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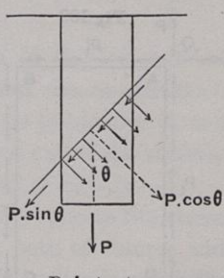
CHAPTER XV.

SHEARING AND TORSION OF ELASTIC MATERIAL.

182. *Distinction between Tangential and Normal Stress.—Equality of Tangential Stress on Planes at Right Angles.*—In the cases we have hitherto considered of simple tension, compression, and bending, the stress on the section under consideration has been at all points normal to the section. But we may take our section inclined at any angle to the stress, and the mutual action is then not normal to the section. The particles on each side of the section partly act on one another in the direction of the section itself, and so constitute a stress analogous to friction, resisting the sliding of one portion relatively to the other. Such a stress is called *tangential* or *shearing stress*, being the stress called into action by shearing.

Fig.138.



Let us return to the case of the stretched bar carrying a load P (Fig. 138). On a transverse section of the bar only a normal stress is produced. Now suppose we take an oblique section, whose normal makes an angle θ with the axis of the bar, and let us resolve the force P into two components, one perpendicular and the other parallel to the section. The normal component $P\cos\theta$ tends to produce a direct separation at the section, producing a tensile stress similar in character to that on a transverse section, but of less intensity.

If A = area of transverse section of bar, then $A\sec\theta$ = area of oblique section; the intensity of the normal stress

$$pn=\frac{P\cos\theta}{A\sec\theta}=\frac{P}{A}\cos^2\theta=p\cos^2\theta,\text{ where }p=\frac{P}{A}.$$

The other component $P \sin \theta$ produces a tangential or shearing stress of intensity

$$p = \frac{P \sin \theta}{A \sec \theta} = p \sin \theta \cos \theta.$$

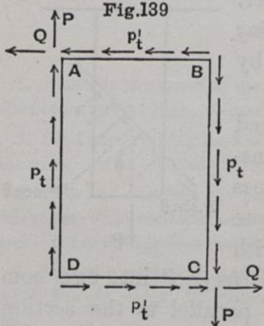
Similarly if the bar is subjected to a compressive instead of a tensile load.

Many materials which offer great resistance to direct compression yield by sliding across an oblique plane. Now p_t is a maximum when $\theta = 45$, this is therefore approximately the angle of separation. The same maximum stress, the value of which is $p/2$, occurs on another plane sloping the other way at an angle of 45°. We sometimes find fracture to occur across two oblique planes; sometimes across one only.

If in $p_t = p \sin \theta \cos \theta$ we change θ into $90 + \theta$, p_t has the same value; so that the intensity of the tangential stresses on two planes at right angles to one another is the same. This is true generally in all cases of stress, as will be seen presently.

183. *Tangential Stress equivalent to a Pair of Equal and Opposite Normal Stresses. Distorting Stress.*—In the example we have just considered we have both shearing and normal stress; but there are cases in which there is only a shearing stress. Let $ABCD$ (Fig. 139) be a rectangular plate of thickness t. Over the surfaces BC and AD suppose a tangential stress to be applied of intensity p_t. Calling b and a the length of the sides of the plate, the total amount of the tangential stress on each side is

Fig.139



$$P = p_t \, . \, bt.$$

To prevent the turning of the plate, suppose the forces P balanced by the application of an uniform stress over the surfaces BA and DC, of intensity p'_t. The amount of the force on each of these sides,

$$Q = p'_t \, . \, a \, . \, t.$$

Since equilibrium is produced, the moment of the couple P must be equal to the moment of the couple Q.

$$\therefore p_t \,.\, bt \,.\, a = p'_t \,.\, at \,.\, b;$$

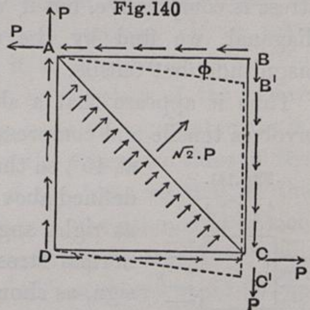
$$\text{or } p_t = p'_t:$$

that is, the intensity of the stress is the same on *BA* as on *AD*.

Shearing therefore cannot exist along one plane only. It must be accompanied by a shearing stress of equal intensity along a plane at right angles. Such a pair of stresses unaccompanied by normal stress constitute a Simple Distorting Stress, so called because it distorts the elements of the body.

Let us now assume, for simplicity, the plate to be square (Fig. 140). The effect of the forces is to produce a change of form, which, in perfectly elastic bodies, is exactly proportional to the shearing force which produces it. The square *ABCD* becomes a rhombus *AB′C′D*, the angle of distortion ϕ being proportional to the stress p_t. We may write

Fig.140



$$p_t = C\phi,$$

where the co-efficient *C* is a kind of Modulus of Elasticity, but of a different nature from that previously employed. The volume of the elastic body *A* is in general practically unaltered. Under the action of the forces it has simply undergone a change of form or figure, and the co-efficient *C* which connects the change of form with the stress producing it, is a co-efficient of elasticity of figure. It is sometimes called the *modulus of transverse elasticity*, but preferably the *co-efficient of rigidity*.

The ordinary (Young's) modulus of elasticity *E* connects the stress and strain in a bar when it undergoes changes both of volume and figure. The co-efficient of rigidity *C* for metallic bodies is generally less than $\frac{2}{5}E$, and for wrought-iron bars may be taken as 10 to $10\frac{1}{2}$ millions.

Let us now take a section of the square plate (Fig. 140) along one of the diagonals and consider the forces which act on the two sides of the triangular upper portion. Resolve these forces parallel and perpendicular to the diagonal. The components of the two *P*'s along the diagonal balance one another, and there will be no tendency for this triangular portion to slide relatively to the other; that is to say, there is no shearing stress on the diagonal section. But the other components, perpendicular to the diagonal, cause the upper triangular portion to press on the lower with a force

$$2\frac{P}{\sqrt{2}} = \sqrt{2} \cdot P.$$

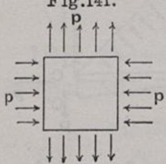
If we divide this force by the area of the diagonal section over which it is distributed, we obtain the intensity of this normal stress,

$$p_n = \frac{\sqrt{2} \cdot P}{\sqrt{2} \cdot at} = p_r$$

On the diagonal section AC which we have been considering, this stress is compressive, but if we take the section along BD, the other diagonal, we find by the same reasoning a stress of the same magnitude, but tensile.

Thus it appears that a shearing stress on any plane necessarily involves tensile and compressive stresses of equal intensity on planes at 45°, so that a simple distorting stress, which was defined above as a pair of shearing stresses on planes at right angles, may also be defined as a pair of normal stresses of equal intensity and of opposite sign, as shown in Fig. 141.

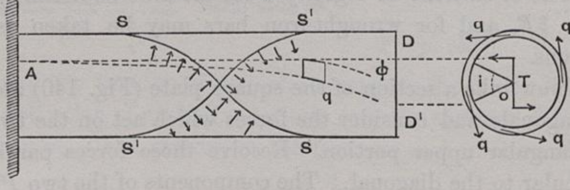
Fig.141.



We now proceed with various examples of this kind of stress, commencing with the case of torsion. Torsion was mentioned as one of the five simple straining actions to which a bar as a whole may be exposed. It is produced by a pair of equal couples applied at the ends of the bar, the axis of the couples being the axis of the bar.

When we consider the nature of the elastic forces called into

Fig.142.



action amongst the particles of the bar, Torsion reduces to a case of Shearing. To understand this, we will begin with a simple case. Imagine a thin tube (Fig. 142) with one end fixed, and the other acted on by a uniform tangential stress of intensity q. Let t be the thickness and d the mean diameter of the tube, then

Sectional area of tube $= \pi dt$ approximately ;
Total shearing force $= q\pi dt$;

and since the force on each unit of area of the section acts approximately at the same distance from the centre of the tube, the total twisting moment $= q\pi dt \times \frac{1}{2}d = \frac{1}{2}q\pi d^2 t$. This twisting moment is balanced by the resistance to turning offered at the fixed end. At any transverse section KK of the tube there will be produced a uniform stress of intensity q.

Let us now consider a small square traced on the surface of the tube, with two sides on two transverse sections. If we take the square small enough we may treat it as a plane square. To balance the shearing stress q, which acts on the sides of the square lying in the transverse planes, a shearing stress of equal intensity is, as explained above, called into action on the other two sides of the square, in the direction of the length of the tube, so that, if the tube were cut by longitudinal slits, the power of resistance to torsion would be as effectually destroyed as if it were cut by transverse slits. But if we make spiral slits at an angle of 45°, as shown at SS in Fig. 142 ; supposing the slits indefinitely fine, and no material removed, the strength of the tube to resist torsion in the direction shown would not be impaired. The material of the tube would then be divided into spirally-bent ribands, which would be in tension along their length, and in compression laterally, the ribands being caused to press against one another. Along a second set of spirals such as $S'S''$, longitudinal compression and lateral tension exist; the lateral forces are indicated in both cases by arrows in the figure.

So much for the state of stress induced in the tube by the torsion. Next as to the change of form which accompanies the stress. The square will be distorted into a rhombus. A straight line AD, drawn on the surface parallel to the axis of the tube passing through the centre of the square, will be twisted into a spiral AD', the angle of the spiral being the angle of distortion of the square. Let θ be that angle, then

$$q = C\theta, \text{ where } C \text{ is the co-efficient of rigidity.}$$

The effect of this is that, relatively to the end A, the end D is twisted round through an angle $DOD' = i$ suppose, called the angle of torsion.

In circular measure $i = \frac{\text{arc } DD'}{r}$ (r = radius of tube). Also arc $DD' = l\theta$, θ being a small angle. Therefore $i = l\theta/r$. Since also $\theta = q/C$, we have the angle of torsion $i = ql/Cr$, in terms of the stress. From this we may express the angle of torsion in terms of the twisting moment producing the torsion.

184. *Torsion of a Shaft.*—We now pass on to the consideration of the torsion of a solid cylindrical shaft. First, let us imagine the shaft to be made up of a number of concentric tubes exactly fitting one another, and let us further imagine that at the end of each tube a suitable twisting moment is applied, so that each tube is twisted round through exactly the same angle. This effect will be produced by applying over the section at the end of each elementary tube a tangential stress, which is proportional to the radius of the tube. If we make $q/r = q_1/r_1$, where q_1 and r_1 refer to the outside tube, then the angle of torsion will be the same for all the tubes, and they will not tend to turn relatively to one another, but all together. We may then suppose them united together again in a solid mass. If the stress applied be proportional to the distance from the centre, the shaft will twist just as if it were a set of tubes, each being subjected to the same stress and strain as if it were an independent tube.

Now in the actual case of the twisting of a solid shaft, all portions from the outside inwards to the centre must turn through the same angle, and hence the shearing stress at any point of the section of the shaft must be proportional to its distance from the centre. This is true except very near the point of application of the twisting moment. Suppose, for example, the twisting moment is applied by means of a wheel keyed on the shaft, then in the immediate neighbourhood of the key-way, the stress will not be as stated, but at a short distance along the shaft the stress distributes itself in the manner described. This is another instance of the general principle already employed in the case of stretching and bending.

The total resistance to torsion of the solid shaft is the sum of the twisting moments of all the concentric tubes into which it may be imagined to be divided. Thus

$$T = \Sigma 2\pi r^2 tq\,;\ \text{in which } q = r\,.\,\frac{q_1}{r_1}.$$

$$\therefore\ T = \Sigma_0^{r_1} 2\pi r^2 tr\frac{q_1}{r_1} = \frac{q_1}{r_1}\Sigma_0^{r_1} 2\pi rt\,.\,r^2,$$

that is, the product of the sectional area of each tube multiplied by the distance squared of the area from the axis of the shaft must be taken and summed. The result is called the Polar Moment of Inertia, which may be written I. Its value is $\frac{1}{2}\pi r_1^4$. Thus

$$T = \frac{q_1}{r_1} I = \frac{q_1}{r}\frac{\pi}{2} r_1^4 = \frac{\pi}{2} q_1 r_1^3.$$

It is not to be supposed that the strength of a shaft of any section to resist torsion is proportional to the polar moment of inertia of that section. In non-circular sections the stress is generally greatest not at the points farthest away from the centre, but more often at those which are nearest the centre. The cases of a rectangle, an ellipse and various other forms have been investigated by M. St. Venant who has obtained the annexed results.*

RELATIVE STRENGTHS OF SHAFTS OF THE SAME SECTIONAL AREA.	
FORM OF SECTION.	STRENGTH.
Circular, - - - - - - - -	1
Square, - - - - - - - -	·8863
Rectangle with sides in the ratio $n : 1$, -	$\sqrt{\frac{2}{n+1/n}} \times$ ·8863
Ellipse with axes in the ratio $n : 1$, - -	$\sqrt{n}$ $(n < 1)$

Dropping the suffixes, taking r to be the outside radius, we can write the moment of resistance to torsion of the shaft,

$$T = \tfrac{1}{2}\pi f r^3,\ \text{or}\ \tfrac{1}{16}\pi f d^3;$$

where f is the co-efficient of strength of the material to resist shear-

* Diagrams and particulars with respect to M. St. Venant's results will be found in Sir W. Thomson's *Treatise on Natural Philosophy*, 1st ed., vol. 1, p. 545.

ing. Thus the strength under torsion is proportional to the cube of the diameter. The formula shows that, assuming f to be the same in each case, the strength of a shaft to resist a twisting moment is double its strength to resist a bending moment. Since $i = ql/Cr$ we can eliminate q, and thus obtain

$$i = \frac{2}{\pi C} \cdot \frac{l}{r^4} \cdot T.$$

185. *Diameter of Shaft to transmit a Given Power.*—Having determined the diameter of shaft required to take a given twisting moment we are now able to obtain a solution of the practical question, What diameter of shaft is required to transmit a given horse-power at a given number of revolutions per minute ?

Let T_0 = *mean* twisting moment transmitted in inch-tons, then $T_0 \times 2\pi N$ = work transmitted per minute in inch tons, where N = revolutions per minute of shaft.

Let HP denote the horse-power to be transmitted, then

$$T_0 \times 2\pi N = \frac{33000 \times 12}{2240} H.P.$$

$$\therefore T_0 = \frac{33000 \times 12}{2240 \times 2\pi} \frac{H.P.}{N}.$$

Now the shaft must be strong enough to take not only the mean but the maximum twisting moment.

We may express the maximum in terms of the mean by writing $T = KT_0$, where K is a co-efficient whose value is different in different cases and T = maximum twisting moment, but

$$T = \frac{\pi}{16} fd^3 \text{ or } d^3 \frac{16T}{\pi f}.$$

$$\therefore d^3 = \frac{16 \times 33000 \times 12}{2\pi^2 \times 2240} \frac{K}{f} \frac{H.P.}{N},$$

and

$$d = 5{\cdot}233 \sqrt[3]{\frac{K}{f} \frac{H.P.}{N}}.$$

The value of f depends in some measure on the fluctuation to which the twisting moment is subject, but under ordinary circumstances should not exceed $3\frac{1}{4}$ tons per square inch (Art. 221) for wrought iron, or, probably, about 5 tons for steel, and $2\frac{1}{4}$ tons for cast iron. The value of K, the ratio of maximum to mean twisting moment, depends on the circumstances discussed in Chapter X. We may assume it equal to $1\frac{1}{2}$ under ordinary circumstances, allowing a small addition for the bending due to the weight of the shaft. On substitution we obtain for wrought iron

$$d = 4\sqrt[3]{\frac{H.P.}{N}}.$$

This formula agrees closely with the best practice in screw-propeller shafting.

When the amount of bending to which the shaft is subject is considerable, as in the case of crank shafts, the diameter determined by this formula is too small. It will be seen hereafter that when all the forces acting on the shaft are known, a value of K can be calculated which gives the effect of bending. If we assume $K = 2$, the co-efficient 4 in the above formula will be replaced by 4·5, and this agrees closely with practice in the crank shafts of marine screw engines. In other cases a still larger value may be necessary.

In the formula for the angle of torsion

$$i = \frac{ql}{Cr};$$

if we replace q by its working value for wrought iron (7,200 lbs.), C by 10,500,000 lbs., and i by the circular measure of 1°, we find

$$l = 12 . 7d,$$

showing that under the working stress the shaft twists through 1° for each $12\frac{3}{4}$ diameters in its length. For many purposes this is much too small, and the dimensions of a shaft then depend on stiffness, not on strength, as in the case of beams (Art. 168). The greatest angle of torsion permissible depends in great measure on the irregularity of the resistance, and no general rule can therefore be laid down for it. If the angle of torsion be given and the length, the diameter will depend on the fourth root of the twisting moment, as shown by the formula of Art. 184. In this, as in other cases where dimensions depend on stiffness, not on strength, steel has no advantage over iron, because the co-efficients of elasticity of the two materials are the same, or nearly so. A hollow shaft is both stronger and stiffer than a solid shaft of the same length and weight.

186. *Distance apart of Bearings.*—The distance apart of the bearings of a shaft depends on the stiffness necessary to resist the bending due to the weight of the shaft itself, and of any pulleys or wheels upon it, together with the tension of belts and other similar forces. If the total load be equivalent to m times the weight of the shaft itself uniformly distributed, the length between bearings for a wrought iron or steel shaft d inches diameter will be given approximately for a stiffness of $\frac{1}{1200}$th by Ex. 7, p. 339.

When, as in screw propeller shafting, the bearings are liable to get out of line, too great stiffness in a shaft will produce great straining actions upon it.

187. *Web of a Beam of I Section.*—Torsion is one of the few cases in practice where a simple distorting stress occurs alone and not in combination with other kinds of stress. It generally happens that a normal stress is combined with it; such, for example, is the case in the web of a beam of I section, to which we next proceed to direct our attention. Taking a transverse section, the normal stress at a point distant y from the neutral axis is given by the formula

$$\frac{p}{y}=\frac{M}{I},$$

and is therefore the same for the same values of M and I, whether the web be thin or thick, while it will be shown presently that the tangential stress is greater the thinner the web, and becomes the most important element when the web is thin.

Let us suppose, for simplicity, the flanges equal, and also that the beam is supported at the ends and loaded in the centre with a weight W.

As we have previously seen, the flanges will sustain the greater

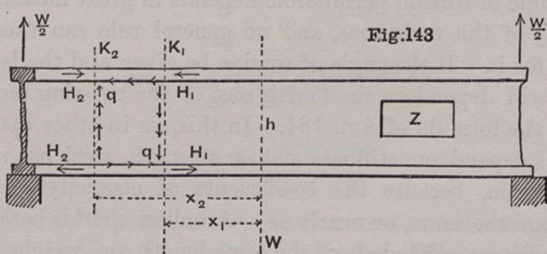


Fig:143

portion of the bending moment, the web carrying only a small portion of it, $\frac{1}{6}$, if the area of the web equals the area of each flange. For simplicity, let us imagine the flanges to take the whole of the bending. Let K_1 and K_2 (Fig. 143) be two transverse sections of the beam at distances x_1 and x_2 from the centre of the beam, $2a$ being the span of the beam, the bending moment at the first section,

$$M = \tfrac{1}{2}W(a - x_1) \text{ and at the 2nd } M_2 = \tfrac{1}{2}W(a - x_2).$$

Now, supposing the flanges to take the whole of the bending the stress H produced on the flanges is given by the formula

$$Hh = M. \quad \text{Thus at } K_1 \text{ we have } H_1 = \frac{W(a - x_1)}{2h},$$

$$\text{and at } K_2 \text{ we have } H_2 = \frac{W(a - x_2)}{2h},$$

and similar forces on the bottom flange only reversed in direction. There will thus be a resultant force $H_1 - H_2$ tending to push the portion K_1K_2 of the flange to the left,

$$H_1 - H_2 = \frac{W(x_2 - x_1)}{2h}.$$

This force is balanced by the resistance of the web to shearing along the line of junction with the flange.

Since H_1H_2 is proportional to the length of K_1K_2, the shearing force per unit of length of web $= W/2h$. If we suppose t to be the thickness of the web, the intensity of the shearing stress will be

$$q = \frac{W}{2ht}.$$

Thus, considering the portion of the web between the sections K_1 and K_2 apart by itself, we see that on the upper and lower horizontal edges of it, where it joins the flanges, it is subject to a shearing stress of intensity q. Now, to balance this stress there must act on the vertical sides KK a shearing stress of equal intensity q. Now, the shearing force for the vertical sections KK is $\frac{1}{2}W$. Supposing the web to be of rectangular section and of height h, then, assuming the whole of the shearing force to be borne by the web, the intensity of the shearing stress on the vertical sections is

$$q = \frac{W}{2ht}.$$

Therefore the assumption that the flanges take the whole of the bending moment is equivalent to supposing the web to take all the shearing. Assuming this, we see that the shearing stress, being uniformly distributed over the vertical section, will be accompanied by an equal shearing stress on any horizontal section. When considered alone, the effect of these shearing stresses on planes at right angles to one another is to produce tensile and compressive stresses on the web in directions making an angle of 45° with the horizontal and vertical planes; and thus the web may be superseded by an indefinite number of diagonal bars inclined at an angle of 45°, thus forming a lattice girder.

If the web is designed so as to be strong enough only to withstand the shearing stress, replacing q by f the co-efficient of strength against shearing f, we find

$$t = \frac{W}{2hf}.$$

The influence of the normal stress due to bending will be considered in the next chapter. Its effect is greatly to increase the strain on the web (see Art. 202), which besides will in most cases exhibit weakness on account of the compressive stress in one of the diagonal directions. If the distance between the flanges is great, the web will be liable to yield by buckling or lateral flexure (see page 317). To prevent this, the web must be stiffened by angle irons rivetted on it. But the girder would then be made heavy, and it is therefore more economical to make large girders with openwork diagonal bracing.

We have in this investigation supposed the beam loaded in the middle, so that the shearing force is uniform throughout the length of each half, and the problem was thus simplified. But the same principles apply if the load be distributed in any manner. The shearing force will then vary from point to point along the beam.

188. *Distribution of Shearing Stress on the Section of a Beam.*—In beams of other types it is still true that the central parts of the beam are subject to shearing, but the total amount of the shearing stress being the same, its intensity is much less, because it is distributed over a greater area. The intensity at the centre of the beam is found as follows for a beam of uniform transverse section.

Suppose the beam supported at the ends and loaded in the middle as before, and take section K_1K_1', K_2K_2'. Let NN be the neutral surface, SS the neutral axis (as in Fig. 122, Art. 153). Above the neutral surface the beam is compressed and below it it is stretched by equal forces. Let these forces be H_1 for the section K_1K_1, and H_2 for the section K_2K_2; then, reasoning as before, the shearing force in the neutral surface must be $H_1 - H_2$, and the intensity of that shearing stress, if b_0 be the breadth at the centre,

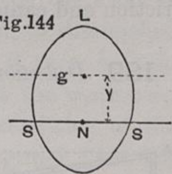
$$q = \frac{H_1 - H}{NN \,.\, b_0}.$$

Now to find H we have, in the notation of Art. 154,

$$H = \Sigma bt \,.\, p = \frac{M}{I} = \Sigma bt \,.\, y = \frac{M}{I} \,.\, kA\bar{y},$$

where kA is the area of that part of the section A which lies above the neutral axis (SLS in Fig. 144), and $\bar{y}$ is the distance of its centre of gravity (g) from that axis. The same result will be obtained if we take that part of the area which lies below the axis. We now have, as before, by substitution,

Fig.144



$$H_2 - H_1 = \frac{W(x_2 - x_1)}{2} \,.\, \frac{k \,.\, A\bar{y}}{I},$$

whence, as usual, replacing I by nAh^2, we find

$$q = \frac{W}{2n} \cdot \frac{k \,.\, \bar{y}}{b_0 h^2}.$$

The total shearing stress on the section is $\frac{1}{2}W$, and therefore the mean intensity is

$$q_0 = \frac{W}{2A}.$$

Thus we obtain the ratio of the shearing stress on the neutral surface to the mean shearing stress on the whole transverse section.

$$\frac{q}{y_0} = \frac{A}{b_0 h} \cdot \frac{k \,.\, \bar{y}}{nh}.$$

In the present case where the beam is loaded in the middle the shearing stress is the same at all points of the neutral surface, but in other methods of loading this will not be the case. The formula however in all cases gives the ratio in question correctly, which will be found to be greater than unity. In fact it is not difficult to see that the shearing stress must be greatest at the neutral surface, and must diminish to zero as we approach the external surface of the beam. The formula then gives the maximum shearing stress on the section.

Let us for example take a rectangular section, then

$$A = b_0 h : k = \tfrac{1}{2} : n = \tfrac{1}{12} : \bar{y} = \tfrac{1}{4}h.$$

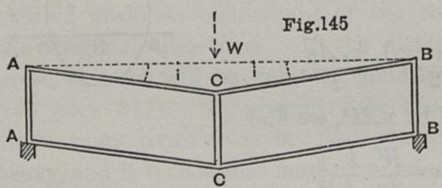
$$\therefore \frac{q}{q_0} = \frac{12}{8} = \frac{3}{2},$$

so that the greatest shearing stress is $1\frac{1}{2}$ times the mean. In like manner in a circular section it may be shown to be as 4 : 3. Other cases are given in the examples at the end of this chapter.

In all cases where a bar is subject to shearing and the sides of the bar are free from tangential stress, the stress on the transverse section will be increased in this way. In pin joints where the pin is an easy fit the only tangential stress on the sides of the pin will be due to friction and cannot be relied on.

189. *Deflection due to Shearing.*—A certain part of the deflection of a beam is due to the distortion of its central parts. Returning to the beam of I section, loaded in the middle, suppose the flanges hinged at the centres, and let vertical stiffening pieces AA, BB, CC, be rigidly connected to the web but hinged to the flanges, then distortion of the web takes place as shown in a very exaggerated way in the figure (Fig. 145), causing a deflection δ of the beam such that

Fig.145



$$\frac{\delta}{\frac{1}{2}l} = i = \frac{q}{C} = \frac{W}{2htC},$$

where C as before is the co-efficient of rigidity, and q the shearing stress is expressed as before.

$$\therefore \delta = \frac{Wl}{4htC} = \frac{ql}{2C}.$$

For wrought iron take $q = 9{,}000$ for the working load and $C = 9{,}000{,}000$, then

$$\delta = \frac{l}{2000},$$

which is about half the working deflection due to bending in ordinary cases.

This calculation however greatly exaggerates the deflection due to shearing even in a beam of I section, for the web cannot in general be so thin as to give a stress of 9,000 lbs. per square inch, and the effect is much less for a uniformly distributed load. Nevertheless in beams of this class the deflection due to shearing is a sensible part of the whole, the more so as in rivetted girders the union of the parts seldom renders them completely rigid. This is the principal reason why large girders show a considerably smaller modulus of elasticity when the deflection is calculated in the usual way than solid bars. In bars this part of the deflection is insensible, the distorting stress being small.

190. *Weakening of Beams by Insufficient Resistance to Longitudinal Shearing of the Web.*—If the central part of a beam be cut away as shown at Z in Fig. 143, the strength of the beam will be diminished and its deflection increased. This wili be true even if there be only a narrow longitudinal slot at the neutral surface, but the weakening is the greater the more material is cut away, the condition of the beam in an extreme case becoming that of an N girder (Art. 25) without diagonal bracing. Imperfect union of the parts of the web along either a longitudinal or vertical section will have the same effect in a less degree. Wooden ships not unfrequently exhibit weakness due to this cause, and to counteract it diagonal riders of iron are introduced to take part of the shearing force. The ordinary formula for resistance to bending cannot be applied in such cases.

191. *Joints and Fastenings.*—Among the most important cases of shearing are those which occur in joints and fastenings of all kinds. Such questions are generally very complex, considered as purely theoretical problems, and the direct results of experience are always required at every step to interpret and confirm theoretical conclusions.

When two pieces butt against each other the pressure is transmitted by contact only, and fastenings are therefore required not for transmission of stress but merely to retain the pieces in their relative positions. With tension it is otherwise; it is still necessary to have surfaces which press against one another, and these can only be obtained by the introduction of fastenings which transmit stress laterally, and are therefore subject to shearing and bending. The parts of a joint should be so proportioned as to be of equal strength. One of the simplest examples is that of a pin joint connecting two bars in tension as in a suspension chain with bar links. Fig. 1. (Plate VIII.) shows a pair of bars of rectangular section connected together by links C and D united as shown by pins passing through eyes at their extremities. In suspension chains there are generally four or five bars placed side by side, but the principle is the same in any case. The pull on the chain is balanced by the resistance to shearing of the pins, which have besides to resist bending. Let d be the diameter of the pins, b the breadth, t the thickness of one of the bars, t' the thickness, b' the breadth of the links which for equality of strength, that is to say, of sectional area, will be connected by the equation

$$2b't'=bt.$$

Let f be the co-efficient of strength for tension, then $\frac{4}{5}f$ (Art. 224) will be the co-efficient for shearing, whence remembering that the maximum shearing stress exceeds the mean in the ratio 4 : 3 as shown above,

$$P=btf=2\pi\frac{d^2}{4}\cdot\tfrac{3}{5}f=\frac{3\pi}{10}fd^2.$$

According to this estimate the area for shearing should be five-thirds the area for tension, but the true ratio is probably not so great: the calculation supposes that the sides of the pin are subject to normal stress alone, whereas the tangential stress due to friction must be considerable. Besides the strength of iron such as is used for pins is greater than that of plates. As the calculation applies only to stress within the elastic limit, it is impossible to test it by experiment. In practice the areas are made nearly equal when nothing else is considered except resistance to shearing. When, however, such a joint is actually pulled asunder it frequently gives way in quite a different manner before shearing commences. Imagine a cylinder pressed down into a semicircular hollow which it very exactly fits, and let the material be elastic and soft compared with the cylinder, then, reasoning as in Art. 115, p. 249, it appears that the stress between the surfaces will be given by the equation

$$p=p_0\cdot\cos\theta,$$

and if P be the pressing force, l the length,

$$p_0\cdot\tfrac{1}{4}\pi dl=P \text{ or } p_0=\frac{4P}{\pi dl}.$$

If the pin fits the eye exactly the pressure will follow this law so long as the tension is small. As the tension increases, however, the pressure becomes more uniformly distributed over the semi-cylinder, because the eye-hole tends to contract laterally as the links of a chain of rings would do under tension. The other extreme supposition would be to suppose it uniformly distributed, then

$$p_0 \cdot dl = P \text{ or } p_0 = \frac{P}{dl}.$$

The actual pressure will be intermediate between these two values. If p_0 be too great the metal crushes under the pressure. The theoretical limit to p_0 will be considered hereafter (Art. 222); for the present it will be sufficient to say that the experiments of Sir C. Fox* have shown that the curved area should be at least equal to the sectional area under tension, that is to say we ought to have

$$\tfrac{1}{2}\pi dl = bt = \tfrac{1}{10}3\pi d^2.$$

To satisfy these conditions we must have for the ordinary case where the thickness of the eye is the same as that of the rest of the bar

$$d = \tfrac{2}{3}b : t = \tfrac{2}{5}b \text{ approximately.}$$

The first of these gives the diameter of pin recommended by Sir C. Fox and other authorities; the second gives the greatest thickness of link for which this diameter gives sufficient resistance to shearing, but the thickness in actual examples of suspension links is generally considerably less. The pin has also to resist bending, but of small amount in the present example. The sides and end of the eye are subject to tension, but it is not uniformly distributed, the question being similar to that of a thick hollow cylinder under internal fluid pressure. The mode in which the eye crushes and then fractures transversely by tension, is shown in Plate VIII., and further described in Chapter XVIII.

In rivetted joints the question is further complicated by the friction between the plates united by the rivets. On the subject of joints and fastenings the reader is referred to Prof. W. C. Unwin's work cited on page 134.

EXAMPLES.

1. Find the diameter of a shaft for a twisting moment of 1000 inch-tons; stress allowed being $3\frac{1}{2}$ tons per square inch. *Ans.* Diameter = 11·3″.

2. From the result of the previous question deduce the diameter of a shaft to transmit 5000 H.P. at 70 revolutions per minute. Maximum twisting moment $= \frac{3}{2}$ the mean. *Ans.* 15·7″.

* Proceedings of the Royal Society, vol. xiv., p. 139.

3. The angle of torsion of a shaft is not to exceed 1° for each 10 feet of length. What must be the diameter for a twisting moment of 100 inch-tons—modulus of transverse elasticity, 10,500,000?

Compare the result with the diameter determined from consideration of strength, taking a co-efficient of $3\frac{1}{4}$ tons. *Ans.* Diameter determined from consideration of stiffness $= 6{\cdot}2''$. Diameter from consideration of strength $= 5{\cdot}2''$.

4. Show that the resilience of a twisted shaft is proportional to its weight.

$$Ans.\ \text{Resilience} = \tfrac{1}{2}Ti = \frac{f^2}{C} \times \frac{\text{Volume}}{4}.$$

5. Compare the strengths of a solid wrought iron shaft and hollow steel shaft of the same external diameter, assuming the internal diameter of the hollow shaft half the external, and the co-efficient for steel $1\frac{1}{2}$ times that for iron.

6. The external diameter of a hollow shaft is double the internal. Compare its resistance to twisting with that of a solid shaft of the same weight and material.

$$Ans.\ \text{Strength is greater in the ratio } \frac{5\sqrt{3}}{6} = 1{\cdot}443.$$

7. A pillar, whose sectional area is $1\frac{1}{2}$ square feet, is loaded with two tons. Find in lbs. per square inch the intensity of the tangential stress on a plane inclined at 15° to the axis of the pillar. *Ans.* Tangential stress $= 5{\cdot}18$ lbs.

8. In a single rivetted lap joint, the pitch of the rivets being three diameters or six times the thickness of the plates, find, 1st, the mean stress on the reduced area; 2nd, the shearing stress on the rivets; and, 3rd, the mean direct stress between rivet and plate: the tension of the joint being 4 tons per square inch of the original area, and the friction between the two surfaces of the plate in contact neglected.

Ans. Mean tension on reduced area - $= 6$ tons.
Shearing stress on rivet - - $= 7{\cdot}6$ tons.
Mean direct stress $\dfrac{4 \times \text{pitch} \times \text{thickness}}{\text{diameter} \times \text{thickness}} = 12$ tons per sq. in.

9. In a beam of I section with flanges and web which may be considered as rectangles, the thickness of each flange is one sixth the outside depth of the beam, and the breadth twice the thickness. The thickness of the web is half that of the flanges: find the ratio of maximum to mean shearing stress on the section. *Ans.* $\frac{18}{7}$.

10. In the last question find the fraction of the whole shearing force which is taken by the web. *Ans.* 80 per cent.

11. If the sectional area of the web of a flanged girder be proportional to the shearing force and the r^{th} power of the depth; find the most economical ratio of span to depth and the limiting span.

If the web be C and each flange A, as on page 317, the whole sectional area is $C + 2A = S$ and the moment of resistance to bending is

$$M = fh(\tfrac{1}{2}S - \tfrac{1}{3}C).$$

Assuming now $C = c\,.\,h^r$, where c is constant,

$$\frac{M}{fh} + \tfrac{1}{3}ch^r = \tfrac{1}{2}S,$$

and therefore, for a given value of M, S is least when

$$M = \tfrac{1}{2} \cdot \frac{r}{r+1} \cdot fSh : \quad C = \frac{3S}{2(r+1)}.$$

In a girder with lattice web the same formula for M holds good, but $S = C(r+1)$.

If now $F = f'C$, where F is the shearing force and f' is a co-efficient much less than the resistance to shearing on account of the necessary stiffening (Art. 187),

$$M = \tfrac{1}{3} \cdot r \frac{f}{f'} \cdot Fh,$$

a formula which will give the required ratio (N) for any given load. If the load be uniformly distributed

$$N = \frac{4r}{3} \cdot \frac{f}{f'}.$$

It is probable that in most cases $r = 2$ nearly, but that the value of f/f' will vary, according to the type of girder, from 2 to 4, being greatest for a continuous web.

The limiting span of a girder of uniform section is readily shown to be

$$L = \frac{4r}{r+1} \cdot \frac{\lambda}{N}. \qquad \text{(Comp. Ex. 13, p. 324.)}$$

The weight of a smaller girder of the same type is found as in Ch. IV.

On the influence of size on the strength of vessels, see papers by Mr. John and the late Mr. Froude in the *Transactions of the Institutions of Naval Architects* for 1874.

12. Show that the weight in lbs. of a shaft to transmit a given horse power at a given number of revolutions is

$$W = 21{,}000 \cdot \frac{K \cdot HP}{N\lambda} \cdot \frac{l}{d},$$

the value of λ being given as in Ch. XVIII, the proper co-efficient of resistance to shearing being used. The rest of the notation is explained on page 362.

The distance to which power can be transmitted by shafting with a given loss by friction is given by Ex. 18, p. 272, when the angle of torsion is immaterial, but in practice is generally limited by the necessity of having sufficient stiffness. The bending and twisting of shafts is considered in Chapters XVII., XVIII.