Werk

Titel: Applied Mechanics

Untertitel: An elementary general introduction to the theory of structures and machines ; Wit...

Autor: Cotterill, James Henry

Verlag: Macmillan

Ort: London

Jahr: 1884

Kollektion: maps

Signatur: 8 PHYS II, 1457

Werk Id: PPN616235291

PURL: http://resolver.sub.uni-goettingen.de/purl?PID=PPN616235291|LOG_0030

OPAC: http://opac.sub.uni-goettingen.de/DB=1/PPN?PPN=616235291

PART V.—TRANSMISSION AND CONVERSION OF ENERGY BY FLUIDS.

231. *Introductory Remarks.*—We now return to the subject of Machines with the object of studying those machines in which fluids are employed as links in a kinematic chain for the purpose of transmitting energy, or as a means by which energy is supplied, stored, or converted.

A fluid is a body in which change of form is produced by the action of any distorting stress, however small, if sufficient time is allowed. In a perfect fluid a sensible change would be produced by a stress of sensible magnitude in an indefinitely short time, but in all actual fluids a time is required which is inversely as the stress—that is, the stress is proportional to the rate of change. This property of fluids is called Viscosity, and is measured by a co-efficient, as will be seen hereafter. The viscosity of a fluid varies greatly in different fluids, and, in the same fluid, is dependent on the temperature. At high temperatures it is much less than at low temperatures. The viscosity of water is exceedingly small.

Fluids are either liquid or gaseous. In liquids the changes of volume are in general small, and no diminution of pressure on the bounding surface will cause their volume to increase beyond a certain limit. Gases, on the other hand, expand indefinitely as the external pressure diminishes.

Liquids are employed in machines either as a simple link in a kinematic chain transmitting energy from some source independent of the liquid, or as a medium by means of which the force of gravity exerts energy. Such machines are called Hydraulic Machines, the fluid employed being in most cases water. On the other hand, gases in general serve as the means by which that form of energy which we call Heat is converted into mechanical energy, capable of being utilized for any required purpose. They may, however, also be employed for the storage and transmission of energy.

The motions of fluids may be studied in two different ways. In the first the Principles of Work and Momentum are applied to the whole mass of fluid under consideration, or to portions which, though small, are yet of visible magnitude; but no attempt is made to conceive, much less to determine, the movements of the smallest particles of which the fluid may be imagined to be made up. This method may be described as the experimental theory, and, as applied to water, forms that part of the subject which is called "Hydraulics." It is based directly on experiment, and requires continual recourse to experiment, just as is the case in questions relating to the friction of solids. Nevertheless, being continually verified by the large-scale experiments of the hydraulic engineer, its results, as far as they go, are as certain as those of any purely experimental subject. On the other hand, an analytical theory has been constructed, by means of which the motions of fluids are determined directly from the laws of motion, without reference to experience. This theory is usually called Hydrodynamics in treatises on mechanics. In the cases in which it is applicable it completely determines the motion of all particles of the fluid, and not merely that of the fluid as a whole.

The first two chapters of this division of our work will be devoted to Hydraulics and Hydraulic Machines, and the third to a brief discussion of the various applications of Elastic Fluids. The transmission and storage of mechanical energy by elastic fluids is often considered as part of hydraulics, because the method of treatment is in many respects similar. In this treatise it will be called "Pneumatics." The relations between heat and mechanical energy form a distinct science called "Thermodynamics," the principles of which will only be referred to when absolutely necessary.

NOTE.—On the value of C in the formula on p. 462 for the discharge of a pipe the reader is referred to the Appendix.

CHAPTER XIX.

ELEMENTARY PRINCIPLES OF HYDRAULICS.

232. *Velocity due to a Given Head.*—When the level of the surface of the water in a reservoir is above surrounding objects, a HEAD of water is said to exist, the magnitude of which is measured, relatively to any point, by the depth (h) of the point below the surface. If the water extend to this point a pressure is produced there which, so long as the water is at rest, is given in lbs. per sq. ft. by the formula

$$p = wh,$$

where w is the weight of a cubic foot of water, that is to say, about $62\frac{1}{2}$ lbs. for fresh water, or 64 lbs. for salt. Since the above formula may be written

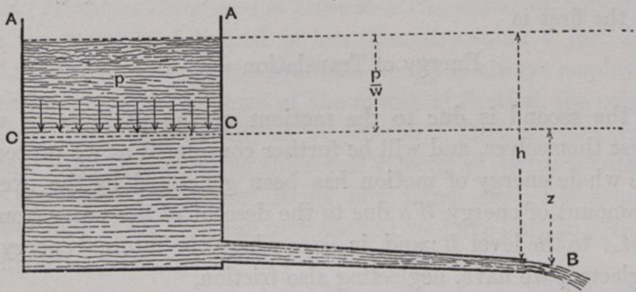
$$h = \frac{p}{w},$$

it appears that a pressure may be measured in terms of the head which would produce it. The fluid is usually water, for which h is reckoned in feet; and 1 lb. per sq. inch is equivalent to 2·3 feet of fresh, or 2·25 feet of salt water. For some purposes, however, mercury is employed, in which case the unit is generally 1 inch. One inch of mercury is equivalent to about ·49 lb. per sq. inch, that is, to a head of 1·1 feet of sea water, or 1·135 of fresh water.

If the surface of the water be exposed to the atmosphere, the pressure p will be in excess of the atmospheric pressure, which must be added to obtain the absolute pressure. The mean value of the atmospheric pressure is 14·7 lbs. per sq. inch, which corresponds to a head of about 33 feet for salt, or 34 feet for fresh water.

A head of water is a source of energy which may be employed in doing work of various kinds, or in simply transferring the water from one place to another. Let us take the second case, and imagine that, by means of a pipe, channel, or passage of any description, the water is delivered at B (Fig. 163), while at the same time, by a stream or otherwise, the surface of the water in the reservoir is kept constantly at the same level AA, so that the head h remains unchanged.

Fig.163.



The motion is then described as Steady, and consists simply in the transfer in each second of a certain weight of water from the stream to the reservoir, while an equal weight traverses the passage, and is delivered at B, the whole mass of water between AA and B remaining constantly in the same condition. The delivery at B may be supposed found by actual measurement; it is usually estimated in gallons per minute or cubic feet per second, as to which it need only be remarked that the gallon weighs 10 lbs., so that a cubic foot per second is about 375 gallons per minute. For large quantities, however, the cubic metre, which weighs about 1 ton, is also employed.

On delivery the water is moving with a certain velocity, but the definition and measurement of this quantity is not so simple. We must now suppose that the centre of gravity of the water delivered in some given time is observed and its velocity noted. This velocity will be the same whatever the time be, and will be a measure of the velocity of the mass of water considered as a whole. In some cases all particles of the water may be moving with this velocity, but in general this is not the case: it is then the mean velocity, and may be described as the "Velocity of Delivery." If the water be discharged by a channel which, near the exit, is of uniform transverse section A, this velocity may also be defined by the equation

$$v = \frac{Q}{A} = \frac{W}{wA},$$

where Q is the discharge in cubic feet per second, and W the weight of this quantity.

The energy of motion of the water may now be separated into two parts, one external and the other internal (Art. 134, page 278), of which the first is

$$\text{Energy of Translation} = \frac{Wv^2}{2g},$$

while the second is due to the motions of the particles of water amongst themselves, and will be further considered as we proceed.

The whole energy of motion has been generated by the exertion of an amount of energy Wh due to the descent of the water from the level AA to the level B; and, in cases where the internal energy may be neglected, we have, neglecting also friction,

$$\frac{v^2}{2g} = h,$$

where h the head is measured to the centre of gravity of the issuing water (page 194).

It has been here supposed that the surface of the water in the reservoir, and after delivery at B, is exposed to the atmosphere, but this is not always the case. Suppose in the figure the reservoir filled to the level CC only, but that the pressure on the surface has any value p, instead of being simply that of the atmosphere. This pressure p may be produced by filling up the reservoir to the level AA where

$$h = z + \frac{p}{w};$$

and as the reservoir is supposed large so that the water is sensibly at rest, except very near the exit, this can produce no change in the motion, which as before is given by

$$v^2 = 2gh = 2g\left(z + \frac{p}{w}\right).$$

In other words, in addition to the actual head z, we have a *virtual* head p/w, due to the difference of pressure p, thus giving a total head h.

The jet of water has been supposed to issue into the atmosphere, but the nature of the medium into which the discharge takes place has little influence, provided its pressure be duly taken into account. It has been proved by experiment that if the pressure of the atmosphere be artificially increased or diminished, the velocity is given by the same formula, modified as explained in the next article. This is also true if the efflux take place into a vessel of water.

233. *Frictional Resistances in General.*—The actual velocity v' with which the water is delivered is less than the value v just found, because a certain part of the available energy is always employed in overcoming certain resistances of the nature of friction, the origin of which we shall see gradually as we proceed. They are measured in two ways: (1) by comparing the actual velocity of delivery with that due to the head; (2) by considering how much energy is employed in overcoming them. In the first method we have only to introduce a co-efficient c given by

$$v' = cv,$$

which is called the Co-efficient of Velocity. It is of course always less than unity, and its value is found by experiment in each special case. In the second we write

$$h - h' = \frac{v'^2}{2g},$$

where h' is the "loss of head" due to friction. The value of h' is most conveniently expressed by connecting it with the *actual* velocity v' with which the water issues. For this purpose we replace h by $v^2/2g$ and v by v'/c, and thus obtain

$$h' = \left(\frac{1}{c^2} - 1\right)\frac{v'^2}{2g} = F\frac{v'^2}{2g},$$

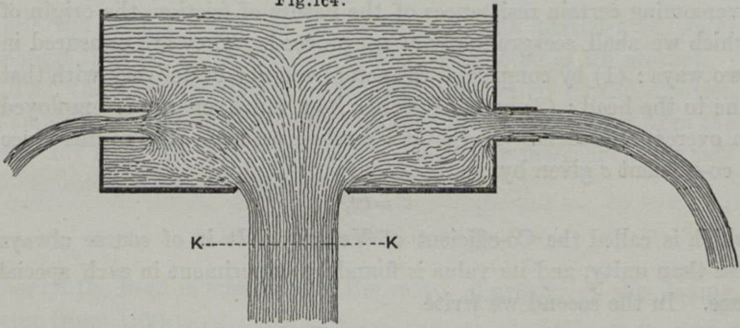
where F is a new co-efficient called the Co-efficient of Resistance connected with the previous one by the equation

$$F = \frac{1}{c^2} - 1.$$

It is found by experience that the values of these co-efficients depend mainly on the form and nature of the bounding surfaces within which the water moves, and, subject to proper limitations, not on the pressure or velocity of the water—a fact which may be expressed by the following law of hydraulic resistance: *The energy lost by frictional* *resistances is a fixed multiple of the energy of motion of the water.* This multiple is the co-efficient F which is sometimes fractional, but is often very large, as we shall see farther on. The physical meaning of this law will be seen hereafter, and the apparent deviations from it which frequently occur will be accounted for.

234. *Discharge from Small Orifices.*—Fig. 164 shows a vessel of water discharging through a circular hole in the bottom which is flat. The hole is small, and its circumference is chamfered below to a sharp edge at the upper surface.

Fig. 164.



On observing the jet of water which issues we see that it is nearly cylindrical but of diameter less than the diameter of the hole. The contraction is complete, so far as can be judged by the eye, at a distance of $d/2$ from the vessel; and by measurement is found to be in the ratio 4 : 5, that is, the sectional area of the jet is to the sectional area of the hole in the ratio 16 : 25.

If the hole be made in the vertical side of the vessel a contracted jet issues in the same way, but under the action of gravity it forms a curve which is very approximately parabolic in form, each particle moving nearly in the same way as a projectile *in vacuo*. This enables us to find the velocity of the efflux (v') by observing a point through which the jet passes, and we thus obtain experimentally the value of the co-efficient c, which appears to be about ·97. The discharge is now given by the formula

$$Q = A_0 \,.\, v' = ckA\sqrt{2gh},$$

where A_0, A are the contracted and actual areas of the orifice, and k

is their ratio which is a fraction called the Co-efficient of Contraction. The discharge therefore depends on the product of the two co-efficients c and k, which may be replaced by

$$C = ck,$$

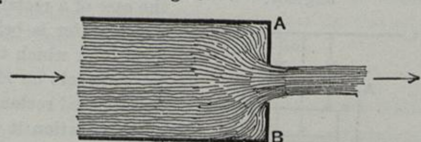
a quantity called the Co-efficient of Discharge.

The value of C can also be determined by direct measurement of the discharge, an observation which can be made with much greater accuracy than those of contraction and velocity on which it depends. In the present case it is found to be ·62, agreeing well with the product ·97 × ·64 of the values given above.

With other forms of orifice the same co-efficients are used, but their numerical values are quite different. In the figure two cases are represented: on the right side of the vessel the water issues through a short pipe the entrance to which from the vessel is square-edged; on the left a similar pipe is employed but it projects inwards instead of outwards. When the pipe projects outwards the water is found to issue in a jet the full diameter of the pipe, that is, k is unity; while, on the other hand, the velocity is much diminished, the value of c being only ·815. When it projects inwards the jet contracts greatly, the value of k being ·5 while the velocity is about the same as in a simple orifice. Thus C instead of being ·62 is ·815 and ·5 in the two cases. The causes of these remarkable differences will be seen hereafter, the results are only given here to illustrate the meaning of the co-efficients under consideration.

235. *Incomplete Contraction.*—The contraction of the issuing jet depends on the average angle at which the moving particles converge towards the orifice before reaching it, and this is the reason why it is so great in the case of a short pipe projecting inwards. If the circumstances be such that the convergence is small the contraction diminishes. Fig. 165 shows a pipe of some size through an orifice in the flat end AB of which water is being forced, issuing into the atmosphere. The co-efficient k is found to depend on the proportion which the area of the original orifice A bears to that of the pipe S, because the smaller S is, the less is

Fig.165.



the angle of the convergence. This has been expressed by an empirical formula due to Rankine which may be written

$$\frac{1}{k}=\sqrt{2\cdot618-1\cdot618\frac{A^2}{S^2}},$$

which will be found to give $k=\cdot618$ when S is infinite, as is nearly the case for a simple orifice as explained above, while for smaller values k increases, becoming unity as it should when $S=A$.

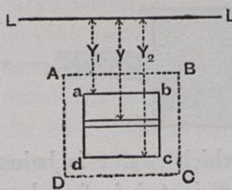
In a similar way if an orifice be near a corner of the vessel the contraction will be diminished. In these cases the contraction is usually described as "incomplete."

236. *Discharge from Large Orifices in a Vertical Plane.*—When the orifices are large, compared with the head and the vessel from which the discharge takes place, the question is more complicated.

If the plane of the orifice be vertical the velocities of the several parts of the fluid are not the same as is the case, so far as can be judged by the eye, when the orifice is small. On the contrary the velocity of that part of the stream which issues from the lower part of the orifice is visibly greater than that proceeding from the upper part. Hence it follows that the centre of gravity of the fluid issuing in a given time, to which the head is measured, is not on the same level as the centre of gravity of the contracted section, but lies below it. The corresponding point on the section may be described as the Centre of Flow. Also the internal energy of motion of the jet is of sensible magnitude and cannot be neglected.

By supposing that each part flows independently of the rest the discharge can be found for an orifice of any shape. For example, take the case of a rectangular orifice $ABCD$ (Fig. 166) from which water is being discharged from a reservoir, the level from which the head is measured being LL. The jet contracts on efflux, and the contracted section may be supposed rectangular. The position and dimensions of this section it will be necessary to suppose known by experiment; let its breadth be b, and let its upper and lower sides be at depths Y_1, Y_2 below LL. Divide the area into strips, and consider any one at depth y, then the velocity will be given by the formula (neglecting friction),

Fig.166.



$$v^2=2gy.$$

The quantity discharged per second will be given by

$$Q = \int_{Y_1}^{Y_2} bv \,.\, dy = b\sqrt{2g}\int_{Y_1}^{Y_2} \sqrt{y} \,.\, dy,$$

which by integration gives

$$Q = \tfrac{2}{3}b \,.\, \sqrt{2g} \,.\, (Y_2^{\frac{3}{2}} - Y_1^{\frac{3}{2}}),$$

which determines the discharge.

The energy of motion of the water discharged per second will be

$$U = w\int_{Y_1}^{Y_2} bv \,.\, \frac{v^2}{2g} \,.\, dy = bw\sqrt{2g}\int_{Y_1}^{Y_2} y^{\frac{3}{2}} . dy,$$

which by integration gives

$$U = \tfrac{2}{5} wb\sqrt{2g} \,.\, (Y_2^{\frac{5}{2}} - Y_1^{\frac{5}{2}}).$$

By dividing U by wQ we get the depth of the centre of gravity of the fluid discharged per second below LL, that is to say, the head h is given by the formula

$$h = \tfrac{3}{5} \,.\, \frac{Y_2^{\frac{5}{2}} - Y_1^{\frac{5}{2}}}{Y_2^{\frac{3}{2}} - Y_1^{\frac{3}{2}}}.$$

The velocity of delivery is

$$V = \frac{Q}{A} = \sqrt{2g} \,.\, \tfrac{2}{3} \,.\, \frac{Y_1^{\frac{3}{2}} - Y_2^{\frac{3}{2}}}{Y_2 - Y_1},$$

and the energy of translation on delivery is

$$U_0 = \frac{WQV^2}{2g},$$

a quantity less than the whole energy WQh by the energy due to internal motions. A common method of treating the question is to measure the head to the centre of the section and then employ the formula

$$V^2 = 2gh$$

with a proper co-efficient of discharge. This method is not exact, for it underestimates both the head and the energy of motion of the water; but its errors partially compensate one another, and its results are made to agree approximately with experiment by the employment of a variable co-efficient. To apply the exact formulæ it is necessary to know the dimensions and position of the contracted section for which the existing experimental data are insufficient. For further particulars on this subject, the reader is referred to Professor Unwin's work cited on page 481.

Again, if the dimensions of the orifice be not small compared with the surface of the water in the vessel from which the discharge takes place, this surface will sink with a velocity V which is of sensible magnitude. If the area of the surface be S and that of the contracted section A_0, the discharge will be

$$Q = A_0 v = SV,$$

an equation which determines V. The water will now have a velocity V before descending through the height h, and the equation of energy is therefore

$$v^2 - V^2 = 2gh,$$

This may be written if we please

$$\frac{v^2}{2g} = h + \frac{V^2}{2g},$$

showing that in addition to the actual head h we must consider the *virtual* head $V^2/2g$ due to the initial velocity of the water. In many hydraulic questions it is inconvenient or impossible to measure the head from still water. It is then measured from some point where the water is approaching the orifice with a velocity V determined by observation. The actual head h must then be increased by the height due to this velocity.

237. *Head relatively to Moving Orifices.*—The passages through which the water is moving may be attached to a ship, locomotive, or other moving structure, in which case the velocity must be reckoned relatively to the structure, and the height due to the velocity must be reckoned as part of the head. If for example in the bow of a vessel moving through the water with velocity V an orifice be opened at the surface level, the water will enter through it, and if unacted on will move within the vessel with velocity V and will possess relatively to the vessel the energy $V^2/2g$ per unit of weight. If it be acted on during entrance by the head due to any difference of level or pressure, so that its velocity is changed from V to v, the corresponding change of energy will measure the work which is done, and therefore the equation $v^2 - V^2 = 2gh$ applies as before.

238. *Steady Flow through Pipes. Conservation of Energy.*—Fig. 167 represents a vessel of water discharging through a large pipe, the section of which varies according to any law. If the pipe "runs full," that is, if it be always completely filled with water, the discharge is

$$Q = A_1 u_1 = A_2 u_2,$$

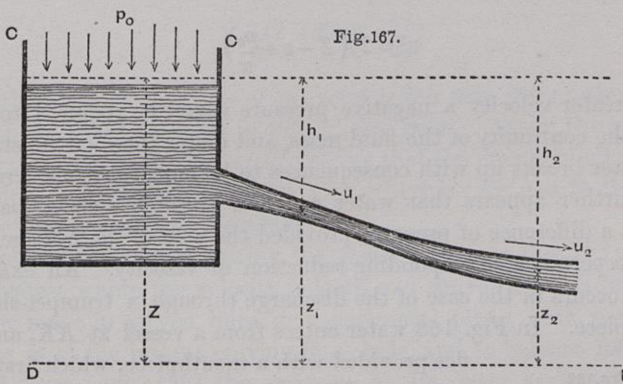
where u_1, u_2 are the velocities through two sections the areas of which are A_1, A_2. Hence the velocity is always inversely as the sectional area, and in an ordinary pipe in which the section is uniform must be the same throughout. Let the pressures be p_1, p_2, and the actual head, that is to say, the depths below the water surface CC, h_1, h_2, then it appears from Art. 234 that

$$\frac{u_1^2}{2g} = h_1 + \frac{p_0 - p_1}{w}; \quad \frac{u_2^2}{2g} = h_2 + \frac{p_0 - p_2}{w},$$

where p_0 is the pressure on the surface CC.

Take now some convenient line DD at a depth Z below the water

Fig.167.



surface CC, and let z_1, z_2 be the elevation of the section above this datum level so that

$$z_1 + h_1 = Z = z_2 + h_2,$$

then the above equations may be written

$$\frac{u_1^2}{2g} + \frac{p_1}{w} + z_1 = Z + \frac{p_0}{w} = \frac{u_2^2}{2g} + \frac{p_2}{w} + z_2.$$

This result shows that if u, p, z be the velocity, pressure, and elevation for any section of the pipe,

$$\frac{u^2}{2g} + \frac{p}{w} + z = \text{Constant}.$$

Each of the terms of this equation represents a particular kind of energy: the first is energy of motion, the third energy of position, the second is energy due to pressure, the origin of which will be further explained in the next chapter. The equation therefore shows that the total energy of the water remains constant as it traverses the pipe, and is accordingly the algebraical expression of the Principle of the Conservation of Energy. It supposes that no energy is lost by frictional resistances, and that any change in the internal motions of the particles amongst themselves may be disregarded. The word "head," the origin of which we have already seen, is frequently employed for the energy per unit of weight. (See Appendix).

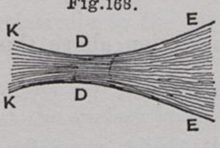
An important consequence of this principle is that where the sectional area of the pipe is least, and consequently the velocity greatest, there the pressure is least. Hence it follows that the velocity cannot exceed a certain limiting value u, found by putting $p=0$. At an elevation z above datum level

$$u^2 = 2g\left(Z - z + \frac{p_0}{w}\right).$$

At a greater velocity a negative pressure would be required to preserve the continuity of the fluid mass, and under these circumstances the water breaks up with consequences to be hereafter considered.

It further appears that water can flow through a closed passage against a difference of pressure, provided the area of the passage vary so as to permit a corresponding reduction of velocity. An example of this occurs in the case of the discharge through a trumpet-shaped mouthpiece. In Fig. 168 water enters from a vessel at KK, an orifice provided with a mouthpiece, which first contracts to DD, and then expands to EE where the jet enters the atmosphere. The pressure at EE is that of the atmosphere, and therefore at DD is less than that of the atmosphere, that is, less than it would be if the trumpet were cut off at the neck. Hence the discharge is increased by the addition of the expanded portion. If the water issued into a vacuum the jet would not expand to fill the wide mouth of the trumpet, which would not in that case have any influence on the discharge. The increased discharge and partial vacuum at DD have been verified by experiment.

Fig. 168.



239. *Distribution of Energy in an Undisturbed Stream. Vortex Motion.*—If the reservoir in the last article be imagined to supply a stream running in a channel of any size either closed or open, that stream, if undisturbed by any of the causes mentioned hereafter, may be supposed made up of an indefinite number of elementary streams, each of which moves as it would do in a closed pipe, as just described, without in any way intermingling with the rest. The forms of these ideal pipes depend solely on the form of the channel in which the stream is confined. The equation

$$\frac{u^2}{2g} + \frac{p}{w} + z = Z + \frac{p_0}{w}$$

applies to the motion in every pipe, and from it we may draw two important conclusions. In the first place, it may be written in the form

$$\frac{p - p_0}{w} = Z - z - \frac{u^2}{2g};$$

and therefore, *the pressure at any point is less than if the water were at rest by the height due to the velocity at that point.* Again, the equation interpreted as in the last article shows that the energy of all parts of the fluid is the same, or, as we may otherwise express it, *the energy of the fluid is uniformly distributed.*

From either way of stating the result it appears that the pressure is greatest where the velocity is least, and conversely. Now, if the water move in curved lines in a horizontal plane, each particle of water is at the instant moving in a circle, and to balance its centrifugal force (Art. 132) the pressure on its outer surface must be greater than that on its inner. It follows therefore that, if a channel is curved so as to alter the direction of the stream, the pressure increases as we go from the inner side of the channel to the outer; while, on the other hand, the velocity is greatest at the inner side and least at the outer. The change is the greater the sharper the bend, for the centrifugal force is greater. In open channels the change at the surface where the pressure is constant is in elevation instead of in pressure.

The magnitude of the change can be calculated in certain cases (see Appendix), of which we can only here consider one which is of special importance. If the particles of water describe circles about a common vertical axis, the elementary streams will form uniform rings, the centrifugal force of which can be calculated as in Art. 145, pp. 293-4. The resultant force on the half ring is—employing the notation of the article cited—given by

$$P = w \cdot 2A \cdot \frac{V^2}{g}.$$

This is balanced by an excess pressure on the outer surface of the half ring, and if that excess be Δp the corresponding resultant force is $\Delta p \cdot 2r$, as shown on p. 305. Equating this to P

$$\Delta p = \frac{w}{g} \cdot \frac{A}{r} \cdot V^2.$$

The ring is supposed of breadth unity, and for A we may write the thickness of the ring, which may be called Δr. Dividing by this, and proceeding to the limit

$$\frac{dp}{dr} = \frac{w}{g} \cdot \frac{V^2}{r},$$

an equation from which the pressure can be found if the law of velocity be given. If the fluid rotated about the axis like a solid mass, V would vary as r; but the case now to be examined is that in which V varies inversely as r, as expressed by the equation

$$Vr = \text{Constant} = k.$$

Substitute and integrate, then replacing k by Vr, it will be found that

$$\frac{p}{w} + \frac{V^2}{2g} = \frac{p_0}{w} + \frac{V_0^2}{2g},$$

where the suffix refers to a given point where the pressure is p_0 and the velocity V_0. This result shows that the energy is uniformly distributed, and we infer that if the direction of a moving current is changed so that the particles of water describe concentric circles, the velocity varies inversely as the distance from the centre.

A mass of rotating fluid is called a "vortex," and in the case just considered the vortex is described as "free," because the motion is that which is naturally produced (comp. Art. 261). A free vortex is necessarily hollow, for to hold the water together a negative pressure would be required near the axis of rotation, but the hollow may be filled up by water moving according to a different law.

240. *Viscosity.*—When the motion of a mass of water is free from sudden changes of direction, loss of energy takes place only through the direct action of viscosity, a property of fluids which it will now be necessary briefly to consider. In Fig. 154, p. 409, a block of plastic material is represented, and it was explained that to produce change of form a certain difference of pressure was necessary, depending on the hardness of the material. In a fluid a similar difference of pressure is necessary to produce a change of form *at a given rate*, and the magnitude of the difference is proportionate to the rate. If u be the rate at which the height of the block is diminishing and the breadth increasing, each reckoned per unit of dimension,

$$p = 2cu,$$

where c is a co-efficient called the "co-efficient of viscosity." Or to express the same thing differently, if ω be the *rate* at which a small rectangular portion of the fluid is distorting, as in Fig. 140, p. 357, q the corresponding distorting stress,

$$q = c \cdot \omega.$$

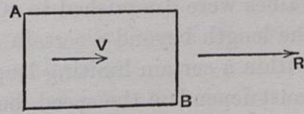
Hence, when a fluid moves, any change of form requires an amount of work to be done which is proportionate to the speed at which the change takes place. In a free vortex the rate of distortion is equal to the angular velocity of the particles round the axis, and varies inversely as the square of the distance; the changes of shape are therefore very rapid near the centre, and energy is consequently dissipated much more rapidly than in the stream from which the vortex is produced.

In the case of water the viscosity is so small that such changes of form as occur in an undisturbed stream are not rapid enough to absorb any large amount of energy. For example, in the discharge from orifices in a thin plate the loss of head is only 5 or 6 per cent. It is only when the water is disturbed by the neighbourhood of a rough surface over which it moves, or otherwise, that large quantities of energy are dissipated and frictional resistances of great magnitude produced.

241. *Surface Friction in General.*—We now proceed to study experimentally some of the more important causes of frictional resistance.

Fig. 169 shows a thin flat plate AB with sharp edges completely immersed in the water. The plate is moving edgeways through the water with velocity V, then a certain resistance R is experienced which must be overcome by an external force. This resistance consists in a tangential action between the plate and the

Fig. 169.



water, and so far is analogous to the friction between solid surfaces but it follows quite different laws, which may be stated as follows:—

(1) The friction is independent of the pressure on the plate.

(2) It varies as the area of the surface in contact with the water.

(3) It varies as the square of the velocity.

These laws are expressed by the formula

$$R = fSV^2,$$

where f is a co-efficient which, as in the friction of solid surfaces, is described as the "co-efficient of friction." The value of this co-efficient depends on the degree of smoothness of the plate. Thus, for example, in some experiments, to be described presently, on thin boards moving through water it was found that the co-efficient was ·004 for a clean varnished surface, and ·009 for a surface resembling medium sand paper, the units being pounds, feet, and seconds.

There are certain limitations to the truth of these laws, as in the case of solid surfaces. In the first place, if the velocity be below a certain limit the water adheres to the surface, and its velocity relatively to the surface is some continuous function of the distance from the surface so that the stream does not break up. This will be further referred to hereafter; for the present it is sufficient to say that the resistance then follows an entirely different law, varying nearly as the velocity instead of the (velocity)^2. The limiting velocity, however, at which this is sensibly the case is so low that in most practical applications the effect may be disregarded. In the second place, it is supposed that the water glides over all parts of the surface, with the same velocity; but if the surface be any considerable length the friction of the front portion of the surface on the water furnishes a force which drags the water forward along with the surface and so diminishes the velocity with which it moves over the rear portion. The friction is thus diminished, and in large surfaces very considerably diminished. Thus Mr. Froude experimenting on a surface 4 feet long, moving at 10 feet per second, found the value of f given above, but when the length was 20 feet and upwards, those values were diminished to ·0025 and ·005 respectively. Increasing the length beyond a certain amount produces very little change, and within a certain limiting length the effect is insensible. These limits must depend on the speed, but no exact observations have been made on this point. The power of the speed to which the friction is proportional has, however, been found to be diminished on long smooth surfaces, as shown below. The skin friction of vessels on which, as we shall see hereafter, the resistance chiefly depends at low speeds, is much diminished by the effect of length.

Experiments on surface friction were made by Colonel Beaufoy. They formed part of an elaborate series of experiments on the resistance of bodies moving through water, carried out during many years in the Greenland Dock, Deptford. Beaufoy employed the formula

$$R = f \, . \, SV^n$$

to represent his results, and for the index n obtained the values 1·66, 1·71, 1·9 in three series of experiments. The standard experiments on the subject are however due to the late Mr. Froude: they were made on boards $\frac{3}{16}$ inch thick, 19 inches deep, towed edgeways through the water. The boards were coated with various substances so as to form the surface to be experimented on.

The following table gives a general statement of Froude's results. In all the experiments in this table, the boards had a fine cutwater and a fine stern end or run, so that the resistance was entirely due to the surface. The table gives the resistances per square foot in pounds, at the standard speed of 600 feet per minute, and the power of the speed to which the friction is proportional, so that the resistance at other speeds is easily calculated.

Nature of Surface.	Length of Surface, or Distance from Cutwater, in Feet.											
	2 Feet.			8 Feet.			20 Feet.			50 Feet.		
	A	B	C	A	B	C	A	B	C	A	B	C
Varnish, ...	2·00	·41	·390	1·85	·325	·264	1·85	·278	·240	1·83	·250	·226
Paraffin, ...	1·95	·38	·370	1·94	·314	·260	1·93	·271	·237	...	...	...
Tinfoil, ...	2·16	·30	·295	1·99	·278	·263	1·90	·262	·244	1·83	·246	·232
Calico, ...	1·93	·87	·725	1·92	·626	·504	1·89	·531	·447	1·87	·474	·423
Fine Sand, ...	2·00	·81	·690	2·00	·583	·450	2·00	·480	·384	2·06	·405	·337
Medium Sand,	2·00	·90	·730	2·00	·625	·488	2·00	·534	·465	2·00	·488	·456
Coarse Sand, ...	2·00	1·10	·880	2·00	·714	·520	2·00	·588	·490	...	...	...

Columns A give the power of the speed to which the resistance is approximately proportional.

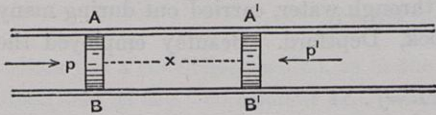
Columns B give the mean resistance per square foot of the whole surface of a board of the lengths stated in the table.

Columns C give the resistance in pounds of a square foot of surface at the distance sternward from the cutwater stated in the heading.

242. *Surface Friction of Pipes.*—When water moves through a pipe the friction of the internal surface causes a great resistance to the flow.

Fig. 170 shows a pipe of uniform transverse section (not necessarily circular) provided with two pistons, AB, $A'B'$, at a distance x, enclosing between them a mass of water. The pistons and included water move forward together with velocity v under the action of a force R, required on account of the friction of the pistons and of the water on the pipe. Omitting piston friction the force R will be given by

Fig.170.



$$R = f \cdot S \cdot v^2 = f \cdot sx\, v^2,$$

where S is the wetted surface and s the perimeter.

If we imagine the pipe full of water moving through it with velocity v, the force R is supplied by the difference of the pressures p, p' on the pistons, and, therefore, if A be the sectional area

$$p - p' = f \cdot \frac{s}{A} \cdot xv^2.$$

The quantity A/s may be replaced by m and is described as the "hydraulic mean depth" of the pipe, a term derived from the case of an open channel to be considered hereafter. In the ordinary case of a cylindrical pipe $m = \frac{1}{4}d$. Further, we may reduce the pressures to feet of water by dividing by w, and thus obtain for the difference of pressure h'

$$h' = f \cdot \frac{x}{m} \cdot \frac{v^2}{w} = f' \cdot \frac{x}{m} \cdot \frac{v^2}{2g},$$

where f' is a co-efficient connected with f by the equation

$$f = f' \cdot \frac{w}{2g}.$$

The value of w, the weight of a cubic foot of water, differs so little from $2g$ that it is unnecessary, for our present purpose (Art. 283), to distinguish between f and f', especially as the value of f is always determined by special experiment on pipes.

This formula for the head necessary to overcome surface friction is continually in use. The formula gives directly the head necessary for a length x of the pipe, when the water, by being enclosed between pistons, is constrained to move over the surface with a given velocity: when the pistons are removed and the water flows freely it represents the facts very imperfectly. The central parts of the stream move quicker than the parts in immediate contact with the pipe, and besides, though the circumstances are different, we cannot be sure that the velocity over the internal surface is not affected in the same way as in the case of a moving surface. The value of f has therefore to be obtained by special experiment, and the result of such experiments are by no means always in accordance with each other. It is found, however, that f lies between the limits ·005 and ·01 according to the condition of the internal surface, and partly also on the diameter and velocity, the value being greater in small pipes than large ones, and at low velocities than high ones. For the present we assume ·0075 as roughly representing the facts when there is no special cause for increased resistance. For a pipe of circular section, length l, we have therefore

$$h' = 4f \cdot \frac{l}{d} \cdot \frac{v^2}{2g},$$

where for $4f$ we commonly assume the value ·03.

243. *Discharge of Pipes.*—The velocity v is the actual velocity with which the water moves, so that $v^2/2g$ is the energy of motion of each pound of the water. The loss of energy by friction is the same as that of raising the water through a height h', and is therefore equal to the energy of motion when

$$\frac{l}{d} = \frac{1}{4f} = 33 \text{ nearly},$$

that is, a length of pipe equal to 33 diameters absorbs an amount of energy equivalent to the whole energy of motion of the water. In pipes of any length, therefore, the effect of friction is very great, so much so that the size of a pipe is principally fixed by the loss of head which can be permitted. It is easily seen that to deliver water with a given velocity the loss varies inversely as the diameter, and that to deliver a given quantity it varies inversely as the fifth power of the diameter; thus, the smallest permissible diameter is fixed almost entirely by the value of h', which may be supposed already known.

The quantity discharged per second is given us by the formula

$$Q = Av = \frac{\pi}{4} d^2 v,$$

and on substitution this becomes

$$Q = \frac{\pi}{4}\sqrt{\frac{g}{2f}} \cdot \sqrt{\frac{h'}{l}} \cdot d^{\frac{5}{2}}.$$

All dimensions are here in feet and Q is in cubic feet per second. If we require gallons per minute for a diameter of d inches, the formula will be

$$G = C \cdot \sqrt{\frac{h'}{l}} \cdot d;$$

where C is a constant the value of which, for $4f = \cdot 03$, is 30, but which is often taken somewhat less (say 27) to allow for contingencies.

244. *Open Channels.*—Returning to Fig. 170, suppose the pipe, instead of being horizontal, is laid at an angle θ (see Fig. 171 next page), so that the difference of level of the two ends is $y = l \cdot \sin\theta$, then the difference of pressure-head is

$$\frac{p - p'}{w} = f \cdot \frac{1}{m} \cdot \frac{v^2}{2g} - y,$$

and therefore may be made zero if the slope of the pipe be

$$\sin\theta = f \cdot \frac{1}{m} \cdot \frac{v^2}{2g} = \frac{h'}{l}.$$

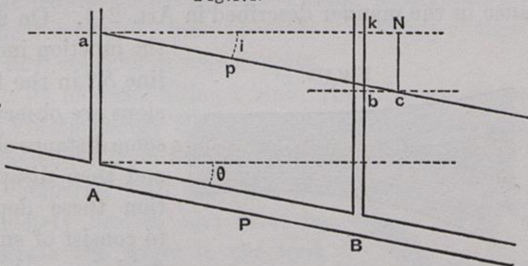
But if the pressure be constant we may remove the upper surface of the pipe and thus obtain the case of an open channel. The quantity m is now the sectional area of the channel divided by the wetted perimeter, and is therefore the actual depth in a very broad shallow channel, but in other cases less in a ratio dependent on the form of section. As before stated it is described as the "hydraulic mean depth" of the channel.

We can now find the velocity and discharge of a stream of given dimensions and fall, provided that we know the value of f, or conversely the size of channel for a given discharge and fall. The value of f, however, varies for the same reasons as in pipes which indeed apply with still greater force, so that the limits of variation are wider. The average value does not differ very widely from ·0075, already adopted for pipes; but to obtain results of even moderate accuracy a special study of the experiments on the subject is necessary, which will not be attempted in this treatise.

245. *Virtual Slope of a Pipe.*—If the pipe be laid at any other angle the pressure will not be constant, and the mode in which it varies is best seen by a graphical construction.

Suppose small vertical pipes Aa, Bb to be placed at the end of A, B of the pipe we are considering (Fig. 171), then (if they enter the water square, without being bent towards the direction of motion) the water will rise in them to a level representing the pressure in feet of water at these points. If there were no friction the level would be the same in both, and the difference (bk in the figure) therefore represents the loss by friction. Now draw a horizontal line through b, and take c on it, so that $ac = AB = l$, then the angle caN is given by the equation

Fig. 171.



$$\sin i = \frac{h'}{l},$$

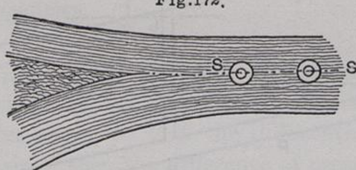
and is therefore the slope of a channel of the same length and hydraulic mean depth which would give the same discharge. This angle is therefore called the VIRTUAL SLOPE of the pipe. At any point P in the pipe, the water would rise to the level of the corresponding point p in the virtual channel, found by taking $ap = AP$. The construction would of course fail if h' were equal to, or greater than l, but this case does not occur in practice; on the contrary, in pipes as in channels the angle i is nearly always small. The virtual slope is frequently one of the data of the question. The line ac is variously described as the "pressure line," "line of virtual slope," or "hydraulic gradient."

The pipe need not be straight; it may be curved or be laid in sections at different slopes, there will still be a continuous hydraulic gradient, provided the diameter be the same throughout; but if the sections be of different diameters each section will have its own slope. In practice care must be taken that the pipe does not rise above its hydraulic gradient, for otherwise there will be a partial vacuum: the pipe then acts as a syphon, which is liable to fail on account of leakage and the presence of air in the water.

246. *Loss of Energy by Eddies and by Broken Water.*—We now proceed to consider other causes of frictional resistance.

In Fig. 172 two streams of water, moving with different velocities, converge towards each other and unite into one. Each stream, so far as can be judged by the eye, moves originally without disturbance in the manner described in Art. 241. On union, however, near the junction indicated by the dotted line *SS* in the figure, small depressions are observed, which move for some distance along with the stream, and then disappear. On examination these depressions are found to consist of small portions of the fluid in a state of rotation, the speed of rotation being greatest at the centre and gradually dying away towards the circumference. A motion of this kind was called a "vortex" in Art. 241, and in the present case is also described as an "eddy"; it is independent of the general motion of the stream, and its energy is therefore of the internal kind. The disappearance of the eddies thus formed is due to viscosity, the effect of which is much greater in the eddy than in the stream as already explained. After the eddies have disappeared the two streams are found to have become a single one, moving with a velocity intermediate between those of the streams which form it, but possessing less energy. Theoretically there is nothing to prevent two streams of a perfect fluid from moving side by side with different velocities, but such a motion is always unstable, and will not long continue without the formation of eddies by a sudden change of direction (Art. 239) in small portions of the fluid which separate from the rest. The instability is greater the more nearly perfect the fluid is. When-

Fig. 172.



ever then water in motion intermingles with water at rest, or moving with a different velocity, internal motions of a complex kind are produced, representing a considerable amount of energy of the internal kind which is virtually lost even before its final dissipation by fluid friction.

Again, in order that a mass of water may form a continuous whole, sufficient pressure must exist on the bounding surface to prevent the pressure at any point within the mass from becoming zero, as explained in Art. 240. If this condition is not satisfied the water breaks up more or less completely, and the result is a confused mass with complex internal motions rapidly disappearing as before by fluid friction. When waves break on a beach, or when paddles strike the water and drive it upwards in a mass of foam, the process takes place on a large scale before our eyes; but the same thing occurs in most cases where the velocity of a mass of water is suddenly changed, and of this we will now consider some examples.

Fig. 173*a* shows a jet of water filling a tank. Here the water pouring in possesses the kinetic energy $Wv^2/2g$ due to the original velocity of the water, and the height from which it falls into the tank. If it be of some size as compared with the tank the water will be completely broken up; if it be small it will penetrate the water in the tank without much apparent disturbance at the surface: in either case the result is a mass of water at rest as a whole, so that its energy is all of the internal kind. If the jet be shut off the water rapidly settles down to rest, the whole energy is then dissipated by fluid friction.

Fig. 173_a

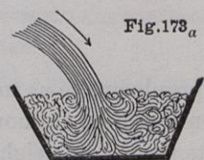
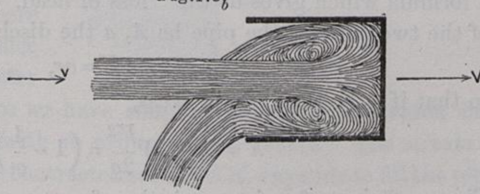


Fig. 173*b* shows a bucket moving horizontally, bottom foremost, with velocity V, while a horizontal jet moving with greater velocity strikes it centrally: the bucket is then filled with broken water which pours out under the action of gravity. In water-wheels a series of buckets are filled in succession, and the broken water carried on with the wheel. Here if the bucket were at rest the loss of energy would be, as before, $Wv^2/2g$; but as it is

Fig. 173_b



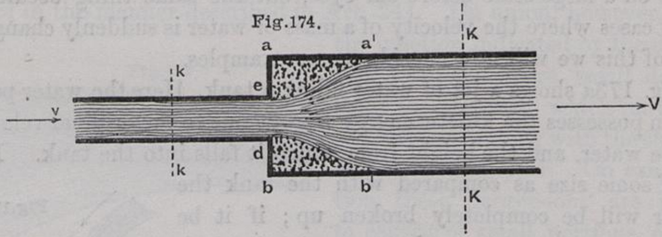
moving with velocity V, the striking velocity on which the breaking depends will be $v - V$, and the loss of energy is

$$U = W\frac{(v-V)^2}{2g},$$

where W is the weight of water acted on in the time considered. Both these cases may be treated as examples of the collision of two bodies considered on page 280, one of the bodies being indefinitely great. The energy of collision is employed in breaking up the water. It is represented in the first instance by internal motions, and subsequently dissipated by fluid friction.

Fig. 174 represents a pipe which is suddenly enlarged from the diameter *cd* to the diameter *ab*. The water is moving through the small part of the pipe with velocity v, and, on passing through *cd*

Fig. 174.



spreads out so as to fill the larger part. At some distance from the enlargement it moves in a continuous mass with velocity V, but in its immediate neighbourhood we have broken water, as in the case of the bucket, from which it only differs in the enclosure of the water in a casing. The loss of energy per unit of weight may be expected to be the same (Art. 252) as before, and is therefore

$$h' = \frac{(v-V)^2}{2g},$$

a formula which gives us the "loss of head." If the sectional areas of the two parts of the pipe be A, a the discharge is

$$Q = AV = av,$$

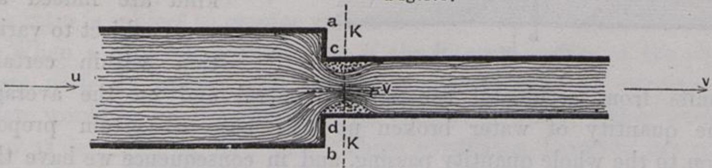
so that if m be the ratio of areas,

$$h' = (m-1)^2\frac{V^2}{2g} = \left(1 - \frac{1}{m}\right)^2\frac{v^2}{2g}.$$

The co-efficient of resistance is therefore $(m-1)^2$ or $(1 - 1/m)^2$, according as the velocity to which it is referred is that in the large pipe or that in the small one.

Instead of the water moving from a small pipe into a large one, we may have the converse case of a suddenly contracted pipe as in Fig. 175. The loss here is due to precisely the same cause, namely a sudden enlargement, which is produced as follows. In the figure the stream of water moving with velocity u contracts on passing through

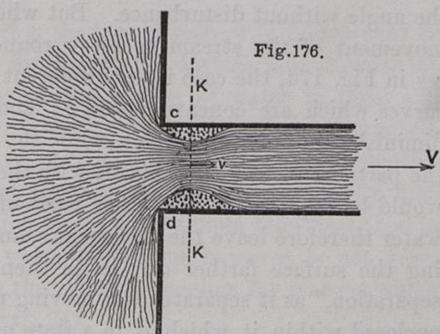
Fig. 175.



cd nearly as it would if the small part of the pipe were removed, as in Fig. 165, p. 449, until it reaches a contracted section KK, and is then moving with a velocity v which is greater than u in the ratio of the area of the large pipe to the *contracted* area KK. The loss of head in this part of the process is not large. After passing KK, however, an expansion takes place to the area of the small pipe, and this is accompanied by breaking up, the space between the contracted jet and the pipe being filled up with broken water.

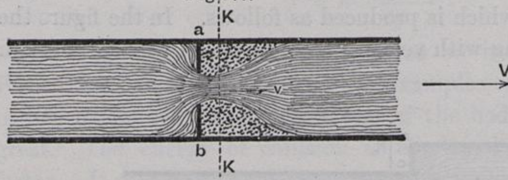
In Fig. 176 we have the extreme case, in which the large pipe is a vessel of any size. We thus obtain the case of a pipe with square edged entrance which has already been referred to in Art. 236. Another modification is that of a diaphragm in a pipe, as in Fig. 177. The small pipe is here larger than the orifice through which the water enters, and in the figure we have simply a single pipe divided into parts by a diaphragm with an orifice in the centre. The stream of water, after passing the contracted section KK, expands to fill the pipe. In cocks when partially closed, a loss of head of the same kind occurs, which may be increased to any extent by closing the cock further.

Fig. 176.



In all these cases the loss of head may be calculated approximately by means of the formula for a sudden enlargement, but the ratio of enlargement is not known exactly, on account of the uncertainty of the value of the co-efficient of contraction to be assumed. Losses of head of this kind are indeed always subject to variation within certain limits from accidental causes; in general and on the average the quantity of water broken up will bear a certain proportion to the whole quantity passing, and in consequence we have the general law of hydraulic resistance stated on page 447, but the ratio may vary from time to time, and cannot be stated with precise accuracy. The causes of this uncertainty will be clearly understood on considering somewhat more closely the manner in which the loss takes place.

Fig. 177.



In Figs. 175, 177 two plane surfaces at right angles meet at a, forming an internal angle, through which water is flowing. The particles of water there describe curves which are all convex towards a, and in conformity with the general principle explained in Art. 239, the pressure must increase and the velocity diminish on going towards a. The water then moves slowly and quietly round the angle without disturbance. But when compelled by the general movement of the stream to move round an external angle such as *kea* in Fig. 174, the case is very different; the particles then describe curves which are concave round e; and consequently the pressure diminishes in going towards e, while the velocity increases. To hold the particles of water in contact with the surface, an infinite pressure would be required in the other parts of the fluid. The particles of water therefore leave the surface at e, and describe a path ea', regaining the surface farther on; ea' is then described as a "surface of separation," as it separates the moving mass of water from a portion enclosed within it which is in a state of violent disturbance. Such are the surfaces shown in Figs. 171-178. It is not, however, to be supposed that these surfaces are sharply defined, and that they permanently separate different masses of water. On the contrary, no such equilibrium is possible; the surfaces are continually fluctuating, and a constant interchange takes place between the so-called "dead" water and the stream. In this intermingling eddies are produced nearly as in the comparatively simple case of two streams given on page 464. The process is always essentially the same, and consists in sudden changes of direction being communicated to parts of the stream which become detached from the rest.

247. *Bends in a Pipe. Surface Friction.*—In some other cases the process of breaking up by which energy is lost is less obvious, and the ratio is subject to greater variations.

When a pipe has a bend in it, if the internal surface of the pipe were perfectly smooth and free from discontinuity of curvature, there would be no disturbance of the current of water, which would flow as described in Art. 241. These conditions, however, are not satisfied by actual bends in pipes, and there is always a loss of head due to them in addition to the loss by surface friction. This loss can only be determined by experiment, but it is easy to conjecture that the loss will be proportional to the angle through which the pipe is bent, and that it will be greater the quicker the bend, that is, the smaller the radius of the bend is as compared with the diameter of the pipe. The extreme case of a bend is a knee, but the loss is not in this case proportional to the angle of the knee, but follows a complex law. For details respecting bends and knees the reader is referred to the treatises cited at the end of this chapter, but some common examples are given in the table (p. 470).

In the case of surface friction the loss of energy is represented in the first instance by eddies formed at the surface and thrown off. In almost all practical cases of the motion of water in pipes and channels, even when to all outward appearance quite undisturbed, the fluid is in fact in a state of eddy motion throughout, and dissipation of energy at every point is going on much more rapidly than would be the case if the motion were of the simple kind described in Art. 241. The quantity of water broken up, however, is not generally in a fixed proportion to the quantity passing, for reasons which are sufficiently indicated in Art. 242.

248. *Summation of Losses of Head.*—The total loss of energy due to a number of hydraulic resistances of various kinds is found by adding together the losses of head due to each cause taken separately. The velocity of the water past each obstacle will not generally be the same for all, and it is then necessary to select some one velocity from which all the rest can be found by multiplication by a suitable factor for each obstacle. If n be this multiplier the loss of head will be

$$h' = \Sigma F n^2 \frac{V^2}{2g},$$

where V is the velocity selected for reference. The value of V is then found for motion under a given head H by the formula

$$(1 + \Sigma F n^2) \frac{V^2}{2g} = H.$$

The various values of F already given are collected with some additions in the annexed table :—

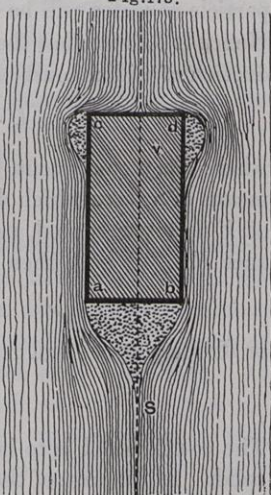
CO-EFFICIENTS OF HYDRAULIC RESISTANCE.

NATURE OF OBSTACLE.	VALUE OF F.	REMARKS.
Orifice in a Thin Plate.	·06	
Square-edged Entrance of a Pipe.	·5	
Sudden Enlargement of a Pipe in the ratio m : 1.	$(m-1)^2$	Referred to Velocity through large part of Pipe.
Bend at Right Angles in a Pipe.	·14	Radius of Bend = 3 × Diameter of Pipe.
Quick Bend at Right Angles.	·3	Radius of Bend = Diameter of Pipe.
Common Cock partially closed.	·75, 5·5, 31	Handle turned through 15°, 30°, 45° from position when fully open.
Surface Friction of a Pipe the length of which is n times the diameter.	$4f.n.$	For a clean Iron Pipe d inches diameter, according to Darcy, $4f = \cdot 02\left(1 + \frac{1}{d}\right)$
Knee in a Pipe at Right Angles.	Unity	In Bends the co-efficient is proportional to the Angle of the Bend, but in Knees the law is much more complex.

249. *Resistance of deeply Immersed Bodies. Ships at Low Speeds.*—The subject of the resistance of ships is outside the limits of this treatise, for the ship moves on the surface of water, exposed to the atmosphere, on which waves are produced; whereas in the branch of mechanics now under consideration, the water is supposed to move within fixed boundaries. A certain part of the subject, however, may properly be considered as belonging to Hydraulics. If a body be deeply immersed in a fluid, that part of the fluid alone which is in its immediate neighbourhood will be affected by its motion, and the question is not essentially different from the cases already considered of the movement of water in pipes and channels.

Fig. 178 shows a parallelopiped *abcd* moving through water in the direction of its length, the face *cd* being foremost. To an observer whose eye travels along with the body the water will appear to move past the solid in a stream of indefinite extent. At some distance away the action of the solid is insensible, but it becomes increasingly great as the solid is approached, and is greatest for that part of the water which moves in immediate contact with it. At *c* and *d* eddies are formed in passing round the corners exactly as is the case at the same points in Figs. 175, 176—the stream in fact is suddenly contracted in the same way as in passing from a large pipe to a small one, the diminution of area in this case being the transverse section of the solid. After this the water moves in actual contact with the solid until it reaches the corners *ab*, when it describes the curves *aS*, *bS*, meeting in *S*, after which it forms a continuous stream as before. The two curves enclose between them a mass of eddying water exactly similar to the eddies at *a* and *b* in Fig. 174—the stream, in fact, suddenly expands, just as in passing from a small pipe to a large one, the increase of area being in this case the sectional area of the solid. The eddies thus formed during the passage of the solid through the water absorb energy, which must be supplied by means of an external force, which drags the body through the water. The eddies

Fig. 178.



at *cd* represent an increased pressure on the front face *cd* of the solid, while those at *aS*, *bS* diminish that at the rear. This kind of resistance to the movement of a body through water is called Eddy Resistance, and may be almost entirely avoided by employing "fair" forms, that is, by avoiding all discontinuity of curvature in the solid itself, and in the junction of its surface with the direction of motion.

A general formula for eddy resistance is derived thus. As already stated the water suffers no sensible disturbance at a certain distance from the solid. If then we imagine a certain plane area A attached transversely to the solid, and moving with it, all the water affected by the solid will pass through this plane, and its quantity will be

$$Q = AV,$$

where V is the velocity. In similar solids this area must be proportioned to the sectional area S of the solid, so that we write $A = cS$, where c is a constant depending on the form. Of this water a certain fraction will be disturbed by eddies, and the velocity of each particle of water will be some fraction of the velocity of the solid. Hence it follows that the energy U generated per second in the production of eddies must be

$$U = c'wQ \cdot \frac{V^2}{2g} = cc'wS \cdot \frac{V^3}{2g},$$

where c' is a co-efficient. Now this amount of energy is generated by means of a force which drags the solid through the water, at the rate of V feet per second, notwithstanding an equal and opposite resistance R. We have then

$$RV = cc'wS \cdot \frac{V^3}{2g},$$

or dividing by V, and replacing cc' by a single constant k,

$$R = k.wS \cdot \frac{V^2}{2g}.$$

The co-efficient k is to be determined by experiment for each form of solid. In the case of the parallelopiped shown in the figure, the value of k depends little on the length, unless it be so short that the eddies at the corners *cd* coalesce with those in the rear of the solid, and it then becomes the same as that of a plate moved flatwise. Further it is nearly the same, if the transverse section be circular instead of square, and does not greatly differ from unity. For the flat plate it is greater and may be taken as 1·25. It must be remarked, however, that resistance of this kind is very irregular, and may vary considerably in the course of the same experiment. Different results are therefore obtained by different experimentalists. By some authorities much larger values are given. The same remarks apply to the case of a sphere for which the value may be taken as about ·4.

In all cases the value of k is independent of the units employed. It is also to a great extent independent of the kind of fluid, being approximately the same for example in air as in water; but this would not hold good for fluids of very different viscosity; nor is it true for high speeds in air, because the compressibility of the air affects the question. The same remarks apply to the co-efficient (F) of hydraulic resistance employed above. It has been found that co-efficients of surface friction are greater in salt water than in fresh in the ratio of the densities of these fluids, as we might anticipate, since surface friction is a kind of eddy resistance.

In well-formed ships the eddy resistance should not be more than 10 per cent. of the total resistance at low speeds, and is frequently less; the principal cause of resistance here is surface friction, which is given by the formula stated in Art. 241. The surface to be considered is the wetted surface, which can be found by direct measurement. It is convenient, however, to have a formula which gives the resistance in terms of the displacement (Δ) of the vessel. If R be the resistance, V the speed, the formula will be

$$R = K \,.\, \Delta^{\frac{2}{3}} V^2,$$

where K is a co-efficient which for speed in knots per hour, displacement in tons, and resistance in lbs., may be taken from ·55 to ·85 according to the type of vessel, if the bottom be in good condition. The speed, however, must not exceed a certain limit on account of the wave resistance, which increases at a much more rapid rate. At low speeds the value of this resistance is small, and it may approximately be considered as compensating for the somewhat slower rate at which the surface friction increases (Art. 241), but at high speeds it becomes a principal part of the resistance, and has to be separately considered. The speed of a wave is proportional to the square root of its length, and the magnitude of the resistance in similar ships depends on the proportion between the length L of the ship, and the length of waves which travel at the same speed. The limit in question is therefore given by the equation

$$V_0 = K' \,.\, \sqrt{L}$$

where for lengths in feet and speeds in knots per hour the co-efficient K' may be taken from ·6 to ·7. (See Appendix.)

Not only is the speed limited to which the resistance formula given above applies, but it must be further remarked that it supposes that the vessel is towed by an external force. If the vessel be propelled by steam power on board, the effective resistance is much greater, because the action of the propeller has (probably always) the effect of increasing the resistance. In screw propulsion this augmentation is very great, being at the rate of 20 to 40 per cent.; the larger value is of common occurrence. In reckoning the engine power required, the resistance must be taken at its augmented value, and the formula of Art. 128, p. 269, employed for the efficiency of the mechanism, which is much less at low speeds than at high speeds, as the formula shows.

250. *Direct Impulse and Reaction.*—The generalized form of the second and third laws of motion, described as the Principle of Momentum in Chapter XI. of this work, may be employed with great advantage when the motion of water in large masses is under consideration, because the total momentum of a fluid mass depends solely on the motion of the centre of gravity (p. 277), and not on the very intricate motions of the parts of the fluid amongst themselves. Further, the energy dissipated by frictional resistances is accounted for by these internal motions, or by the mutual actions of the fluid particles, and the total momentum is therefore independent of these resistances. Hence it follows that results may be obtained which are true notwithstanding any frictional resistances, and in some cases the loss of energy by them may be determined *a priori*. Also the pressures on fixed surfaces may be found which do no work, and to which therefore the principle of work does not directly apply.

Fig.179.

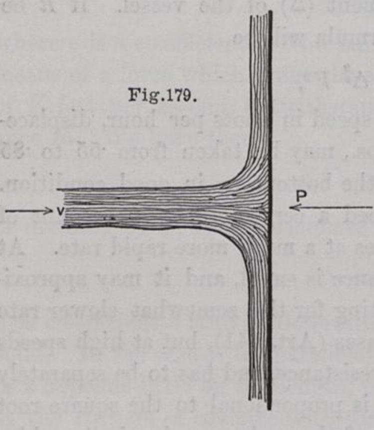


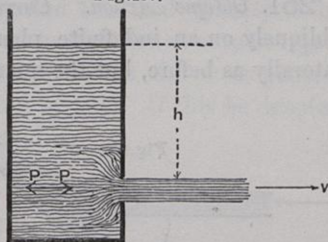
Fig. 179 shows a jet of water striking perpendicularly a fixed plane of infinite extent, and exerting on it a pressure P. The magnitude of this pressure is found by considering that the plane exerts an equal and opposite pressure on the water, which changes its velocity. The water, originally moving with velocity v, spreads out laterally, and any motion which it possesses is parallel to the plane. In time t the impulse is Pt, and the change of momentum is Mvt, where M is the mass of water delivered per second. Equating these we have

$$P = Mv = \frac{W}{g}.\,v,$$

where W is the weight of water delivered per second.

If the plane be smooth, and gravity be neglected, the motion of the water will be continuous; but if it be rough to any extent, so that breaking-up occurs, the result will still be correct, provided only the roughness be symmetrical about the axis of the jet. And the action of gravity parallel to the plane does not affect the question.

Fig. 180.



In Fig. 180 we have the converse case of water issuing from a vessel with a lateral orifice. Here the water, which originally was at rest, issues with velocity v, and the momentum generated in time t is Mvt. To produce this momentum a corresponding impulse is required, which is derived from the resultant horizontal pressure P of the sides of the vessel upon the water. We have as before

$$P = Mv = \frac{Wv}{g}.$$

A pressure equal and opposite to P is exerted by the water on the vessel: this is described as the "reaction" of the water; and, if the vessel is to remain at rest, must be balanced by an external force supplied by the supports on which it rests.

A remarkable connection exists between the change of pressure on the sides of the vessel consequent on the motion and the co-efficients of contraction and resistance.

First, suppose the water at rest, the orifice being closed, then the value of P is zero, and the pressure on the area of the orifice is $w \,.\, A \,.\, h$, the notation being as in Art. 236. When the orifice is opened the pressure on that side is diminished, first, by the quantity $w \cdot A \cdot h$; secondly, by an unknown diminution S due to the motion of the water (p. 455) over the surface near the orifice. Now

$$P = S + w \cdot A \cdot h = \frac{wA_0 v'^2}{g} = 2wA_0 (h - h'),$$

the notation still being as in the article cited. Replacing A_0 by kA we obtain

$$S = wA\{2k(h - h') - h\} = wAh\left(\frac{2k}{1+F} - 1\right).$$

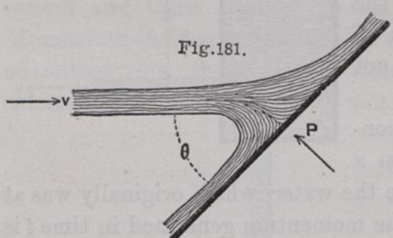
Since S is always positive the least value of k is

$$k = \frac{1+F}{2}.$$

If there be no frictional resistances $k = \cdot 5$, and this is the smallest value k can have under any circumstances. For a small pipe projecting inwards as in Fig. 164, p. 448, these conditions are approximately realized, the water being at rest over the whole internal surface of the vessel.

251. *Oblique Action. Curved Surfaces.*—When a jet impinges obliquely on an indefinite plane (Fig. 181), the water spreads out laterally as before, but the quantity varies according to the direction. In the absence of friction the velocity of individual particles is the same as that of the jet in whatever direction the water passes. At the same time the velocity of the whole mass of water parallel to the plane cannot be altered by the action of the plane, and is therefore $v \cdot \cos\theta$, where θ is the angle the jet makes with the plane. It immediately follows that any small portion of water diverging from K the centre of the jet at an angle ϕ with the jet must be balanced by another portion diverging in the direction immediately opposite, and the quantities so diverging must be in the ratio $1 - \cos\phi : 1 + \cos\phi$, being inversely as the changes of velocity parallel to the plane. But if the circumstances be such that breaking-up takes place, the motion of the water parallel to the plane will be undetermined, and in general there will be a tangential action on the plane of the nature of friction.

Fig. 181.

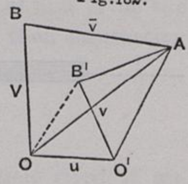


The normal pressure on the plane is in all cases the same, being given by the formula

$$P = Mv \cdot \sin\theta = \frac{W}{g} \cdot v \cdot \sin\theta.$$

If the surface on which the water impinges be curved it is necessary to know the average direction and magnitude of the velocity with which the water leaves the surface. In the absence of friction, as already noticed, the velocity of the individual particles is unaltered unless the water be enclosed in a pipe so that the pressure can be varied—a case for subsequent consideration; the direction, however, will depend on the way in which the water is guided. In cases which occur in practice it will generally be found either that the whole of the water is guided in some one direction, or that it leaves the surface in all directions symmetrically.

Fig. 182.



Taking the first case, suppose the original velocity (v) of the water to be represented by OA (Fig. 182), and the final velocity to be diminished to V by friction, and altered in direction so as to be represented by OB. Then the change of velocity in the most general sense of the word (p. 275) is represented by AB. If this be denoted by $\bar{v}$ the change of momentum per second is

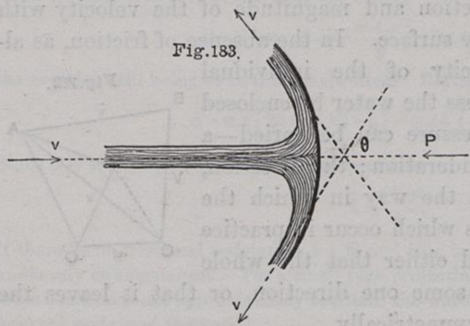
$$P = \frac{W\bar{v}}{g}.$$

The resultant pressure on the surface is parallel to AB and numerically equal to P.

In applications to machines the curved surface is frequently a vane which is not fixed, but moves with a given velocity; the pressure can then be found by a simple addition to the diagram. Through O draw OO', representing the velocity (u) of the moving surface in direction and magnitude, then $O'A$ represents the velocity with which the water strikes the surface. Considering the vane as fixed, the velocity is now estimated with which the water would leave it, and $O'B'$ drawn to represent it; the change is now AB' instead of AB. If the absolute velocity is required with which the water leaves the surface, it may be found simply by joining OB', which will completely represent it: the change of velocity being AB', whether the velocities are absolute or relative to the moving surface.

The cup vane ACA (Fig. 183), against which a small jet of water impinges centrally, may be taken as an example where the water spreads in all directions symmetrically. If OA be tangent to the vane at A, making an angle θ with the centre line of the jet, the water leaves the vane in the direction OA with unaltered velocity (neglecting friction). The resultant pressure P is in the direction of the jet, and the velocity in that direction is altered from v to $v \cos\theta$ in the opposite direction, so that the change of velocity is $v(1+\cos\theta)$. Thus we have

Fig. 183.



$$P = \frac{Wv}{g}(1+\cos\theta).$$

252. *Impulse and Reaction of Water in a Closed Passage.*—When the water is moving in a closed passage the resultant pressure to be considered in applying the principle is not merely that on the sides of the passage, but also that on the ideal surfaces which separate the mass of water we are considering from the complete current. In the previous cases the pressure of the atmosphere on the free surface bounding the fluid was the same throughout, and was balanced by an equal pressure of the surface against which it impinges, which is not included in the preceding results. This is now no longer the case.

An important example is that of the sudden enlargement in a pipe already referred to in Art. 246. In Fig. 174, page 466, take ideal sections KK, kk of the large and small portions of the pipe, and consider the whole mass of water between them. This mass is acted on (1) by the pressure (p) on the transverse section kk, (2) by the pressure (P) on the transverse section KK, and (3) by the pressure of the sides of the pipe. If we resolve in the direction of the length of the pipe, the only part of (3) which we need consider is the pressure (p') on the annular surface ac, bd, the area of which is $A-a$, and the whole resultant pressure is therefore $PA - pa - p'(A-a)$ in the direction opposite to the motion of the water. Now let W be the weight of water delivered in one second, then in that space of time W passes from the small pipe, where its velocity is v, to the large pipe, where it has a velocity V, so that if we equate the resultant pressure to diminution of momentum

$$PA - pa - p'(A-a) = \frac{W}{g}(v-V) = \frac{wAV(v-V)}{g},$$

a formula which may be written

$$\frac{P}{w}-\frac{p}{w}=\frac{V(v-V)}{g}+(p'-p)(1-\frac{1}{m}),$$

m being as in Art. 246 the ratio of enlargement. Let now H be the total head in the large pipe and h in the small one, then subtracting $(v^2-V^2)/2g$ from both sides and re-arranging the terms

$$h-H=\frac{(v-V)^2}{2g}+(p-p')(1-\frac{1}{m}).$$

Comparing this result with that obtained in the article cited, it appears that the value of the loss of head there given is a necessary consequence of supposing $p=p'$, but cannot otherwise be correct. That the pressure in the broken water at *ac*, *bd* is nearly equal to the pressure in the small pipe may be considered probable *a priori*, independently of the experimental verification which the formula has received.

EXAMPLES.

1. The injection orifices of the jet condenser of a marine engine are 5 feet below the surface of the sea, and the vacuum is 27 inches of mercury: with what velocity will the water enter the condenser, supposing three-fourths the head lost by frictional resistances? Also find the co-efficients of velocity and resistance and the effective area of the orifices to deliver 100,000 gallons per hour. *Ans.* Velocity = 23·6′ per second; Area = 27 sq. inches.

2. Water is discharged under a head of 25′ through a short pipe 1″ diameter with square-edged entrance; find the discharge in gallons per minute. *Ans.* $66\frac{1}{3}$.

3. Water issues from an orifice the area of which is ·01 sq. feet in a horizontal direction and strikes a point distant 4′ horizontally and 3′ vertically from the orifices. The head is 2′ and the discharge 25 gallons per min.; find the co-efficients of velocity, resistance, contraction, and discharge. *Ans.* c = ·816, F = ·5, k = ·57, C = ·57.

4. The wetted surface of a vessel is 7,500 sq. feet, find her skin resistance at 8 knots and the H.P. required to propel her, taking the resistance to vary as V^2 with a co-efficient of ·004. *Ans.* Resistance = 5,600 lbs., H.P. = 137.

5. The diameter of a screw propeller is 18′, the pitch 18′, and the revolutions 91 per min. Neglecting slip find the H.P. lost by friction per sq. feet of blade at the tips, taking a co-efficient ·008 to include both faces of the blade. *Ans.* Friction = 65 lbs. per sq. feet. H.P. = 10·6.

6. Two pipes of the same length are 3″ and 4″ diameter respectively: compare the losses of head by skin friction (1) when they deliver the same quantity of water, (2) when the velocity is the same. *Ans.* Ratio = 4·21 and 1·33.

7. Water is to be raised to a height of 20′ by a pipe 30′ long 6″ diameter: what is the greatest admissible velocity of the water if not more than 10 per cent. additional power is to be required in consequence of the friction of the pipe? *Ans.* $8\frac{1}{2}'$ per sec.

8. Two reservoirs are connected by a pipe 6″ diameter and three-fourths of a mile long. For the first quarter mile the pipe slopes at 1 in 50, for the second at 1 in 100, while in the third it is level. The head of water over the inlet is 20 feet and that over the outlet 9 feet. Neglecting all loss except that due to surface friction, find the discharge in gallons per min., assuming $f = \cdot 0087$. *Ans.* $v = 3\cdot 43'$ per sec. Discharge = 253 gallons per min.

9. A river is 1000′ wide at the surface of the water, the sides slope at 45°, and the depth is 20′; find the discharge in cubic feet per sec. with a fall of 2′ to the mile, assuming $f = \cdot 0075$. *Ans.* 154,000.

10. A tank of 250 gallons capacity is 50′ above the street. It is connected with the street main, the head in which is 52′ by a service pipe 100′ long: find the diameter of the pipe that the tank may be filled in 20 min. What must the head in the main be to fill the tank in 5 min. with this service pipe? *Ans.* $d = 1\cdot 6''$. Head in main = 82′.

11. Water is discharged by a vessel from a long pipe: show that the discharge is the same for all pipes of the same length with the discharging extremity in the same horizontal line. Draw the hydraulic gradient and examine the case of a syphon.

12. In question 2 suppose the pipe instead of being short to be 25″ long, find the discharge, assuming for surface friction $f = \cdot 01$. *Ans.* 52.

13. A horizontal pipe is reduced in diameter from 3″ to $\frac{1}{2}''$ in the middle, the reduction being very gradual. The pressure head in the pipe is 40′, what would be the greatest velocity with which water could flow through it, all losses of head being neglected? *Ans.* 1·4′ per sec.

14. A pipe 2″ diameter is suddenly enlarged to 3″. If it discharge 100 gallons per min., the water flowing from the small pipe into the large one, find the loss of total head and the gain of pressure head at the sudden enlargement. State the two values of the co-efficient of resistance.

Ans. Loss of head = $8\frac{1}{2}''$. $F = 1\cdot 59$ or $\cdot 31$.
Gain of pressure = 1′ 2″.

15. In the last question suppose the water to move in the reverse direction. Find the loss of head and the change of pressure consequent on the sudden contraction, assuming the co-efficient of contraction to be ·66.

Ans. Loss of head = $7\frac{1}{2}''$.
Diminution of pressure = 2′ $5\frac{3}{4}''$.

16. A horizontal pipe 30′ long is suddenly enlarged from 2″ to 3″ and then suddenly returns to its original diameter. Length of each section = 10′. Draw the hydraulic gradient when the pipe is discharging 100 gallons per min. into the atmosphere, assuming as coefficient of surface friction $4f = \cdot 03$. Find the total loss of head. *Ans.* Total loss of head = 10′ $2\frac{1}{2}''$.

17. A pipe contains a diaphragm with an orifice in it the area of which is one-fifth the sectional area of the pipe. Find the co-efficient of resistance of the diaphragm, assuming the contraction on passing through the orifice the same as that on efflux from a vessel through a small orifice in a thin plate. *Ans.* $F = 46$.

18. Find the loss of head in inches due to a bend through 45° of radius 6″ in a pipe 2″ diameter, the velocity of the water being 12′ per sec. *Ans.* ·2″.

19. A plane area moves perpendicularly through water in which it is deeply immersed; find the resistance per sq. feet at a speed of 10 miles per hour. Deduce the pressure of a wind of 20 miles per hour using the same co-efficient. *Ans.* Resistance = 269 lbs. Wind pressure = 1·312 lbs.

20. Compare the resistance of an area moving flatwise through the water with its resistance moving edgewise so far as due to surface friction, the co-efficient for which is ·004. *Ans.* Ratio = 312.

21. In question 1 suppose the ship moving at 10 knots and the orifice of entry so arranged as to cause no additional resistance: find the velocity of delivery. *Ans.* Additional head = 4·42′; velocity = 25′ per sec.

22. Water is supplied by a scoop to a locomotive tender at a height of 7′ above the trough. Assuming half the head lost by frictional resistances, what will be the velocity of delivery when the train is running at 40 miles per hour, and what will be the lowest speed of train at which the operation is possible? *Ans.* 39′ per sec.; $14\frac{1}{2}$ miles per hour.

23. A stream of water delivering 500 gallons per min. at a velocity of 15 feet per sec. strikes an indefinite plane (1) direct, (2) at an angle of 30°; find the pressure on the plane.

24. Employ the principle of momentum to prove the formula on page 455 for the resultant centrifugal force of one-half a rotating ring of fluid.

REFERENCES.

For further information on subjects connected with the present chapter, the reader is referred to a treatise on Hydraulics by Professor W. C. Unwin, M.I.C.E., forming part of the article Hydro-Mechanics in the edition of the "Encyclopædia Britannica" now (1883) in course of publication.

CHAPTER XX.

HYDRAULIC MACHINES.

253. *Hydraulic Motors in General*—Hitherto the energy exerted by means of a head of water has been supposed to be wholly employed in overcoming frictional resistances, and in generating the velocity with which the water is delivered at some given point. We now proceed to consider the cases in which only a fraction of the head is required for these purposes; the remainder then becomes a source of energy at the point of delivery by means of which useful work may be done. A machine for utilizing such a source is called an Hydraulic Motor.

Hydraulic energy may exist in three forms, according as it is due to motion, elevation, or pressure. In the first two cases it is inherent in the water itself, being a consequence of its motion or its position as in the case of any other heavy body. In the third it is due to the action of gravity or some other reversible force, sometimes on the water itself, but oftener on other bodies, as, for example, the load on an accumulator ram. The water is then only a transmitter of energy and not directly the source of it. As, however, the energy transmitted is proportional to the weight of water delivered, just as in the two other cases, the water is as before described as possessing energy. The energy per unit of weight is called "head," as sufficiently explained in the preceding chapter, and the "total head" is the sum of the "velocity head," the "actual head," and the "pressure head."

Hydraulic motors are classed according to the mode in which the water operates upon them, which may be either by weight, or by pressure, or by impulse, including in the last term also "reaction."

254. *Weight Machines.*—To utilize a head of water, consisting of an actual elevation (h) above a datum level at which the water can be delivered and disposed of, a machine may be employed in which the direct action of the weight of the water, while falling through the height h, is the principal motive force.

The common overshot water-wheel (Fig. 2, plate III. p. 152) may be taken as a type. Here the driving pair is a simple turning pair, and the driving link is the force of gravity upon the falling water which acts directly on buckets open to the atmosphere. If G be the delivery in gallons per minute, the energy exerted in foot-pounds per minute is

$$E = 10Gh.$$

The head h is here measured from the level of still water in a reservoir which supplies the wheel. If v be the velocity of delivery to the wheel, the portion $v^2/2g$ is converted into energy of motion before reaching the buckets and operates by impulse. In a wheel of this class, therefore, the water does not operate wholly by weight. The speed of the wheel is limited to about 5 feet per second by the centrifugal force on the water, which, if too great, causes it to spill from the buckets. It will be seen hereafter that the velocity of the water should be about double this, so that v is about 10 feet per second, and the part of the fall operating by impulse is therefore about 1·5 feet. The remainder operates by gravitation, but a certain fraction is wasted by spilling from the buckets, and emptying them before reaching the bottom of the fall. More than one half the head operating by impulse is always wasted (Art. 260), and this class of wheels is therefore only suitable for falls exceeding 10 feet. The great diameter of wheel required for very high falls is inconvenient, but examples may be found of wheels 60 feet diameter and more. The efficiency of these wheels under favourable circumstances is ·75, and is generally about ·65.

In "breast wheels" the buckets are replaced by vanes which move in a channel of masonry partially surrounding the wheel. The water is admitted by a moveable sluice through a grating of fixed blades in the upper part of the channel. The channel is thus filled with water, the weight of which rests on the vanes and furnishes the motive force on the wheel. There is a certain amount of leakage between the vanes and the sides of the channel, but this loss is not so great as that by spilling from the buckets of the overshot wheel. The efficiency is found by experience to be as much as ·75. As the diameter of the wheel is greater than the fall a breast wheel can only be employed for moderate falls.

In both these machines the water virtually forms part of the piece on which it acts. This link of the kinematic chain forms one element of the driving pair, while that attached to the earth forms the other. In the overshot wheel the water is contained in open buckets, in the breast wheel it is contained in a closed chamber or channel. A third class of weight machines is referred to farther on under the head of pumps.

255. *Hydraulic Pressure Machines in Steady Motion.*—A water wheel of great diameter is a slow-moving cumbrous machine, and for heads of 100 feet and upwards it is therefore necessary to employ a pressure or an impulse machine. Such machines are also often more convenient for low falls.

In pressure machines the driving link is compressed water, which is forced between the elements of the driving pair by some source of energy which supplies the necessary head. The head is sometimes an actual elevation either natural or artificial: in the docks at Great Grimsby the hydraulic machinery is operated from a tank placed on a tower 200 feet high. It is however difficult to get a considerable pressure in this way, and an apparatus called an Hydraulic Accumulator is therefore generally resorted to. Two forms occur, of which one is shown in Pl. IX. In the first a plunger or ram is forced into a cylinder by heavy weights placed in a plate-iron cage suspended from it and stayed by iron rods. The accumulator is supplied by pumps generally worked by steam, which is the ultimate source of the energy, the accumulator merely serving the purpose of a store of energy which can be drawn on at pleasure. For ordinary hydraulic machinery the pressure is limited to 750 lbs. per square inch from the difficulty of obtaining pipes of sufficient strength and of working slide valves under heavy pressures. In machines for riveting and other special purposes, however, pressures of 1500 lbs. per square inch and upwards are employed. The accumulator then consists of a cylinder B (Fig. 1, Pl. IX., p. 497), loaded with ring weights EE, sliding on a fixed spindle F, divided into two lengths, of which the upper portion is of smaller diameter than the lower.

In either form the accumulator provides a store of compressed water which can be supplied by suitable pipes to any number of machines, placed often at considerable distances. A head of 1700 feet is thus readily obtained, and for special purposes much more: differences of level may therefore be disregarded as of small importance, and the water considered as operating wholly by pressure.

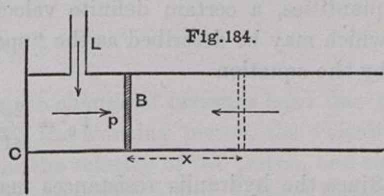


Fig. 184.

The driving pair of the machine forms a chamber of variable size which is alternately enlarged by the pressure of the water, and contracted to expel it. In most cases it is a simple cylinder C and piston B (Fig. 184): the water is admitted by a port from a pipe L, transmitting it from the accumulator at pressure p. Let the piston move through a space x, let A be its area, then

$$\text{Energy exerted} = pAx = p \,.\, X,$$

where X is the volume swept through by the piston. If w be as usual the weight of a cubic foot, wV is the weight of water which enters the cylinder as the piston moves through the distance x, and therefore

$$\text{Energy exerted per lb. of water} = \frac{p}{w} = \text{pressure head in cylinder.}$$

This might have been anticipated from what was said in the last chapter as to the meaning of the term "head," and in fact it is equally true if the driving pair be not a simple piston and cylinder, but of any other kind.

The head in the cylinder is less than that in the accumulator, on account of the friction in the supply pipe and other frictional resistances, and it is on the action of these resistances that the working of the machine depends. Let V be the velocity of the piston in its cylinder, p_0 pressure in accumulator, F the co-efficient of hydraulic resistance *referred to the velocity of the piston* (Art. 248), then, neglecting differences of level, also the heights due to velocities of working and accumulator pistons,

$$\frac{p_0 - p}{w} = F \,.\, \frac{V^2}{2g}.$$

If the machine be moving steadily the pressure p will be equal to the useful resistance which the piston is overcoming, increased by the friction of the piston in its cylinder. Thus p and p_0 will be known quantities, a certain definite velocity V_0 will then be determined, which may be described as the "speed of steady motion": it is given by the equation

$$V_0^2 = \frac{2g}{wF}(p_0 - p).$$

Since the hydraulic resistances may be increased to any extent at pleasure by the turning of a cock, it follows, that the speed of an hydraulic pressure machine can be regulated at pleasure. Further, if the resistance to the movement of the piston be diminished, the speed will increase only by a limited amount, and can, under no circumstances, be greater than is given by

$$\overline{V}_0^2 = p_0 \frac{2g}{wF},$$

which can be regulated as before. The surplus energy is here absorbed by the frictional resistances, and an hydraulic pressure machine therefore possesses the very important, and, for many purposes, valuable characteristic that *it contains within it its own brakes.*

256. *Hydraulic Pressure Machines in Unsteady Motion.*—Although the speed of a pressure engine cannot exceed a certain limit, which is easily found, yet it does not follow that that limit will ever be reached. When the engine starts, the piston and the water in the pipes have to be set in motion, the force required to do this is so much subtracted from that available to overcome resistances. A considerable time therefore elapses before a condition approaching steady motion can be obtained.

In Fig. 170, p. 460, water is supposed flowing through a pipe with a velocity u. Two pistons at a distance x enclose water between them, as in Art. 242, then the difference of pressure $p_1 - p_2$ in the case of steady motion is simply balanced by the surface friction, but in unsteady motion is partially employed in accelerating the flow of the water. Neglecting friction the acceleration g' will be given by the formula

$$(p_1 - p_2)\, A = W \,.\, \frac{g'}{g},$$

where A is the sectional area of the pipe and W is the weight of the water between the pistons. Replacing W by $Ax \cdot w$, as in the preceding article,

$$\frac{p_1 - p_2}{w} = x \cdot \frac{g'}{g},$$

which gives a simple formula for the change of pressure head due to inertia. Now if nA be the area of the working piston, the velocity of the water in the pipe is n times the velocity of the piston, and the accelerations are necessarily in the same ratio; and hence it follows that the difference of pressure head between cylinder and accumulator due to an acceleration g' of the piston is for a length of pipe l

$$\frac{p_1 - p_2}{w} = nl \cdot \frac{g'}{g}.$$

The inertia of the piston itself requires a certain pressure to accelerate it. Let q_0 be the "pressure equivalent to that weight" found, as in Art. 109, page 235, then the pressure due to inertia is

$$q = q_0 \cdot \frac{g'}{g};$$

then, dividing by w,

$$\frac{p_1 - p_2}{w} + \frac{q}{w} = \left(nl + \frac{q_0}{w} \right) \frac{g'}{g} = \lambda \frac{g'}{g},$$

where λ is a certain length. This may be described as the "length of working cylinder equivalent to the inertia of the moving parts," and may always be readily calculated for any given engine. (See Appendix.) The pressure in feet of water necessary to overcome inertia will then always be given by the simple formula

$$\text{Pressure due to inertia} = \lambda \frac{g'}{g}.$$

It will now be seen that the weight of water in the pipes and cylinders is so much added to the weight of the piston, that in the pipes being multiplied by the ratio of areas of cylinder and pipe. A water-pressure engine is therefore a machine with very heavy moving parts, a circumstance which greatly limits its speed irrespectively of frictional resistances. The smaller the pipes the heavier the parts virtually are, and this must be considered as well as friction (p. 461) in fixing their diameter.

It will be advisable to consider a particular case more in detail. Suppose, as is sometimes the case in practice, that a water-pressure engine is employed to turn a crank, and let us suppose that the crank shaft rotates nearly uniformly as in Ch. IX., then the difference between the pressure in the accumulator and that transmitted to the crank pin may be represented graphically thus:—

Let V be the velocity of the crank pin and let the stroke be $2a$ or AB in the diagram (Fig. 185). Set up

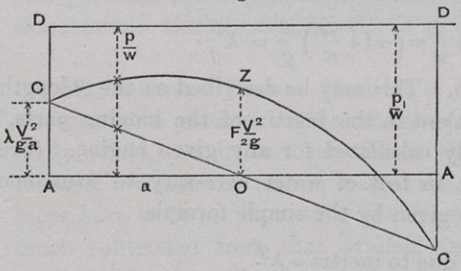
$$CA = \lambda \cdot \frac{V^2}{ga},$$

and draw the sloping line COC. Then, as in Art. 109, already cited, the ordinate of that line represents the pressure necessary to overcome the inertia of the piston and the water connected with it. Again, set up

$$OZ = F\frac{V^2}{2g} = CA \cdot \frac{Fa}{2\lambda},$$

and on the oblique base COC draw the parabola CZC, then (comp. Arts. 20, 109) the ordinate of this parabola will represent the pressure necessary to overcome the hydraulic resistances at every point. If then the horizontal line DD be drawn at a height representing the pressure in the accumulator, the intercept between that line and the parabola will represent the pressure transmitted to the crank pin at each point of the stroke. The slope of CC and the height of the parabola increase rapidly with the speed, which must never be great enough to cause the parabola to touch DD, otherwise a violent shock will occur. The same effect will be produced by any falling off in the useful resistance: the angular acceleration of the crank shaft then raises the central part of the line CC and with it the line of frictional resistances. It should be observed that the curve of frictional resistances may also be taken to represent the kinetic energy of the piston, both these quantities being proportional to the square of the velocity of the piston. It is therefore the graphical integral of the curve of acceleration (Ch. IX.).

Fig.185.



The simple example here given will serve as an illustration of the great variations of pressure which occur in water-pressure engines and their consequent liability to shocks. For which reason escape valves or air chambers must be provided to relieve the pressure when it becomes excessive. Unless the resistance be very uniform an additional accumulator is required as near as possible to the machine.

257. *Examples of Hydraulic Pressure Machines.*—Water-pressure engines form a large and interesting class of hydraulic motors of which a few examples will now be given.

(1) In direct-acting lifts a weight is raised by the direct action of fluid pressure on a ram the stroke of which is equal to the height lifted. The weight here rests on a cage or platform fixed to the upper end of the ram and sliding in guides. The water is frequently supplied from a tank at a moderate elevation, so that the pressure head diminishes as the lift rises, This is a very convenient arrangement for the purpose, as it supplies an additional pressure at the bottom of the stroke where it is required to overcome inertia at starting, and a diminished pressure at the top where the lift requires to be stopped. The useful resistance is here constant and the pressure head would be represented by the ordinates of a sloping line. A diagram of speed and acceleration may be constructed by a process similar to that given in the last article.

(2) A direct-acting lift necessarily occupies a great space, and the stroke of the working cylinder is therefore often multiplied by the use of blocks and tackle as shown in Fig. 2, Plate IX. The cylinder may be placed in any convenient position, and the chain passes from the blocks over fixed pullies to the cage which is suspended from it. The friction of the pullies is here considerable, and there is a liability to breakage; but for convenience the arrangement is one which is frequently employed.

(3) In hydraulic cranes the working cylinder is sometimes placed below and sometimes occupies the crane post which is tubular. The stroke is multiplied by tackle as in the previous case, the chain passing through the crane post and over fixed pullies to the extremity of the jib. An example is shown in Fig. 2, Plate IX., p. 497.

(4) A water-pressure engine may be employed to turn a crank. Three working cylinders inclined at 120° are frequently used as shown in Fig. 1, Pl. X., p. 497. They are single-acting and drive the

same crank as in the small steam engines of the same type employed where great speed is required. The water is admitted to the outer ends of the cylinders, so that the piston rods are always in compression.

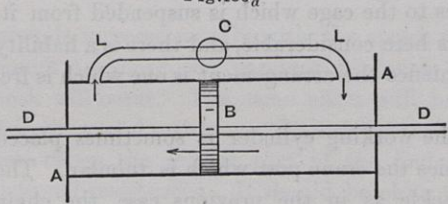
(5) The hydraulic mechanism applied to work heavy guns on board ship consists of a cylinder in which works a piston attached to a rod, the sectional area of which is one-half that of the cylinder. If water be admitted at both ends of the cylinder the piston moves outwards, but if to the inner end only, it moves inwards. The motive force in either case is the same, being due to the difference of areas. This apparatus serves also as a brake of the kind described in the next article. For details and illustrations the reader is referred to the *Gunnery Manual.*

258. *Hydraulic Brakes.*—It has been sufficiently explained that hydraulic resistances absorb an amount of energy which varies as the square of the speed. A hydraulic machine therefore may be employed as a brake, and it is in this way that large amounts of surplus energy are most easily disposed of. Moreover, by its use the speed of any machine to which it is applied is readily controlled.

An hydraulic brake is constructed by interposing a mass of fluid between the elements of a pair so that any motion of the pair causes a breaking-up of the fluid with a corresponding resistance.

A common case is that of a sliding pair consisting of a piston and cylinder filled with water or oil, which passes from one side of the piston to the other whenever the piston moves. Two examples of this apparatus are shown in skeleton in Figs. 186*a*, 186*b*. In the first (Fig. 186*a*) the piston rod *DD* projects through both cylinder covers, and communication is made between the two ends of the cylinder by a pipe *LL* provided with a cock *C*, which can be closed at pleasure. At *D* the rod is attached to the piston rod of a steam cylinder employed to obtain the very considerable force necessary to work the starting and reversing gear of large marine engines. The resistance of this brake is zero when the piston begins to move, but

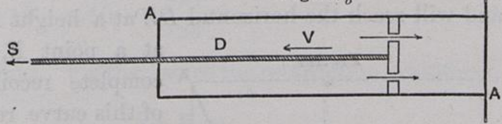
Fig. 186*a*



increases as the square of the speed, and thus effectually prevents it from moving too rapidly. The maximum speed is controlled by turning the cock. For a detailed description of this gear the reader is referred to a treatise on the *Marine Engine*, by Mr. Sennett.

In the second (Fig. 186*b*) the water passes from one end of the cylinder through orifices in the piston itself. This is the common "compresser" or Service Buffer.* The piston rod in this case passes out at one end only of the working cylinder, and is attached to the gun, the recoil of which is to be checked. The theory of this apparatus is of some interest, and will now be briefly considered.

Fig.186$_b$



Let n be the ratio of the area of the piston to the *effective* area of the orifices, then the loss of head must be

$$\frac{p_1 - p_2}{w} = (n-1)^2 \frac{V^2}{2g},$$

where V is the speed of piston and p_1, p_2 are the pressures on the two sides of the piston. Hence the pull

$$S = wA(n-1)^2 \frac{V^2}{2g}$$

on the piston rod is necessary to overcome the hydraulic resistance at this speed. The gun is gradually brought to rest by this resistance, aided by the friction of the slide.

At the instant of firing a certain amount of kinetic energy is generated in the gun, given by the formula

$$\text{Energy of Recoil} = k \cdot \frac{WV_0^2}{2g}, \quad \text{(Art. 135, p. 279)}$$

where V_0 is the initial velocity of recoil, and k is a co-efficient (about 1·2) introduced on account of the inertia of the powder gases. As the gun recoils its velocity diminishes, and if P_0 be the friction of the slide the retarding force will be

$$S + P_0 = wA(n-1)^2 \frac{V^2}{2g} + P_0.$$

The maximum value of S will be found by writing V_0 for V, and may be denoted by S_0.

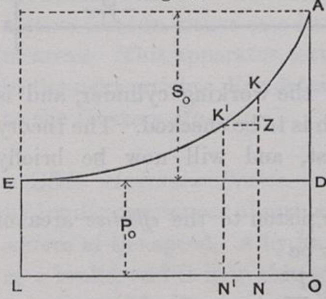
* *Manual of Gunnery for Her Majesty's Fleet*, p. 68.

To represent this graphically, in Fig. 187 draw a curve in which the ordinate KN at any point N represents the retarding force after the gun has recoiled through the space ON from the point O, at which the action of the powder pressure ceases, and the gun has its maximum velocity V_0. This curve will start from a point A such that

$$OA = S_0 + P_0,$$

and will reach the horizontal DE at a height P_0 above the base line at a point E, such that OL is the complete recoil. The area $OAEL$ of this curve represents the energy of recoil which has all been absorbed by the frictional resistance of the slide and the hydraulic resistance of the compressor. Further, the area $KNN'K'$ between two ordinates will represent the diminution of energy as the gun recoils through the space NN' between them, a circumstance which enables us to construct the curve, for if VV' be the velocities of the recoiling gun at NN' respectively,

Fig.187.



$$\text{Area } KNN'K' = k.\frac{W(V^2 - V'^2)}{2g}.$$

But if SS' be the corresponding values of S,

$$KZ = S - S' = wA(n-1)^2\frac{V^2 - V'^2}{2g};$$

and if the ordinates be taken near together the area in question will be nearly $KN \,.\, NN'$. We have therefore, by division,

$$\frac{KZ}{KN} = NN' \,.\, \frac{wA(n-1)^2}{kW}.$$

That is, if a number of equidistant ordinates be drawn near together the ratio of consecutive ordinates is constant. The curve may be roughly traced from this property; it is identical with the curve already drawn in Art. 123, p. 262, except that it is a linear instead of a polar curve.

The mean resistance to recoil is given by the equation

$$(\bar{S} + P_0)l = \text{Energy of Recoil},$$

where l is the distance traversed. It would, of course, be advantageous to have a uniform resistance to recoil, because the maximum pressure in the compressor would be diminished and less strain thrown on the gear. This is the object of the various modified forms of the compressor, in which the orifices are not of constant area, but become smaller as the recoil proceeds. In order that the resistance may be constant we must have

$$\bar{S} = wA(n-1)^2 \frac{V^2}{2g},$$

so that $(n-1)V$ is constant. Further, since the retardation is uniform,

$$V^2 = 2g \cdot \frac{\bar{S} + P_0}{W} x,$$

where x is the distance from the end of the recoil. It appears therefore that the orifices should vary in such a way that $(n-1)^2 x$ should be constant. Descriptions of two forms of compressor, with varying orifices, will be found in the *Gunnery Manual.*

Instead of a sliding pair we may employ a turning pair. This is the common "fan" or "fly" brake used to control the speed and absorb the surplus energy of the striking movement of a clock, or in other similar cases. A friction dynamometer (p. 290) was designed by the late Mr. Froude for the purpose of measuring the power of large marine engines, in which the ordinary block or strap surrounding a shaft or drum is replaced by a casing in which a wheel works. Vanes attached to the wheel and the fixed casing thoroughly break up a stream of water passing through the casing. Any amount of energy may thus be absorbed without occasioning any considerable rise of temperature. Siemens' combined brake and regulator has been mentioned already (p. 288).

259. *Transmission of Energy by Hydraulic Pressure.*—Energy may be distributed from a central source, and transmitted to considerable distances with economy by hydraulic pressure. The delivery in gallons per minute of a pipe d'' diameter is

$$G = 27 \sqrt{\frac{h'}{l}} \cdot d^{\frac{5}{2}} \qquad \text{(Art. 243.)}$$

Assume now that the pipe supplies an hydraulic machine at a distance of l feet from an accumulator in which h is the head. Further, suppose that n per cent. is lost by friction of the pipe, then the power transmitted in foot-lbs. per minute is

$$10Gh = 270\ h\sqrt{\frac{nh}{100l}} \cdot d^{\frac{5}{2}},$$

and the distance to which N horse power can be transmitted with a loss of n per cent. is in feet

$$l = \frac{h^3 d^5 n}{1{,}500{,}000 N^2} \qquad \text{(nearly)}.$$

With the usual pressure in accumulators of 750 lbs. per square inch, or 1700 feet of water, this gives the simple approximate formula

$$l = 3300\,\frac{d^5 n}{N^2}.$$

Thus for example, 100 horse power may be transmitted by a 5″ pipe to a distance of 4 miles, or 10 horse power by a 1′ pipe to a distance of 220 yards, with a loss by friction not exceeding 20 per cent. The diameter of pipe is limited by considerations of strength and cost.

The power of a motor supplied by a given pipe does not increase indefinitely as its speed increases, but is greatest when one-third of the head is lost by friction.* The maximum possible power is therefore given by the formula

$$H.P. = 220\sqrt{\frac{d^5}{l}} \qquad \text{(approximately)}.$$

This is of course two-thirds the value of N in the preceding formula.

260. *Pumps.*—If the direction of motion of an hydraulic motor be reversed by the action of sufficient external force applied to drive it, while, at the same time, the direction of the issuing water is reversed so as to supply the machine at the point from which it originally proceeded, we obtain a machine which raises water instead of utilizing a head of water. Every hydraulic machine therefore may be employed to raise water as well as to do work, and most of them actually occur in this form; they are then called PUMPS, though in some cases this name would not be used in practice. Much of what has been said about motors applies equally well to pumps: the principal difference lies in the fact that the useful resistance which the pump overcomes is always reversible, whereas in the motor this is

* This result was pointed out to the writer by Mr. Hearson. It appears to be little known.

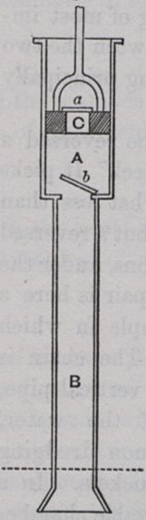
not necessarily the case. The principles of action and the classification of hydraulic machines are, in the main, the same in both cases. Some points omitted while considering motors as being of most importance in pumps, and certain differences of action between the two will now be briefly noticed. Certain machines occurring principally as pumps will be mentioned.

(1) If the direction of motion of an overshot wheel be reversed a machine is obtained which is known as a "Chinese Wheel." It picks up water in its buckets and raises it to a height somewhat less than the diameter of the wheel. This machine is little used, but a reversed breast wheel is frequently employed in drainage operations, under the name of a "scoop," or "flash" wheel. The working pair is here a turning pair, but in the chain pump we find an example in which one of its elements is a chain passing over pulleys. The chain is endless and is provided with flat plates fitting into a vertical pipe, the lower end of which is below the surface of the water, and through which the water is raised. In the common dredging machine the closed channel (p. 484) is replaced by buckets. In a third class of weight machines the water occupies a movable chamber and forms with it a kinematic pair with only one solid element, while it forms, with the link attached to the earth, a working pair which has also but one solid element. The Archimedian screw, and certain varieties of "scoop" wheel, in which the water enters the scoop at the circumference of the wheel and is delivered at the centre, are examples of this kind.

(2) The most common forms of pumps are the "lift" or "force" pumps, which consist of a chamber which expands to admit the water to be lifted and contracts in the act of lifting; they are therefore pressure machines like those considered in Art. 255-6, but reversed. The name "pump" originally applied to these machines alone.

Fig. 188 shows a common lift pump. A is a cylinder at a certain height h_1 above the water to be raised, B is a piston working in the cylinder by the action of which the water is lifted. The piston has orifices in it which permit the water to pass through. The orifices are closed by a valve, as is also the opening at the bottom of the cylinder. These valves are simple "flaps" which open on hinges to permit the water to pass upwards, but close the passage to motion in the opposite direction, thus acting as a ratchet (p. 171). Assuming the piston at the bottom of its stroke, at rest close to the bottom of the cylinder, let it be supposed to rise; the valve a will rise and allow air to pass if any. After several strokes the air will be nearly exhausted, and if h_1 be not too great the empty space will be filled with water raised from the tank by atmospheric pressure. Thus the water will pass into the cylinder closely following the piston. At the top of the stroke the piston commences to descend, a closes and b opens, allowing the water to pass above the piston. This water is now raised by the piston to any required height. In force pumps the process is the same, but the water passes out through an orifice in the bottom of the cylinder instead of through the piston; the raising of the water above the level of the cylinder is done in the down stroke instead of the up.

Fig. 188.



The difference between this action and that of a pressure motor lies mainly in the valves, which here open and close automatically by the action of the water, instead of by external agency. Further, the pump wholly or partly works by *suction*, a method by no means peculiar to pumps, for it also occurs in motors, but not so frequently. The height of the water barometer is 34 feet, but the height to which a pump will work by suction is not so great. When the piston is at the bottom of its stroke there must, for safety, always be a certain clearance space below. This space always contains air, the pressure of which diminishes as the piston rises, but cannot be reduced to zero. Further, a certain pressure is required to overcome the weight and friction of the valve before it opens. At least 3 feet of the lift is absorbed in this way, and generally considerably more. To obtain a high vacuum for scientific purposes, air pumps are specially designed to meet these difficulties. Also, leakage must be allowed for and the diminution on account of friction and inertia, which will be considerable if the speed be too great or the pipes too small, as will be understood on reference to Arts. 255-6, all of which applies to pumps as much as to motors. It is hardly necessary to observe that power is neither gained nor lost by the use of suction; it simply enables the working cylinder to be placed above the water to be lifted, an arrangement which is in most cases convenient. The limit in practice is about 25 feet.

PLATE IX.

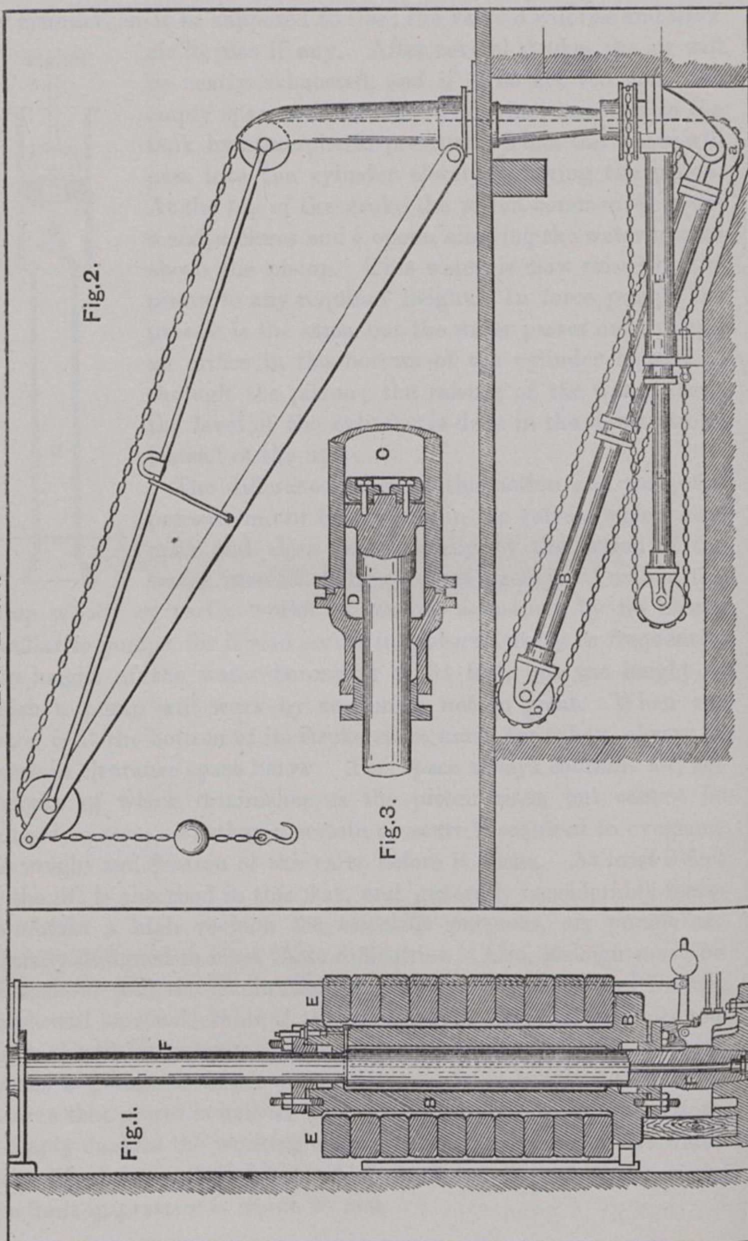


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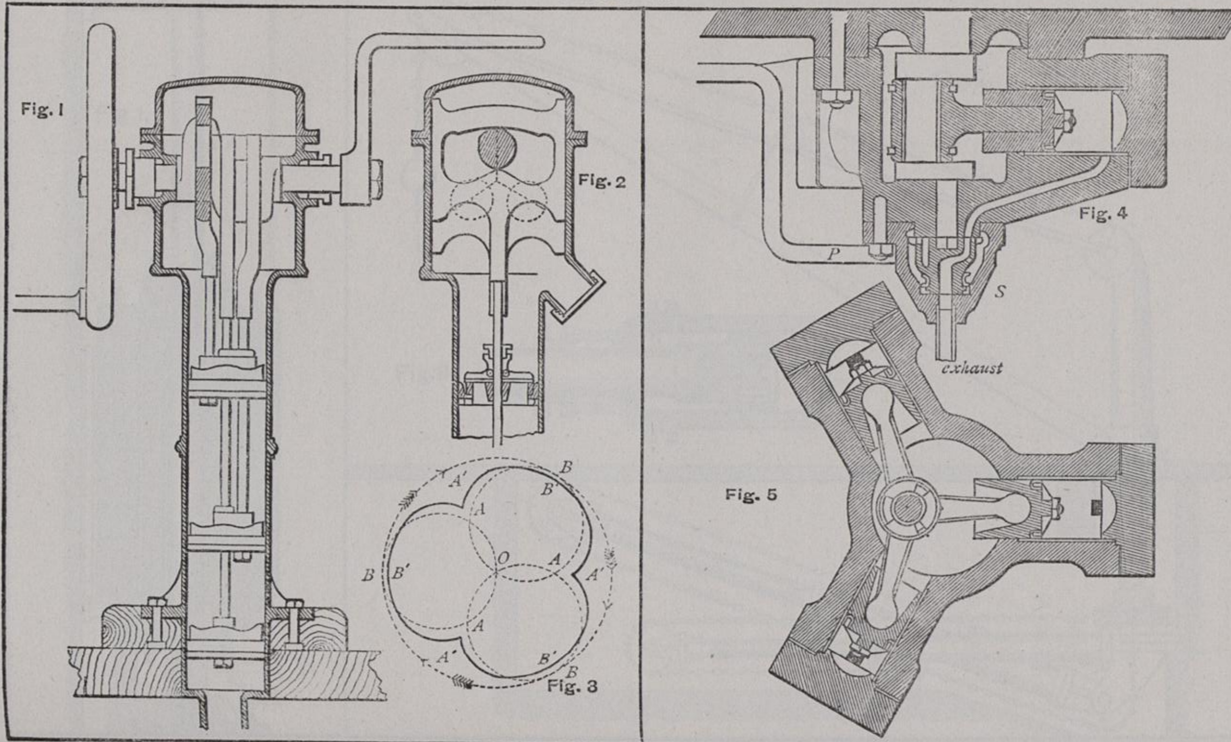
PLATE X.



To face page 497.

Pumps are commonly, but not always, single-acting; they are worked by the direct action of a reciprocating piece, or by means of a rotating crank. In the first case, when independent, a piston acted on by steam or water pressure is attached to a prolongation of the pump plunger: a crank and fly-wheel is often added, as in Fig. 4, Plate II., p. 121, to control the motion and define the stroke. When driven by the crank three working cylinders, placed side by side with a three-throw crank, are commonly used, in order to equalize the delivery, and so to avoid the shocks due to changes of velocity. An air-chamber, forming a species of accumulator, may also be used with the same object. An arrangement of pumps, as applied by Messrs. Donkin & Co. to raise water from a well 200 feet deep and force it to a height of 143 feet above the engine-house, may be mentioned as an example. A set of lift pumps at the bottom of the well worked by "spear" rods from the surface, are combined with a set of force pumps in the engine house itself. The speed of these pumps is about 80 feet per minute, and they deliver about 600 gallons per minute. Pumps almost always have a certain "slip," that is, they deliver less water than corresponds to the piston displacement and number of strokes: in this example the slip was 12 per cent. The efficiency of the pumps and mechanism of the engine was found to be 66 per cent. by careful experiments.*

In raising water from great depths in mines, force pumps at the bottom of the mine are used, worked by heavy "spear" rods from a beam engine at the surface. The weight of the rod supplies the motive force during the downward stroke of the pump; while the engine, which is single-acting, raises the rods again during the downward stroke of the steam piston.

DESCRIPTION OF PLATES IX. AND X.

In order further to illustrate the action of water-pressure machines Plates IX. and X. have been drawn.

Fig. 1, Plate IX., shows the differential accumulator described on page 494.

In Fig. 2 is represented a hydraulic crane, designed by Sir W. Armstrong, for lifting weights of 2 to 3 tons. In it the hydraulic power is applied to rotate the crane as well as to lift the weight.

In order to effect the lift the high-pressure water from the accumulator is admitted to the cylinder *A*, and forces out the plunger *B*. There are two pulleys at *a* and two at *b*. One end of the chain is secured to the cylinder *A*, it is led round *b*, then round

* *Minutes of Proceedings of the Institution of Civil Engineers*, vol. 66.

a, again round *b*, then under the second pulley at *a* up through the hollow crane post on to the weight as shown. The effect of this arrangement is that any movement of the plunger *B* is at the hook multiplied four times.

If *B* is simply a plunger working in a stuffing box, then the expenditure of energy is always the same whatever weight is being lifted, and the amount must be equal to that which corresponds to lifting the maximum possible weight.

This is an objection which is common to all such machines. The surplus energy is expended in overcoming frictional resistances (p. 486). To mitigate this evil, in cranes of high power the plunger has a piston end, which fits a bored cylinder, and is provided with a cup leather, as shown in Fig. 3. The sectional area of the plunger is about one-half that of the cylinder. If a light weight is to be lifted, water is admitted to both sides of the piston, and the difference of the pressures, equal to what would be exerted on a simple plunger, is available for effecting the lift. When it is required to lift a heavy weight water is admitted to the side *C* only of the piston, the annular space *D* being put in communication with the atmosphere. Thus the full pressure due to the area of the piston is exerted with the corresponding expenditure of water.

For the purpose of rotating the crane a pair of cylinders, *E*, are provided, of which one only is shown in the figure. The thrusting out of the plunger *F* of one of them by the pressure of the water causes the other to be drawn in by means of a chain which passes around a recessed pulley secured to the crane post.

In Plate X., Figs 1 and 2 show the construction of Downton's Pump, so much used on board ship. In the barrel work three buckets with flap valves, as shown in Fig. 2. The rods to which the upper and second buckets are attached are necessarily out of centre. The rods to the lower buckets pass through deep stuffing boxes in the buckets above, and thus the buckets are maintained from canting seriously. The movement of the buckets is effected by a three-throw crank, the crank pins, which are not round, being set at 120° apart. These pins fit and work in a curved slot in the bucket rod heads. Assuming the admission of no air but water only from below, the discharge of the pump will at each instant equal the displacement of the fastest upward moving bucket. Accordingly the rate of discharge may be represented by a curve, as in Fig. 3. If the slot in the rod head were straight and the pin round, then, the crank moving uniformly, in direction shown, the velocity of discharge would be represented by the radii from *O* to the dotted curve *BABABA*, which is made up of parts of three circles, the position of the radius being that of either of the three cranks. The effect of the curved slot is to diminish the maximum and increase the minimum discharge, as shown by the full curve *B'A'B'A'B'A'*.

Figs. 4 and 5 of this Plate are sections of the hydraulic engine referred to on page 489, employed to rotate a capstan. It need only be further added that a single rotating valve *V* suffices for admission and exhaust of all three cylinders. The high-pressure water is supplied by the pipe *P* to the passage *S* surrounding the valve and exhausted from the cylinders through the central passage.

EXAMPLES.

1. In estimating the power of a fall of water it is sometimes assumed that 12 cubic feet per second will give 1 H.P. for each foot of fall: what efficiency does this suppose in the motor? *Ans.* ·72.

2. An accumulator ram is 9 inches diameter, and 21 feet stroke: find the store of energy in foot-lbs. when the ram is at the top of its stroke, and is loaded till the pressure is 750 lbs. per square inch? *Ans.* 958,000 foot-lbs.

3. In a differential accumulator the diameters of the spindle are 7 inches and 5 inches; the stroke is 10 feet: find the store of energy when full, and loaded to 2,000 lbs. per square inch. *Ans.* 377,000 foot-lbs.

4. A direct-acting lift has a ram 9 inches diameter, and works under a *constant* head of 73 feet, of which 13 per cent. is required by ram friction and friction of mechanism. The supply pipe is 100 feet long and 4 inches diameter. Find the speed of steady motion when raising a load of 1,350 lbs., and also the load it would raise at double that speed?

Ans. Speed = 2 feet per second.
Load = 150 lbs.

5. In the last question, if a valve in the supply pipe is partially closed so as to increase the co-efficient of resistance by $5\frac{1}{2}$, what would the speed be?

6. Eight cwt. of ore is to be raised from a mine at the rate of 900 feet per minute by a water-pressure engine, which has four single-acting cylinders, 6 inches diameter, 18 inches stroke, making 60 revolutions per minute. Find the diameter of a supply pipe 230 feet long, for a head 230 feet, not including friction of mechanism. *Ans.* Diameter = 4 inches.

7. Water is flowing through a pipe 20 feet long with a velocity of 10 feet per second. If the flow be stopped in one-tenth of a second, find the intensity of the pressure produced, assuming the retardation during stoppage uniform. *Ans.* 62 feet of water.

8. If λ be the length equivalent to the inertia of a water-pressure engine, F the co-efficient of hydraulic resistance, both reduced to the ram, v_0 the speed of steady motion: find the velocity of ram, after moving from rest through a space x against a constant useful resistance. Also find the time occupied.

$$Ans.\ v^2 = v_0^2\left(1 - \epsilon^{-\frac{Fx}{\lambda}}\right); \quad t = \frac{\lambda}{F \, . \, v_0} \log_\epsilon \frac{v_0 + v}{v_0 - v}.$$

9. An hydraulic motor is driven from an accumulator, the pressure in which is 750 lbs. per square inch, by means of a supply pipe 900 feet long, 4 inches diameter; what would be the maximum power theoretically attainable, and what would be the velocity in the pipe at that power? Find approximately the efficiency of transmission at half power. *Ans.* H.P. = 240; $v = 22$; efficiency = ·96 nearly.

10. A gun recoils with a maximum velocity of 10 feet per second. The area of the orifices in the compressor, after allowing for contraction, may be taken as one-twentieth the area of the piston: find the initial pressure in the compressor in feet of liquid. *Ans.* 621.

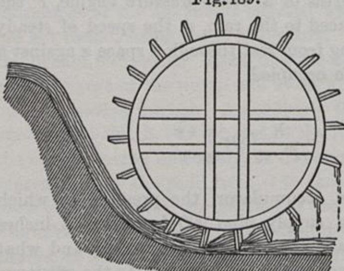
11. In the last question assume weight of gun 12 tons; friction of slide 3 tons; diameter of compressor 6 inches; fluid in compressor water: find the recoil. *Ans.* 4 feet $2\frac{1}{2}$ inches.

12. In the last question find the mean resistance to recoil. Compare the maximum and mean resistances each exclusive of friction of slide. *Ans.* Total mean resistance = 4·4 tons. Ratio = 2·5.

261. *Impulse and Reaction Machines in General.*—The source of energy may be a current of water or the head may be too small to obtain any considerable pressure, and it is then necessary to have some means of utilizing the energy of water in its kinetic form. A machine for this purpose operates by changing the motion of the water and utilizing the force to which the change gives rise. If the water strikes a moving piece and is reduced to rest relatively to it, the machine works by "impulse," and if it be discharged from a moving piece, by "reaction." There is no difference in principle between these modes of working, and both may occur in the same machine. In either case the motive force arises from the mutual action between the water and the piece which changes their relative motion. Machines of this class are also employed for high falls when the low speed of pressure machines renders their use inconvenient or impossible. The water is then allowed to attain a velocity equivalent to a considerable portion of the head immediately before entering the machine, so that its energy is, in the first instance, wholly or partially converted into the kinetic form.

The simplest machine of this kind is the common undershot wheel, consisting of a wheel (Fig. 189) provided with vanes against which the water impinges directly. Let the velocity of periphery of the wheel be V, then the water after striking the vanes is carried along with them at this velocity. If, then, the original velocity of the water be v, the diminution of velocity due to the action of the vanes will be $v - V$. Let W be the weight of water acted on per second, then the impulse on the wheel must be

Fig. 189.



$$P = \frac{W(v - V)}{g},$$

but if A be the sectional area of the stream,

$$W = Avw,$$

this being the weight of water per second which comes in contact with all the vanes taken together.

$$\therefore\ P = \frac{w}{g} A v\ (v - V).$$

The power of the wheel is PV foot-lbs. per second, and the energy of the stream is $Wv^2/2g$, therefore

$$\text{Efficiency} = \frac{2V\,(v - V)}{v^2}.$$

This is greatest when $V = \frac{1}{2}v$ and its value is then ·5, showing that the wheel works to best advantage when the speed of periphery is one-half that of the stream, but that the efficiency is low, never exceeding ·5.

Such wheels may be seen working a mill floating in a large river, or in other similar circumstances, but they are cumbrous and, allowing for various losses, not included in the preceding investigation; their efficiency is not more than 30 per cent. In the early days of hydraulic machines, they were often used for the sake of simplicity or, as in the example shown in the figure, from a want of comprehension of their principle.* In mountain countries, where unlimited power is available, they are still found. The water is then conducted by an artificial channel to the wheel, which sometimes revolves in a horizontal plane. When of small diameter their efficiency is still further diminished.

In overshot wheels and other machines operating chiefly by weight the head corresponding to the velocity of delivery is partly utilized by impulse, and the speed of the wheel is determined by this consideration. In all cases of direct impulse, if h is that part of the head operating by impulse, the speed of maximum efficiency is

$$V = \tfrac{1}{2}\sqrt{2gh} = 4\sqrt{h},$$

or in practice somewhat less, and at that speed at least half that head is wasted.

The great waste of energy in this process is due partly to the velocity V with which the water moves onward with the wheel, and partly to breaking-up during impulse. It is in fact easy to see that

* See Fairbairn's *Millwork and Machinery*, from which this figure is taken, vol. I., p. 149.

one-fourth the head is wasted by each of these causes. To avoid it, the water must be received by the moving piece against which it impinges without any sudden change of direction, and must be discharged at the lowest possible velocity, effects which may be produced by a suitably-shaped vane curved so as to deflect the water gradually and guide it in a proper direction. The principle on which such a vane is designed may be explained by the annexed diagram. In Fig. 190, AB is a vane, moving with velocity V in a given direction, against which a jet strikes. Drawing a diagram of velocities, let Oa represent v, the velocity of the jet, and let OO' represent V. Then as before (p. 477) $O'a$ represents the velocity of the jet relatively to the vane, and, in order that the water may impinge without shock, the tangent to the vane at A must be parallel to $O'a$. The vane is now curved so as gradually to deflect the water, in doing which there is a mutual action between the jet and the vane which produces the motive force which drives the wheel. If the water leave the vane at B, its velocity relatively to the vane is represented by $O'b$ drawn parallel to the vane at B, and somewhat less than $O'a$ in magnitude, to allow for friction, unless the water be enclosed in a passage, when it will bear some given proportion to $O'a$.

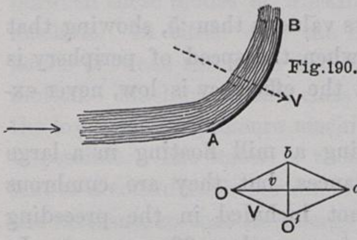


Fig.190.

The absolute velocity with which the water moves at B is now represented by Ob, and this may be arranged to deliver the water in a convenient direction with a velocity just sufficient to clear the wheel and no more. The efficiency may then theoretically be unity, and, practically, after allowing for losses, may be increased to ·65 or ·7. Vanes of this kind were applied to water wheels by Poncelet. The wheel, in this case, revolves in a vertical plane, and the water, on impinging at A, ascends to B: it then descends under the action of gravity, and is discharged at the same point A at which it entered, so that $O'b$ is approximately equal and opposite to $O'a$.

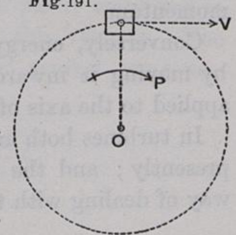
In all impulse and reaction machines there is a speed of maximum efficiency which, as in the simple case first considered, is given by the formula

$$V = k\sqrt{2gh},$$

where k is a fraction depending on the type of machine.

262. *Angular Impulse and Momentum.*—The most important of these machines are those in which the change of motion produced in the water is a motion of rotation, and it is needful to consider that form of the principle of momentum which is applicable to such cases.

Fig. 191.



In Fig. 191, W is a weight describing a circle round O with velocity V; then the product of its momentum by the radius r is called the "moment of momentum" of the weight about O. If O represent an axis to which W is attached rigidly, we may imagine it turning under the action of a force P at a radius R. The moment of P multiplied by the time during which it acts is called the "moment of impulse."

During the action of P the weight will move quicker and quicker, and the motion is governed by the principle expressed by the equation

Moment of Impulse = Change of Moment of Momentum.

If L be the moment of P, then, taking the time as one second,

$$L = \text{Change of Moment of Momentum per second.}$$

This equation is true, not only for a single weight and a single force, but also for any number of weights and any number of forces. As in other forms of the principle of momentum it is also true, notwithstanding any mutual actions or any relative movements of the weights or particles considered. Further, any radial motions which the particles possess may be left out of account, for they do not influence the moment of momentum. A particular case is when $L = 0$, then the moment of momentum remains constant, a principle known as the Conservation of Moment of Momentum. The terms "moment of momentum" and "moment of impulse" are often replaced by "angular momentum," "angular impulse."

A weight rotating about an axis is capable of exerting energy in two ways. First, it may move away from the axis of rotation, overcoming by its centrifugal force a radial resistance which it just overbalances. Secondly, it may overcome a resistance to rotation in the shaft to which it is attached. In either case the work done will be represented by a diminution in the kinetic energy of the weight.

If the shaft be free, the diminution of kinetic energy must be equal to the work done by the centrifugal force, and it may be proved in this way, that if V be the velocity of rotation of the weight, r the radius,

$$Vr = \text{constant},$$

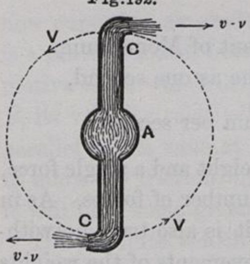
an equation equivalent to the conservation of the momentum of momentum.

Conversely, energy may be applied to a rotating weight either by moving it inwards against its centrifugal force, or by a couple applied to the axis of rotation.

In turbines both modes of action occur together as we shall see presently; and the principle of momentum is the most convenient way of dealing with the question.

263. *Reaction Wheels.*—Fig. 192 shows a reaction wheel in its simplest form. CAC is a horizontal tube communicating with a vertical tubular axis to which it is fixed, and with which it rotates. Water descends through the vertical tube, and issues through orifices at the extremities of the horizontal tube so placed that the direction of motion of the water is tangential to the circle described by the orifices. The efflux is in opposite directions from the two orifices, and a reaction is produced in each arm which furnishes a motive force. There are two methods of investigating the action of this machine which are both instructive. Frictional resistances are, in the first instance, neglected.

Fig. 192.



(1) Let the orifices be closed, and let the machine revolve so that the speed of the orifices in their circular path of radius r is V. Centrifugal action produces a pressure in excess of the head h existing when the arms are at rest, the magnitude of which is $V^2/2g$, and this pressure is so much addition to the head, which now becomes

$$H = h + \frac{V^2}{2g}.$$

This quantity H may also be considered as the head "relative to the moving orifices" estimated as in Art. 239

When the orifices are opened, the water issues with velocity v given by

$$v^2 = 2gH = V^2 + 2gh;$$

thus the water issues with a velocity greater than V, and after leaving the machine has the velocity $v - V$ relatively to the earth. The energy exerted per lb. of water is h, and this is partly employed in generating the kinetic energy corresponding to this velocity. The remainder does useful work by turning the wheel against some useful resistance, so that we have per lb. of water

$$\text{Useful Work} = h - \frac{(v-V)^2}{2g} = \frac{V(v-V)}{g},$$

and, dividing by h,

$$\text{Efficiency} = \frac{V(v-V)}{gh} = \frac{2V}{v+V}.$$

(2) A second method is to employ the principle of the equality of angular impulse and angular momentum already given in Art. 262. Originally the water descends the vertical tube without possessing any rotatory motion, but after leaving the machine it has the velocity $v - V$; its angular momentum is therefore for each lb. of water

$$\text{Angular Momentum} = \frac{(v-V)}{g} \,.\, r.$$

Now according to the principle the angular momentum generated per second is also the angular reaction on the wheel which, when multiplied by V/r, the angular velocity of the wheel, gives us the useful work done per second. Performing this operation, and dividing by the weight of water used per second, we get per lb. of water

$$\text{Useful Work} = \frac{V(v-V)}{g}.$$

This is the result already obtained, and the solution may now be completed by adding the kinetic energy on exit.

From the result it appears that the proportion which the waste work bears to the useful work is $v - V : 2V$, which diminishes indefinitely as v approaches V; but in this case the velocities become very great, since $v^2 - V^2$ is always equal to $2gh$. The frictional resistances then become very great, so that in the actual machine there is always a speed of maximum efficiency which may be investigated as follows :—

Let F be the co-efficient of hydraulic resistances referred to the orifices, then

$$(1+F)\frac{v^2}{2g}=H=h+\frac{V^2}{2g}.$$

The useful work remains as before, and therefore

$$\text{Efficiency}=\frac{2V(v-V)}{v^2-V^2+F.v^2},$$

a fraction which can readily be shown to be a maximum when

$$v-V=V\sqrt{\frac{F}{1+F}},\text{ or } v=V\left\{1+\sqrt{\frac{F}{1+F}}\right\},$$

which value of v, when substituted in the preceding equation, will give the value of V in terms of h for maximum efficiency. The existence of a speed of maximum efficiency is well known by experience with these machines. In general it is found to be about that due to the head, so that

$$V^2=2gh,$$

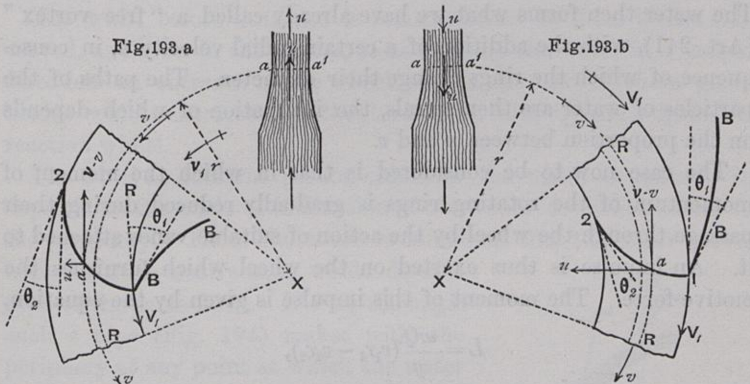
a value which corresponds to $F=\cdot125$, and gives an efficiency of ·67. This is about the actual efficiency of these machines under favourable circumstances; of the whole waste of energy two-thirds, that is two-ninths of the whole head, is spent in overcoming frictional resistances, and the remaining one-third, or one-ninth the whole head, in the kinetic energy of delivery.

The reaction wheel in its crudest form is a very old machine known as "Barker's Mill." It has been employed to some extent in practice as an hydraulic motor, the water being admitted below and the arms curved in the form of a spiral. These modifications do not in any way affect the principle of the machine, but the frictional resistances may probably be diminished.

264. *Turbine Motors.*—A reaction wheel is defective in principle, because the water after delivery has a rotatory velocity in consequence of which we have seen a large part of the head is wasted. To avoid this, it is necessary to employ a machine in which some rotatory velocity is given to the water before entrance in order that it may be possible to discharge it with no velocity except

that which is absolutely required to pass it through the machine. Such a machine is called in general a TURBINE, and it is described as "outward flow," "inward flow," or "parallel flow," according as the water during its passage through the machine diverges from, converges to, or moves parallel to, the axis of rotation.

Fig. 193*a* shows in plan and section part of an annular casing forming a wheel revolving about an axis XX through which water is



flowing, entering at the centre and spreading outwards. The water leaves the wheel at the outer circumference. Fig. 193*b* is similar, but the flow is inward instead of outward.

If we consider a section aa made by a concentric cylinder of length y and radius r, the flow will be

$$Q = u \, . \, 2\pi r y,$$

where u is the radial velocity or, as we may call it, the "velocity of flow." The area of the section $(2\pi r y)$ may conveniently be called the "area of flow." The value of Q is everywhere the same, and therefore ury must be constant. It is generally desirable to make u constant or nearly so, and then the form of the casing is such that ry is constant. Whether this be so or not, the value of u can always be calculated at any radius for a given wheel with a given delivery.

The water which at any given instant is at a given distance r from the axis may be considered as forming a ring RR, which rotates while at the same time it expands or contracts according as the flow is outward or inward. The velocity of the periphery of this ring may be described as the "velocity of whirl," and if it be called v, the moment of momentum of a ring, the weight of which is W, is

$$M=\frac{W}{g}\,.\,vr.$$

If the wheel has no action on the water, this quantity cannot be altered, and we must then have

$$vr=\text{Constant}.$$

The water then forms what we have already called a "free vortex" (Art. 241), with the addition of a certain radial velocity u, in consequence of which the rings change their diameter. The paths of the particles of water are then spirals, the inclination of which depends on the proportion between u and v.

The case now to be considered is that in which the moment of momentum of the rotating rings is gradually reduced during their passage through the wheel by the action of suitable vanes attached to it. An impulse is thus exerted on the wheel which furnishes the motive force. The moment of this impulse is given by the equation,

$$L=\frac{wQ}{g}(v_1r_1-v_2r_2),$$

where wQ is the weight of all the rings passing through the machine in a second, and the suffixes 1, 2 refer to entrance and exit respectively, as indicated in the figures for the two cases of outward flow and inward flow. The turbine works to best advantage when the water is discharged without any whirl, that is when $v_2=0$, and putting aside friction the only loss then is that due to the velocity of flow u, which may be made small by making the wheel of sufficient breadth at the circumference where the water is discharged.

In practice there are of course always frictional resistances, but, for given velocities, the impulse on the wheel is not altered by them, so that the moment of impulse is always given by the above equation. Suppose, now, h the *effective* head found from the actual head by deducting (1) the height due to the velocity of delivery, (2) the friction of the supply pipe and passages in the wheel, (3) the loss (if any) by shock on entering the wheel; then

$$\text{Work done per second} = wQh.$$

But, if V_1 be the speed of periphery of the wheel at the radius r_1 where the water enters, V_1/r_1 is the angular velocity of the wheel, and $L . V_1/r_1$ is the work done per second. We have then for the case where there is no whirl at exit

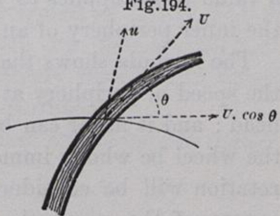
$$V_1 v_1 = gh.$$

The effective head h in this formula includes (1) a part equivalent to the useful work, and (2) a part equivalent to the frictional resistances to the rotation of the wheel, such as friction of bearings and friction of the water surrounding the wheel (if any) on its external surface. This last item is often described as "disc friction."

The whirl before entrance is communicated by fixed blades BB, curved, as shown in the figures, so as to guide the water in a proper direction on entrance to the wheel. It is the use of these guide blades which characterizes the turbine as distinguished from the reaction wheel.

The whirl at different points, either in the wheel or outside it, depends on the angle of inclination of the vanes or guide blades to the periphery. These blades are so numerous that the water moves between them nearly as it would do in a pipe of the same form. If θ be the angle such a pipe (Fig. 194) makes with the periphery at any point at which the water is flowing through it with velocity U, the radial and tangential components of that velocity will be $U. \sin\theta$ and $U. \cos\theta$. The first of these is always the velocity of flow u, whether the pipe be fixed or whether it be attached to the revolving wheel. In the fixed pipe the second is the velocity of whirl which we may call v', and we have for motion before entrance

Fig.194.



$$v' = u . \cot\theta.$$

In the moving pipe, however, it is the velocity of whirl relatively to the revolving wheel, and this is $V - v$, therefore

$$V - v = u . \cot\theta.$$

Suppose the vanes of the wheel are radial at the circumference where the water enters. In order that the water may have no velocity of whirl relatively to the wheel on entrance, and that the water may enter without shock, we must then have $v' = V_1$, that is, the value of θ for the fixed guide blade at entrance should be given by

$$\tan\theta_1 = \frac{u}{V_1}.$$

Further, the water should be discharged without whirl, that is, v should be zero at the circumference where the water leaves the wheel, hence

$$\tan\theta_2 = \frac{u}{V_2}.$$

The inclination of the fixed blades at entrance, and of the vanes at entrance and exit is thus determined. At intermediate points it would be desirable that it should so vary that vr should diminish uniformly from entrance to exit in order that the action of all parts of the vane upon the water may be the same. This condition would completely determine the form of the vane, but, in practice, any "fair" form would be a sufficient approximation.

Supposing the vanes thus designed $v_1 = V_1$, and the speed of periphery of the wheel at the circumference where the water enters is then given by the simple formula

$$V_1 = \sqrt{gh},$$

a value which applies to the outer periphery of an inward-flow and the inner periphery of an outward-flow turbine (see Appendix).

The formula shows that the turbine works to best advantage when the speed of periphery at entrance is that due to *half* the *effective* head: and it never can be advantageous to run it quicker. But, if the wheel be wholly immersed in water, the frictional resistance to rotation will be considerable, and as that resistance varies as the square of the speed the wheel may be run slower without much reduction of efficiency, or, it may be, even with an increase.

Many forms of outward-flow turbines exist, of which the best known was invented by Fourneyron, and is commonly known by his name. The inward-flow or vortex turbine was invented by Prof. James Thomson. For descriptions and illustrations of these machines the reader is referred to the treatises cited at the end of this chapter.

265. *Turbine Pumps.*—Impulse and reaction machines are always reversible, and every motor may therefore be converted into a pump by reversing the direction of motion of the machine and of the water passing through it. If, for example, in the reaction-wheel of Fig. 192 we imagine the wheel to turn in the opposite direction with velocity V, while by suitable means the water is caused to move in the opposite direction with velocity $v - V$, so as to enter the orifices with velocity v, it will flow through the arms to the centre and be delivered up the central pipe. The only difference will be that the lift of the pump will not be so great as the fall in the motor on account of frictional resistances. So, any turbine motor is at once converted into a turbine pump by reversing the direction of its motion and supplying it with water moving with a proper velocity. An inward-flow motor is thus converted into an outward-flow pump, and conversely.

No inward-flow pump appears as yet to have been constructed, though it has occasionally been proposed. The "centrifugal" pump so common in practice is, of course, always an outward-flow machine.

The earliest idea for a centrifugal pump was to employ an inverted Barker's mill, consisting of a central pipe dipping into water connected with rotating arms placed at the level at which water is to be delivered. This machine, which must be carefully distinguished from the true reversed Barker's Mill mentioned above, operates by suction. Its efficiency, which may be investigated as in Art. 263, is very considerable (Ex. 4, p. 520), but there are obvious practical inconveniences which prevent its use in ordinary cases. The actual centrifugal pump is a reversed inward-flow turbine.

All that was said about motors in the last article applies equally well to pumps, and the same formula

$$Vv = gh$$

applies, V being the speed of rotation of the wheel, now usually called the "fan" and v that of the water, both reckoned at the outer periphery where the water issues. The quantity h is now the *gross* lift found by adding to the actual lift, the head corresponding to the velocity of delivery, the friction of the ascending main, the friction of the suction pipe and passages through the wheel into the main, and the losses by shock at entrance and exit.

A pump, however, works under different conditions from a motor, and corresponding differences are necessary in its design. The energy of a fall can by proper arrangements be readily converted, wholly or partially, into the kinetic form without any serious loss by frictional resistances, and the water can, therefore, be delivered to the wheel with a great velocity of whirl to be afterwards reduced by the action of the wheel to zero. When such a motor is reversed, the water enters without any velocity of whirl, and leaves with a velocity, the moment of momentum corresponding to which represents the couple by which the wheel is driven. To carry out the reversal exactly, this velocity ought to be reduced to as small an amount as possible in the act of lifting. Now the reduction of a velocity without loss of head is by no means easy to accomplish, and (see Appendix) always requires some special arrangement.

In Thomson's inward-flow turbine, when reversed, the water is discharged with a velocity of whirl which is equal to the speed of periphery V, and given by the formula

$$V = \sqrt{gh}.$$

The corresponding kinetic energy represents at least half the power required to drive the pump, and if it be wasted, as was the case in some of the earlier centrifugal pumps constructed with radial vanes, the efficiency is necessarily less than ·5, and in practice will be at most ·3. To avoid this loss, the wheel must revolve in a large "vortex chamber," at least double the diameter of the wheel from the outer circumference of which the ascending main proceeds. The water before entering the main forms a free vortex, and its velocity is reduced one-half as it spreads radially from the wheel; three-fourths the kinetic energy is thus converted into the pressure form. The speed of periphery in pumps of this class is that due to *half* the *gross* lift. Assuming their efficiency as ·65, the gross lift is found by an addition of 50 per cent. to the actual lift.

Many examples of vortex-chamber pumps exist, but they are comparatively rare, probably because the machine is more cumbrous; in practice a different method of reducing the velocity of discharge is generally employed. Instead of the vanes being radial at the outer periphery, they are curved back so as to cut it at an angle θ, given by the formula (p. 509)

$$V - v = u \,.\, \cot \theta,$$

the velocity of whirl is thus reduced from V to kV, where k is a fraction, and the speed is then

$$V = \sqrt{g\frac{h}{k}}.$$

If the efficiency be supposed ·65, and the velocity be reduced in this way to one-half its original value, this gives about $10\sqrt{H}$ for the speed where H is the actual lift. The greater speed is a cause of increased friction as compared with the vortex-chamber arrangement, but on the other hand the friction of the vortex is by no means inconsiderable, and this is so much subtracted from the useful work done.

The centrifugal pump in this form was introduced by Mr. Appold in 1851, and is commonly known by his name.

Another important point in which the pump differs from the motor is in the guidance of the water outside the wheel. In the motor there are four or more fixed blades which guide the water to the wheel; but in the pump the outer surface of the chamber surrounding the wheel forms a single spiral guide blade. The whole of the water discharged from the wheel rotates in the same direction, and in order that the discharge may be uniform at all points of the circumference the sectional area of this chamber should increase uniformly from zero at one side of the ascending main to a maximum value at the other side. In some of the earlier designs of centrifugal pumps it was supposed that some of the water would rotate one way and some the other, but in fact all the discharged water rotates with the wheel, and the passage should be so designed as to permit this, the area corresponding to the proposed velocity of whirl. There are, however, examples in which the water is discharged in all directions into an annular casing, and guided by spiral blades parallel to the axis of rotation. (See a paper by Mr. Thomson, *Min. Proc. Inst. C.E.*, vol. 32.)

Centrifugal pumps work to best advantage only at the particular lift for which they are designed When employed for variable lifts, as is constantly the case in practice, their efficiency is much reduced and does not exceed ·5. It is often much less.

Few centrifugal pumps utilize more than a small fraction of the energy of motion possessed by the water at exit from the wheel, and an investigation of their efficiency on the supposition that this energy is wholly wasted is therefore of considerable interest.

Let h_0 be the actual lift, and let all frictional losses except that specified be neglected; then, if u be the velocity of flow, and v the velocity of whirl at exit, the loss of head is $(u^2+v^2)/2g$, and the gross lift is

$$h = h_0 + \frac{u^2+v^2}{2g}.$$

Substituting this value of h in the formula for V, and replacing u by its value $(V-v)\tan\theta$, we obtain

$$Vv - gh = gh_0 + \frac{v^2 + (V-v)^2\tan^2\theta}{2}.$$

Adding $\frac{1}{2}V^2$ to each side, and re-arranging the terms,

$$\tfrac{1}{2}V^2 = gh_0 + \tfrac{1}{2}(V-v)^2 . \sec^2\theta,$$

a formula from which we find

$$\text{Efficiency} = \frac{h_0}{h} = \frac{V^2 - (V-v)^2\sec^2\theta}{2Vv}$$

$$= \sec^2\theta\left(1 - \tfrac{1}{2}\left(\frac{v}{V} + \frac{V}{v}\sin^2\theta\right)\right).$$

This result shows that the efficiency is greatest when

$$v = V . \sin\theta;$$

and on substitution we find

$$\text{Maximum efficiency} = \sec^2\theta(1 - \sin\theta) = \frac{1}{1 + \sin\theta}.$$

The speed of maximum efficiency is found from the equation

$$\tfrac{1}{2}V_0^2 = gh_0 + \tfrac{1}{2}V_0^2\sec^2\theta . (1 - \sin\theta)^2,$$

which gives

$$V_0^2 = (1 + \operatorname{cosec}\theta)gh_0.$$

The proper velocity of flow is

$$u_0 = V_0 . \tan\theta(1 - \sin\theta),$$

and the area of flow through the periphery of the wheel should be made to give this velocity with the intended delivery.

At any other speed V the velocity of flow will be given by

$$u^2 = (V^2 - 2gh_0)\sin^2\theta,$$

and the efficiency may be found by the preceding formula.

In the best examples of centrifugal pumps working at a suitable lift, it is probable that enough of the energy of motion on exit from the wheel is utilized to provide for the various frictional resistances neglected in this investigation. But in ordinary cases this will not be true, and the efficiency is much reduced. The theory, taking into account all the resistances, is too intricate to enter on here.

When a centrifugal pump is started the fan is filled with water which, in the first instance, rotates as a solid mass with the fan. If the radius of the inner periphery be m times that of the outer where m is a fraction, it will not commence to deliver water till the speed reaches the value

$$V^2(1 - m^2) = 2gh_0.$$

But when once started, the speed may be reduced below this value without stopping the delivery, provided that some of the energy of motion on exit from the wheel is utilized. This has been observed to occur in practice, and it will serve as a test of efficiency.

266. *Limitation of Diameter of Wheel.*—For a given fall in a motor or lift in a pump the diameter of wheel in a turbine is limited, because the frictional resistances increase rapidly with the diameter.

Let u be the velocity of flow, d the diameter, b the inside breadth of wheel at exit; then the delivery in cubic feet per second is

$$Q = ub\pi d.$$

Now, if the breadth b be too small as compared with the diameter, the surface friction of the passages through the wheel will be too great, as in the case of a pipe the diameter of which is too small for the intended delivery. Thus b is proportional to d: also, we have seen that u is proportional to V_1, that is to $\sqrt{h}$, and it follows therefore, by substitution for b and u, that

$$Q = Cd^2 \sqrt{h},$$

where C is a co-efficient.

If the wheel be wholly immersed in the water the surface friction (Ex. 8, p. 521) is relatively increased by increasing the diameter. On investigating how great the diameter may be without too great a loss we arrive at the same formula.

Where it is of importance to have as large a diameter as possible to reduce the number of revolutions per minute, the diameter of wheel in a pump or a turbine is therefore found by the formula

$$d = \sqrt{\frac{G}{c\sqrt{h}}}.$$

If G be the delivery in gallons per minute, h the actual fall in feet, d the external diameter also in feet, the value of c for an outward-flow turbine is about 200. In a centrifugal pump the value is probably not very different, h being now the actual lift, but it varies in different types of pump.

Centrifugal pumps cannot generally be employed for high lifts, partly because it becomes increasingly difficult to utilize the energy of motion on exit from the wheel, and partly on account of disc friction. The fan rotates more than twice as fast as the wheel of a turbine, and the disc friction is consequently more than four times as great.

267. *Impulse Wheels.*—The formula

$$V_1^2 = gh,$$

which gives the speed of a turbine wheel in terms of the effective head, also gives the velocity of whirl at entrance, and therefore shows that, of the whole head employed in driving the wheel and producing the velocity of flow, one-half operates by impulse. The remainder operates by pressure, and this class of turbines is therefore not simple impulse, but impulse-pressure machines. It is necessary therefore that the wheel should revolve in a casing, and that the passages should be always completely filled with water. The diameter of wheel is then limited as explained in the last article, and for a small supply of water and a high fall the number of revolutions per minute becomes abnormally great. This consideration and the necessity of adaptation to a variable supply of water render it often advisable to resort to a machine in which the passages are actually or virtually open to the atmosphere. The whole of the energy of the fall is then converted into the kinetic form before reaching the wheel, and consequently operates wholly by impulse.

A wheel of this kind approaches closely in principle to the Poncelet water wheel mentioned in Art. 261, but is often still described as a "turbine," because the water is guided by fixed blades before reaching the wheel. A common example is the Girard turbine. The flow of the water is here parallel to the axis of the wheel, spiral guide blades are ranged round the circumference of a cylinder like the threads of a screw in order to give the necessary whirl to the water before entrance. The wheel is provided with a similar set of spiral vanes curved in the opposite direction, which reduce it to rest as it passes through. In the French *roue à poire* the wheel is conical, the water enters at the circumference, and, guided by spiral vanes, descends to the apex where it is discharged.

268. *Propellers.*—The subject of propellers is outside the limits of simple hydraulics for reasons already indicated when considering the resistance of ships (Art. 249). Nevertheless they may be regarded as hydraulic machines, and their connection with the machines already referred to forms a proper subject for consideration.

Every propeller operates by means of the mutual action between it and the water on which it acts consequent on a change of velocity which it produces in the water of the sea; it is therefore an impulse and reaction machine applied so as to produce a propelling force which drives the vessel through the water. Since the resistance of the vessel is directly astern, the change of motion produced is sternward so far as it is of any utility for the purpose. Some forms of propeller, as, for example, the screw, give lateral motions to the water, but the energy thus employed is wasted.

(1) The simplest form of propeller is that in which the water is drawn into the vessel through orifices of proper size, and projected by means of a centrifugal pump through two orifices in the side of the vessel so placed that the water issues directly astern. The reaction of the issuing jet furnishes a propelling force on the vessel. The problem here is just the same as that of the simple reaction wheel already considered in Art. 263: the fact that the orifices move in straight lines instead of in a circle making no difference in the propelling reaction. Hence, if R be the resistance of the vessel, v the velocity of discharge, V the velocity of the vessel,

$$R = \frac{wQ}{g}(v - V).$$

The engine power is employed in the first instance in creating a head h, but, supposing this known, the question is unaltered, and therefore neglecting frictional resistances,

$$\text{Efficiency} = \frac{2V}{v + V} = \frac{1}{1 + e}.$$

If we call the counter-efficiency $1 + e$, and if we further suppose A to be the *joint* area of the orifices, and $K \,.\, \Delta^{\frac{2}{3}} V^2$ the resistance of the ship (Art. 249), we shall find by substitution that for all speeds

$$K \,.\, \Delta^{\frac{2}{3}} = \frac{wA}{g}(1 + 2e)\ 2e,$$

a formula which shows that the efficiency is increased by increasing the size of the orifices, and enables us to calculate the size for a given efficiency.

In every propeller, in the absence of frictional resistances and of any disturbance due to the passage of the vessel through the water, *the efficiency of a propeller is greater the greater the quantity of water on which it operates.* In the case of the jet propeller which we are now considering, this general conclusion is modified by the effect of frictional resistances. If we make the same supposition as to these as in the reaction wheel, the efficiency will be found, as before, to be greatest for a certain value of v/V, and this will correspond to a certain definite area of orifice. The circumstances being somewhat different, a different way of expressing the frictional resistances would probably more closely represent the facts, but the general conclusion must be the same. It is of little use to consider this more closely, as the disturbance of the water passing near the vessel, produced by drawing a large quantity of water through the orifices of entry, must modify in an unknown way the resistance of the ship. Hence the best magnitude and position of the orifices of entry and efflux can only be found by careful experiments. Such experiments have not hitherto been carried out, but it may be considered probable that the jet propeller cannot compete with other forms of propeller so far as efficiency is concerned. If it is ever introduced, it will be for the sake of facility in manœuvring or some other similar reason.

(2) If the action of paddles be observed, two streams of water are seen proceeding from the floats which play the part of the jets in a jet propeller. The most efficient kind are those in which the floats turn about axes on which they are mounted in such a way as to enter and leave the water without any shock. The streams are then simple jets of area not very different from that of the floats, and are driven back with a velocity which is about the same as that of the floats themselves. If, then, v be the speed of the paddles as calculated from their diameter and revolutions, V the speed of the ship, the propelling reaction is given by the same formula as for jets, namely,

$$R = \frac{w}{g} Q.(v - V),$$

the velocities being in feet per second, and Q the quantity acted on in cubic feet per second. The energy exerted per second is, however, now Rv, and therefore

$$\text{Efficiency} = \frac{V}{v}.$$

For given values of v and V the efficiency is less than that of the simple jet when the frictional resistances are left out of account. The reason of this is that the value just found includes the loss due to breaking up as the paddles strike the water and drive it upwards in a mass of foam before it settles down to the nearly undisturbed motion of the streams. In smooth water, when the paddles are not too deeply immersed, the efficiency however of paddles is far greater than that of the jet, because the area of orifice which can practically be employed is so limited in the jet and the frictional losses in the pump and passages so great.

(3) In rough water the efficiency of paddles is much reduced, and this is also the case when the immersion varies in consequence of alterations in the displacement of the vessel due to consumption of fuel on a voyage or other causes. In sea-going vessels, therefore, the paddles are replaced by a screw propeller.

In this case the action of the propeller is much more complex: the water has a rotatory velocity communicated to it as well as a sternward velocity, and these velocities are different for each portion of the screw blade. Further, the water in which the screw works has very complex motions due to the action of the sides of the ship upon it, a circumstance which affects the resistance of the ship as well as the action of the propeller. For these reasons the screw is not so efficient, other things being the same, as well constructed paddles. On the other hand the quantity of water acted on is large, and the action is not greatly influenced by the circumstances just mentioned as reducing the efficiency of paddles.

On comparing a screw propeller with the machines already considered, it will be perceived that it is a parallel-flow impulse wheel reversed, with two important modifications. First, the fixed guide blades are omitted. It is true that it has been proposed to employ such blades in order to avoid the loss occasioned by the transverse velocity given to the water which is useless for propelling purposes, but the gain of efficiency, if any, is very small. Secondly, the number of blades in the wheel is large; whereas in a screw propeller there are often only two, and in all cases they occupy much less than half the whole circumference.

Both these modifications are due to the same cause, and arise from the fact that in the propeller the changes of velocity produced by the action of the blades are small compared with the whole velocity of the water through the propeller, whereas in the wheel they are large. Hence the frictional resistances in the propeller are disproportionately great and render it necessary to reduce the surface exposed to the water as much as possible.

This reasoning also shows that the frictional losses in the centrifugal pump and passages of a jet propeller must be great. Such a pump should receive the water at the velocity of the ship, and gradually increase its velocity to that attained on efflux from the orifices. The frictional resistances will depend on the whole velocity, but the propelling reaction on the difference of velocities. Or if the alternative be adopted of checking the motion of the water on entry, and afterwards giving it the whole velocity of efflux, the reduction of velocity will be difficult to accomplish without considerable loss, and the propelling reaction will not be greater than before. All unnecessary changes of velocity are a cause of loss.

EXAMPLES.

1. In a reaction wheel the speed of maximum efficiency is that due to the head. In what ratio must the resistance be diminished to work at four-thirds this speed and what will then be the efficiency? Obtain similar results when the speed is diminished to three-fourths its original amount.

Ans. Efficiency = ·63 or ·64.
Ratio = ·84 or 1·14.

2. Water is delivered to an outward-flow turbine, at a radius of 2 feet, with a velocity of whirl of 20 feet per second, and issues from it in the reverse direction at a radius of 4 feet, with a velocity of 10 feet per second. The speed of periphery at entrance is 20 feet per second, find the head equivalent to the work done in driving the wheel. *Ans.* 24·22 feet.

3. In a Fourneyron turbine the internal diameter of the wheel is $9\frac{1}{2}$ inches, and the outside diameter 14 inches. The effective head (p. 509) is estimated at 270 feet: find the number of revolutions per minute. *Ans.* 2200.

NOTE.—These data are about the same as those of a turbine erected at St. Blasien in the Black Forest.

4. An inverted Barker's mill (p. 511) is used as a centrifugal pump. If the coefficient of hydraulic resistances referred to the orifices be ·125, show that the speed of maximum efficiency is that due to twice the lift, and find the maximum efficiency. *Ans.* Maximum efficiency = ·75.

5. A centrifugal pump delivers 1500 gallons per minute. Fan 16 inches diameter. Lift 25 feet. Inclination of vanes at outer periphery to the tangent 30°. Find the breadth at the outer periphery that the velocity of whirl may be reduced one-half, and also the revolutions per minute, assuming the *gross* lift $1\frac{1}{2}$ times the actual lift. *Ans.* Breadth = $\frac{3}{4}$ inch. Revolutions = 700.

6. In the last question find the proper sectional area of the chamber surrounding the fan (p. 513) for the proposed delivery and lift. Also examine the working of the pump at a lift of 15 feet. *Ans.* 24 sq. inches.

7. A jet of water, moving with a given velocity, strikes a plane perpendicularly. Find how much of the energy of the jet is utilized in driving the plane with given

speed. Determine the speed of the plane for maximum efficiency, and the value of the maximum efficiency. *Ans.* Speed of maximum efficiency = one-third that of jet. Maximum efficiency $= \frac{8}{27}$.

8. Assuming the ordinary laws of friction between a fluid and a surface, and supposing that any motion of the fluid due to friction does not affect the question: find the moment of friction (L), and the loss of work per second (U), when a disc of radius a rotates with speed of periphery V.

$$Ans.\ L = f \cdot \frac{2\pi}{5} \cdot a^3 V^2; \quad U = f \cdot \frac{2\pi}{5} \cdot a^2 \cdot V^3.$$

9. If the rotating disc in question 8 be surrounded by a free vortex of double its diameter, show that the loss by friction of the vortex on the flat sides of the vortex chamber is $2\frac{1}{2}$ times the loss by friction of the disc.

10. The resistance of the *Waterwitch* at 8 knots is 5,500 lbs., the orifices of her jet propeller are each 18 inches by 24 inches, what must be the delivery of her centrifugal pump in gallons per minute to propel her at this speed, and what will be the efficiency, neglecting frictional resistances.

Ans. Velocity of efflux = 29·3 per second.
Delivery = 66,000 gallons per minute.
Efficiency = ·63.

11. In the last question find the H.P. required for propulsion, assuming the efficiency of the pump and engines ·4. *Ans.* 525.

12. If the jet propeller of the *Waterwitch* be replaced by feathering paddles, what will be the area of the stream driven back for a slip of 25 per cent. Find the efficiency and the water acted on in gallons per minute.

Ans. Joint area of streams = 34 square feet.
Efficiency = ·75.
Delivery = 236,000 gallons per minute.

REFERENCES.

The subject of hydraulic machines is very extensive, and it is impossible within the limits of a single chapter to do more than give a general idea of their working. For descriptive details and illustrations the reader is referred, amongst other works, to

GLYNN. *Power of Water.* Weales' Series.
FAIRBAIRN. *Millwork and Machinery.* Longman.
COLYER. *Water-Pressure Machinery.* Spon.
BARROW. *Hydraulic Manual.* Printed by authority of the Lords Commissioners of the Admiralty.

The theory of hydraulic machines is further developed in Professor Unwin's work on Hydraulics, already cited on page 481, and the various special treatises therein mentioned, also in Rankine's treatise on the *Steam Engine and other Prime Movers.*

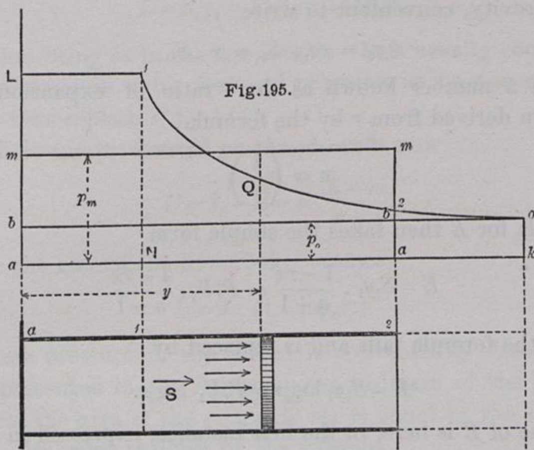
CHAPTER XXI.

PNEUMATICS AND THERMODYNAMICS.

269. *Preliminary Remarks.*—An elastic fluid under pressure is a source of energy which, like a head of water in hydraulics (p. 445), may be employed in doing work of various kinds by a machine, or simply in transferring the fluid from one place to another. In hydraulics we commence with the case of simple transfer, but the density of gases is so low that, unless the differences of pressure considered are very small, the inertia and frictional resistances of the fluid employed in a pneumatic machine have little influence: it is the elastic force, which is the principal thing to be considered. In studying pneumatics, therefore, we commence with machines working under considerable differences of pressure and then pass on to consider the flow of gases through pipes and orifices together with those machines in which the inertia and frictional resistances of the fluid cannot be neglected.

270. *Expansive Energy.*—The special characteristic of an elastic fluid is its power of indefinite expansion as the external pressure is diminished. While expanding, it exerts energy of which the fluid itself is, in the first instance, the source, whereas the energy exerted by an incompressible fluid is transmitted from some other source. Expansive energy is utilized by enclosing the fluid in a chamber which alternately expands and contracts; the common case being that of a cylinder and piston.

Fig. 195 represents in skeleton a cylinder and piston enclosing a mass of expanding fluid. Taking a base line aa' to represent the stroke, set up ordinates to represent the total pressure S on the piston in each position; a curve $1Q2$ drawn through the extremities of these ordinates is the Expansion Curve. Reasoning as in Art. 90, p. 195, the area of this curve represents the energy exerted as the piston moves from the position 1, where the expansion com-



mences, to the position 2, where it terminates. One common case was considered in the article cited, namely, that in which the expansion curve is a common hyperbola. This is included in the more general supposition,

$$Sy^n = S_1y_1{}^n = S_2y_2{}^n,$$

where y is the distance of the piston from the end a of its stroke, and n is an index which, for the particular case of the hyperbola, is unity. Most cases common in practice may be dealt with by ascribing a proper value to n; for air it ranges between 1 and 1·4, and for steam it is roughly approximately unity. The suffixes indicate the points at which the expansion commences and terminates.

If, now, E be the energy exerted during expansion,

$$E = \int_{y_2}^{y_1} Sdy = S_1y_1{}^n\int_{y_1}^{y_2} y^{-n}dy = S_1y_1{}^n.\frac{y_2{}^{1-n} - y_1{}^{1-n}}{1-n}.$$

This formula may be written in the simpler form

$$E = \frac{S_1y_1 - S_2y_2}{n-1},$$

in applying which, the terminal pressure S_2 is supposed to have been previously found from the equation

$$S_2 = S_1 . \left(\frac{y_1}{y_2}\right)^n.$$

It is, for brevity, convenient to write

$$y_2 = ry_1; \; S_2 = x \, . \, S_1,$$

where r is a number known as the "ratio of expansion," and x is a fraction derived from r by the formula

$$x = \left(\frac{1}{r}\right)^n.$$

The formula for E then takes the simple form

$$E = S_1 y_1 \, . \, \frac{1 - rx}{n - 1} = P_1 V_1 \, . \, \frac{1 - rx}{n - 1}.$$

If $n = 1$ the formula fails and is replaced by

$$E = S_1 y_1 \log_\varepsilon r = P_1 V_1 \log_\varepsilon r.$$

The value of E is here, in the first instance, expressed in terms of the total pressure on the piston, but as in Art. 255, p. 485, we may replace S by PA, and Ay by V, so that $S_1 y_1$ is replaced by $P_1 V_1$. In "rotatory" engines and pumps the expanding chamber is not a simple cylinder and piston, but is formed from a turning pair. Or, more generally, the chamber pair may be formed from any two links of a kinematic chain which it may be convenient to select for the purpose. In its last form the formula is applicable in every case. If the expansion curve be not given in the form supposed, the value of E is determined graphically by measuring the area of the curve, in doing which, when the chamber is not a simple cylinder, the base of the diagram must represent the volume swept out by the chamber pair, and the ordinates the pressures per unit of area.

271. *Transmitted Energy.*—The energy exerted by an elastic fluid consists not merely of that derived from the expansive power of the fluid pressing against the piston, but also of that which is transmitted in the same way as would be the case if it were incompressible. The fluid is supplied from a reservoir, which may either be an accumulator in which it is stored by the action of pumps, or a vessel in which, by the action of heat, it is generated or its elasticity increased.

In any case, so long as the cylinder remains in communication with the reservoir the fluid enters at nearly constant pressure, and energy is exerted on the piston just as in the water-pressure engine. During this period of admission the energy exerted is

$$L = S_1 y_1 = P_1 V_1 = 144 p_1 V_1,$$

the notation being as in the last article. It is usually convenient to express volumes in cubic feet and pressures in lbs. per square inch. We must thus replace P by $144p$.

The whole energy exerted on the piston is now

$$U = L + E = L \,.\, \frac{n - rx}{n - 1},$$

which for the case of the hyperbola becomes

$$U = L \,.\, (1 + \log_\epsilon r.)$$

The mean pressure on the piston is conveniently denoted by p_m, and is represented in the figure by the ordinate of the line mm so drawn that the area of the rectangle ma is equal to the area of the diagram. Its value is given by the formulæ

$$p_m = \frac{p_1}{r} \,.\, \frac{n - rx}{n - 1}\,;\; p_m = p_1 \,.\, \frac{1 + \log_\epsilon r}{r}.$$

A reservoir filled with an elastic fluid at high pressure is an accumulator, the absolute amount of energy stored in which is the expansive energy or the total energy according as the pressure is not, or is, maintained by the addition of fresh fluid in place of that discharged, the expansion being supposed indefinite in either case. With the law of expansion already supposed, when n is greater than unity, rx vanishes when the expansion curve is prolonged indefinitely. The total absolute energy is then

$$U_1 = P_1 V_1 \,.\, \frac{n}{n - 1},$$

where V_1 is the whole volume of fluid considered.

When n is not greater than unity, U_1 is infinite. In practice, however, there is always a "back" pressure P_0 on the working piston, or, more generally, on the sides of the chamber in which the fluid is enclosed. In overcoming this, the work $P_0 r V_1$ is done, and nothing is gained by prolonging the expansion beyond the point 0 at which the terminal pressure P_2 has fallen to P_0. The corresponding ratio r_0 is given by the formula

$$\log r_0 = \frac{1}{n}\log\left(\frac{1}{x_0}\right) = \frac{\log P_1 - \log P}{n}.$$

The available energy is found by writing $r = r_0$, $x = x_0$ in the value of U, and subtracting $P_0 r_0 V_1 = P_1 V_1 r_0 x_0$. The result is

$$\frac{n}{n-1} \cdot P_1 V_1 (1 - r_0 x_0) = U_1 - U_0,$$

being the difference of the values of U when the expansion commences at 1 and at 0. It is always finite, and is graphically represented by the area $L10bb$.

In the transmission and storage of energy by elastic fluids this quantity plays the same part as the "pressure-head" in hydraulics, to which indeed it reduces if n be supposed very great, r_0 unity, and V_1 the volume of a lb. of water. It is the energy of a given quantity of fluid due to a given difference of pressure. The term "head" may be used for this when the quantity considered is a mass of 1 lb. When $n = 1$

$$U_1 - U_0 = P_1 V_1 \log_\epsilon r_0.$$

When the reservoir is not kept full the only available energy is the expansive energy, less the work done in overcoming P_0 through the volume $V_0 - V_1$. This is graphically represented by the curvilinear triangle $N01$, and is most conveniently given by the formula

$$A.E = U_1 - U_0 - (P_1 - P_0) V_1.$$

272. *Cycle of Mechanical Operations in a Pneumatic Motor—Mechanical Efficiency.*—Motors operating by the pressure of an elastic fluid may be described generally as Pneumatic Motors. They are either supplied from an accumulator as in hydraulic motors of the same class, or they may be heat-engines serving as the means by which heat energy is utilized. In either case the mechanism of the motor is the same, and consists of a chamber which expands to admit the fluid and contracts to discharge it, with a proper kinematic chain for utilizing the motion of the chamber pair.

In water-pressure engines the contraction to expel the water from the chamber is not considered, because all pressures are reckoned above the atmosphere, and the pressure in the accumulator is so great that small differences of pressure may be disregarded. With elastic fluids it is commonly different: the "exhaust" of the chamber must be taken into account.

Returning to Fig. 195, suppose that the piston has reached the end of its stroke, the cylinder is then filled with fluid of a certain pressure p_2 which may be supposed known. Let now a valve be opened allowing the cylinder to communicate with the atmosphere, or with a reservoir containing fluid at a lower pressure p_0. The fluid in the cylinder then rushes out into the reservoir, and the pressure in the cylinder speedily subsides to p_0; the fluid expands in this process, but its expansive energy is wasted in producing useless motions in the air which afterwards subside by friction. After subsidence let the piston be moved back by an external force applied to it which supplies the energy necessary to overcome the "back" pressure p_0. The fluid is discharged from the chamber, and so long as the communication with the exhaust reservoir is open the pressure remains constantly p_0. We represent this on the diagram by drawing a horizontal line *bb*, the ordinate of which is p_0. The work done in overcoming back pressure is $144p_0V_2$ and is represented on the diagram by the rectangle *ba*; this is so much subtracted from the energy exerted by the motor.

Thus the volume of the chamber goes through a cycle of changes alternately expanding and contracting. During expansion energy is exerted, the corresponding mean pressure p_m is the "mean forward pressure." During contraction work is done, and the corresponding mean pressure is the "mean back pressure." The difference between the two is the "mean effective pressure" which measures the useful work done, as shown by the equation

$$\text{Useful work} = (p_m - p_0)144V_2,$$

and is graphically represented by the area of the closed figure *L*12*bb*.

In most cases the moveable element of the chamber pair divides the chamber into two parts, one of which expands while the other contracts, and conversely: the motor is then described as "double acting." The force acting on the moving piece is then the difference between the forward pressure in one chamber and the back pressure in the other, and when the stress on the parts of the machine is to be considered this is the effective pressure upon which the stress depends (p. 240). For all other purposes, however, the back pressure is to be taken as just explained.

If the pressures p_1, p_0 in the supply and exhaust reservoirs be given, and also the form of the expansion curve, the only waste of energy in this process arises from incomplete expansion. Imagine the expansion curve prolonged to the point o where it meets the back pressure line, and suppose the stroke lengthened so as to reach this point, then additional work would be done by the fluid which would be represented graphically by the area of the curvilinear triangle $2ob$. This area represents energy lost by unbalanced expansion, and to avoid it the expansion must be "complete," that is, the fluid must be allowed to expand till its pressure has fallen to p_0, the pressure in the exhaust reservoir, a condition seldom fulfilled in practice, because the loss by friction and other causes becomes disproportionately great. Leaving this out of account, a pneumatic motor is capable of exerting only a certain maximum amount of energy, quite irrespectively of the nature of its mechanism, but dependent only on the pressures between which it works and the nature and treatment of the fluid. A motor which reaches this maximum power may be described as *mechanically* perfect, and the ratio of the actual useful work done to the theoretical maximum may be described as the MECHANICAL EFFICIENCY of the motor.

In practice the mechanical efficiency is diminished not only by incomplete expansion, but also by a portion of the fluid being retained in the "clearance" space of the chamber after the exhaust is completed. This, however, is a detail which cannot be considered here. The theoretical maximum is clearly the same as the store of energy in the fluid used, already found in the last Article, and denoted by $U_1 - U_0$. The consumption of fluid (neglecting clearance) is one cylinder full, at the terminal pressure, in each stroke.

273. *Pneumatic Pumps.*—A pneumatic like an hydraulic motor may be reversed by applying power to drive it in the reverse direction, and the machine thus obtained is a Pump which takes fluid at a low pressure and compresses it into a reservoir at high pressure.

The cycle in the pump is the same as the cycle in a motor, but the operations take place in reverse order. As the chamber expands fluid is drawn in from the low-pressure reservoir and energy is exerted on the piston by the original "back" pressure: as the chamber contracts the fluid is compressed till it reaches the pressure p, when a valve opens and admits it to the high-pressure reservoir. There is, however, this important difference, namely, that the process of unbalanced expansion in the motor cannot be reversed; and therefore, if the pump is to operate on the same weight of fluid, the volume of the working cylinder must be enlarged so that the expansion curve may start from o. If this be supposed, the compression curve will, for the same fluid treated in the same way, be identical with the expansion curve of the motor. If there were no unbalanced expansion the motor would be exactly reversible, and the condition of a motor being mechanically perfect may therefore be described by saying that it must be mechanically reversible. The difference of working of the valves in pumps and motors has already been referred to in Art. 260.

Air pumps are employed either to compress atmospheric air to a high pressure or to exhaust a chamber. In the second case the atmosphere is the high pressure reservoir into which the exhaust air is compressed. In the old "atmospheric" engine the steam was employed merely to produce a vacuum, the motive force being the pressure of the atmosphere: this machine is therefore a reversed air pump.

In all pneumatic motors a pump is required to replace the fluid in the supply reservoir. Unless the motor be a heat engine this pump must be driven by external agency, and the whole process is one of distribution, transmission, and storage of energy, as in water-pressure engines. In Whitehead torpedoes indeed, and in some other similar cases, the reservoir is not kept full: but the motor then works with constantly diminishing power till the store of energy is exhausted. In condensing steam engines an air pump is employed to exhaust the condenser.

The work done in pumping is found by the formulæ of the last article. Examples will be found at the end of this chapter.

274. *Indicator Diagrams.*—The pressure existing in the chamber of a pneumatic machine may be graphically exhibited by means of an instrument called an Indicator. In steam engines especially its use is indispensable to enable the engineer to study the action of the steam.

Figs. 1 and 2, Plate XII., show an indicator in elevation and section. S is a drum revolving in a vertical axis, A is a cylinder communicating with the steam cylinder, the pressure in which is to be measured. P is a pencil connected by linkwork with a small piston H so as to move with it up or down in a vertical line. The piston is pressed down by a spring which measures the pressure, while the drum, by means of a cord passing over pulleys and connected with the steam piston, revolves through arcs exactly proportional to the spaces traversed by it. A card is folded round the drum, and as the engine moves a curve is traced by the pencil upon it which shows the pressure at each point of the stroke. In practice many precautions are necessary to secure accuracy in the diagram; the more so the higher the speed, because the friction and inertia of the parts of the indicator, together with unequal stretching of the cord and inaccuracy in the reducing motion connecting the drum with the steam piston, may give rise to serious errors. To diminish the effect of inertia the stroke of the indicator piston is made short and multiplied by linkwork.

In the example shown (Crosby's patent) the spring applied to the drum to keep the cord tight has a tension which increases as the drum rotates from rest. This increase compensates for the inertia of the drum, and is said to give a more nearly uniform tension of the cord.

Fig. 196 shows an indicator diagram taken in this way from the high-pressure cylinder of a compound engine.

BB is the atmospheric line drawn on the card by the indicator

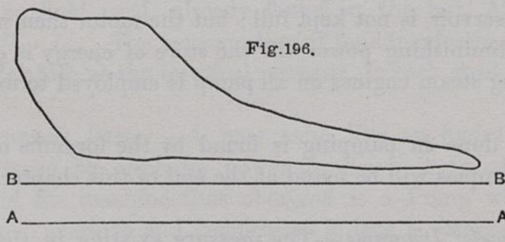
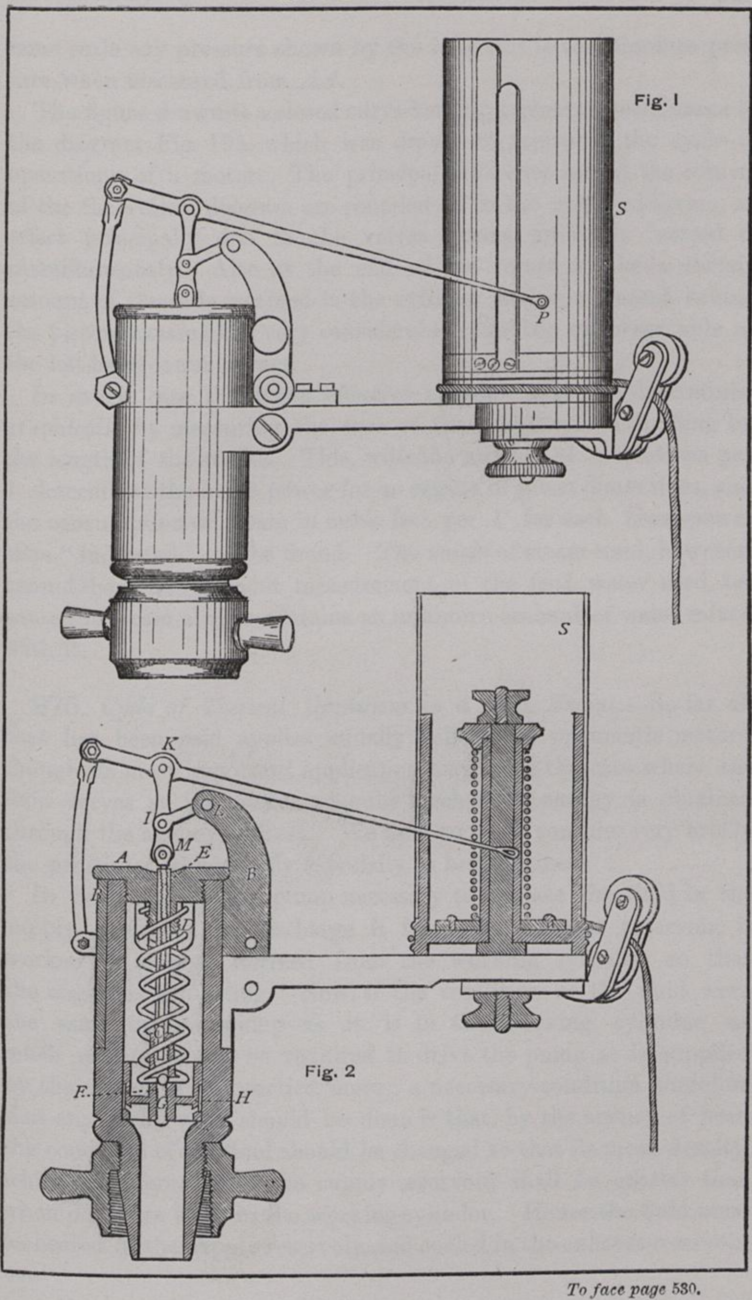


Fig. 196.

pencil when the cylinder communicates with the atmosphere. AA is the vacuum line laid down on the diagram at a distance below BB, which represents the pressure of the atmosphere, as found by the barometer, reckoned on the scale of pressures. Then on the

PLATE XI.



To face page 530.

same scale any pressure shown by the indicator is the absolute pressure when measured from *AA*.

The figure drawn is a closed curve bearing a general resemblance to the diagram Fig. 195, which was drawn to represent the cycle of operations of a motor. The principal difference is that the corners of the theoretical diagram are rounded off in the actual diagram, an effect principally due to the valves closing gradually instead of instantaneously. Also at the end of the return stroke a certain amount of steam is retained in the cylinder and compressed behind the piston, causing the very considerable rounding off observable at the left-hand lower corner.

In every case the mean effective pressure may be determined graphically by measuring the area of the diagram and dividing by the length of the stroke. This, with the number of revolutions per 1′ determines the horse power for an engine of given dimensions, and the consumption of steam in cubic feet per 1′ for each horse-power thus "indicated" can be found. The *weight* of steam used, however, cannot be found without measurement of the feed water used, because the steam always contains an unknown amount of water mixed with it.

275. *Cycle of Thermal Operations in a Heat Engine.*—So far all that has been said applies equally well to all pneumatic motors, though its most important application may be to the case where the fluid serves as the means whereby mechanical energy is obtained through the agency of Heat. We now go on to consider very briefly the principles which apply especially to heat engines.

In heat engines the pump necessary to replace the fluid in the supply reservoir, or discharge it from the exhaust reservoir, is worked by energy derived from the working cylinder, so that the engine is self-acting. Now, if the condition of the fluid were the same in the pump as it is in the working cylinder, as much energy would be required to drive the pump as is supplied by the motor, or, in practice, more; a necessary condition therefore that any useful work should be done is that, by the agency of heat, the condition of the fluid should be changed so that its mean density, while being forced into the supply reservoir, shall be greater than when doing its work in the working cylinder. Hence the fluid must be heated in the supply reservoir, and cooled in the exhaust reservoir, and therefore, in every heat engine, in addition to the cycle of mechanical operations there is a cycle of thermal operations consisting of an alternate addition and subtraction of heat; the heat in question being supplied by a body of high temperature and abstracted by a body of low temperature.

In non-condensing steam engines the pump is the feed pump which supplies the boiler with the fluid in the state of water; in the boiler heat is supplied which converts it into steam of density many hundred times less than that of water. The pump is, in this case, very minute, and requires a trifling amount of energy to work it. In condensing engines we have, in addition, the air pump.

In air engines the compressing pump is generally a conspicuous part of the apparatus and requires a large fraction of the power of the motor to drive it; because the changes of density due to the alternate heating and cooling are comparatively small.

276. *Mechanical Equivalent of Heat.*—Heat and mechanical energy are mutually convertible: a unit of heat corresponding to a certain definite amount of mechanical energy which is called the "MECHANICAL EQUIVALENT" of heat. In British units the thermal unit is equivalent to 772 ft.-lbs.

The statement here made is the First Law of the science of Thermodynamics, and it shows that quantities of heat may be expressed in units of work, and, conversely, quantities of work in units of heat. In dealing with questions relating to heat and work, a common unit of measurement must be selected. In most cases the thermal unit is adopted, and quantities of work reduced to such units by division by 772.

In heat engines the cycle of thermal operations consists of an alternate addition of heat (Q), and subtraction of heat (R), so that, if kU be the useful work,

$$kU = Q - R,$$

that is, the work is done at the expense of an equivalent amount of heat which disappears during the action of the engine. In steam engines this has been tested experimentally by measuring the heat supplied in the boiler and the heat discharged from the condenser. The difference should be, and in fact is found to be, the thermal equivalent of the work done by the engine. The ratio kU/Q is called the "absolute," or sometimes, for reasons we shall see presently, the "apparent" efficiency of the engine. It is always a small fraction: in the best steam engines, for example, it does not exceed ·13, losses connected with the furnace and boiler not being included. (Comp. Art. 278.) The quantity U is here the theoretical maximum (p. 528) for a pneumatic motor working between the given limits of pressure, and k is the "mechanical" efficiency.

277. *Mechanical Value of Heat. Thermal Efficiency.*—In stating the first law of thermodynamics nothing is said about the temperature at which the heat is used. In other words, the mechanical equivalent of heat is just the same whether the temperature be low or high. Yet common experience tells us that the value of heat for mechanical purposes depends very much on this circumstance. The heat discharged from the condenser of a condensing steam engine, or with the exhaust steam of a non-condensing engine, is of little value for the purposes of the engine. So obvious is this fact that the first attempts at connecting the work done by a heat engine with the heat supplied to it may be partly described as attempts to show that temperature, not quantity, was equivalent to energy, heat being supposed as indestructible as matter.

It is now known, however, that difference of temperature is not in itself energy, but merely an indispensable condition that heat may be capable of being converted into work. The power of a heat-engine depends on difference of temperature being greater, the greater that difference is; but in all cases only a fraction of the heat supplied is converted into mechanical energy.

In the converse operation of converting mechanical energy into heat it is possible, by employing it in overcoming frictional resistances, to obtain an amount of heat equal to the energy employed, but such processes are always irreversible. The only way of converting heat into work is by means of a heat engine in which the rejection of heat at low temperature is as essential as the supply of heat at high temperature.

Difference of temperature is wasted if heat be allowed to pass from a hot body to a cold one without the agency of steam, air, or some other body, the density of which is changed by its action. When once wasted it cannot be recovered, a fact of common experience which is expressed by stating the Second Law of Thermodynamics.

Heat cannot pass from a cold body to a hot one by a purely self-acting process.

By a "self-acting" process in this statement is meant any process of the nature of a perpetual motion which is independent of any external agency. By the employment of mechanical energy drawn from external bodies, heat may be made to pass from a cold body to a hot one, the amount of energy required being greater the greater the difference of temperature. And the method recently introduced of raising steam, without the use of a furnace, by means of heat derived from the exhaust steam condensed in a solution of caustic soda, shows that energy derived from chemical affinity may serve the purpose. But, if no energy is employed, no heat will pass.

Difference of temperature must therefore be carefully utilized, and since the smallest difference of temperature is sufficient to cause heat to pass from a source into the air or steam which exerts energy, it at once follows that the process of conversion of heat into work will be most efficient if all the heat be supplied while the fluid has the temperature of the source of heat, and all the heat rejected while it has the temperature of the body which subtracts heat. These are the conditions of maximum efficiency, and if they are satisfied it is possible to show that a mechanically perfect motor (p. 528) supplied with heat Q will exert the energy

$$U = Q \cdot \frac{t_1 - t_2}{t_1 + 461},$$

t_1, t_2 being the temperatures Fahrenheit of addition and subtraction of heat. This is true whatever be the nature of the heat engine employed for the purpose, and no more heat can be converted into work under any circumstances. A heat engine which satisfies these conditions may be described as "thermally perfect."

If two bodies be at the same temperature heat may be made to flow in either direction from one to another, the actual direction being determined by a difference which may be made as small as we please: that is, the process is *reversible*. Hence the conditions of maximum thermal efficiency may also be described by saying that the cycle of thermal operations must be "thermally reversible." And the condition that an engine may be both mechanically and thermally perfect may be completely described by stating that the engine is reversible.

Whichever way we adopt of stating the result it follows at once that a unit of heat has a certain definite MECHANICAL VALUE given by the equation

$$M = 772 \cdot \frac{t_1 - t_2}{t_1 + 461}.$$

where t_1, t_2 are the temperatures between which it can be used.

278. *Thermal Efficiency.*—If an engine be mechanically perfect the work done per unit of heat will be simply the mechanical value, if the conditions of maximum efficiency are satisfied. In general, however, some of the heat will be supplied at a lower temperature than the source of heat, and some will be abstracted at a higher temperature than that of the refrigerator. When this is the case difference of temperature is wasted and there is a corresponding loss of thermal efficiency. If the temperature is known at which the air or steam is, while it is being supplied with a certain quantity of heat, or while a certain quantity of heat is being abstracted, the mechanical value of that heat can be found corresponding to that temperature. This quantity represents the work actually done since the engine is supposed mechanically perfect, and the same calculation being made for all the heat supplied or abstracted, the total actual work will be known. Dividing this by the total quantity of heat the actual work (U) per unit of heat will be known. The ratio

$$e = \frac{U}{M}$$

may be described as the "THERMAL EFFICIENCY" of the engine.

In practice the engine will not be either mechanically or thermally perfect: its efficiency will then be the product (ek) of the mechanical efficiency and the thermal efficiency. The efficiency thus calculated is estimated relatively to an engine which is mechanically and thermally perfect, and may be described as the "relative" or "true" efficiency, as distinguished from the "absolute" or "apparent" efficiency defined in a former article.

To estimate the efficiency of a heat engine without any reference to the temperatures between which the heat can be used is very misleading. The true efficiency of the best condensing steam engines is about 50 per cent., instead of 13 per cent. as it appears to be merely from the quantity of heat used. In comparing engines of different kinds, however, the same limits of temperature should be employed. (See Appendix.)

279. *Compound Engines.*—The working fluid may be discharged from one contracting chamber into a second which simultaneously expands. In many cases an intermediate reservoir is employed, which receives the fluid from the first chamber and supplies it to the second; the two chambers are then virtually separate, and form two distinct motors, the power of which can be separately calculated. The sum of the two is the power of the compound motor; it is necessarily the same as if the fluid had been used with the same expansion curve between the same extreme pressures in a single chamber; except that the frictional resistance of the passages between the chambers and the intermediate reservoir represents a certain loss of energy in the compound motor which does not occur in the simple one. When there is no intermediate reservoir there is no distinct period of admission or expansion in the low-pressure chamber, but the power may still be determined graphically for each chamber, and the results added.

In every case the energy of the fluid is the same, and cannot be affected by the mechanism employed to utilize it, unless its density or elasticity be altered by contact with the sides of the chamber in which it is enclosed. In steam engines, however, the action of the sides of the cylinder has great influence by condensing steam as it enters the cylinder. The liquefied steam is re-evaporated towards the end of the stroke as the temperature of the steam falls, but the process is nevertheless a very wasteful one. The action is greater the greater the degree of expansion employed, because the range of temperature is greater, and the gain by expansion is thus in great measure neutralized or even converted into a loss. By employing two cylinders instead of one the expansion is divided into two parts each of moderate amount, and liquefaction may be in great measure avoided. Compound engines are therefore being used more and more wherever economy of fuel is a consideration, and in marine practice have almost superseded the simple engine.

The principal losses in steam engines are (1) a mechanical loss due to incomplete expansion, and (2) a thermal loss due to liquefaction. One of these cannot be diminished without increasing the other; but considerable economy may be effected by the use of a "steam jacket," by the employment of superheated steam, and by compounding.

280. *Internal Energy. Internal Work.*—The distinction between internal work and external work was pointed out in Art. 92, p. 199, and the corresponding distinction between internal and external energy of motion in Art. 134, p. 278. These distinctions are principally important in fluids, because the extreme mobility of their parts renders internal motions, of great magnitude, of common occurrence. We have already seen in Chap. XIX. how energy is dissipated by the internal action of liquids; in gases the same dissipation occurs, and is even more important.

In liquids the absorption of energy is completely irreversible, but in gases it is not so. We may have internal energy as well as internal work: the greater part of the expansive energy of a gas being due to internal actions.

The state of an elastic fluid is completely known when its pressure and volume are known, but these quantities are capable of any variation we please within wide limits, provided only that we have unlimited power of adding or subtracting heat. If, however, a third quantity, the temperature, be considered, it will be found that the three are always connected together by a certain equation depending on the nature of the fluid, so that when any two are given the third is known. For example, in the so-called "permanent" gases such as dry air, the equation is very approximately

$$PV = c \, . \, T,$$

where T is the temperature reckoned from the "absolute" zero, a point 461° F. below the ordinary zero of Fahrenheit's scale, and c is a constant which for pressures (P) in lbs. per square foot and volumes (V) in cubic feet per lb. has, for dry air, the value 53·2. The "state" of the fluid is completely known if any two of these three quantities are known, but not otherwise.

To produce a given change of *state* a certain definite amount of work must be done in overcoming molecular resistances; this is the internal work, and is the same under all circumstances. But in gaseous fluids, the molecular forces being reversible, may tend to give rise to the change of state, and then we have internal energy instead of internal work. Taking the first case: if the change be at constant volume this internal work will be the total work done; but in general the volume changes, and in consequence external work is done, the amount of which depends not merely on the change of state, but also on the way in which that change is carried out. The total amount of work is the sum of the internal and external work: it is done by the agency of heat energy supplied from without, so that we write

Heat Expended = Internal Work + External Work,

the three quantities being expressed in common units.

For the application of this equation to questions relating to the formation of steam the reader is referred to a treatise on the Steam Engine by the present writer. We have now to consider the flow of gases through pipes and orifices, for which purpose the equation is written

Expansive Energy = Internal Energy + Heat Supply,

or, in other words, of the whole expansive energy of the fluid, a part is derived from internal molecular forces, and apart from heat supplied from without.

If no heat is supplied from without the expansive energy is equal to the internal energy: this case is called "adiabatic" expansion, obtained by writing $n = 1{\cdot}4$ in the formulæ of Art. 270. More generally, it is shown in treatises on thermodynamics that the internal work done in changing the temperature of a lb. of air from T_1 to T_2 is $I_2 - I_1$, where

$$I = K_v \, . \, T = 2{\cdot}5 \; PV,$$

K_v being the specific heat at constant volume, which is $2{\cdot}5 \; c$. Hence the equation becomes for a heat supply Q

$$E = 2{\cdot}5 \; c \; (P_2V_2 - P_1V_1) + Q.$$

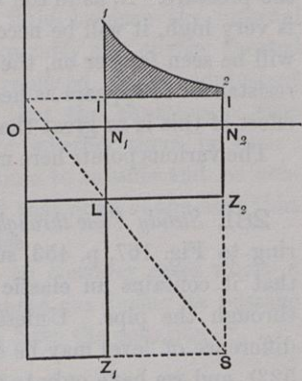
This is the fundamental equation from which all cases may be derived.

If heat be supplied to a permanent gas at a uniform rate as the temperature falls, it may be shown that the law of expansion is $PV^n =$ constant, as supposed in Art. 270, and this is generally permissible with sufficient approximation. The expansion index n then depends upon the proportion which Q bears to E. If $Q = E$ the expansion is hyperbolic, and the whole of the expansive energy is derived from heat supplied from without.* The manner in which the expansive

* For further explanation of the statements here made the reader is referred to Cotterill's *Steam Engine*, chapter IV.

energy (E) depends on the heat supply (Q) is well seen by the annexed diagram (Fig. 197). Let, as before, the ordinates of the point 1 represent the pressure and volume before expansion and those of the point 2 after expansion, 1, 2 being the expansion curve. Set downwards N_1Z_1, N_2Z_2 each equal to $2\frac{1}{2}$ the corresponding pressure ordinates, and complete the rectangles OZ_1, OZ_2. Then complete the rectangle Z_1Z_2, and draw the diagonal SL to meet the vertical through O. Finally through the intersection draw II horizontally: then the rectangle IN_2 will be found to be the difference of the rectangles OZ_1, OZ_2, and therefore represents the internal energy exerted during expansion. Thus the area $12II$ (shaded in the figure) represents the heat supply: which will depend not only on the points 1, 2, that is, on the change of state of the air, but also on the form of the expansion curve, that is, on the way in which the change takes place.

Fig. 197



When the pressure of air is changed the changes of temperature are generally so great that, unless the process be very rapid, they are accompanied by a flow of heat to or from external bodies. The amount of heat thus abstracted from, or supplied to the air, is unknown, and the index n cannot, therefore, be found accurately. An approximate value is assumed according to circumstances in each special case.

When air is compressed into a reservoir for the purpose of storing energy, its temperature cannot remain permanently above that of surrounding bodies, and the process will be most economical when the temperature is kept as low as possible by surrounding the pump with cold water. We then assume $n=1$, but the result of the calculation will generally be too small. The heat to be abstracted is the thermal equivalent of the absolute amount of work done in compression alone. The energy exerted in pumping is *greater* than this by the amount of energy transmitted by the air (if any) which leaves the reservoir, and *less* by the energy exerted by the back pressure which here serves as a source of energy. (See Art. 271, p. 525.)

When air expands from a reservoir in which it is stored at a moderate pressure it receives heat from without, but the amount is uncertain and cannot be relied on. We therefore assume $n=1\cdot4$, though the result of the calculation will be too small. The ratio of results for $n=1\cdot4$ and $n=1$ will be a measure, though a crude and imperfect one, of the efficiency of the process of storage and transmission of energy by compressed air. The efficiency is less the higher the pressure. If, as in the Whitehead torpedo, the pressure employed is very high, it will be necessary to reduce it by wire-drawing. As will be seen farther on, the mechanical energy dissipated by internal resistances re-appears as heat which maintains the temperature. The effect of this is so great that the process is (probably) economical.

The various points here mentioned are illustrated by Ex. 1-4, p. 552.

281. *Steady Flow through a Pipe. Conservation of Energy.*—Referring to Fig. 167, p. 453, suppose that the reservoir is closed, and that it contains an elastic fluid at high pressure which is flowing through the pipe. Unless the change of pressure be very small, difference of level may be disregarded as relatively unimportant (p. 522), and we have only to consider differences of pressure, while, on the other hand, we must now remember that, when the pressure changes, energy is exerted by expansion as well as by transmission. The energy transmitted from the reservoir *to* any point where the pressure is P and volume V is P_0V_0, where the suffix indicates the state of the fluid in the reservoir. Of this the amount PV is transmitted through the point, and the difference $P_0V_0 - PV$ together with the expansive energy E is employed in generating the kinetic energy which the gas possesses in consequence of the velocity u with which it is rushing through the pipe at the point considered. Thus, if the motion be steady,

$$\frac{u^2}{2g} = 3\cdot5\,(P_0V_0 - PV) + Q,$$

where Q is the heat (if any) supplied during the passage from the reservoir to the point. If no heat be supplied,

$$\frac{u^2}{2g} + 3\cdot5\,PV = \text{Constant},$$

an equation which may also be written

$$\frac{u^2}{2g} + K_p.T = \text{Constant},$$

where K_p is the specific heat at constant pressure. If we have to do with any elastic fluid other than a permanent gas, $3\cdot 5\ PV$ must be replaced by $I + PV$, where I is the internal energy, and if the question be such that the elevation of the point considered has any sensible influence, the term z must be added as in the corresponding case of an incompressible fluid.

The equation for a compressible fluid, however, is much more general than that for an incompressible fluid, because the internal energy is taken into account, and consequently any energy exerted in overcoming frictional resistances is replaced by an equivalent amount of heat generated. It follows that the equation is true whether there be frictional resistances or whether there be none: provided that the internal motions have time to subside and be converted into heat by friction, and provided that none of the heat thus generated is transmitted to external bodies.

It sometimes happens that we have to consider cases where a quantity of heat Q is supplied to a permanent gas during its passage from a point 1 to a point 2, we shall then have the equation

$$Q = K_p\ (T_2 - T_1) + \frac{u_2^2 - u_1^2}{2g},$$

an equation which is true, however great the variations of pressure or temperature are, and whether or not there are frictional resistances.

282. *Velocity of Efflux of a Gas from an Orifice.*—The most important applications of the equation for the steady flow of a gas are to the discharge of air or steam from an orifice and to the flow of air through long pipes.

In the first case the frictional resistances are small and are consequently neglected. It will be desirable to give a method of treating the question which is independent of the general equation.

In Fig. 198a $o12k$ represents the expansion curve for a small portion of the gas as it rushes out of the reservoir A (Fig. 198b) in which it is confined through a small orifice into the atmosphere. The jet contracts at issue to a contracted section kk, nearly as in the case where the fluid is incompressible, and then, in general, expands again in some such way as is shown in the figure. The velocity through the contracted section may be denoted by u, and the pressure there by P. The area of the contracted section is connected with the area of the orifice by the equation

$$A = kA_0,$$

as on page 449, k being a co-efficient.

Each small portion of the fluid expands from the state represented by the point o on the diagram to that represented by K:

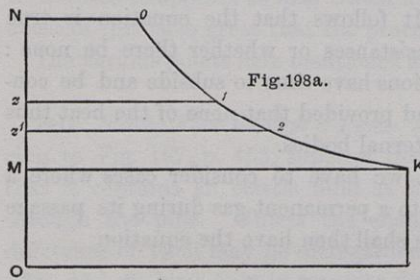


Fig.198a.

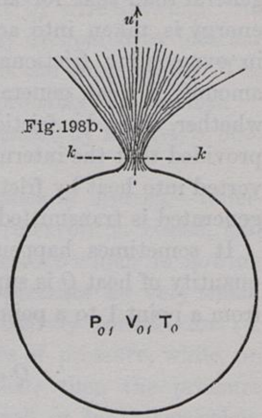


Fig.198b.

in some intermediate state it will be represented by a point 1 on the expansion curve, and immediately after by 2, a point near to 1. Let u_1, u_2 be the corresponding velocities, then

$$\frac{u_2^2 - u_1^2}{2g} = \frac{P_1 - P_2}{w} = V.\delta P,$$

where w is the mean density and V the mean specific volume represented graphically by the mean of $z1$, $z'2$. Hence $V.\delta P$ is represented by the area of the strip cut off by these ordinates. Dividing the whole area into strips, the area of each strip represents the corresponding change in $u^2/2g$, so that the total area represents the final value of this quantity. We have then

$$\frac{u}{2g} = \text{Area } No\kappa M = \int_P^{P_0} VdP = h.$$

The quantity h thus found and graphically represented is the "head" due to difference of pressure, as fully explained in Art. 271.

Assuming the expansion curve $PV^n =$ Constant, as before,

$$\frac{u^2}{2g} = \frac{n}{n-1}(P_0 V_0 - PV) = \frac{nc}{n-1}(T_0 - T).$$

Now, if the expansion be adiabatic $n = 1\cdot4$, and $nc/(n-1)$ is equal to K_p, so that the result might have been written down at once from the general equation of the preceding article.

Employing the notation of Art. 270, but replacing the suffix 1 by the suffix 0, the velocity of efflux is given by the formula

$$\frac{u^2}{2g} = \frac{n}{n-1} . P_0 V_0 \, (1 - rx).$$

283. *Discharge from an Orifice.*—The weight of gas discharged per second from an orifice of contracted area A is now found from the formula

$$W = \frac{Au}{V},$$

where V is the specific volume of the gas at the instant of passing through the contracted section, and therefore supposing A unity the weight per unit of area is given by

$$W^2 = 2g \, . \, \frac{n}{n-1} \, . \, \frac{P_0 V_0}{V} \, (1 - rx).$$

For V we now write rV_0 and finally obtain

$$W^2 = 2g \, . \, \frac{n}{n-1} \, . \, \frac{P_0}{V_0} \, . \, \frac{1 - rx}{r^2}.$$

In applying this formula x must be supposed known and r calculated from it by the equation on p. 524.

It will be found on examination that as x diminishes from unity W increases to a maximum value and then diminishes again to zero. That is, if the pressure in the throat of the jet at the contracted section be diminished the discharge does not increase indefinitely, but reaches a maximum and then decreases. On substitution for r in terms of x it will be seen that for a given pressure (P_0) in the reservoir W is greatest when $x^{\frac{2}{n}} - x^{\frac{n+1}{n}}$ is greatest.

This will be found to be the case when

$$x = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}.$$

The expansion is adiabatic, and the values of n with the resulting values of x for maximum discharge are shown in the annexed table.

NATURE OF GAS.	VALUE OF n.	VALUE OF x.
Dry Air, ...	1·408	·527
Superheated Steam, ...	1·3	·546
Dry Saturated Steam, ...	1·135	·577
Moist Steam, ...	1·1	·582
	1	$\frac{1}{\sqrt{\epsilon}} = \cdot 6$

The discharge is therefore a maximum when the external pressure is from ·5 to ·6 the pressure in the reservoir. For dry air it will be found on substitution that the maximum discharge per second per unit of contracted area is

$$W_m = \frac{3\cdot 9\, P_0}{\sqrt{P_0 V_0}} = \frac{P_0}{1\cdot 87 \sqrt{T_0}},$$

and for dry steam

$$W_m = \frac{3\cdot 6\, P_0}{\sqrt{P_0 V_0}}.$$

The pressure P_0 was originally supposed expressed in lbs. per square foot, but it may now be taken as lbs. per square inch in the numerator of these fractions, in which case W_m will be the discharge per square inch.

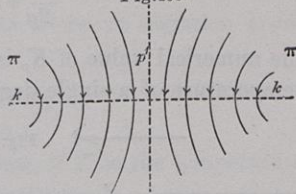
The diminution of the discharge on diminution of the external pressure below the limit just now given, is an anomaly which had always been considered as requiring explanation, and M. St. Venant had already suggested that it could not actually occur. In 1866 Mr. R. D. Napier showed by experiment that the weight of steam of given pressure discharged from an orifice really is independent of the pressure of the medium into which the efflux takes place; and in 1872 Mr. Wilson confirmed this result by experiments on the reaction of steam issuing from an orifice.*

The explanation lies in the fact that the pressure in the centre of the contracted jet is not the same as that of the surrounding medium.

* *Discharge of Fluids*, by R. D. Napier. Spon, 1866. *Annual of the Royal School of Naval Architecture for* 1874.

The jet after passing the contracted section suddenly expands, and the sudden change of direction of the fluid particles gives rise to centrifugal forces which cause the pressure to increase on passing from the surface of the jet to the interior on the principle explained on page 455. This will be better understood by reference to the annexed figure (Fig. 199) which shows a longitudinal section of the jet at the point where the contraction of transverse section is greatest. The particles describe curves the radius of curvature of which increases from a small minimum value at the surface k to an infinite value at the centre. The pressure p, increases from that of the medium (π) at k to a maximum p' at the centre, the increase being very rapid at first and afterwards more gradual. The problem is therefore far more complicated than we have supposed, each small portion of the jet having its own pressure and (consequently) its own velocity and density.

Fig. 199.



The results of experiment however suggest that an approximate solution may be obtained by the assumption of a mean pressure in the throat of the jet, with a corresponding mean velocity; this mean pressure being that which gives maximum discharge in every case in which that quantity is greater than π. At lower pressures it is to be assumed equal to π.

Adopting this hypothesis we see that whenever steam is discharged from a boiler the pressure in which is greater than, about, 25 lbs. per square inch absolute, or 10 lbs. above the atmosphere, the formula given above for maximum discharge is to be used. If we assume the mean value 252 for $\sqrt{P_0 V_0}$ this gives $p_1/70$ for the weight discharged from an orifice per square inch of effective area per second, the pressure p_1 being the absolute pressure in the boiler expressed in lbs. per square inch. Contraction and friction must be allowed for by use of a co-efficient of discharge, the value of which however is more variable than that of the corresponding co-efficient for an incompressible fluid. Little is certainly known on this point.

283. *Flow of Gases through Pipes.*—Returning to the general equation, we have now to examine the case where air or steam flows through a pipe of considerable length. As in the case of water, the frictional resistances are then so great that most of the head is taken up in overcoming them. The velocities of the fluid are therefore not excessive, and the value of $u^2/2g$ varies comparatively little.

Now in the equation

$$\frac{u^2}{2g} + K_p \, . \, T = \text{Constant}$$

the numerical value of K_p is about 184, and therefore a variation of temperature of a single degree will correspond to a great change in $u^2/2g$; it may therefore be assumed that the temperature remains very approximately constant, provided only that the difference of pressure at the two ends of the pipe is not too great compared with its length.

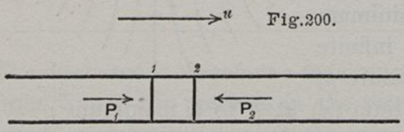


Fig. 200.

In Fig. 200 suppose 1, 2 to be two sections of the pipe at a distance Δx so small that, in estimating the friction, the velocity may be taken at its mean value u; then the force required to overcome friction is

$$R = f \, . \, s \, . \, \Delta x \, . \, u^2,$$

where s is the perimeter and f the co-efficient of friction, defined as on page 460. Replacing this by a new co-efficient f' as in the passage cited,

$$R = f' \, \frac{w}{2g} \, . \, s \, \Delta x \, u^2,$$

in which equation w means the weight of unit-volume of the gas. Now, it was pointed out on page 473 that surface friction was a kind of eddy resistance, and that in the case of water it was proportional to the density. This leads us to suppose that in fluids of varying density, not f but f' is a constant quantity. Replacing w by its equivalent $1/V$, we obtain

$$R = f \, . \, \frac{s\Delta x}{V} \, . \, \frac{u^2}{2g}.$$

We now apply the principle of momentum which will be expressed by the equation

$$(P_1 - P_2)A = W \, . \, A\frac{u_2 - u_1}{g} + R,$$

where W is the weight of gas flowing through the pipe per unit of area per second, and the suffixes refer to the two sections in question, the area of which is A. Now, the motion through the pipe being steady, W is the same throughout, so that

$$\frac{u}{V} = W = \text{Constant}.$$

By substitution for W and writing H for $u^2/2g$, an equation is obtained in the differential form

$$-V \,.\, \delta P = \delta H + f \,.\, \frac{\delta x}{m} \,.\, H,$$

m being the hydraulic mean depth. Now, if T be the temperature, which, as remarked above, is sensibly constant,

$$P = \frac{cT}{V} = W \,.\, \frac{cT}{u} ; \therefore \delta P = -W \,.\, \frac{cT}{u^2} \,.\, \delta u.$$

Substitute again for W and u, we then find

$$-V \,.\, \delta P = \frac{1}{2} cT \,.\, \frac{\delta P}{H}.$$

On substitution, the differential equation becomes integrable by dividing by H, and we obtain

$$\frac{1}{2} cT \left\{ \frac{1}{H_0} - \frac{1}{H} \right\} = \log_\epsilon \frac{H}{H_0} + f \,.\, \frac{l}{m},$$

where l is the length of the pipe, and H_0, H the values of $u^2/2g$ at entrance and exit respectively. In application of this equation the term containing the logarithm is small as compared with the rest, and may generally be omitted; also

$$\frac{H}{H_0} = \frac{p_0^2}{p^2},$$

a ratio which is known if the pressures are the data of the question.

The value of the co-efficient is found by experience to be nearly the same as for water, that is, about ·007. In the case of steam cT is to be replaced by the nearly constant product PV, which is to be taken from a table for this quantity so as to obtain a mean value according to the pressure considered.

The equation just found must not be applied to cases in which the difference of pressure is too great compared with the length of the pipe. The friction is then not great enough to prevent the velocity from becoming excessive; the temperature then sensibly falls, instead of remaining constant as supposed in the calculation. An equation can be found which takes account of the fall of temperature when necessary, but in such cases as commonly occur in practice, the supposition of constant temperature is amply sufficiently approximate. When the difference of pressure is small the equation will be found to reduce to the hydraulic formula for flow in a pipe. This case will be considered presently.

The head is given by the formula

$$h = PV \,.\, \log_\epsilon \frac{p_0}{p} = PV \log_\epsilon r,$$

and the power expended in forcing the air through is Wh or $PAu \,.\, \log_\epsilon r$ ft. lbs. per 1″.

284. *Flow of Gases under Small Differences of Pressure.*—When the differences of pressure are small and no heat is added or subtracted, a gas flows in the same way as a liquid of the same mean density. In the case of air the mean specific volume is found from the equation

$$V = \frac{cT}{P} = \frac{T}{40},$$

the units being feet and pounds, the mean pressure that of the atmosphere, and the temperature measured on Fahrenheit's scale from the absolute zero. At 59° this gives $V = 13$ cubic feet, but the actual volume will vary slightly from variations in the mean pressure.

The small differences of pressure with which we have now to do are commonly measured by a syphon gauge in inches of water. One inch of water is equivalent to a pressure of 5·2 lbs. per sq. ft.

If now ΔP be the difference of pressure in lbs. per sq. ft., i the corresponding number of inches of water, the head due to it will be, as in Art. 283,

$$h = V \,.\, \Delta P = \frac{T}{7{\cdot}7} \,.\, i.$$

The velocity due to this head, or, what is the same thing, the volume discharged per sq. ft. of *effective* area per second in the absence of frictional resistances, is in cubic feet

$$u = \sqrt{2gh} = 2{\cdot}89\sqrt{Ti},$$

and the weight-discharge in pounds per second

$$W = \frac{u}{V} = 115 \cdot 6 \sqrt{\frac{i}{T}}.$$

At 59° one inch of water gives a head of 67·5 feet and a discharge of 65·9 cubic feet, or 5·07 lbs. per second; but at 539° the head is 130 feet and the discharge 91·3 cubic feet, or 3·67 lbs., results which show that the effect of a given difference of pressure is entirely different according to the temperature of the flowing air. This is a point which must always be borne in mind in applying hydraulic formulæ to the flow of gases. Frictional resistances are taken into account by the employment of a co-efficient as in hydraulics, and as elsewhere explained, there is reason to believe that the values of these co-efficients are the same, except so far as they may be dependent directly or indirectly on the co-efficient of contraction (p. 463). Co-efficients of contraction are more variable in air than in water, but their average value does not differ widely in the two cases, and may provisionally be assumed the same.

In particular, it is well established that the formula for the discharge of a pipe in cubic feet per second (p. 462),

$$Q = k\sqrt{\frac{h'}{l}} \cdot d^{\frac{5}{2}},$$

applies to air as well as to water with the same value of the co-efficient k, that is (d in feet) about 40. The head h' is calculated, as just explained, according to the temperature of the air, for a given difference of pressure.

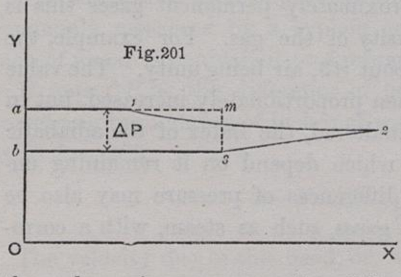
It is sometimes necessary to consider the flow of some gas other than atmospheric air. In approximately permanent gases this is easily done if we know the density of the gas. For example, the density of common coal gas is about ·43, air being unity. The value of c in the formula $PV = cT$ is then proportionately increased, but in other respects the formulæ are unaltered, the index of the adiabatic curve and the constants 2·5, 3·5 which depend on it remaining unaltered. The formula for small differences of pressure may also be employed for the non-permanent gases, such as steam, with a corresponding modification.

Pneumatic machines in which the variation of pressure is small are analogous to hydraulic machines, and most of what was said in the last chapter is applicable to them. The common fan, for example, is a centrifugal pump, the lift of which is the difference of pressure reckoned in feet of air, that is, at ordinary temperatures, about 67 feet for each inch of water. The speed of periphery is $\sqrt{gh}$ in feet per second, where h is the lift increased, as explained in the case of the pump, on account of frictional resistances and the curving-back of the vanes.

Fans are employed to produce a current of air for the purpose of ventilating a mine, ship, or structure of any kind. In mines they are often 30 feet diameter or more. The pressure required is here small and the speed moderate. They are also used to produce a forced draught in torpedo boats, or the blast of a smithy fire. The pressure is then 5 inches of water and more, corresponding to a lift of 300 feet and upwards. The speed of periphery is consequently excessive, and for the comparatively great pressures required for a foundry cupola or a blast furnace, it is necessary to resort to some other form of blowing machine. The speed of periphery might perhaps be reduced by the use of fixed guide blades to give the air a rotatory motion before entering the fan.

285. *Varying Temperature. Chimney Draught.*—If the temperature of the flowing air is varied by the addition or subtraction of heat, its density will be altered during the flow, and it is then necessary to know the mean density, in order that we may be able to calculate the "head" due to a given difference of pressure as measured by the water gauge.

In Fig. 201 OX, OY are axes of reference parallel to which ordinates are drawn as usual to represent volumes and pressures. A given difference of pressure ΔP is represented by the difference ab of a pair of ordinates. The original volume of the air is represented by $a1$. Suppose now that in flowing through a passage of any description the air is heated, as for example in passing through a furnace,



the volume increases greatly while the pressure falls slightly; this will be represented by the curve 12, terminating at a point 2. The form of the curve will depend on the law of heating, and will be very different according to the state of the fire; if the bars of the grate be blocked by clinker and the surface of the fire be free from special obstruction, most of the frictional resistance and corresponding fall of pressure will occur before the air is heated, and the curve will slope rapidly near 1 and slowly afterwards; while, conversely, if the fire be covered with fresh fuel and the grate bars clear, the reverse may be true. After being heated let the air pass through a boiler tube, by which heat is abstracted, till it reaches the chimney: the volume then diminishes greatly while the pressure falls slightly, as shown by the curve 23, terminating at a point 3, such that $b3$ represents the volume of the air in the chimney. The form of the curve 23 will depend on the law according to which the tube abstracts heat. The area of the whole figure $a123b$ represents the "head" due to the whole difference of pressure ΔP, and it will now be obvious that this head will vary according to circumstances which cannot be precisely known. Thus the mean density cannot be found except by empirical formulæ derived by direct experience, and consequently applicable only to the special cases for which they have been determined. It has hitherto been most usually assumed in the case of a furnace and boiler that the mean density was that of the air in the chimney, which amounts to supposing that the forms of the curves 12, 23 are such that the area of the rectangle $a3$ is equal to the area of the whole figure. This is the supposition employed by Rankine, and in many cases it appears to lead to correct results.

In every case of the flow of heated air it must be carefully considered what the mean density will probably be. Its value can often be foreseen without difficulty. It is only in the case of long passages, where the air suffers great frictional resistance while being heated or cooled, that it is uncertain what value to adopt.

The draught which draws air through a fire may be produced artificially or by the action of a chimney. In the latter case there is a difference of pressure within and without the chimney at its base due to the difference of weight of a column of air of the height of the chimney at the temperature of the chimney and at that of the atmosphere. Radiation causes the temperature of the air to be less in the upper part of the chimney and so diminishes the draught, and frictional resistances have the same effect. If these be disregarded the draught in inches of water will be

$$i = 7 \cdot 7 \left\{ \frac{1}{T_0} - \frac{1}{T} \right\} l,$$

where T_0 is the temperature of the atmosphere, T that of the chimney, while l is the height of the chimney in feet. The temperatures are here reckoned from the absolute zero.

If, for example, the temperature of the chimney be 539° F., and that of the atmosphere 59° F., the height of chimney required for a draught of 1 inch of water will be about 141 feet, or in practice more on account of friction and radiation.

The effect of this draught in drawing air through a furnace or through passages of any kind will vary according to the circumstances which have just been explained.

EXAMPLES.

1. Find the store of energy in the reservoir of a Whitehead torpedo. Capacity 5 cubic feet. Pressure 70 atmospheres.

Ans. If $n = 1$ 2,420,000 ft.-lbs., or 3,130 thermal units.
$n = 1{\cdot}4$ 1,092,000 ,, or 1,414 ,,
Ratio of results = ·45.

2. In the last question the air is supplied to the torpedo engines by a reducing valve, so that the pressure in the supply chamber remains constantly at 13 atmo spheres: find the available energy.

Ans. If $n = 1$ 1,900,000 ft.-lbs.
$n = 1{\cdot}4$ 1,346,000 ,,

NOTE.—The difference between these results and the preceding is the effect of wire-drawing (resistance of valve). The efficiency of the process may be taken as 1346/2420 or ·56. The supply chamber is supposed small.

3. Air is stored in a reservoir the pressure in which is maintained always nearly at 10 atmospheres: find the store of energy per cubic foot of air supplied from the reservoir.

Ans. If $n = 1$ 48,700 ft.-lbs.
$n = 1{\cdot}4$ 35,700 ,,
Ratio = ·733.

4. A chamber of 100 cubic feet capacity is exhausted to one-tenth of an atmosphere: find the work done, assuming $n = 1$.

Here if the chamber be imagined to contract, compressing the air still remaining in it, the energy exerted will be due to the pressure of the atmosphere, and the difference between this and the work done in compression will be available for other purposes. In exhausting this is reversed. *Ans.* 142,000 ft.-lbs.

5. Find the mechanical efficiency of an engine so far as due to incomplete expansion (ratio r): assuming the expansion hyperbolic.

Ans. If R be the ratio of complete expansion,

$$\text{Efficiency} = \frac{1 + \log_\varepsilon r - \dfrac{r}{R}}{\log_\varepsilon R}.$$

6. In the last question obtain numerical results for a condensing engine, taking the back pressure at 2 lbs. and boiler pressure 60 lbs.

Ans. Ratio of expansion,	1	2	5	10,
Efficiency, - -	·284	·48	·72	·87.

7. Find the comparative mechanical efficiencies in a condensing and a non-condensing engine. Back pressure in condensing engine 2, in non-condensing 16. Boiler pressure 60 and 100. Ratio of expansion 5 in both cases.

The engines must here be supposed to have the same lower limit of pressure of 2 lbs.; and the result for the non-condensing engine includes the loss by the actual back pressure being 16 lbs. *Ans.* ·72, ·46.

8. Find the loss by wire-drawing between two cylinders from one constant pressure of 60 lbs. to another constant pressure of 40 lbs. Expansion hyperbolic. *Ans.* ·405 PV.

9. One vessel contains A lbs. of fluid at a given pressure P_A, and a second B lbs. of the same fluid at a lower pressure P_B. A communication is opened between the vessels, and fluid rushes from A to B: find the loss of energy.

The loss here is the difference between the energy exerted by A lbs. expanding from V_A to V, and the work done in compressing B lbs. from V_B to V: where V_A, V_B are the specific volumes of the fluid in A and B, and V that of the fluid after equilibrium has been attained, found from the formula

$$V = \frac{AV_A + BV_B}{A + B}.$$

Hence the loss is very approximately

$$\text{Loss} = \frac{AB}{A+B} \cdot \frac{(V_B - V_A)(P_A - P_B)}{2}.$$

10. In a compound engine the receiver is half the volume of the high-pressure cylinder, and at release the pressure in the cylinder is 25 lbs. per square inch, while that in the receiver is 15 lbs. per square inch: find the loss of work per lb. of steam. Obtain the results also when the receiver is double instead of one half the cylinder.

Ans. Case I., 1638 ft.-lbs.
Case II., 3873 ,,

11. In a condensing engine find the mean effective pressure and the consumption of steam in cubic feet per I.H.P. per minute at the boiler pressure: being given, back pressure 3, boiler pressure 60 lbs. per square inch (absolute), ratio of expansion 5.

Ans. Mean effective pressure = 28·33 lbs. per square inch.
Consumption of steam = 1·62 cubic feet per minute.

12. If the volume of 1 lb. of dry steam at the boiler pressure be taken in the preceding question as 7 cubic feet and the liquefaction in admission 20 per cent.: find the weight of steam consumed in lbs. per I.H.P. per hour. *Ans.* 17·5.

13. Find the mechanical value of a unit of heat, the limits of temperature being 600° and 60°; 300° and 100°; 400° and 212°.

Ans. 393, 203, 169 ft.-lbs.

14. The limits of temperature in a heat engine are 350° and 600°; find the thermal efficiency when two-thirds of the whole heat supplied is used between 300° and 100°, one-sixth between 200° and 100°, and one-sixth between 250° and 100°. *Ans.* ·705.

15. In question 6, on account of a gradual increase in the liquefaction the thermal efficiency at the several ratios of expansion mentioned is assumed as ·9, ·85, ·7, ·5: find the true efficiency. *Ans.* ·256, ·408, ·504, ·435.

16. In a compound engine the pressure of admission is 100 lbs. per square inch, the steam is cut off at one-third in the high-pressure cylinder, the ratio of cylinders is $2\frac{1}{2}$: the back pressure is 3 lbs. per square inch, the large cylinder 40 inches diameter, and the speed of piston 400 feet per second. Find the H.P., neglecting wire-drawing and sudden expansion. *Ans.* 567.

17. In the last question suppose that the engine has a very large intermediate reservoir, and that the cut-off in the low-pressure cylinder is ·5; find the pressure in the reservoir, neglecting wire-drawing, also the loss per cent. by sudden expansion at exhaust from the high-pressure cylinder, and the percentage of power developed in the two cylinders.

Obtain the results also for a cut-off of one-third in the low-pressure cylinder.

Ans.

	Cut off $\frac{1}{2}$.	Cut off $\frac{1}{3}$.
Pressure in reservoir, ...	26·7	40
Loss by sudden expansion per cent., ...	·8	·7
Percentage of power in high-pressure cylinder ...	46·5	32·4
" " low-pressure " ...	52·6	67·6

18. Compare the efficiencies of the simple and compound engine, assuming the liquefaction the same at the best ratio of expansion, which is 5 in the simple engine and 7 in the compound engine, while in the latter 5 per cent. of the work is lost by wire-drawing between the cylinders. Back pressure and boiler pressure in both cases 3 lbs. and 84 lbs. respectively.

Ans. Gain by compounding $2\frac{1}{2}$ per cent.

19. In question 16, instead of supposing the whole expansion represented by a single hyperbolic curve, assume that at the end of the stroke in the high-pressure cylinder the steam is dry, while at the end of the stroke in the low-pressure cylinder the steam contains 10 per cent. water. Obtain the required result for the cut-off ·5 and find the weight of steam used (exclusive of jacket steam) in lbs. per I.H.P. per hour. Also obtain the results when the steam at the end of the stroke in the high-pressure cylinder contains 30 per cent. water, all other data remaining the same.

Ans.

	Case I.	Case II.
Pressure in reservoir, ...	22·5	14·9
Percentage of power in high-pressure cylinder,	55	37·5
" " low-pressure "	55	62·5
Lbs. of steam per I.H.P. per hour, ...	13	16·5

NOTE.—The results of this question may be readily obtained by use of tables of the properties of steam. They show clearly the great influence on the working of a compound engine of the relative liquefaction in the cylinders. If liquefaction be permitted in the high-pressure cylinder the compound engine loses its advantage.

20. Air at a pressure of 1000 lbs. per sq. inch and a temperature of 539° expands to 6 times its volume without gain or loss of heat; find the pressure and temperature at the end of the expansion. *Ans.* $p = 81$, $t = 27°$.

21. In the last question suppose the air at the end of the expansion to have a pressure equal to $1\frac{1}{2}$ times that given by the adiabatic law, and heat to be supplied at a uniform ratio as the temperature falls; find the index of the expansion curve and

the work done during expansion. Compare the heat supplied with the work done and find the specific heat.

Ans. $n = 1{\cdot}174$. Specific heat $= 1{\cdot}3$.
Work done $= 1{,}134{,}000$ ft.-lbs. Ratio $= {\cdot}435$.

22. Air is contained in a vessel at a pressure of 25 lbs per sq. inch and temperature 70°. What will be the velocity with which the air issues into the atmosphere (pressure 15 lbs. per sq. inch)? Also find the discharge and the head.

Ans. $h = 13{,}420$: $u = 930$ ft. per second.
$W = 34{\cdot}26$ lbs. per sq. inch of orifice per minute.

23. In the last question find the initial pressure corresponding to maximum discharge for all external pressures less than that of the atmosphere. Find this discharge. *Ans.* Pressure $= 28{\cdot}5$ lbs. per sq. inch.
Discharge $= 39\frac{1}{2}$ lbs per sq. inch per minute.

24. What weight of steam will be discharged per minute from an orifice 2 inches diameter, the absolute boiler pressure being 120 lbs. per sq. inch. Coefficient of discharge ·7. *Ans.* 227 lbs.

25. Air flows through a pipe 6 inches diameter and 4000 feet long; the initial pressure is 20 and the final pressure 15 lbs. per sq. inch; temperature 70°; find the velocities and the discharge. $4f = {\cdot}03$.

Ans. Velocity at entrance = 39 feet per second.
,, exit = 52 feet ,,
Discharge = 4 lbs. ,,

26. In the last question find the loss of head and the H.P. required to keep up the flow. *Ans.* $h' = 8124$ feet. H.P. = 59.

27. Steam at 50 lbs. rushes through a pipe 3 inches diameter and 100 feet long with velocity at entrance of 100 feet per second; find the loss of pressure. $4f = {\cdot}03$.

Ans. 1·6.

REFERENCES.

For descriptive details and illustrations of the mechanism of steam engines the reader is referred amongst other works to

THURSTON. *History of the Growth of the Steam Engine.* International Scientific Series. Kegan Paul.

SENNETT. *The Marine Steam Engine.* Longman.

RIGG. *Practical Treatise on the Steam Engine.* Spon.