CHAPTER XVI.

IMPACT.

192. *Preliminary Remarks. General Equation of Impact.*—Hitherto the forces applied to the body or structure under consideration have been imagined to have been originally very small, and to have increased gradually to their actual amount. This is seldom exactly the case in practice, while it frequently happens that the load is applied all at once, or that it has a certain velocity at the instant it first comes in contact with the body. Such cases may all be included under the head of IMPACT, and will form the subject of the present chapter.

When a body in motion comes into contact with a second body against which it strikes, a mutual action takes place between them, which consists of a pair of equal and opposite forces, one acting on the striking body, the motion of which it changes, the other on the body struck which it in general moves against some given resistance. Certain changes of figure and dimension, or, in other words, strains are likewise produced in both bodies, in consequence of the stress applied to them.

The simplest case is where the impact is direct and the resistance to motion has some definite value, as, for example, where a pile is driven by the action of a falling weight. Here let R be the resistance which the pile offers to be driven; that is to say, the load which, resting steadily on the pile, would just cause it to commence to sink; let W be the falling weight, h the height from which it falls, x the space through which the pile sinks in consequence of the blow; then the mutual action between the pile and the weight at the instant of impact consists of a pair of equal and opposite forces R.

The whole height through which the weight falls is $h+x$, and the space through which the resistance is overcome is x; hence, equating energy exerted and work done, we have

$$W(h+x)=Rx.$$

This equation shows that any resistance, however great, can be overcome by any weight, however small; and also, that the force of the blow, as measured by the space the pile is driven, is proportional to its energy. We have however assumed that the whole energy of the blow is employed in driving the pile, whereas some of it will always be expended in producing vibrations and in damaging the head of the pile and the bottom of the weight. As the pile is driven deeper, the resistance to being driven increases and at length becomes equal to the crushing stress of the material: the pile then sinks no farther, the whole of the energy of the blow being wasted in crushing.

This last is also the case of impact of a flying shot against a soft plastic substance, which exerts during deformation a definite force uniform or variable which brings the weight to rest in a certain space. Suppose V the velocity of the shot, x the space, and R the mean resistance which the substance offers, then the kinetic energy of the shot is $WV^2/2g$, while the work done is Rx, equating which

$$W.\frac{V^2}{2g}=Rx.$$

Here the whole energy of the blow is spent in producing changes of figure in the body struck; but if the striking body had been soft, and the body which is struck hard and immoveable, the energy of the blow would have been employed in producing change in the shape of the striking body. Thus we may write down as the general equation of impact—

Energy of blow = Work done in overcoming the resistance to movement of the body struck.
+ Work done in internal changes in the striking body
+ Work done in internal changes in the body struck.

Which of these three terms is the most important will depend on the relative magnitude of the resistance to movement, and the crushing stress of the materials of the two bodies. If either body have a sensible motion after impact, the corresponding kinetic energy must be taken account of in writing down the equation, as will be seen farther on.

193. *Augmentation of Stress by Impact in Perfectly Elastic Material.*—We now proceed to apply the equation to the case which most immediately concerns us, namely, that of impact on perfectly elastic material, including in this the effect of a load which is applied all at once.

We will suppose a structure or piece of material of any kind resting on immoveable supports, and struck by a body harder than itself, so that we may neglect all changes produced in the striking body. Generally in both bodies there will also be produced vibrations, of the nature of those constituting sound, which absorb a certain amount of energy, but this we shall neglect. The whole energy of the blow then is supposed expended in straining the structure, or piece of material, struck by the blow.

Now the effect of impact is to produce a mutual action S, which represents a force applied to the structure at some definite point. In consequence of this the structure suffers deformation, and the point of application moves through a space x. The resistance to deformation is proportional to x, because the limit of elasticity is not exceeded; it therefore commences by being zero, and increases gradually till the velocity of the striking body is wholly destroyed. The mean value of the resistance is therefore one half its maximum value. During the first part of the period occupied by the impact the mutual action S is greater than the resistance, and during the second part less, as will be explained fully presently; but, when the maximum strain has been produced, the mean value during the whole period must be exactly equal to the mean resistance, the weight and the structure being at rest. The state of rest is only momentary, for the strained structure will immediately, in virtue of its elasticity, commence to return to its original form; but, for the moment, a strain has been produced, which is a measure of the effect of the blow, and which must not exceed the powers of endurance of the material.

Let now R be the maximum resistance, and let the blow consist in the falling of a weight W, through a height h above the point where it first comes in contact with the structure; then $h + x$ is the whole height fallen through, and it follows from what has been said that

$$W(h + x) = \tfrac{1}{2}Rx.$$

The resistance R may also be described as the "equivalent steady load," being the load which, if gradually applied at the point of impact, would produce the same stress and strain which the structure actually experiences. We most conveniently compare it with W by supposing that we know the deflection δ which the structure would experience if the striking weight W were applied as a steady load at the point of impact; we then have, since the limits of elasticity are not exceeded,

$$\frac{x}{\delta}=\frac{R}{W}.$$

Substituting the value of x we get

$$\frac{R^2}{W^2}=\frac{2R}{W}+\frac{2h}{\delta}.$$

Let the height h be n times the deflection δ, then solving the quadratic, the positive root of which alone concerns us,

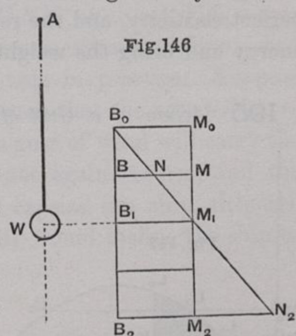
$$R = W(1 + \sqrt{2n + 1}),$$

an equation which shows how the effect of a load is multiplied by impact.

194. *Sudden Application of a Load.*—A particular case is when $h = 0$, then $R = 2W$. So that if a load W is suddenly applied to a perfectly elastic body, from rest, not as a blow, it will produce a pressure just twice the weight. This case is so important that we will consider a special example in detail.

Let a long elastic string be secured at A. If a gradually increasing weight be applied the string will stretch, and the weight descend. Let the load required to produce any given extension be represented by the ordinates of the sloping line B_0NN_2 (Fig. 146). Next, instead of applying a gradually increasing load, let a weight W represented by B_0M_0 be applied all at once to the unstretched string. The string will of course stretch, and the weight descend. When it has reached B (Fig. 146) the tension of the string pulling upwards, being represented by BN, will be less than W acting downwards. Moreover, in the descent B_0B_1 an amount of energy has been exerted by the weight represented by the area of the rectangle B_0M_0MB.

Fig.146



At the same time the work which has been done in stretching the string is represented by the area of the triangle B_0NB. The excess of energy exerted over work done has been employed in giving velocity to the descending weight, and is stored as kinetic energy in the weight.

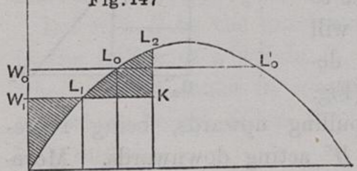
On reaching B, the tension of the string is just equal to the weight, but the stretching does not cease here. The weight has now its greatest velocity, which corresponds to an amount of kinetic energy represented by the triangle $B_0M_0M_1$. Although any further extension of the string causes the upward pull of the string to be greater than the weight W, yet the weight will go on descending until the energy that it has exerted is equal to the work done in stretching the string; then the kinetic energy will be exhausted and the weight will be brought to rest. This will occur when the area of the triangle $B_0N_2B_2$ equals the area of the rectangle $B_0M_0M_2B_2$, that is when $B_2N_2 = 2B_2M_2$, or $B_0B_2 = 2B_0B_1$.

We thus see that the tension of the string produced by the sudden application of the load is twice that due to the same load steadily applied.

The string will not remain extended so much as B_0B_2, for now the upward pull of the string, exceeding the weight, will cause it to rise. On reaching B_1 it will have the same velocity upwards that it had on first reaching B_1 downwards. This will carry it to B_0, from which it will again fall, and so on. Practically, the internal friction due to imperfect elasticity, and the resistance of the air, will soon absorb the energy and bring the weight to rest at B_1.

195 *Action of a Gust of Wind on a Vessel.*—Another interesting example of the way in which the sudden application of a load augments its effect is furnished by the case of a vessel floating upright in the water and acted on by a sudden gust of wind, a question which, though not strictly belonging to this part of the subject, involves exactly the same principle.

Fig. 147



First, suppose no wind pressure, but that a gradually increasing couple is applied to heel the vessel.

If along a horizontal line (Fig. 147) angles of heel be marked off, such as ON, and for those points ordinates such as NL are set up to represent on some convenient scale the magnitude of the couple required to produce that angle of heel, a curve OL will be obtained, which we have already (p. 198) called the curve of *Statical Stability* of the ship.

Now suppose a steady wind pressure to be gradually applied. It will produce on the masts and sails a definite moment, on account of which the ship will incline to a certain angle, such that the ordinate of the curve of stability corresponding to that angle will represent the moment of the wind pressure. So long as the wind is constant, she will remain inclined at that angle. Next suppose the same wind pressure to be suddenly applied all at once, as by a gust to the ship floating upright at rest. The ship will heel over, and until she is inclined to some extent the wind moment will be greater than the righting moment, and the excess will cause the ship to acquire an angular velocity. Accordingly, when she arrives at the angle of heel corresponding to the moment of wind pressure on the stability curve, she does not come to rest, but inclines farther, until the energy exerted by the wind pressure is all taken up in overcoming the righting moment through the angle of inclination. The work thus done is represented by the area of the curve of stability standing above the angle of heel reached.

Let OW_1 represent the magnitude of the wind moment. The ship will incline until the area $OL_2N_2 =$ area OW_1KN_2, or area $OW_1L_1 =$ area L_1L_2K; that is, if the moment of wind pressure remains undiminished as the ship heels, which will hardly be true in practice. Suppose the moment of wind pressure OW_0 to be such that the area $OW_0L_0 =$ the area $L_0L_2L_0'$. In this case the sudden gust of wind will carry the ship to such an angle ON_0' that she will not again return; and the smallest additional pressure of wind will capsize the ship, although that same wind pressure applied gradually would incline the ship to the angle ON_0 only.

196. *Impact at High Velocities. Effect of Inertia.*—Returning to the general case of impact against a perfectly elastic structure (Art. 193), let us now take the other extreme case in which the height through which the weight falls is great compared with the deflection

δ due to the same weight gradually applied; then, since n is great, our equation becomes

$$R = W . \sqrt{2n} = W\sqrt{\frac{2h}{\delta}},$$

which may be written in either of the forms

$$R = \sqrt{\frac{2W}{\delta}} \cdot \sqrt{Wh} \quad (1); \quad \text{or } x = \sqrt{2h\delta} \qquad (2.)$$

The first form shows that the stress produced by the impact is proportional to the square root of the energy of the blow, and the second, that the deflection occasioned by the fall of a given weight is proportional to the square root of the fall, or, what is the same thing, to the velocity of impact. These results are exact when the impact is horizontal, and the last has been verified by experiment. It is to be remembered that the limits of elasticity are supposed not to be exceeded: when a rail or carriage axle is tested by a falling weight, as is very commonly done, the energy of the blow is generally much in excess, and the piece of material suffers a great permanent set, the resistance is then approximately constant instead of increasing in proportion to the deflection. The effect of the blow is then more nearly directly proportional to its energy. It will be seen presently how small a blow matter is capable of sustaining without injury to its elasticity.

The effect of a blow, on a structure or piece of material as a whole, is diminished, on account of its inertia, by an amount which is greater the greater the velocity of impact, but which varies according to the relative mass and stiffness of its parts. In the act of yielding the parts of the body are set in motion, and the force required to do this is frequently greater than the crushing strength of the materials, so that a part of the energy of the blow is spent in local damage near the point of impact.

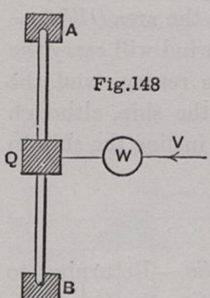


Fig.148

Figure 148 shows a narrow deep bar AB, the ends of which rest in recesses in the supports, which prevent them from moving horizontally, but do not otherwise fix them. The bar carries a weight Q in the centre, against which a second weight W moving horizontally strikes with velocity V. The bar being very flexible horizontally, the weight Q at the first instant of impact moves as it would do if free; that is, the two weights move onwards together with a common velocity v fixed by the consideration that the sum of the momenta of the two weights is the same before and after impact, so that

$$WV = (W + Q)v.$$

The energy of the two weights after impact is

$$(W + Q)\frac{v^2}{2g} = \frac{W^2}{W + Q} \cdot \frac{V^2}{2g},$$

showing that the energy of the blow has been diminished in the proportion $W : W + Q$. The loss is due to the expenditure of energy in damage to the weights.

If now, instead of a weight Q attached to the centre of a flexible bar, we suppose the bar less flexible and of weight Q, the effect of the blow is diminished by the same general cause, but not to the same extent: the diminution cannot be calculated exactly, but may be estimated by replacing Q in the preceding formula by kQ, where k is a fraction to be determined by experiment. In a series of elaborate experiments made by Hodgkinson on bars struck horizontally by a pendulum weight, it was found that k was $\frac{1}{2}$.

We are thus led to separate the energy of a blow into two parts:

$$E_1 = \frac{W^2}{W + kQ} \cdot \frac{V^2}{2g} : \quad E_2 = \frac{k \, . \, WQ}{W + kQ} \cdot \frac{V^2}{2g}.$$

The first of these strains the structure or piece of material as a whole, and the second does local damage at the point of impact. Hence the great difference which exists between the effect of two blows of the same energy, one of which is delivered at a low, and the other at a high velocity. At high velocities most of the energy is expended in local damage; at low velocities most is expended in straining the structure as a whole.

If the body which is struck be in motion, instead of resting on immoveable supports, as in Fig. 148, the energy of the blow will be diminished. This case has been considered in Ch. XI., p. 280, where it is shown that the energy of the collision is

$$E = \frac{WQ}{W + Q} \cdot \frac{V^2}{2g},$$

where V is the relative velocity of the bodies. Of this a part—represented, as before, by replacing Q by kQ—is spent in local damage and the rest in straining the structure as a whole.

The exceptional case where, as in the collision of billiard balls, the limit of elasticity is not exceeded at the point of impact, need not be here considered. The energy of local damage is, then, not wholly dissipated in internal changes: a part is recovered during the restitution of form which occurs in the second part of the process of impact, and increases the action on the structure as a whole. In ideal cases the whole may be thus recovered, but, in practice, a portion is always employed in producing local vibrations, and finally dissipated by internal friction.

197. *Vibrating Loads. Synchronism.*—The load on a structure may vary from time to time, continuously, or otherwise, and its effect will then, in general, be greatly augmented. Some simple examples will now be considered.

Returning to the case of the weight suspended from an elastic string (Fig. 146, p. 377); suppose in the first instance the weight at rest, then the corresponding extension (δ) is B_0B_1 in the figure and the position of the weight is B_1. Next imagine the weight raised vertically and suddenly released, it will oscillate about B_1 as a mean position. In any position B the tension of the string is represented by BN and the weight by BM, so that NM represents an unbalanced force which draws the weight downwards when it is above B_1, and upwards when below. Now NM is proportional to BB_1, and the weight therefore moves under a force always proportional to its distance from B_1.

This kind of motion is known as a "simple harmonic motion"; we have already had an example in the case of a piston moving in its cylinder; for in Ch. IX., p. 235, it was shown that the force necessary to move the piston varies as the distance from the centre of the stroke. In fact Fig. 99, p. 234, may be taken to represent the motion, the velocity of the weight in any position being represented by QN. From the formulæ given in the article cited it is easy to show that the time of a double vibration of the weight is given by

$$t = 2\pi\sqrt{\frac{\delta}{g}},$$

being the same as that of the small oscillations of a pendulum of length B_0B_1. It is dependent only on the elasticity of the string and the magnitude of the weight, not on the extent of the vibration.

The vibrations of any structure may be distinguished into general and local, that is into the vibrations of the structure as a whole, and the vibrations of its parts. All such vibratory motions are of the same general character, as in the simple case just described they take place in certain definite times depending on the inertia and elasticity of the structure and its parts.

Next suppose the weight (Fig. 146) oscillating about B_1, and let B be the extreme upward position. At the instant when the weight is at B imagine a small downward force P applied; the effect of this will be that the weight descends to a position B_2 before coming to rest, such that $B_1B_2 > BB_1$, instead of being equal to BB_1, as would otherwise be the case. Then suppose P removed, the weight will rise to a point as much above B as B_1B_2 is greater than BB_1. Again suppose P applied, then the weight will descend below B_2, and this process may be continued indefinitely. Thus it appears that a load P, however small, if applied and removed at intervals, *corresponding to the natural period of vibration of the weight W*, will produce a vibration of continually increasing extent, thus augmenting indefinitely the tension of the string, which will soon break, however small the original load W and its fluctuation P. If the weight P be applied as before at B, but removed and replaced at a different interval, the vibration will still augment, in the first instance, but the augmentation will be limited, and will be succeeded by a diminution, and so on indefinitely.

In the foregoing simple example numerical results could readily be obtained if necessary; in actual structures and machines the circumstances are much more complex, and calculations are therefore generally difficult, but the same general principles hold good. Whenever the load on a structure fluctuates the stress due to it is greater than that which corresponds to the maximum load: and the augmentation is greater the more nearly the period of fluctuation approaches the period of vibration of the whole structure, or of that part of it immediately affected by the load. Vibrations of the same period are often described as "synchronous."

As examples of a fluctuating load may be mentioned—

(1) When a company of soldiers march in regular time over a suspension bridge vibrations of the flexible structure are set up which are constantly augmented by synchronism. On a girder bridge the augmentation would be comparatively small, the period of vibration of the bridge being generally very different.

(2) In certain torpedo boats the vibration due to the action of the screw is excessive at one particular speed. This is an effect of synchronism between the revolutions of the screw and the period of bending vibrations of the boat in a horizontal plane.

(3) When a ship rolls broadside on to a series of equal waves the rolling is increased by the action of the waves, and is greatest when the period of the waves is equal to the period of rolling of the ship in still water.

One case of a fluctuating load can be completely worked out without much difficulty, and the result has been applied to various purposes. This is where the load fluctuates according to the harmonic law already considered for an elastic string. The calculation cannot be given here, but it may be mentioned that it is in this way that the late Mr. Froude arrived at his well known conclusions respecting the rolling of ships amongst waves.*

198. *Impact when the Limits of Elasticity are not Exceeded. Resilience.*—The effect of impact on perfectly elastic material may also be dealt with by considering the amount of energy stored up in the body in consequence of the deformation which each of its elementary parts have suffered. We have already seen that when a piece of material is subjected to a simple uniform longitudinal stress of intensity p, the amount of work U done by the stress is

$$U = \frac{p^2}{2E} \times \text{Volume}.$$

Let w be the weight of a unit of volume of the material, and W the weight of the body considered, then we may write our equation

$$U = W . H,$$

where H is a certain height given by

$$H = \frac{p^2}{2Ew},$$

and the whole elastic energy of the body may be measured by this height, which is the distance through which the body must fall to do an equivalent amount of work.

If for p we write f the elastic strength of the material, then we obtain what we have already called the Resilience of the body, and H becomes what we may call the "height due to the resilience," which,

* *Transactions of the Institution of Naval Architects*, vol. ii.

for each material, has a certain definite value, given in feet in Table II., Ch. XVIII., for various common materials.

Now in cases of impact where the limit of elasticity is not exceeded, the whole energy of the blow is spent in straining the material or structure, and hence that energy must not, in any case, exceed the resilience. Thus, on reference to the table, it will be seen that in ordinary wrought iron the height is given as 2 ft. 2 in., from whence it follows that in the most favourable case a piece of iron will not stand a blow of energy greater than that of its own weight falling through twenty-six inches, without being strained beyond the elastic limit. If the parts of the body are subject to torsion, about 50 per cent. may be added to these numbers, but, on the other hand, they are subject to large deductions on account of the inequality of distribution of stress within the body. Only a portion of the body is subjected to the maximum stress, the rest is strained to a less degree, and consequently has absorbed a less amount of the energy of the blow. Thus, for example, a beam or circular section, even though it be of "uniform strength" (Art. 161), has only one fourth the resilience of a stretched bar of the same weight, because it is only the particles on the upper and lower surfaces which are exposed to maximum stress, the central parts having their strength only partially developed.

We now draw two very general and important conclusions.

(1) When a body or structure is exposed to a blow exceeding that represented by its own weight falling through a very moderate height, a part, or the whole, is strained beyond the elastic limit.

(2) When a body or structure is not of uniform strength throughout, the excess of material is a cause of weakness.

On reference to Table II., Ch. XVIII., it will be seen that exceptions occur to the first principle in the case of the hardest and strongest steel, and in wood and some other substances of organic origin of low specific gravity; but, as a rule, the property of ductility or plasticity is essential to resistance to impact. Bodies which do not possess it are generally brittle. In good ductile iron and soft steel the non-elastic part of the resistance to impact will be seen hereafter to be at least 1000 times the elastic part, assuming both equally developed through all parts of the material. These remarks apply to a single blow; the effect of repetition will be considered hereafter.

As an example of the application of the second principle we may mention the bolts for armour plates invented by the late Sir W. Palliser. In these bolts the shank is turned down to the diameter of the base of the thread so as to be of equal strength throughout. (See Ex. 4, p. 308.)

EXAMPLES.

1. A hammer weighing 2 lbs. strikes a nail with a velocity of 15 feet per 1 inch driving $\frac{1}{8}$ inch, what is the mean pressure overcome by the nail? *Ans.* 673 lbs.

2. If the load on a stretched bar is suddenly reversed so as to produce compression, show that the stress will be trebled.*

Energy stored in stretched bar will on the release of the load be employed in compression, and in addition the load will be exerted through a distance = original extension + compression. The two together must be equal to the work done in compressing the bar.

Note—Such sudden reversal as is here supposed rarely if ever occurs in practice.

3. A load of 1000 lbs. falls through 1″ before commencing to stretch a suspending rod by which it is carried. If the sectional area of the rod is 2 sq. in., length 100″, and modulus of elasticity 30,000,000, find the stress produced.

Stress = 17,828 lbs. per sq. in.

4. A load of 5000 lbs. is carried by the rod of the preceding question, and an additional load of 2000 lbs. is suddenly applied; what is the stress produced?

Stress = 4500 lbs. per sq. in.

5. A beam will carry safely 1 ton with a deflection of 1 inch; from what height may a weight of 100 lbs. drop without injuring it, neglecting the effect of inertia? *Ans.* 11·2 inches.

6. The maximum stability of a vessel is 4000 foot-tons. The curve of stability is represented sufficiently approximately by a triangle, such that the angle of maximum stability is $1/n$ the angle of vanishing stability. Find the moment which, applied suddenly and of uniform amount to the ship upright and at rest, would just capsize her.

Area OCD = area DAE. $OB = N . ON$, and $AN = 4000$;

$$CD = \frac{OB}{N}\frac{OC}{4000}, \text{ and } DE = OB\frac{4000 - OC}{4000}.$$

g the areas in terms of OC and OB we get

$$OC \text{ the capsizing moment} = \frac{4000\sqrt{n}}{1 + \sqrt{n}}.$$

* This result which appears little known was pointed out to the writer by Mr. Hearson. Some examples on impact will be found in Prof. Alexander's treatise on Applied Mechanics, part I.