CHAPTER XVIII.

MATERIALS STRAINED BEYOND THE ELASTIC LIMIT. STRENGTH OF MATERIALS.

211. *Plastic Bodies*—If the stress and strain to which a piece of material is exposed exceed certain limits its elasticity becomes imperfect, and ultimately separation into parts takes place. We proceed to consider what these limits are in different materials under different circumstances: it is to this part of the subject alone that the title "Strength of Materials" is, strictly speaking, appropriate.

Reference has already been made (Art. 147) to a certain condition in which matter may exist, called the Plastic state, which may be regarded as the opposite of the Elastic state, which has been the subject of preceding chapters. In this condition the changes of size of a body are very small, as before; but if the stress be not the same in all directions the difference, if sufficiently great, produces continuous change of shape of almost any extent. Some materials are not plastic at all under any known forces, but many of the most important materials of construction are so, more or less, under great inequality of pressure.

Fig.154

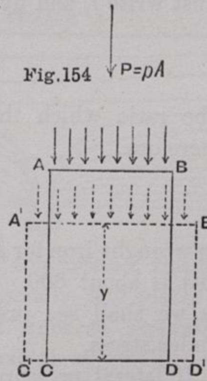


Fig. 154 shows a block of material which is being compressed by the action of a load P applied perfectly uniformly over the area AB. Let the intensity of the stress be p, then so long as p is small the compression is small and proportional to the stress; but when it reaches a certain limit the block becomes visibly shorter and thicker.

This limit depends on the hardness of the material, and the value of p may be called the "co-efficient of hardness." In an actual experiment the friction of the surfaces between which the block is compressed holds the ends together, so that it bulges in the middle, as in Fig. 158, p. 419, which represents an experiment on a short cylinder of soft steel. In the ideal case the sectional area remains uniform, changing throughout inversely as the height, as expressed by the equation

$$Ay = A_1 y_1,$$

where A is the area and y the height of the block.

In a truly plastic body p the intensity of the stress remains constant, and therefore the crushing load P varies as A, that is inversely as y. This is the same law as that of the compression of an elastic fluid when the compression curve is an hyperbola, and we therefore conclude (Art. 90) that the work done in crushing is

$$U = Py \,.\, \log_\varepsilon r = pAy \log_\varepsilon r = pV \log_\varepsilon r,$$

where r is the ratio of compression and V the volume. Certain qualities of iron and soft steel will endure a compression of one-fourth or even of one-half the original height, and amounts of energy are thus absorbed which are enormous compared with the resilience of the metal. To illustrate this, suppose that plasticity begins as soon as the limit of elasticity f is overpassed, then for p we must write f, and by Art. 96 the resilience for a volume V is

$$\text{Resilience} = \frac{f^2}{2E} \,.\, V.$$

The ratio which the work just found bears to the resilience is therefore

$$\text{Ratio} = \frac{2E}{f} \,.\, \log_\varepsilon r.$$

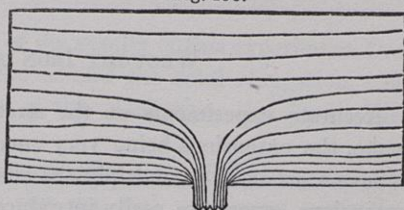
In wrought iron for a compression of one fourth the height ($r = 1{\cdot}333$) this is about 800. The actual ratio must be much greater, because, as we shall see presently, the hardness of the material increases under stress.

If lateral pressure of sufficient magnitude be applied to the sides of the block, the longitudinal force being removed, the effect is elongation instead of compression, contraction of area instead of expansion. The magnitude of the lateral pressure is found by imagining a tension applied both longitudinally and laterally of equal intensity. Such a tension has no tendency to alter the form of the block, being equivalent to fluid pressure, but it reduces the lateral pressure to zero, while it introduces a longitudinal tension of the same amount, which has the same value as the longitudinal compression of the preceding case. We see then that in every case a certain definite difference of pressure is required to produce change of shape in a plastic body, the direction of the change depending on the direction of the difference. The work done is found by the same formula as before, r meaning now the ratio of elongation.

In the process of drawing wire the lateral pressure is applied by the sides of the conical hole in the draw-plate, which are lubricated to reduce friction, and the force producing elongation in the wire is the sum of the tensile stress applied to draw the wire through the hole and the compressive stress on the sides. The work done is given by the same formula as before, p being now the sum in question.

212. *Flow of Solids.*—When a plastic body changes its form the process is exactly analogous to the flow of an incompressible fluid, which indeed may be regarded as a particular case. In the solid the distorting stress at each point at which the distortion is going on has a certain definite value which in the fluid is zero. The experimental proof of this is furnished by the experiments of M. Tresca, of which Fig. 155 shows an example. Twelve circular plates of lead are placed one upon another in a cylinder, which has a flat bottom with a small orifice at its centre. The pile of plates being forcibly compressed, the lead issues at the orifice in a jet, and the originally flat plates assume the forms shown in the figure. The lines of separation, indicating the position of particles of the metal originally in a transverse section, are quite analogous to the corresponding lines in the case of water issuing from a vessel through an orifice in the bottom. Tresca's experiments were very extensive, and showed that all non-rigid material flowed in the same way. Lead ap-

Fig. 155.



proaches the truly plastic condition; the difference of pressure necessary to make it flow being always about the same. Tresca ascribes to it the value of 400 kilogrammes per square centimetre, or about 5,700 lbs. per square inch; * but it is probably subject to considerable variations.

The manufacture of lead pipes, the drawing of wire, and all the processes of forging, rolling &c., by which metals are manipulated in the arts, are examples of the Flow of Solids.

213. *Preliminary Remarks on Materials. Stretching of Wrought Iron and Steel.*—Materials employed in construction may roughly be divided into three classes. The first are capable of great changes of form without rupture, and, when possessing sufficient strength to resist the necessary tension, may be drawn into wire. This last property is called ductility, and this word may be used to describe the class which we shall therefore call Ductile Materials. The second, being incapable of enduring any considerable change of this kind, may be described as Rigid Materials. The third are in many cases not homogenous, but may be regarded as consisting of bundles of fibres laid side by side, they may therefore be described as Fibrous Materials; they are generally of organic origin.

We shall commence with the consideration of ductile materials, and more especially of

Wrought Iron and Steel.

Accurate experiments on the stretching of metal are difficult to make, the extensions being very small and the force required great. If levers are used to multiply the effect of a load or to magnify the extensions, errors are easily introduced. If the levers are dispensed with, a great length of rod is necessary and a heavy load the manipulation of which involves difficulties. The best modern testing machines operate by hydraulic pressure, and the elongations are measured by micrometers. The experiment we select for description was made by Hodgkinson on a rod of wrought iron ·517 inch diameter,

* The co-efficient employed by Tresca, and called by him the "co-efficient of fluidity," is half that used in the text. It is the magnitude of the distorting stress necessary to produce flow.

49 feet 2 inches long, loaded by weights placed in a scale pan* suspended from one end. The load applied was increased by equal increments of 5 cwt. or 2667·5 lbs. per square inch of the original sectional area of the bar; each application of the load being made gradually, and the whole load removed between each. At each application and removal the elongation was measured so as to test the increment of elongation, both temporary and permanent, occasioned by each load. If the rod were perfectly elastic the temporary increments should be equal and the permanent elongations (usually called "sets") zero.

STRETCHING OF A WROUGHT IRON ROD, 49 FEET 2 INCHES LONG.

Load.		Elongation in Inches.	Increment of Elongation.	Permanent Set.
2667·5 × 1	2667·5	·0485	·0485	
,, × 2	5335	·1095	·061	
,, × 3	8003	·1675	·058	·0015
,, × 4	10,670	·224	·0565	·002
,, × 5	13,338	·2805	·0565	·0027
,, × 6	16,005	·337	·0565	·003
,, × 7	18,673	·393	·056	·004
,, × 8	21,340	·452	·059	·0075
,, × 9	24,008	·5155	·0635	·0195
,, × 10	26,675	·598	·0825	·049
,, × 11	29,343	·760	·162	·1545
,, × 12	32,010	1·310	·550	·667

The annexed table shows part of the results of this experiment, the first column giving the load, the second the total elongation, the third the successive increments of the elongation, the fourth the total permanent set.

On examining the table we see that, after some slight irregularities at the commencement due to the material not being perfectly homogeneous, the increments of elongation are nearly constant till we reach the eighth load of 21,340 lbs. per square inch, after which the increments show a rapid increase. Further, the permanent set,

* Being one of the best of its kind of old date this experiment has often been quoted. For the original description see the *Report of the Commissioners appointed to enquire into the Application of Iron to Railway Structures*. For a notice of some important experiments on stretching recently made the reader is referred to the Appendix.

which at the commencement is very minute and increases very slowly, at the same point shows a sudden increase indicating that the observed increase is almost wholly due to a permanent elongation of the bar, the temporary increase following approximately the same law as before. Notwithstanding this the bar is not torn asunder till a much greater load is applied. The table shows the results up to a load of 32,000 lbs. per square inch, but rupture did not occur till a load of 53,000 lbs. was applied. The extension at the same time increased to nearly 21 inches, being more than forty times its amount at the elastic limit.

We conveniently represent the results graphically by setting off the elongations as abscissæ along a base line with corresponding ordinates to represent the stress, thus obtaining a curve of "Stress and Strain" (Fig. 156). The curve will be seen to be nearly straight up to a stress of 22,000 lbs. and then to bend sharply, becoming nearly straight in a different direction. A curve of permanent set may also be constructed which is seen to follow the same general law.

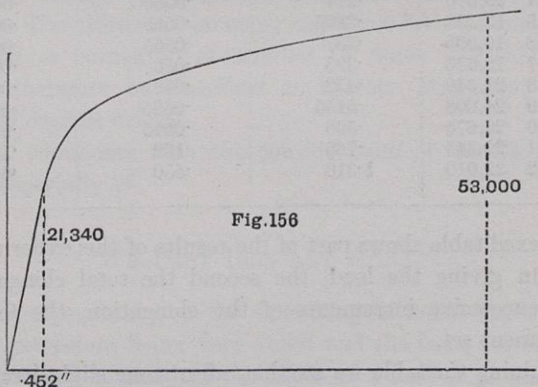


Fig.156

This experiment may be taken as a type of many hundreds of such experiments which have been made on iron and the softer kinds of steel, showing that in these materials a tolerably well-defined limit exists, within which the material is nearly perfectly elastic (compare Art. 127); the small deviations are more due to the want of perfect homogeneity in the bar than to actual defect in the elasticity. They usually diminish greatly if the experiment be tried a second time on the same bar. The position of the limit of elasticity and the value of the modulus of course vary. Some examples will be given presently.

Accompanying the increase of length of the bar we find a contraction of area; within the elastic limit, however, this is so small as to escape observation. Outside the limit it becomes visible, consisting in the first instance of a more or less uniform contraction at all or nearly all points, followed by a much greater contraction at one or sometimes two points where there happens to be some local weakness.* Within the elastic limit the density of the bar diminishes, but by an amount so small that the fact is rather known by reasoning than determined by experiment. Outside the limit there is a permanent diminution which is perceptible, though still very small.

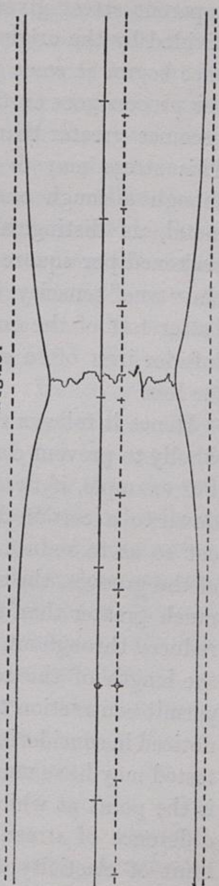


Fig.157

Thus beyond the elastic limit the bar draws out, changing its form like a plastic body without sensible change of volume. The bar finally tears asunder at the most contracted section, as shown by the annexed figure (Fig. 157) representing an experiment by Mr. Kirkaldy on a bar of iron 1 inch diameter, in which the contraction of area was 61 per cent., and the elongation 30 per cent., ultimate strength 58,000 lbs. per square inch of original area, 146,000 lbs. per square inch of fractured area. The contraction of section in good iron and soft steel is 50 or 60 per cent.

214. *Real and Apparent Tensile Strength of Ductile Metals.*—Thus the process of stretching an iron bar beyond the limit of elasticity till it breaks is an example of the "flow of solids," the iron behaving to a certain extent like a plastic body. There is, however, this

* On this point see *Preliminary Experiments on Steel by a Committee of Civil Engineers*, London, 1868. On account of the uncertainty of the amount of contraction at various points, the ultimate extension is an imperfect measure of the ductility of the iron, even when the pieces are of the same length and sectional area.

difference, that a constantly increasing stress is necessary to produce continuous flow, which increase is supplied partly by increase of the stretching load, partly by the contraction of area. The actual stress at each instant on the contracted area is much greater than the apparent stress given in the table, which is merely the total load divided by the original sectional area. Hence when contraction has once begun at some point of local weakness it continues there, and the process goes on till the stress per square inch of the reduced area becomes greater than the metal will bear, when fracture takes place. This stress may to a certain extent be regarded as a measure— though a rough and imperfect one — of the true tenacity of the metal, as distinguished from the "apparent" tenacity which is reckoned per square inch of the original area. For many purposes the "true" tenacity, in good iron more than double the apparent, is a better test of the quality of the iron than the actual breaking load, inferior iron often showing a high apparent tenacity but contracting far less.

Hence it follows that if the form of the piece be such as partly or wholly to prevent contraction the apparent strength will be increased. For example, if two pieces of the same bar be taken and one turned down to a certain diameter, while in the other narrow grooves are cut so as to reduce the diameter to the same amount at the bottom of the grooves, the strength of the grooved piece will be found to be much greater than that of the piece the diameter of which has been reduced throughout, and this can only be explained by observing that the length of the reduced part of the grooved bar is insufficient to permit contraction to any considerable extent. This is a point to be noticed in considering experimental results.* The form of the specimen tested may have much influence. Further, since the limit of elasticity is the point at which flow commences, and since the flow is due to difference of stress, it follows that the same causes must raise the limit of elasticity, and thus we are led to the conclusion that there are two elements constituting strength in a material, first, tenacity and, secondly, rigidity. In some materials, such as these we are now considering, the tenacity is much greater than the rigidity, and in them the limit of elasticity will depend on the rigidity, and will have different positions according to the way the stress is applied.

* See *Experiments on Wrought Iron and Steel*, by Mr. Kirkcaldy, p. 74. 1st edition. Glasgow, 1862.

It will lie much higher, and the apparent strength will be much greater when lateral stress is applied to prevent contraction.

215. *Increase of Hardness by Stress beyond the Elastic Limit.*—In clay and other completely plastic bodies a certain definite difference of pressure is sufficient to produce flow: in iron, copper, and probably other metals, however, as we have just seen, this is not the case, the metal acquiring increased rigidity in the act of yielding to the pressure. Thus the effect of stress exceeding the elastic limit is always to raise the limit, whether the stress be a simple tensile stress or whether it be accompanied by lateral pressure. All processes of hammering, cold rolling, wire drawing, and simple stretching have this effect. If a bar be stretched by a load exceeding the elastic limit and then removed, on re-application of a gradually increasing load we do not find a fresh drawing out to commence at the original elastic limit, but at or near the load originally applied.* If the load be further increased drawing out re-commences. Hence, whenever iron is mechanically "treated" in any way which exposes it to stress beyond the elastic limit, contraction is prevented and the apparent strength is increased: for example iron wire is stronger than the rod from which it is drawn; when an iron rod is stretched to breaking the pieces are stronger than the original rod. It is not certain that the real strength of materials is always increased by such treatment; perhaps in some cases the contrary, for we know that the modulus of elasticity and specific gravity are somewhat diminished.† On the other hand there are cases in which the increase of strength is greater than can be accounted for in this way. On annealing the iron it is found to have resumed its original properties, a circumstance which indicates that the increased rigidity is due to a condition of constraint which is removed by heating the metal till it has assumed a completely plastic condition. In considering the effect of impact, the diminution of ductility occasioned by the application of stress beyond the elastic limit is a most important fact to be taken into account (see Art. 226). Working

* Styffe *On Iron and Steel*, p. 68.

† The raising of the limit of elasticity by mechanical treatment of various kinds has long been known: in the case of simple stretching the effect appears to have been first noticed by Thälen in a paper, a translation of which will be found in the *Philosophical Magazine* for September, 1865.

iron or steel hot has generally the effect of increasing both its strength and its ductility.

216. *Compression of Ductile Material.*—In a perfectly elastic material compression is simply the reverse of tension, the same changes of dimension being produced by the same stress, but in the reverse direction. Also in a plastic body a given difference of stress produces flow, whether the stress be tensile or compressive; hence in ductile metals we should expect to find the modulus of elasticity and the limit of elasticity nearly the same in compression as in tension. These conclusions are borne out by experiment. In the case of wrought iron and steel experiments on the direct compression of a bar are more difficult to carry out than experiments in tension, the bars are necessarily of limited length, and must be enclosed in a trough to prevent lateral bending; minute accuracy is therefore hardly attainable. A considerable number have, however, been made, from which it appears that the modulus of elasticity and the limit of elasticity are nearly the same in the two cases.*

EXPERIMENT BY SIR W. FAIRBAIRN ON A BLOCK ·72 INCH DIAMETER OF SOFT BESSEMER STEEL.

TOTAL LOAD $= P$.	HEIGHT OF BLOCK $= y$.	CRUSHING STRESS $p = \frac{Py}{Ay_1}$.
0	·997	0
16·7	·92	37·8
20·1	·865	42·9
23·3	·797	45·9
26·3	·731	47·4
29·5	·672	48·9
32·6	·613	49·4
35·8	·574	50·6
39·3	·535	51·9
41·0	·505	50·8

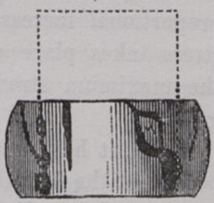
REMARKS.—The apparent ultimate tensile strength of this steel was 36 tons, its limit of elasticity 22 tons per square inch. Modulus of elasticity 30,300,000 lbs. Ratio of contraction ·41. Real tensile strength 88·5 tons.

* Perhaps the best set of experiments are those made by the "Committee of Civil Engineers." See their report already cited, p. 7-13.

The metal yields beyond the limit by a process of flow of the same character as in tension, but expanding laterally instead of contracting. This is especially seen in experiments made by the late Sir W. Fairbairn in 1867, and somewhat earlier by Mr. Berkeley, on the compression of short blocks of steel. In both, the blocks were pieces of round bars, of height somewhat greater than the diameter, and the results were very similar.

The annexed table gives the results of one of Sir W. Fairbairn's experiments. Column 1 gives the actual load laid on; column 2 the corresponding height of the block, both given directly by the experiments. Column 3 is calculated by dividing the product of load and height by the original sectional area and height, and represents the crushing stress per square inch of the mean sectional area. If the block did not bulge in the centre on account of friction holding its ends together (Fig. 158), this would be the actual crushing stress, which, however, must in fact be less. The table shows that after a compression of about one-third the crushing stress remains nearly constant at about 50 tons per square inch. The experiment terminated at a compression of one-half. This kind of steel then is perfectly elastic up to 22 tons per square inch, is partially plastic between 2 and 50, and behaves as a plastic body under a difference of stress of 50 tons per square inch.

Fig. 158.



In ductile materials fracture takes place under compression by longitudinal cracks as shown in Fig. 158, which represents an experiment on a different quality of steel. The amount of compression which different materials will bear is very different according to their malleability; it is generally difficult to fix upon the ultimate strength, as it depends on the mode in which the experiment is made. In iron and steel it is somewhat less than the apparent tensile strength.

The compression of iron blocks has been less thoroughly studied than that of steel, but it is known that the results are similar although the strength and the ultimate ratio of compression are much less. Set becomes sensible at about 10 tons per square inch, and the ultimate strength is from 40 to 50,000 lbs. per square inch if lateral flexure be prevented.

217. *Bending beyond the Elastic Limit.*—Since wrought iron and steel are nearly perfectly elastic when the stress applied is not too great, it follows that the formula already obtained for the moment of resistance to bending of a bar must be true for these materials so long as the stress does not exceed the elastic limit determined by tension experiments of the kind just described. Experience fully confirms this conclusion, for the deflection obtained by experiment agrees well with that found from formulæ previously given with the same value of the modulus. As soon, however, as the maximum stress exceeds this limit, it is no longer true that the stress at different points of the transverse section varies as the distance from the neutral axis. It does not increase so fast, because the extension and compression at points near the surface is not accompanied by a proportional increase of stress. Hence, a partial equalization of stress takes place over the transverse section, and consequently the maximum stress for a given moment of resistance is not so great.

Again, it has been repeatedly explained in the earlier part of this book that the lateral connection of the several layers into which we imagine a beam divided has no influence on the stress produced by bending so long as the limit of elasticity is not exceeded. But when the limit is passed, the connection between those layers which are most stretched and compressed with those layers which have not yet lost their elasticity prevents their contraction and expansion, and so raises the limit of elasticity in accordance with the general principle explained in Art. 215. Thus, the limit of elasticity lies higher, and the apparent strength is greater in bending than in tension. In Fairbairn's experiment quoted above the same steel was tested in tension, compression, and bending. The elastic limit in bending was 30 tons, in tension 22 tons. The magnitude of the difference will depend on the form of transverse section, and on the ductility of the material. According to Mr. Barlow it may reach 50 per cent. in a rectangular section.* The case of çast iron will be referred to farther on.

Putting aside the effect of lateral connection, it may be interesting to make a calculation of the effect of equalization, by supposing that under a bending moment very slowly and steadily applied beyond the elastic limit, the metal behaves like a truly plastic material

* *Phil. Trans.*, 1855-57.

throughout the transverse section, so that the stress is uniform. Referring to the formula on page 313, we have

$$\Sigma pybt = M,$$

in which we must now, instead of assuming that p varies as y, suppose p a constant. Then

$$M = 2p \,.\, A\bar{y},$$

where A is the area of the part of the section which lies on either side of the neutral axis and $\bar{y}$ the distance of its centre of gravity from that axis. For the same value of the modulus this gives a moment of resistance in a rectangular section 50 per cent. greater than if the material had been elastic. How far any apparent increase of strength due to equalization or lateral connection may be regarded in practice is uncertain. A failure of elasticity must have taken place at certain points in order that there may be any increase at all, and in cases where the load is frequently reversed the bar must be weakened. (See Art. 225.)

Cast Iron and other Rigid Materials.

218. *Stretching of Cast Iron.*—The phenomena attending rupture by tension of cast iron are essentially different from those described above for the case of ductile metals. This will be sufficiently shown by an experiment, also made by Hodgkinson, on a bar of this material 50 feet long, 1·159 inch diameter. The experiment was made in the same way as that already described on the wrought-iron rod,* and the results are shown in the annexed table. The first four loads were applied as before, by increments of 5 cwt., here equivalent to 531 lbs. per square inch; the whole load, after measurement of the elongation, being

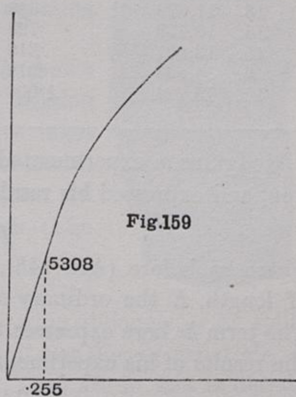


Fig.159

completely removed, and the permanent set measured. After the fourth load the increment was 10 cwt., and this was carried on till the bar broke at a stress of 16,000 lbs. per square inch.

* *Report of Commissioners on the Application to Railway Structures*, p. 51.

The third column as before shows the increments of elongation, which, after a stress of 5,308 lbs. per square inch, or $\frac{1}{3}$ the breaking load, has been reached, show a gradual increase till actual rupture occurs. The results of the experiments are graphically exhibited in the annexed diagram (Fig. 159) of stress, strain, and permanent set. The form of the curve is different from that of wrought iron, showing no point of maximum curvature, because in this material the bar does not draw out.

STRETCHING OF A CAST-IRON BAR 50 FEET LONG, 1·159 INCH DIAMETER.

LOAD IN LBS. PER SQUARE INCH.	ELONGATION IN INCHES.	INCREMENT OF ELONGATION.	PERMANENT SET.
1. 531	·024	·024 × 2 = ·048	Perceptible.
2. 1,062	·0495	·0255 × 2 = ·051	·0015
3. 1,592	·0735	·024 × 2 = ·048	·002
4. 2,123	·09828	·0247 × 2 = ·0514	·0045
5. 3,185	·1485	·0503	·0105
6. 4,246	·200	·0515	·0155
7. 5,308	·255	·055	·022
8. 6,370	·313	·058	·028
9. 7,431	·374	·061	·037
10. 8,493	·435	·061	·046
11. 9,554	·504	·069	·056
12. 10,616	·572	·068	·067
13. 11,678	·648	·076	·0795
14. 12,739	·728	·080	·095
15. 13,801	·816	·088	·1115
16. 14,863	·912	·096	·132
17. 15,924	1·000	·088	—

Hodgkinson experimented on a large variety of different kinds of iron, and expressed his results by a formula, which may be written

$$p = Ee(1 - ke),$$

where, as before (Art. 148), p is the stress, e the extension per unit of length, E the ordinary modulus of elasticity, and k a constant. The term ke here expresses the defect of elasticity of the bar. From the results of his experiments we find the average values

$$E = 14{,}000{,}000\ ; \quad k = 209.$$

Cast iron, however, is a material of variable quality, and the value of these constants may have a considerable range. Up to one third the breaking load it may be regarded as approximately perfectly elastic, but the limit is by some authorities placed much higher.

219. *Crushing of Rigid Materials.*—In the ductile metals the effects of compression are nearly the reverse of those of extension, as has been sufficiently shown in previous articles, but in cast iron this is by no means the case. Hodgkinson experimented in this question with great care and accuracy, testing pieces of iron of exactly the same quality under compression and tension to enable a comparison to be made. The bars were enclosed in a frame and tested by direct compression. Hodgkinson expressed his results by a formula, which may be written

$$p = Ee(1 - ke),$$

the symbols having the same meanings as before, and the values may be taken as

$$E = 13,000,000\ ;\ \ k = 40.$$

The smaller value of k indicates that the elasticity under compression is much less imperfect under the same stress. Short cylinders of the metal were also crushed, and the crushing load found to be five times the tensile strength or more.

It thus appears that in compression cast iron is six times stronger than in tension, and this is true not merely of the ultimate resistance, but in great measure also of the elastic resistance, for the elasticity of the metal is not sensibly impaired until one third the crushing load is reached.

Fig. 160.

The manner in which crushing occurs is shown in the accompanying figure; instead of bulging out like a ductile metal, oblique fracture takes place on a plane inclined at 45° or rather less to the axis, being (approximately) the plane on which the shearing stress is a maximum (Fig. 160).

Great resistance to compression, as compared with tension, and sudden fracture by shearing obliquely or by splitting longitudinally are characteristics of all non-ductile materials, of which cast iron may be taken as a type. They are in fact materials the tenacity of which is much less than the rigidity.

220. *Breaking of Cast-Iron Beams.*—When a cast-iron bar is bent till the tensile stress at the stretched surface exceeds one third the tensile strength of the material, the defective elasticity of the metal causes a partial equalization of stress on the transverse section as in the case of wrought iron. Besides this, the elasticity being much more perfect under compression than under tension, the equalization is greater on the stretched side than on the compressed side, and the neutral axis moves towards the compressed edge of the beam. For both these reasons the moment of resistance to bending is greater for a given maximum tensile stress than it would be if the material were perfectly elastic. Thus it follows that if the co-efficient in the ordinary formula for bending be assumed equal to the tensile strength of the material, the calculated moment of resistance will be less than the actual moment of rupture of the beam by an amount which is greater for a rectangular section than for an I section. The discrepancy is found by experiment to be very great, the calculated moment for a rectangular section being less than one half, while for an I section it is about equal that found by experiment. The causes just pointed out only partially account for this, especially as Mr. Barlow's experiment cited above appears to show that no considerable deviation of the neutral axis takes place, and it is probable that the lateral connection of the several layers of the beam has (near the breaking point) a sensible influence on the strength of the parts of the beam exposed to tensile stress, a question we shall return to farther on.

Shearing and Torsion. Compound Strength.

221. *Shearing and Torsion.*—We now pass on to cases where the ultimate particles of the material are subject not to a simple longitudinal stress, but to stress of a more complex character. The simplest case is that of a simple distorting stress where the stress consists of a pair of shearing stresses (Fig. 140) on planes at right angles, or what is the same thing (Art. 183) of a pair of equal and opposite longitudinal stresses (Fig. 141) on planes at right angles. Examples of this kind of stress occur in shearing, punching, and twisting. Experiments on shearing are subject to many difficulties and are often not conducted in such a way as to satisfy the conditions necessary for uniformity of distribution of stress on the section. Moreover they necessarily give the ultimate resistance only without reference to the limit of elasticity. The whole process of shearing and punching is very complex, being at the commencement of the operation usually accompanied by a flow of the metal similar to that already referred to. Thus, when a hole is punched in a thick plate the punch sinks deep into the plate before the actual punching takes place, the metal being displaced by lateral flow, and the piece ultimately punched out being of less height than the thickness of the plate.*

Separation takes place in the first instance by the formation of fine cracks inclined at 45° to the plane of shearing. In soft materials the surfaces slide past each other and separate, but in harder materials there is a strong tendency to the formation of an oblique fracture. In wrought iron and steel the ultimate resistance to shearing is probably about three fourths the ultimate resistance to tension of the same material. The question of a theoretical connection between the elastic strengths in the two cases is considered further on.

Experiments on torsion are not numerous, and many of those which exist are not experiments on simple twisting, but on a combination of bending and twisting. Such experiments would be of great value if accompanied by corresponding experiments on simple twisting and bending made on similar pieces of material. It is known however that in the ductile metals the elastic resistance to torsion is less than the resistance to tension. A series of experiments on torsion made by Prof. Thurston give some interesting results.† Curves are drawn the abscissæ of which represent angles and the ordinates twisting moments, and the form of these curves shows that in some cases defective homogeneity causes a great deficiency in the elasticity at small angles of torsion. In general, however, the curves closely resemble the ordinary curve of stress and strain, already given for a stretched bar, being nearly straight up to a certain point and then curving towards the axis.

In twisting, as in bending, after passing the elastic limit, the stress at each point of the section, instead of varying as the distance from the centre as it must do in perfectly elastic material, varies much

* On this subject see M. Tresca's paper cited above, and two articles in the Journal of the Franklin Institute.

† See Paper on *Materials of Machine Construction*, read before the American Society of Civil Engineers, 1874. No diameters are given, except for the woods, so that the stress corresponding to the limit of elasticity cannot be found.

more slowly so as to become partially equalized. Hence the twisting moment corresponding to a given maximum stress is greater than it would be if the elasticity were perfect. In the case where the equalization is perfect it is easy to show that the twisting moment is increased in the proportion 4 : 3, a result first given in 1849 by Prof. J. Thomson. The curves given by Thurston show that in many cases an approximately constant twisting moment was reached indicating that nearly complete equalization must have existed. On the case of cast iron see Art. 223.

222. *Theories of Compound Strength.*—A simple distorting stress is included in the more general case of three simple longitudinal stresses of any magnitudes acting on planes at right angles. To this, indeed, all cases of stress can be reduced, and if we knew the powers of resistance of a material to three such stresses simultaneously, all questions relating to strength of materials could (at least theoretically) at once be answered. Unfortunately experiments fitted to decide the question have not hitherto been made, and in consequence hypotheses have explicitly or implicitly been resorted to.

First, it is often tacitly supposed that the powers of resistance of a material to a simple longitudinal stress are unaffected by the existence of a lateral stress. For example, if a material bears 10 tons per square inch under a simple stretching force, it is assumed that when formed into a pipe and exposed to internal fluid pressure it would also bear 10 tons on the square inch if the pipe were homogeneous and free from joints, notwithstanding the fact that the material is exposed to stress (Art. 150) tending to tear it transversely as well as longitudinally. It is, however, far from probable that this can be the case, at any rate as regards the elastic strength. In ductile materials, the limit of elasticity of which depends as we have seen on rigidity, any lateral force must raise or lower the elastic limit according as it acts in the same direction as the longitudinal stress or in the opposite direction.

Secondly, it may be supposed that the maximum elongation or contraction of a material in a given direction must be a certain definite quantity, irrespective of any elongation or contraction in any other direction. This theory leads to results which are more probable than the preceding, and as it has been much employed by Continental writers we shall give some examples.

Let us take a piece of wrought iron and imagine that when exposed to a simple stretching force its limit of elasticity corresponds to a stress of 10 tons per square inch, accompanied by an elongation of $\frac{1}{1200}$th of its length. The second theory asserts that the maximum admissible elongation is still $\frac{1}{1200}$th, even though the sides of the bar be acted on by any force, the effect of which will be that quite a different longitudinal stress will be required to produce that elongation.

The relations between stress and strain are expressed by the equations (Art. 206)

$$Ee_1 = p_1 - \frac{p_2 + p_3}{m};$$

$$Ee_2 = p_2 - \frac{p_1 + p_3}{m};$$

$$Ee_3 = p_3 - \frac{p_2 + p_1}{m}.$$

The first theory supposes that p_1 can never exceed 10 tons, and the second that e_1 can never exceed $\frac{1}{1200}$th (or Ee_1 10 tons), whatever p_2, p_3 are. In the case of a thin pipe under internal fluid pressure $p_3 = 0$ (nearly), $p_2 = \frac{1}{2}p_1$ (Art. 150); thus assuming $m = 4$ we have on the second theory

$$10 = p_1 - \frac{p_1}{8}, \text{ or, } p_1 = 11{\cdot}43,$$

so that the material will bear under these circumstances a stress of 11·43 tons per square inch as safely as it bears 10 tons under simple tension, and this value, therefore, may be assumed for the co-efficient in the formula which gives the corresponding internal pressure. In like manner in the case of a thin sphere the material will bear a stress of $13\frac{1}{3}$ tons per square inch, being an increase of 30 per cent.

223. *Connection between the Co-efficients of Strength for Shearing and Tension.*—On either theory the resistance to a simple distorting stress may be found in terms of the resistance to simple tension, for such a stress consists (p. 358) of a pair of equal and opposite simple stresses of equal intensity. In the first case the resistances to tension and shearing ought to be equal, in the second since, writing $p_2 = p_1$, we find

$$Ee_1 = p_1 + \frac{p_1}{m},$$

$$\text{or, } p_1 = \frac{m}{m+1} Ee_1,$$

it follows that the resistance to shearing is $\frac{m}{m+1}$ or about four fifths the resistance to tension, a result on the whole borne out by experience. It should be remarked that the theory only professes to give a connection between the elastic resistances in the two cases, the equations only holding good for perfectly elastic material, which, moreover, must be supposed isotropic. The ultimate resistance to torsion of cast iron is much greater than its resistance to tension, which is probably due to the same causes as in the case of bending.

Now, rigid materials on this theory are imagined to give way to longitudinal compression, when the lateral expansion produced by the compression is the same as would be produced by a simple tensile stress; from which it appears that the elastic resistance to compression should be from three to four times the elastic resistance to tension, as may easily be supposed to be the case.

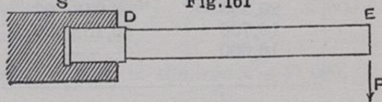
A third theory, more easily conceivable *a priori*, is to suppose that each material is capable of enduring, without injury to its elasticity, a certain definite change of volume and a certain definite change of shape. We thus have two co-efficients of elastic strength analogous to the two fundamental constants which express the other elastic properties of isotropic matter. On this theory, if the resistance to a simple distorting stress in any plane be independent of the existence of any other kind of stress whether fluid or otherwise, as in fact is the case before the limit is reached, it would follow that this resistance must be one half the resistance to a longitudinal stress. It is probable that some theory of this kind may ultimately prove to be the true one; but, in the absence of the necessary experimental data, the second theory may be provisionally assumed, as its results have not as yet been contradicted by experience. It is applied by first finding the principal stresses as in Ch. xvii., and then deducing the principal strains as just now explained. The greatest of these strains multiplied by E may be described as the "equivalent simple tensile stress," and should not exceed the limit prescribed by the strength of the material.

REPETITION AND IMPACT.

224. *Wöhler's Experiments on Alternate Bending and Twisting.*—In bodies which satisfy the definition of perfect elasticity a load within the elastic limit produces no permanent change, unless perhaps some thermodynamic effect, and it follows from this that after removal the body is completely uninjured, so that the load may be repeated indefinitely. Experience confirms this conclusion. The balance spring of a watch bends and unbends more than a million times a week for years together: and the parts of a machine if originally sufficiently strong, remain so to all appearance for an indefinite time. But, if the load be beyond the elastic limit, permanent changes are produced, and there is every reason to believe that a slow deterioration of strength, due perhaps to some kind of internal abrasion, is ultimately destructive. The most definite information on this point is furnished by the experiments of M. Wöhler published in 1870. Bars were loaded in various ways and the load wholly or partially removed: the process was repeated till the bar broke: the number of repetitions necessary for this purpose being counted was found to depend, first, on the maximum stress and, secondly, on the fluctuation of stress.

First suppose the stress alternately tensile and compressive of equal intensity. Wöhler tried this both in bending and in twisting. Figure 161 represents a round bar DE, with one end enlarged and fitted into a socket in a revolving shaft S. At the free end E a load P was applied, which produced at D, the point of maximum bending, a stress of intensity found by the usual formula. The shaft being set in motion the piece of material was bent alternately backwards and forwards once in each revolution. A number of pieces being tried successively with gradually diminishing loads, the revolutions necessary to produce fracture were found to increase as shown by the annexed table for the case of wrought iron. The pieces broken were exactly similar, and we therefore find a regular increase in the number of revolutions necessary to produce fracture as the stress diminishes. It is already very large at 18,700 lbs. per square inch, and at 16,600 the piece cannot be broken at all. We may therefore place the resistance to alternate bending of this kind of iron at about 17,000 lbs. per square inch, while for cast steel of various qualities it was found to range from 25,000 to 30,000, and for copper 10,400. These results do not differ much from the limit of elasticity of the materials in question

Fig.161



as determined in the usual way by experiments on tension. Indeed we have here the most satisfactory definition of the limit of elasticity. If we attempt to define the elastic limit as the stress at which the material ceases to possess the properties of a perfectly elastic body, we are embarrassed by the small and variable deviations which we find under almost any load, which only gradually and at very different loads under different circumstances pass into the large differences characteristic of the non-elastic state. The resistance to unlimited alternate stress however is a definite quantity which, so far as we know, is independent of the causes which produce these variations.

ALTERNATE BENDING OF A BAR OF AXLE IRON FURNISHED BY THE PHŒNIX COMPANY IN 1857.

STRESS IN LBS. IN SQ. INCH.	REVOLUTIONS.	REMARKS.
33,300	56,430	The last of these pieces was unbroken after more than 132 million revolutions.
31,200	99,000	
29,100	183,145	
27,000	479,490	
25,000	908,800	The ultimate tensile strength of this iron was 47,000 lbs. per square inch and the elongation about 20 per cent.
23,000	3,632,588	
20,800	4,918,000	
18,700	19,187,000	
16,600	——	

Similar experiments were made with a different apparatus on alternate twisting. They were less extensive, but led to the important conclusion that the strength of the qualities of steel for which they were tried was four fifths that of the same steel under alternate bending. From this it is inferred that the proof resistance to shearing is four fifths the proof resistance to tension, as required by a theory of strength already referred to. (See Art. 223.)

225. *Influence of Fluctuation of Stress.*—It had already been shown by Prof. J. Thomson, in a paper published in 1848,* that twisting or bending a bar beyond its elastic limit in one direction must increase its powers of resistance to a second strain in the same direction, and diminish it to a strain in the opposite direction.

* *Cambridge and Dublin Mathematical Journal.*

Accordingly we find that when a bar is strained in one direction only its powers of resistance to unlimited repetition are greatly increased. Wöhler made very extensive experiments on stretching, bending, and twisting of pieces of iron and steel to a given maximum stress, the load being wholly or partially removed at each repetition. The number of repetitions necessary for fracture was found to vary, not only according to the magnitude of the maximum stress, but also according to the fluctuation. It was greater when the load was only partly removed than when it was wholly removed. Some results are given in the annexed table, which shows the limits between which the stress varied when fracture was just not produced by unlimited repetition.

Resistance to Unlimited Repetition of Bending.

Nature of Fluctuation.	Fluctuation of Stress.			
	Iron.		Steel.	
Alternating, ...	+17,000 ;	−17,000	+29,000 ;	−29,000
Load wholly removed,	31,000 ;	0	50,000 ;	0
Load partially removed,	45,000 ;	25,000	83,000 ;	36,500

Remark.—The ultimate tensile strength of the iron was 47,000, and of the steel, 106,000.

Any greater fluctuations with the given maximum stress, or any greater maximum stress with the given fluctuation, produced fracture. Experiments on stretching and twisting led to similar results, and it should be especially noticed that in cases of unlimited repetition the resistance to stretching is the same as the resistance to bending, but the resistance to twisting less. In the case of cast iron the resistance to stretching with complete removal of load was found to be 10,400, but no experiments on bending or twisting were made.

Thus it appears that the ultimate strength of a material is very different according to the fluctuation in the load to which it is exposed; the same iron, which will bear only 17,000 lbs. per square inch when bent alternately backwards and forwards, will bear 31,000 when bent in one direction only, and 45,000 when the stress varies between 25,000 and 45,000. Several formulæ have been devised to represent the results of the experiments, of which one will now be given.* Let p_0 be the ultimate tensile strength of a material and Δ the fluctuation, then the actual ultimate strength under unlimited repetition will be

$$p = \tfrac{1}{2}\Delta + \sqrt{p_0(p_0 - \tfrac{3}{2}\Delta)}.$$

When $\Delta = 2p$ we get the case of alternate stress with which we commenced, where $p = \frac{1}{3}p_0$, and when $\Delta = p$ we have the case of repeated stress in one direction with complete removal at each repetition. The formula gives the same results as the experiments in the extreme cases, and may be expected to be approximately correct in intermediate cases.

226. *Impact.*—In Wöhler's experiments the load was applied without shock. In cases of impact also there is reason to believe that within the limit of elasticity a material will bear unlimited repetition. Thus in Hodgkinson's experiments on beams struck by a pendulum weight, it was found that if the blow produced less than one third the ultimate deflection, the beam would sustain more than 4,000 blows without apparent injury, a plate of lead being introduced to prevent local damage.

In most cases of impact, however, the elastic limit is exceeded, and the destructive effect of repetition is then much greater than when the load is gradually applied. In the ductile metals the resistance to impact is at first very great, as has already been sufficiently explained; but every time the limit of elasticity is overpassed the hardness of the metal is increased, so as to make it less able to resist the second blow. This may be illustrated by a diagram in which OQQ' is a curve of stress and strain, Q the original elastic-limit, $Q'N$ the stress produced by the first blow, so that the area $OQQ'N$ represents the energy of that blow. The

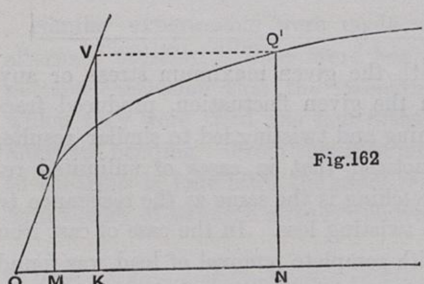


Fig.162

* *Elements of Machine Design*, by Prof. W. C. Unwin, p. 25.

effect of the blow is to raise the limit from the stress QM to the stress $Q'N$ nearly. Hence the curve of stress and strain now becomes OVS, where V is the new limit, and the material will only bear a blow the energy of which is the triangle OVK, without the original stress $Q'N$ being exceeded. Thus by constant repetition of blows, which originally only produced a stress not much exceeding the elastic limit, a much greater stress may be produced. It is believed that this is in the main the explanation of the destructive effect of repeated blows and continuous severe vibration : pieces of material exposed to which are found to have a short life. The effect may be further augmented by synchronism (Art. 197, p. 382).

Co-efficients of Strength and Factors of Safety.

227. *Factors of Safety and Co-efficients of Working Strength.*—Before we can apply theoretical formulæ to the determination of the dimensions of actual structures and machines, it is necessary to know the value of the co-efficients of strength to be used, and this is always a matter which requires great care and attention to the circumstances under which certain dimensions are found to be sufficient by long practical experience. In the first instance it depends on the ultimate strength of the material, and may be expressed by dividing that quantity by a Factor of Safety. But the ultimate strength varies as we have seen, and the word "factor of safety" is used with various meanings.

The primary meaning of the expression is the divisor necessary to provide a margin of strength for unknown contingencies such as the following.

(1.) The ultimate strength of a piece of material is uncertain, for two pieces of material of the same description and manufacture are not always equally strong. The liability to variation is much greater in some materials than others, for example in cast iron than in wrought iron. The strength of stone varies so much that, in carrying out any important work, experiments are frequently made on the stone to be employed in it.

(2.) The piece of material may be subject to corrosion or other influence, which in course of time diminishes its strength.

(3.) Errors of workmanship are unavoidable, and in some instances may greatly increase the stress to which the material is exposed. This, for example, is the case in pillars, the factor of safety for which must always be greater than for other parts of a structure.

(4.) The magnitude of the load and its mode of application is generally more or less uncertain. This however may be provided for by assuming a maximum load.

The factor required to provide for contingencies such as these may be called the "real" factor of safety, but by an addition to its value it may be made to provide against contingencies which can if necessary be exactly foreseen and calculated. Assuming all the forces acting on a structure to be known it is possible to find the stress on each part of it, but the calculation may be too complex to be often used, or its result may be known approximately under similar circumstances. Hence it often happens that the dimensions of a piece are determined by a formula involving only part of the straining forces which act on it, and the rest are provided for by an increased factor of safety. Thus the real stress on the metal of a screw bolt, when the effect of screwing up is taken into account, is double the total tension per square inch of the gross sectional area. If that bolt be used for a cylinder cover exposed to steam pressure the total tension will be much greater than that due to the pressure of the steam. These two circumstances taken together may be taken into account by the use of a factor of safety three or four times greater than the real one. Such cases are common in practice, but the factor to be used must then be determined by comparison with good examples under similar circumstances.

Again, it is necessary that a piece should be stiff enough as well as strong enough, and when formulæ for strength are used in such cases it is often necessary to employ very large and very arbitrary factors of safety. Here however the difficulty arises from an erroneous method of calculation.

228. *Values of Co-efficients.*—In parts of machines subject to alternating straining actions we know by Wöhler's experiments that the ultimate strength is somewhat less than the elastic strength under simple tension, being for wrought iron and soft steel about one third the ultimate tensile strength. The load on such parts will rarely be applied without shock, the effect of which cannot precisely be determined. In ordinary cases it will be sufficient to

treat this case as if the load were suddenly applied by using a further divisor of 2. We thus obtain the working strength by using a total factor of safety of 6. For wrought iron this gives a co-efficient of 4 tons, or 9,000 lbs. per square inch, which is known by experience to give sufficient strength where all the straining actions are taken into account. In long struts a factor of 8 or more must be used for reasons already sufficiently explained, and this is also necessary where, as has been the case, till lately with steel, the material is not completely reliable. For timber the usual factor is 10. The co-efficient for shearing and torsion is to be taken provisionally as four-fifths that for tension and bending, that is for wrought iron $3\frac{1}{4}$ tons per square inch; but from the incompleteness of experimental data it is not certain that this value is not too large.

In structures the fluctuation of the straining actions is in general much less, and the ultimate strength by Wöhler's experiments is much greater. Yet the working strength employed is not very different. In the first place, it is rarely permissible to exceed the elastic limit on account of the permanent deformation which ensues. In the second place, the whole of the straining actions on each piece of the structure, especially the effect of imperfect joints, are rarely included in calculations. For example, the friction of pin joints may, under unfavourable circumstances, add 60 per cent. to the maximum stress on the links of a suspension chain (Ex. 4, p. 440). Hence the working strength for wrought iron rarely exceeds $4\frac{1}{2}$ or 5 tons per square inch. In reckoning the load Rankine recommended that the "dead" load should be divided by 2 and added to the "live" load in order to obtain the effective live load. More recently on the strength of Wöhler's experiments it has been proposed to find the ultimate strength of each piece under the maximum stress and fluctuation of stress to which it is subject, and divide by a constant factor of safety. There can be no doubt that a smaller co-efficient is necessary the greater the fluctuation, but the principle of a constant factor is open to question: it appears to lead either to co-efficients which are smaller than are known to be safe, or else to values above the limit of elasticity.

229. *Fibrous Materials. Ropes.*—Fibrous materials are those which may be regarded as made up of fibres, usually of organic origin, more or less closely united by cohesion or interlacing. The relative movements of the fibres are hindered by forces of the nature of friction, which are much less than the molecular forces to which the tenacity of a homogeneous solid body is due. Hence the strength and stiffness of a piece of material are much less than those of the fibres of which it is made up.

In most kinds of woods the fibres are arranged longitudinally, and the material is therefore especially characterized by its low resistance to division into parts longitudinally. Thus the resistance to longitudinal shearing of fir timber is only 600 lbs. per square inch, whereas its tenacity is about 20 times this amount, approaching that of cast iron. So, again, crushing takes place by longitudinal splitting under a stress little more than half the tenacity. Further, the condition of the material greatly influences the lateral cohesion of the fibres and thus affects its strength and elasticity. In timber which has been artificially dried the elasticity is nearly perfect up to the breaking point, whereas in the green state the elasticity is imperfect and the strength greatly reduced. Hence the importance of seasoning timber so as to be moderately dry.

The ordinary formulæ, however, will apply in all cases where the stress is a simple longitudinal stress, the direction of which is that of the fibres; that is to say, in tension, compression, and ordinary cases of bending. They will only fail when the bending is accompanied by crushing and shearing of considerable intensity, as when short pieces are acted on by transverse forces.

In cloth and similar materials two sets of fibres at right angles are united by interlacing. Resistance to tension is thus obtained with almost complete flexibility.

In ropes of all kinds the fibres are ranged in spiral curves in the process of manufacture, and their tension then produces lateral pressure, the friction arising from which is sufficient for union. The strength of a rope, though very great compared with its weight, is only one third that of the yarn of which it is spun, and on a similar principle the strength of large cables is less than that of the smaller ropes called "hawsers" of which they are made up. The strength of a rope is usually expressed by the formula

$$T = \frac{C^2}{k},$$

where C is the girth of the rope in inches, T the tension in tons, and

k a constant. The old rule in the navy was to take $k=5$ to obtain the breaking weight of a rope, but the table now employed gives $k=3{\cdot}3$, that is, a strength 50 per cent. greater. In small ropes k may be even less. The safe working load is not more than one sixth the breaking load. In iron wire ropes $k=1$, or for ropes above 6 inches girth somewhat more. The strength of wire ropes is more than doubled by the employment of steel. The safe working load may be taken as one fifth their breaking load.

TABLE I.—WEIGHT AND WORKING STRENGTH OF VARIOUS MATERIALS.

MATERIAL.	WORKING STRENGTH.				WEIGHT PER YARD IN POUNDS.			Working Strength in Feet of Material. $=\lambda$	
	Stress in Tons per Square Inch.		Area in Square Inches per Ton.		Per Ton of Stress under Working Load.		Per Square Inch of Area.		
	T.	C.	T.	C.	T.	C.		T.	C.
Cast Iron, ...	1·5	4·5	·667	·222	6	2	9	1120	3360
Wrought Iron, ...	4·5	4·5	·222	·222	2·22	2·22	10	3024	3024
Ordinary Soft Steel,	7	7	·143	·143	1·43	1·43	10	4700	4700
Ordinary Steel Wire,	13	...	·077	...	·77	...	10	9000	...
Copper Wire, ...	4	...	·25	...	2·9	...	11·5	2320	...
Deal, ...	·5	·3	2	3·3	1·5	2·5	·75	4480	2700
Oak, ...	·75	·45	1·33	2·22	1·33	2·22	1	5040	3024
Granite, ...	...	·3	...	3·33	...	11·66	3·5	...	576
Brickwork, ...	...	·06	...	15·6	...	39	2·5	...	160
Hemp Ropes, ...	·6	...	1·67	...	2·5	...	1·5	2700	...
Iron Wire Ropes, ...	2	...	·5	...	2·6	...	5·25	2600	...
Steel Wire Ropes,	5	...	·2	...	1·1	...	5·5	6000	...

TABLE II.—ELASTICITY AND RESILIENCE.

MATERIAL.	ELASTIC STRENGTH.						ELASTICITY (Tons and Inches).		RESILIENCE UNDER TENSION.	
	Stress in Tons per Square Inch.			Strain.			Young's Modulus.	Rigidity.	Foot Pounds per Cubic Foot.	Height in Feet and Inches
	T.	C.	S.	T.	C.	S.				
Cast Iron, ...	3	9	...	·000375	·001125	...	8000	...	185	4″·5
Wrought Iron, ...	9	9	7	·0007	·0007	·0014	13000	5000	1060	2′ 2″
Soft Steel, ...	15	15	12	·0012	·0012	·0024	13000	5200	2900	6′
Hard Steel, ...	25	25	20	·002	·002	·004	13000	5200	8000	16′ 6″
Tempered Steel, ...	50	...	...	...	...	...	15000(?)	...	34500	72′
Strongest Steel Wire,	150	...	...	·0115	...	...	13000	...	276000	577′
Fir, ...	$1\frac{1}{2}$	...	...	·0021	...	...	700	35	2150	58′
Oak, ...	2	...	...	·0028	...	...	700	35	4300	86′

TABLE III.—ULTIMATE STRENGTH AND DUCTILITY.

MATERIAL.	Ultimate Strength in Tons per Square Inch. T.	C.	S.	Elongation per Cent.	$\frac{\text{Tension} \times \text{Elongation}}{2}$
Iron Bars, ...	25	22	18	20	250
Iron Plates, ...	22	19	16	10	110
Soft Steel (·15 to ·3 per cent. of carbon),	30	...	$22\frac{1}{2}$	25	375
Medium Steel (·3 to ·5 per cent. of carbon)	35	...	27	15	262
Hard Steel (·5 to ·75 per cent. of carbon),	45	...	...	8	180
Cast Iron, ...	$7\frac{1}{2}$	45	12	...	...
Lead, ...	$1\frac{1}{2}$	...	...	...	...
Sheet Copper, ...	$13\frac{1}{2}$	...	...	...	...
Cast Copper, ...	$8\frac{1}{2}$	...	...	...	...
Oak, ...	$5\frac{1}{2}$	...	1	...	...
Fir, ...	$5\frac{1}{2}$	...	·27	...	...

230. *Tables of Strength.*—For a detailed account of the properties of materials the reader is referred to the authorities cited above and at the end of this chapter. A convenient summary is given in Rankine's *Useful Rules and Tables.* It will be here sufficient to give a few examples.

Table I. gives the weight and working strength of a variety of materials. From what has been said in preceding articles it appears that the working strength varies according to circumstances. Hence the values given in the table may be exceeded, and sometimes greatly exceeded when special care is taken in the selection of material, in the estimation of strains, and in the execution of the work. On the other hand cases occur in which they are too large, and, it may be, greatly too large if due care is not exercised. The first two columns give the safe load in tons per square inch of sectional area, the second two the area necessary to sustain a load of 1 ton, in tension (T) and compression (C) respectively. The next two give the weight of 1 yard length of a bar which will sustain 1 ton, and the numbers therein given are therefore the comparative weights of bars of equal strength. The same comparison is effected in a different way in the last two columns, which give the length in feet of a bar or column the weight of which is equal to the working load on its transverse section. It is on this quantity, which is denoted by λ in Arts. 40, 41, pp. 90, 92, that the limiting dimensions of a structure depend. It is used for this purpose in Ex. 13, p. 324, and Ex. 11, p. 372. It will be observed that weight for weight timber is stronger than wrought iron.

Table II. gives the elastic properties of certain materials in tension (T), compression (C), and shearing (S). It has been sufficiently explained in preceding chapters.

Table III. shows the ultimate strength and ductility of materials in common use. The first three columns give the ultimate resistance to tension, compression, and shearing. The fourth gives the elongation expressed as a percentage of the original length, which, if the length of the pieces experimented on be a constant multiple of the diameter, forms a measure of the ductility. The ultimate strength and ductility of steel vary according to the amount of carbon it contains in such a way that the sum of the two remains nearly constant, other things being equal. Thus in the examples given in the table the sum is about 53. In steel compressed in a fluid state by Sir J. Whitworth's process the constant sum is about one third greater. The last column gives half the product of the ultimate tensile stress and the elongation, a quantity which is sometimes used as a measure of the powers of resistance to impact. The actual amount of work done in stretching a bar till it breaks is much greater than this, as is seen on considering the form of the curve of stress and strain. A more exact measure of the resistance to impact would be furnished by an experiment on a short block such as that described on page 418.

EXAMPLES.

1. Show that the modulus of rupture of a material is 18 times the load which will break a bar of the material 1 inch square and 1 foot long: the bar being supported at the ends and the load applied at the centre.

N.B.—The modulus of the rupture is the value of the co-efficient in the ordinary formula for bending when the load is that found by experiment to break the beam.

2. A balcony, 6 feet long and 4 feet broad, is supported by a pair of cast-iron beams fixed in the wall at one end. The beams are of rectangular section, 2 inches broad, and depth near the wall 4 inches. What load per square foot will the balcony bear, the stress on the iron being limited to 1 ton per square inch? Also, how should the depth vary for uniform strength along the length of the beam?

Ans. Equating the greatest bending moment to the maximum moment of resistance to bending we find the load which the balcony will bear

$$= 41{\cdot}5 \text{ lbs. per square foot.}$$

As to the depth of the beam: for uniform strength $\frac{M}{I}y$ must be constant from which we find that the depth at any point of the beam must be proportional to the distance from the outer end of the beam; so that the lower side of the beam should be a sloping plane.

3. A paddle shaft is worked by a pair of engines with cranks at right angles. Supposing the steam pressure constant, and the resistance of each wheel equal and uniform, and obliquity of connecting rod neglected; compare the co-efficients of strength to be used in calculating the diameter of the paddle and intermediate shafts.

Ans. The uniform moment of resistance of the paddle wheel $=\frac{1}{2}$ the mean turning moment of the two engines. The twisting moment of the paddle shaft, when either crank is on the dead centre, $=\frac{1}{2}$ maximum twisting moment of one engine. At the same instant this is the twisting moment on the intermediate shaft. When the other crank is on the dead centre the twisting moment on intermediate shaft is the same in magnitude, but reversed in direction, and when the two cranks make angles of 45° with the dead centres the twisting of the paddle shaft $=\frac{1}{2}$ the maximum combined twisting moment of the two engines, that is $\sqrt{2}$ times its amount when either crank is on the dead centre; but the twist is in the same direction always. Therefore on the paddle shafts the stress alternates between x and $\sqrt{2x}$, and on the intermediate shaft between x and $-x$.

Hence applying formula

$$p=\tfrac{1}{2}\bar{p}+\sqrt{p_0(p_0-\tfrac{3}{2}\bar{p})},$$

we have for paddle shaft,

$$\bar{p}=\cdot 414\,x\,;\ p=1\cdot 414\,x\,;\ \therefore\ \bar{p}=\cdot 292p\,;$$

substituting, we obtain

$$p=\cdot 888\,p_0.$$

For intermediate shaft, $\bar{p}=2x$; $p=x$; $\bar{p}=2p$; and $p=\frac{1}{3}p_0$.

If the stress on the paddle shaft alternates to zero, by the wheels rolling out of the water, or by the stopping of the engine, then $p=\cdot 6p_0$.

4. A suspension chain is constructed with bar links united by pin joints; the diameter of the pins is two-thirds the breadth of the link (p. 371). If the bridge vibrate show that the maximum stress on the links may be increased by deviation (p. 341) due to friction of pins (p. 248) in the ratio $1+2f:1$, where f is the co-efficient of friction.

AUTHORITIES ON STRENGTH OF MATERIALS.

In addition to the works expressly cited in this chapter may be mentioned—

HODGKINSON. *Experimental Researches on the Strength and other Properties of Cast Iron.* Weale. 1846.

WEYRAUCH. *Iron and Steel.* New York. 1877.

BARLOW. *Strength of Materials.* Lockwood.

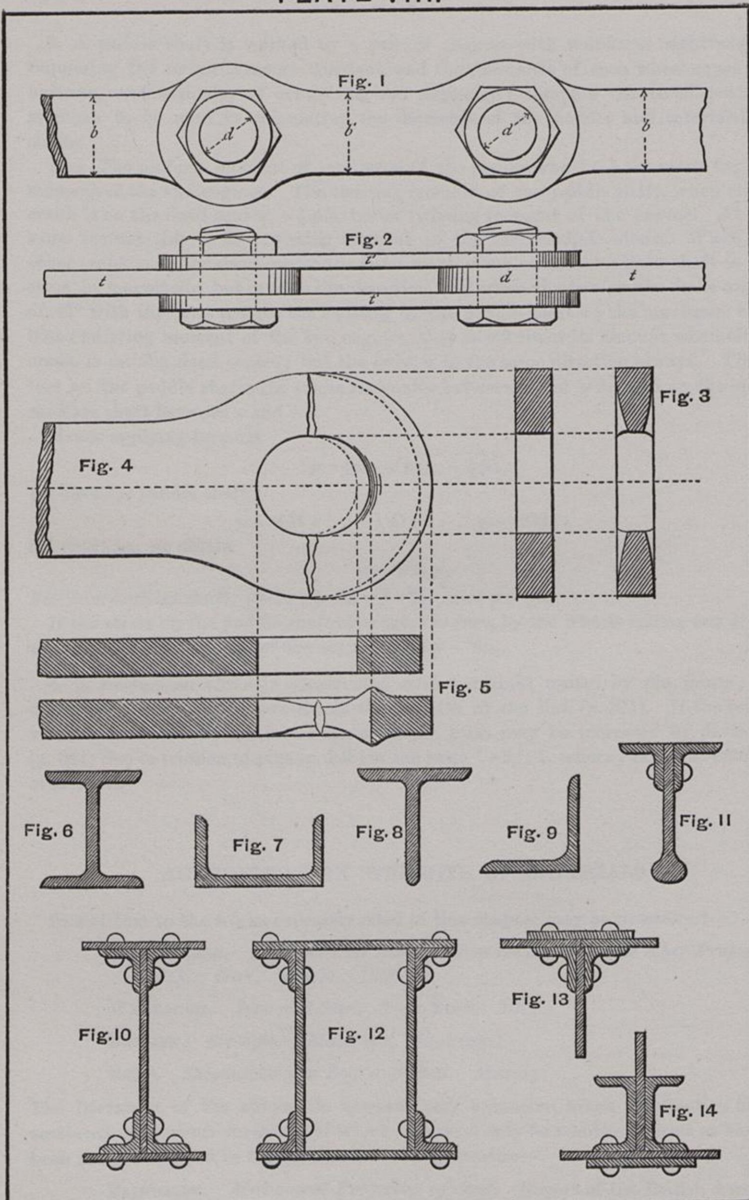
REED. *Shipbuilding in Iron and Steel.* Murray.

The literature of the subject is however very extensive, much information being scattered in various memoirs, of which two need only be mentioned here as having been much employed in the preparation of this treatise—

FAIRBAIRN. *Mechanical Properties of Steel.* Report of the British Association for 1867.

WÖHLER. *Die Festigkeits-Versuche.* Berlin. 1870.

PLATE VIII.



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DESCRIPTION OF PLATE VIII.

To illustrate various questions considered in Chaps. XII. and XV. Plate VIII. has been drawn.

Figs. 1, 2 represent the pin joint connecting two bars in tension, discussed in Art. 191, p. 369. Figs. 3, 4, 5 show the way in which the joint yields when the pins are too small. In Fig. 4 the original dimensions of the eye and eyehole are shown by dotted lines, while the full lines show what they become after yielding. Fig. 3 gives transverse sections of the eye before and after failure, showing the thinning out due to lateral contraction during stretching beyond the elastic limit. After this contraction has reached a certain limit the metal tears asunder, as shown in Fig. 4. The longitudinal section (Fig. 5) shows the corresponding spreading out at the top of the hole due to compression beyond the elastic limit. This lateral expansion is partially prevented in riveted joints, and (p. 410) this may be the reason why direct stress in them is of less importance. The failure of pin joints in this way furnishes a good example of the "flow of solids."

The remaining figures of this plate are intended to give some idea of the manner in which iron girders are constructed. Figs. 6, 7, 8, 9 are transverse sections of "H iron," "channel iron," "tee iron," and "angle iron:" these are rolled in one piece and, in combination with plates, form the materials from which large girders are built up. For small beams such as floor joists H iron or tee iron of the requisite depth and sectional area may be used. Figs. 10, 12 are sections of two of the commonest forms of built-up girders. In the first the web is a single plate to which angle irons are riveted to form the flanges, further strength being obtained by an additional covering plate. The second is similar, but the web consists of a pair of plates, a form known as a "box beam." Fig. 11 is commonly used in shipbuilding as a deck beam or otherwise: a "bulb iron" here forms the web and lower flange, while the upper flange is formed by a pair of angle irons as before. Figs. 13, 14 give examples of girders of more complex construction employed where greater strength is necessary: one flange only is shown in section in each case. For further details the reader is referred to the treatises by Mr. Hutchinson and Sir E. Reed, cited on pages 60 and 440.