For example, let the section be circular, then $n = \frac{1}{16}$, $q = \frac{1}{2}$, and we find

$$p = \frac{H}{A}\left(1 + \frac{8z}{h}\right),$$

from whence it appears that a deviation from the axis of $\frac{1}{16}$th the diameter of a rod increases the effect of a thrust or pull 50 per cent. Similarly it can be shown that if the line of action of the force lie outside the middle fourth of the diameter of a circular section, or the middle third of a rectangular section, the maximum stress will be more than double the mean, and at certain points the stress will be reversed. In designing a structure, then, the greatest care must be exercised that the line of action of a thrust or pull lies in the axis of the piece which is subjected to it; to effect which, the joints, through which such straining actions are exerted, must be so designed that the resultant stress at the joint is applied at the centre of gravity of the section of the piece. This is a condition which cannot always be satisfied, and allowance in any case must be made for errors in workmanship. In practical construction it is the joints which require most attention, being most often the cause of failure. In frames which are incompletely braced the friction of pin joints causes the line of action of the stress to deviate from the axis. (See Ch. XVIII.)

The effect is increased in the case of a thrust and diminished in the case of a pull by the curvature of the piece, which increases or diminishes z. Fig. 133 shows the axis of a column, under the action of a weight W, suspended from a short cross piece of length a. The column bends laterally, as shown in an exaggerated way in the figure. The inclination of AB to the horizontal is so small that the difference between the actual and the projected length of AB may be disregarded; the bending moment at O is therefore $W(a + \delta)$, where δ is the lateral deviation AN of the top of the pillar. This deviation we will in the first instance suppose small compared with a, and then determine the condition that this may actually be the case. Neglecting it, the axis of the pillar is bent by the uniform bending moment Wa into a circular arc of radius R, and as in Art. 163

Fig.133.



$$\delta \,.\, 2R = l^2,$$

substituting for R its value (Art. 155) we get

$$\delta = \frac{Ml^2}{2EI} = \frac{Wal^2}{2EI};$$

whence we find

$$\frac{\delta}{a} = \frac{Wl^2}{2EI}.$$

The condition, then, that the lateral deviation should be small is that W should be much less than $2EI/l^2$, and if this condition be satisfied the stress will not be much increased beyond that indicated by the formula given above. The very important cases in which W is large will be treated presently.

In the case of a pull this restriction on the use of the formula need not be attended to, the effect of the deviation being to diminish the stress.

174. *Effect of a Thrust on a Loaded Beam.—Case II.* Uniformly loaded beam supported at the ends and subject to compression.

Let the load be W and the thrust H, then

$$p = \frac{H}{A}\left\{1 + \frac{q}{n} \cdot \frac{\frac{1}{8}Wl}{Hh}\right\}.$$

For example, let the section be rectangular, then $q = \frac{1}{2}$, $n = \frac{1}{12}$, and we find

$$p = \frac{H}{A}\left\{1 + \frac{3l}{4h} \cdot \frac{W}{H}\right\}.$$

Let us further suppose the ratio of depth to span one sixteenth, then

$$p = \frac{H}{A} \cdot \left(1 + 12\frac{W}{H}\right) = \frac{W}{A}\left(12 + \frac{H}{W}\right),$$

which shows how greatly the effect of a thrust is increased by a moderate bending moment.

If the deflection be supposed 1 inch in 100 feet then H will in consequence produce an additional bending action at the centre equal to $Hl/1200$, which will be equivalent to an addition to W of $H/150$. For safety H ought not to exceed $3W$, and the stress due to the bending action of the uniform load on the beam will then be increased about 25 per cent. by the effect of the thrust. This calculation shows why it is often necessary to support a beam at points not too far apart by suitable trussing even when support is not required to give sufficient stiffness. Theoretically a proper "camber" given to the beam will counteract the bending action, and, conversely, a small accidental deflection will increase it.

175. *Remarks on the Application of the General Formula.*—The formula given above in Art. 172 is much used in questions relating to the stability of chimneys, piers, and other structures in masonry and brickwork. The stress on horizontal sections of such structures varies uniformly or nearly so, and the formula then shows where the stress is greatest and also where it becomes zero, tension usually not being permissible. It must be borne in mind however that the bending is frequently unsymmetrical, so that the axis of the bending moment will not coincide with the neutral axis of the bending stress on the section (Art. 162). The stability of blockwork and earthwork structures is a large subject which will not be considered in this treatise.

176. *Straining Actions due to Forces Normal to the Section.*—The reasoning of this section shows that when a structure is acted on by forces some or all of which have components normal to a given section, the straining actions due to the normal components will in general depend on the relative yielding of the several parts of the section (Art. 42). These normal components however can always be reduced to a single force, acting through any proposed point in the section, and a couple, and if the point be properly chosen according to the nature of the structure at the section that single force will be a simple thrust or pull; thus in the cases we have mentioned the point is the centre of gravity of the section. Having done this the couple will be so much addition to the bending action. An important example of this is the case of a vessel floating in the water in which the horizontal longitudinal component of the fluid pressure generally produces bending, the arm of the bending couple being the distance of the intersection of the line of action of the resultant with the section considered, from the neutral axis of the "equivalent girder."

177. *Maximum Crushing Load of a Pillar.*—When the compressing force is sufficiently great it produces a strong tendency to bend the pillar even though there be no lateral force. We have already seen that the condition that this shall not be the case is that W shall be small compared with the quantity $2EI/l^2$, and we now proceed to inquire the effect produced when W has a larger value. All these cases come under the head of what is called Crushing by Bending, and are very common and important in practice.

As in the case of the deflection of a beam the question is much more simple when the pillar bends into an arc of a circle, which it will do in various cases explained in Art. 163. The case which we select is that in which the sectional area remains constant and the thickness varies. Such a pillar is of uniform strength when very slightly bent, and when more bent the weakest point is at the base. As the breadth becomes great at the summit this form could not be practically applied without modification, but the conclusions derived by considering it may be applied with slight modifications to the cases which occur in practice. *

When the load is applied exactly at the centre the elevation of such a pillar is a semi-ellipse with vertex at the summit; when not exactly at the centre the ellipse is truncated. For the present purpose it is not necessary to consider this point further, as the form is not intended for practical application.

Assuming then the form of the bent pillar to be a circular arc we have as before

$$\delta = \frac{Ml^2}{2EI},$$

but we have now, since we cannot neglect δ,

$$M = W(a + \delta).$$

Hence by substitution we find

$$\delta = \frac{W(a+\delta)l^2}{2EI},$$

where I is the moment of inertia at the base, from which we find

$$\delta = \frac{a}{\dfrac{2EI}{Wl^2} - 1}.$$

This result shows that the pillar bends laterally more and more

* The case where the thickness is uniform has been considered by Dr. Young in his *Natural Philosophy* (see Young's works, Peacock's edition, p. 139), who shows that the outline is a circular arc, as follows at once from Art. 161. The compressive stress however near the summit of the pillar is then very great.

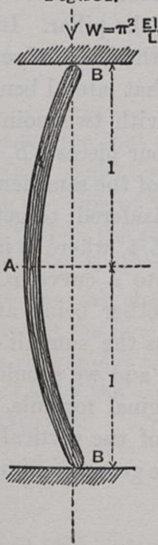
as W increases, and breaks with some value of W which we will find presently by substitution in the formula of Art. 172.

First, however, observe that if $a = 0$, that is, if the line of action of the load pass through the centre of the pillar at its summit, then $\delta = 0$ unless the denominator of the fraction be also zero, that is, unless

$$W = 2 \cdot \frac{EI}{l^2}.$$

The interpretation of this is, that if W be less than the value just given the pillar will not bend at all, but if disturbed laterally will return to the upright position when the disturbing force is removed. If W have exactly that value then, when put over into any inclined position the pillar will remain there in a state of neutral equilibrium, while the smallest increase of W above this limit will cause the

Fig.134.



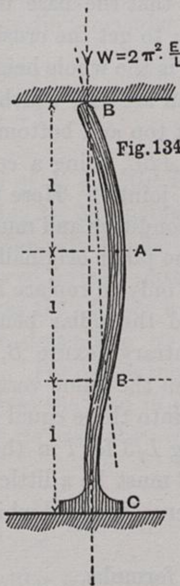
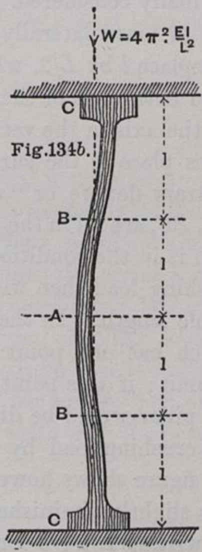


Fig.134a.

Fig.134b.



pillar to bend over indefinitely and so break. Thus the foregoing equation may be regarded as giving the crushing load of the pillar under certain conditions to be defined more exactly presently.

If the pillar had not bent into the arc of a circle as has been just supposed, we should have arrived at exactly the same formula except that the co-efficient 2 is replaced by a not very different number depending on the circumstances of the particular case. If the transverse section be uniform then the pillar bends into a curve of sines and we must replace 2 by $\pi^2/4$ or 2·47, thus obtaining

$$W = \frac{\pi^2}{4} \cdot \frac{EI}{l^2},$$

a formula which having been first obtained by Euler is known as Euler's Formula. It applies directly to a column fixed firmly in the ground and entirely free at the upper end; it can however easily be modified to suit the cases more common in practice where the ends of the column are constrained to lie in the same vertical line. There will be three such cases shown in Figs. 134, 134*a*, 134*b*.

In the first the ends of the pillar are rounded and it bends laterally in the curve BAB; each half AB is then in the position of the pillar originally considered, except that the base instead of the summit is free to move laterally, hence to get the crushing load we have only to replace l by $L/2$, where L is the whole height of the pillar. In the third both ends of the column are flat, which has the effect of retaining the axis in the vertical at top and bottom, so that lateral bending takes place in the curve $CBABC$, being a curve with two points of contrary flexure or "virtual joints." Here the four pieces CB, BA, AB, BC, are all in the same condition and must be of the same length; each is in the condition of the pillar originally considered; to get the crushing load then we have only to replace l by $L/4$, where L is the whole length. In the second the pillar bends into a curve $BABC$ which has one point of contrary flexure B, the other being at the summit; if this point were in the same vertical as the summit then the pillar would be divided into three equal parts and we should get the crushing load by writing $L/3$ for l in the original formula. As the figure shows however, B must be a little out of the vertical, and this slightly diminishes the crushing load which we get approximately by writing $L/2\sqrt{2}$ for l.

We thus obtain the three formulæ,

$$W = \pi^2 \cdot \frac{EI}{L^2}; \; W = 2\pi^2 \cdot \frac{EI}{L^2}; \; W = 4\pi^2 \cdot \frac{EI}{L^2},$$

for the three cases in question with a uniform section. If the pillar be bent into a circle as described above, then π^2 is to be replaced by 8.

178. *Manner in which a Pillar crushes. Formula for Lateral Deviation.*—The value of W here found is the maximum load which a pillar, free to deflect laterally, can sustain under any circumstances; but, in order that it may actually be sustained, the pillar must be perfectly straight, the material must be perfectly homogeneous, and the line of action of the load must be exactly in the axis. These conditions cannot be accurately satisfied, and consequently a lateral deflection is produced, which increases indefinitely as the load approaches the theoretical maximum. This may be expressed by supposing that a is not zero, but some known quantity depending on the degree of accuracy with which the conditions are satisfied, and which may be called the "effective" deviation; since, when the pillar is straight and homogeneous, it will be the actual deviation of the line of action of the load from the axis. Let W_0 be the theoretical maximum load as calculated from the preceding formulæ and W the actual load, then

$$\delta = \frac{a}{\dfrac{W_0}{W} - 1} = a \cdot \frac{W}{W - W_0} \text{ (p. 344.)};$$

thus we see that a load of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ the theoretical maximum produces a lateral deflection of $1a$, $2a$, $3a$, increasing the deviation of the load from the axis of the column to $2a$, $3a$, $4a$. These numbers are only exact when the pillar is so formed as to bend into the arc of a circle, when this is not the case they follow a more complicated law of the same general character depending on the type of pillar and the nature of the deviation. For our purpose the simple case is sufficient. It is convenient to express the load in pounds per square inch of the area (A) of the pillar at its base, then we may write with the notation of Art. 155

$$p_0 = \frac{W_0}{A} = \pi^2 \cdot nE \cdot \frac{h^2}{L^2};$$

for the case where the pillar is rounded at both ends, the number π^2 being replaced by $2\pi^2$ or $4\pi^2$ in the two other cases of the last article. Similarly writing $p = W/A$ for the actual load on the pillar, we get by substitution

$$\delta = a \cdot \frac{p}{p_0 - p}, \quad \text{or } a + \delta = a \cdot \frac{p_0}{p_0 - p}.$$

The deviation is accompanied by an increase in the maximum stress (f) on the transverse section, which is given by the formula

$$f = \frac{H}{A}\left(1 + \frac{q}{n} \cdot \frac{M}{Hh}\right) \qquad \text{(p. 340)},$$

from which we get, replacing H by W and M by $W(a + \delta)$,

$$f = p\left(1 + \frac{qa}{nh} \cdot \frac{p_0}{p_0 - p}\right),$$

a result which shows that f increases indefinitely as p approaches p_0, so that the pillar must break before the theoretical maximum is reached, however small the original deviation is. The greatest value of f must be the elastic strength, for as soon as this is past an additional lateral deviation at the most compressed part will occur, sooner or later accompanied by rupture.

The formula may be written in the more convenient form,

$$\left(\frac{f}{p} - 1\right)\left(1 - \frac{p}{p_0}\right) = \frac{qa}{nh},$$

in which it is worth while to observe that the right-hand side is unity for the deviation necessary to produce double stress when the pillar is so short that no sensible augmentation of the deviation is produced by lateral bending. In materials like cast iron which have a low tenacity, very long pillars give way by tension on the convex side; the formula then becomes

$$\left(\frac{f'}{p} + 1\right)\left(1 - \frac{p}{p_0}\right) = \frac{qa}{nh},$$

where f' is the tensile stress at the elastic limit. The two formulæ give the same result if

$$p = \frac{f - f'}{2}.$$

For loads greater than this the first formula applies, and for small loads the second. In pillars flat, but not fixed at the ends, without capitals f' may be zero.

179. We thus see that if a pillar were absolutely straight and homogeneous it would crush, by direct compression if p_0 were greater than f, and by lateral bending if p_0 were less than f, the crushing load being the least of these two quantities; but that the smallest deviation will be augmented by lateral bending, so that the actual crushing load will be less than the least of these quantities. Experience confirms this conclusion. When a long pillar is loaded we do not find that it remains straight till a certain definite load p_0 is reached, and then suddenly bends laterally. We find, on the contrary, that a perceptible lateral deflection is produced by a small load, which gradually increases as the load is increased, till rupture takes place, showing, as we might anticipate, that some small deviation existed originally. And as that deviation evidently depends upon accidental circumstances it is impossible, from imperfection of data, to find the actual crushing load of a pillar for those proportions of height to thickness, for which its effect is greatly augmented by a small deviation. The augmentation is on the whole greatest when

$$f = p_0 = \pi^2 \,.\, n \,.\, E \,.\, \frac{h^2}{L^2};$$

that is, when

$$\frac{L}{h} = \sqrt{\frac{\pi^2 n E}{f}}.$$

This gives, by taking the values of E and f from Table II., Ch. XVIII.

Wrought Iron,	$L = 38\sqrt{\pi^2 n} \,.\, h = 30h$	(Circular Section).
Soft Steel,	$L = 29\sqrt{\pi^2 n} \,.\, h = 23h$	,,
Hard Steel,	$L = 23\sqrt{\pi^2 n} \,.\, h = 18h$	,,
Cast Iron,	$L = 20\sqrt{\pi^2 n} \,.\, h = 16h$	,,

In the case of cast iron there is a difficulty in determining the value of f, but if we suppose that the elasticity of the material is not greatly impaired at half the ultimate crushing load, we get the value given. The case of timber is exceptional, and will be referred to further on. For pillars fixed or half-fixed at the ends the number π^2 is to be replaced by $4\pi^2$ or $2\pi^2$ as before.

Let us assume this condition satisfied, and let us imagine the pillar loaded with three fourths the theoretical maximum crushing load, then by substitution we find, $qa/nh = \frac{1}{3} \,.\, \frac{1}{4}$, or since $n/q = \frac{1}{8}$ for a circular section,

$$\frac{a}{h} = \frac{1}{96},$$

from which it will be seen how small a deviation will cause the pillar to crush under three fourths the theoretical maximum load, when the proportion of height to thickness is that just given. With a pillar of double this height deviation has little influence, and with a pillar of one third this height lateral flexure has little influence on the resistance to crushing.

On the whole, then, it would seem that the most rational way of designing pillars would be to calculate the theoretical maximum load, and then adopt a factor of safety depending on the value of the deviation found from the above formula; it is obvious that in some cases a much larger deviation may be considered likely than in others. For the case of thin tubes see Ch. XVIII.

180. *Gordon's Formula.*—The greater part of our experimental knowledge respecting the strength of pillars is due to Hodgkinson.* His results show that in cast-iron pillars with flat ends, the length of which exceeds 100 diameters, the theoretical maximum is closely approached, while with shorter lengths the strength falls off considerably, as might be expected. In other respects the theoretical laws are approximately fulfilled, the principal difference being that columns with one or both ends rounded are somewhat stronger relatively to columns with flat ends than theory would indicate, an effect which may be partly due to imperfect fixing of the ends. Various empirical formulæ have been given to express the results of experiment on the crushing of pillars. That which has been most used is commonly known as Gordon's. It is so constructed as to agree in form with the theoretical formulæ in the extreme cases in which those formulæ give correct results. As modified by Rankine, only replacing r^2, the square of the radius of gyration, by nh^2, in the notation of this work the formula is

$$\frac{W}{A} = \frac{f}{1 + \dfrac{l^2}{cnh^2}},$$

which becomes, when l/h is small,

$$W = Af,$$

and when l/h is large,

$$W = \frac{cnfAh^2}{l^2} = cf \cdot \frac{I}{l^2};$$

while for intermediate values it gives intermediate results.

* *Phil. Trans.*, 1840, Part II. An abridgment is given in Hodgkinson's work on Cast Iron, cited at the end of Chapter XVIII.

If we compare this last with Euler's formula for a column with flat ends, we get

$$c = 4\pi^2 \frac{E}{f},$$

and this may be called the "theoretical" value of the constant c. The values actually used for c are somewhat different, being deduced from such experiments as have been made, and the results for different forms of section are not always consistent. Rankine gives

VALUE OF CONSTANTS.

	Value of f.	Value of c.
Wrought Iron, . . .	36,000	36,000
Cast Iron, ...	80,000	6,400
Dry Timber, ...	7,200	3,000

These values refer to struts fixed at the ends and to the crushing load. If one end be rounded, the value of c must be divided by 2, and if both ends are rounded, by 4. A large factor of safety must be employed, for reasons already sufficiently indicated.

Rankine's formula has been very extensively tested for the case of wrought columns of large size of various transverse sections, constructed of riveted plates, and has been found to give good results.*

In the case of timber Hodgkinson found, from a limited number of experiments on struts of oak and red pine of small dimensions, a formula which agrees with the formula for the theoretical maximum crushing load when the value of E in that formula is taken as about 900,000 lbs. per square inch. It is possible that the low lateral tenacity of this material increases its flexibility under a heavy crushing load. The formula gives a crushing stress greater than the direct resistance to crushing of the material when L is less than $20h$, which seems hardly probable, and the lower values given by Gordon's formula appear preferable. In the case of steel the value of f may be expected to be increased and the value of c diminished in the ratio of the direct resistance to crushing of steel and wrought iron respectively.

Calculations made by Gordon's formula may be tested by calculating the deviation a by the formula on p. 348; the magnitude of this will be to some extent a measure of the safety of the proposed load.

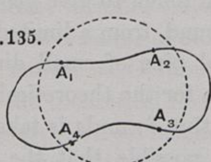
* "Minutes of Proceedings of the Institution of Civil Engineers," vol. liv.

In all cases of struts of large size subject to a heavy load, special care is necessary in considering all the circumstances—if a deflection be occasioned by the unsupported weight of the strut itself, or if, as is often the case, it be constructed of riveted plates, a large margin of safety is desirable. So also in pieces forming part of a machine in which a bending action may be produced by inertia and friction, or which are subject to shocks, the simple thrust alone is often a very imperfect measure of the stress to which they are subject.

Returning to the case of a long slender column we observe that the resistance to crushing depends solely on the stiffness and not on the strength being proportional to the modulus of elasticity. Hence a long column is stronger when made of wrought iron than when made of cast iron, although with short columns the reverse is true. It appears from Gordon's formula that for a ratio of length to diameter of about $26\frac{1}{2}$ the two materials are equally strong. In very long columns steel is not stronger than iron, for its modulus of elasticity is not very different; in shorter lengths however the greater resistance to direct crushing of steel gives it an advantage.

181. *Collapse of Flues.*—There are other cases of crushing by bending. An important one is that of the yielding of a thin tube under *external* fluid pressure. The strength of a tube under external fluid pressure is as different from that of a tube under internal pressure as the strength of a bar under compression is different to its strength under tension.

Fig. 135.



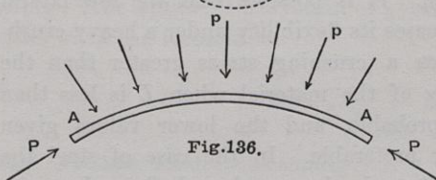


Fig. 136.

A tube perfectly uniform in thickness made of perfectly homogeneous hard material and subject to perfectly uniform normal pressure externally, would theoretically maintain its form until it yielded by the direct crushing of the material. But when the pressure exceeds a certain limit the tube is in a state of unstable equilibrium, and any deviation from perfect accuracy in the above conditions will cause the tube to yield by collapsing, the collapsing being accompanied by bulging. If the tube is very long it will collapse in the manner shown in Fig. 135, the circumference dividing itself up into four arcs two of which are concave outwards and the other two convex. A want of exactness in the construction will in practice generally prevent the collapsing from being symmetrical. Each portion of tube between the points A is under the action of forces applied at the ends towards one another, which crush it by lateral bending just as a long column is crushed. Just before collapsing, each segment AA (Fig. 136), of length s say, will be under the action of a thrust P suppose, applied at the ends tangentially. Equilibrium is maintained by fluid pressure of intensity p on the convex side. When the pressure exceeds a certain limit the equilibrium is unstable, some accidental circumstance determining the position of the point A of contrary flexure, and the consequent length s of any arc.

The thrust per inch length of the tube may be taken as approximately proportional to p. Thus if t = thickness of tube, we may expect that the collapsing pressure would be given by a formula like that which expresses the crushing load of a long slender rod of rectangular section, namely, $p = k't^3/s^2$ where k' is an unknown co-efficient. All other things being equal, the diameter alone varying, the length s of an arc AA would be proportional to the diameter of the tube d, and, under those circumstances, the collapsing pressure would probably vary with t^3/d^2. But the length of the tube, as well as the diameter, influences the value of s. In all practical cases, as in all those on which experiments were made, the ends of the tube are rigidly constructed, and very much support the tube in the neighbourhood from collapsing; thus the proximity of the ends has an important effect in determining the length of the arcs into which the circumference divides itself. If the length of the tube is decreased a limit will be reached below which the tube on collapsing divides itself up into six arcs, three concave and three convex, as shown in Fig. 137. Then the length of each arc will bear a smaller proportion to the diameter than in the long tube. A still shorter tube will, when it collapses, divide it into eight arcs, and so on. Thus the length s is in some way dependent on the length of the tube. The correctness of this reasoning is borne out by experiments made by Fairbairn and others. In Fairbairn's experiments the tubes were made of riveted wrought-

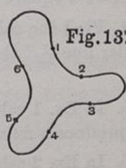


Fig. 137.

iron plates. The ends were made rigid by a strong stay placed within the tube, keeping the ends apart. The tube thus constructed was placed in a larger cylinder of wrought iron and external pressure was applied by forcing water in. The pressure being gradually increased the tube will at last suddenly collapse, making a noise which indicates the instant of the occurrence. The results of the experiments showed that the collapsing pressure may be approximately expressed by the formula

$$p=k\frac{t^2}{ld},$$

the dimensions being all in inches, the co-efficient $k=9,672,000$. This formula must not be used for extreme cases nor for tubes of thickness less than $\frac{3}{8}$ inch.

Since a short tube is so much stronger than a long one, we have an explanation of the advantage of riveting a T iron ring around a boiler furnace tube, which amounts to a virtual shortening of the length of the tube. Other formulæ have been proposed, some of which represent the results of experiment more closely, but the materials at present available do not admit of the construction of a satisfactory formula. *

EXAMPLES.

1. Find the thickness of metal of a cast-iron column fixed at the ends, 1 foot mean diameter, 20 feet high, to carry 100 tons. Factor of safety, 8. *Ans.*—Thickness 1″.

2. Find the crushing load of a wrought-iron pillar 3″ diameter, 10 feet high, free at the ends. *Ans.*—Crushing load = 66,218 lbs. = 30 tons nearly.

3. If in last question the pillar were of rectangular section of breadth double the thickness, what sectional area would be required for equal strength? *Ans.*—Sectional area = 9·4 square inches instead of 7 square inches as before.

4. Find the collapsing pressure, according to Fairbairn's formula, of a cylindrical boiler flue $\frac{7}{16}$″ thick, 48″ diameter, and 30 feet long. *Ans.*—Collapsing pressure = 107 lbs.

5. In Ex. 1 calculate the deviation of the line of action of the load from the axis to produce a maximum stress of 10,000 lbs. per square inch. *Ans.*—1·8″.

6. In Ex. 2 calculate the deviation to produce a maximum stress of 9,000 lbs. per square inch with a load of 11,000 lbs. or of 22,000 lbs. *Ans.*—1·2″ or $\frac{1}{12}$″.

* See a paper by Professor W. C. Unwin, *Minutes of the Proceedings of the Institution of Civil Engineers*, from which the preceding remarks are partly taken. Some other cases of crushing by bending will be given in the Appendix.