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# CHAPTER XIX.

## ELEMENTARY PRINCIPLES OF HYDRAULICS.

**232.** *Velocity due to a Given Head.*—When the level of the surface of the water in a reservoir is above surrounding objects, a HEAD of water is said to exist, the magnitude of which is measured, relatively to any point, by the depth ($h$) of the point below the surface. If the water extend to this point a pressure is produced there which, so long as the water is at rest, is given in lbs. per sq. ft. by the formula

$$p = wh,$$

where $w$ is the weight of a cubic foot of water, that is to say, about $62\frac{1}{2}$ lbs. for fresh water, or 64 lbs. for salt. Since the above formula may be written

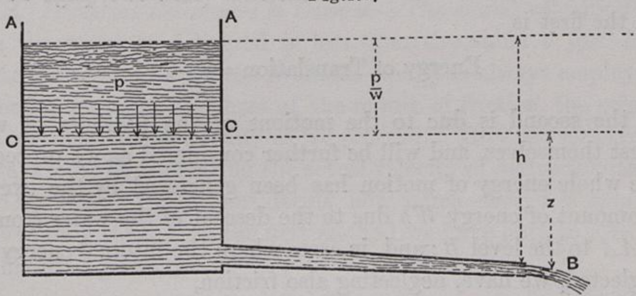
$$h = \frac{p}{w},$$

it appears that a pressure may be measured in terms of the head which would produce it. The fluid is usually water, for which $h$ is reckoned in feet; and 1 lb. per sq. inch is equivalent to 2·3 feet of fresh, or 2·25 feet of salt water. For some purposes, however, mercury is employed, in which case the unit is generally 1 inch. One inch of mercury is equivalent to about ·49 lb. per sq. inch, that is, to a head of 1·1 feet of sea water, or 1·135 of fresh water.

If the surface of the water be exposed to the atmosphere, the pressure $p$ will be in excess of the atmospheric pressure, which must be added to obtain the absolute pressure. The mean value of the atmospheric pressure is 14·7 lbs. per sq. inch, which corresponds to a head of about 33 feet for salt, or 34 feet for fresh water.

A head of water is a source of energy which may be employed in doing work of various kinds, or in simply transferring the water from one place to another. Let us take the second case, and imagine that, by means of a pipe, channel, or passage of any description, the water is delivered at $B$ (Fig. 163), while at the same time, by a stream or otherwise, the surface of the water in the reservoir is kept constantly at the same level $AA$, so that the head $h$ remains unchanged.

Fig. 163.




The motion is then described as Steady, and consists simply in the transfer in each second of a certain weight of water from the stream to the reservoir, while an equal weight traverses the passage, and is delivered at $B$, the whole mass of water between $AA$ and $B$ remaining constantly in the same condition. The delivery at $B$ may be supposed found by actual measurement; it is usually estimated in gallons per minute or cubic feet per second, as to which it need only be remarked that the gallon weighs 10 lbs., so that a cubic foot per second is about 375 gallons per minute. For large quantities, however, the cubic metre, which weighs about 1 ton, is also employed.

On delivery the water is moving with a certain velocity, but the definition and measurement of this quantity is not so simple. We must now suppose that the centre of gravity of the water delivered in some given time is observed and its velocity noted. This velocity will be the same whatever the time be, and will be a measure of the velocity of the mass of water considered as a whole. In some cases all particles of the water may be moving with this velocity, but in general this is not the case: it is then the mean velocity, and may be described as the "Velocity of Delivery." If the water be discharged by a channel which, near the exit, is of uniform transverse section $A$, this velocity may also be defined by the equation

$$v = \frac{Q}{A} = \frac{W}{wA},$$

where $Q$ is the discharge in cubic feet per second, and $W$ the weight of this quantity.

The energy of motion of the water may now be separated into two parts, one external and the other internal (Art. 134, page 278), of which the first is

$$\text{Energy of Translation} = \frac{Wv^2}{2g},$$

while the second is due to the motions of the particles of water amongst themselves, and will be further considered as we proceed.

The whole energy of motion has been generated by the exertion of an amount of energy $Wh$ due to the descent of the water from the level $AA$ to the level $B$; and, in cases where the internal energy may be neglected, we have, neglecting also friction,

$$\frac{v^2}{2g} = h,$$

where $h$ the head is measured to the centre of gravity of the issuing water (page 194).

It has been here supposed that the surface of the water in the reservoir, and after delivery at $B$, is exposed to the atmosphere, but this is not always the case. Suppose in the figure the reservoir filled to the level $CC$ only, but that the pressure on the surface has any value $p$, instead of being simply that of the atmosphere. This pressure $p$ may be produced by filling up the reservoir to the level $AA$ where

$$h = z + \frac{p}{w};$$

and as the reservoir is supposed large so that the water is sensibly at rest, except very near the exit, this can produce no change in the motion, which as before is given by

$$v^2 = 2gh = 2g\left(z + \frac{p}{w}\right).$$

In other words, in addition to the actual head $z$, we have a *virtual* head $p/w$, due to the difference of pressure $p$, thus giving a total head $h$.

The jet of water has been supposed to issue into the atmosphere, but the nature of the medium into which the discharge takes place has little influence, provided its pressure be duly taken into account. It has been proved by experiment that if the pressure of the atmosphere be artificially increased or diminished, the velocity is given by the same formula, modified as explained in the next article. This is also true if the efflux take place into a vessel of water.

**233.** *Frictional Resistances in General.*—The actual velocity $v'$ with which the water is delivered is less than the value $v$ just found, because a certain part of the available energy is always employed in overcoming certain resistances of the nature of friction, the origin of which we shall see gradually as we proceed. They are measured in two ways: (1) by comparing the actual velocity of delivery with that due to the head; (2) by considering how much energy is employed in overcoming them. In the first method we have only to introduce a co-efficient $c$ given by

$$v' = cv,$$

which is called the Co-efficient of Velocity. It is of course always less than unity, and its value is found by experiment in each special case. In the second we write

$$h - h' = \frac{v'^2}{2g},$$

where $h'$ is the "loss of head" due to friction. The value of $h'$ is most conveniently expressed by connecting it with the *actual* velocity $v'$ with which the water issues. For this purpose we replace $h$ by $v^2/2g$ and $v$ by $v'/c$, and thus obtain

$$h' = \left(\frac{1}{c^2} - 1\right)\frac{v'^2}{2g} = F\frac{v'^2}{2g},$$

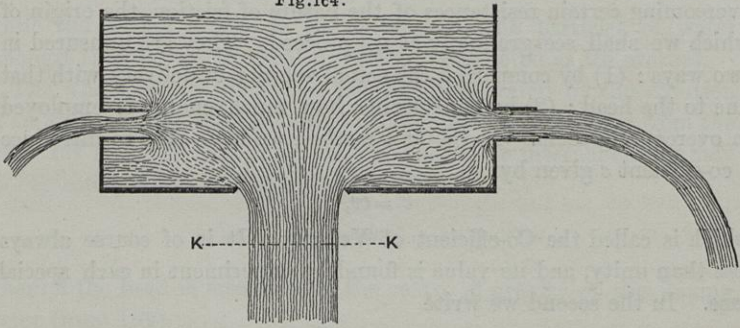
where $F$ is a new co-efficient called the Co-efficient of Resistance connected with the previous one by the equation

$$F = \frac{1}{c^2} - 1.$$

It is found by experience that the values of these co-efficients depend mainly on the form and nature of the bounding surfaces within which the water moves, and, subject to proper limitations, not on the pressure or velocity of the water—a fact which may be expressed by the following law of hydraulic resistance: *The energy lost by frictional* *resistances is a fixed multiple of the energy of motion of the water.* This multiple is the co-efficient $F$ which is sometimes fractional, but is often very large, as we shall see farther on. The physical meaning of this law will be seen hereafter, and the apparent deviations from it which frequently occur will be accounted for.

**234.** *Discharge from Small Orifices.*—Fig. 164 shows a vessel of water discharging through a circular hole in the bottom which is flat. The hole is small, and its circumference is chamfered below to a sharp edge at the upper surface.

Fig. 164.




On observing the jet of water which issues we see that it is nearly cylindrical but of diameter less than the diameter of the hole. The contraction is complete, so far as can be judged by the eye, at a distance of $d/2$ from the vessel; and by measurement is found to be in the ratio $4:5$, that is, the sectional area of the jet is to the sectional area of the hole in the ratio $16:25$.

If the hole be made in the vertical side of the vessel a contracted jet issues in the same way, but under the action of gravity it forms a curve which is very approximately parabolic in form, each particle moving nearly in the same way as a projectile *in vacuo*. This enables us to find the velocity of the efflux ($v'$) by observing a point through which the jet passes, and we thus obtain experimentally the value of the co-efficient $c$, which appears to be about ·97. The discharge is now given by the formula

$$Q = A_0 \,.\, v' = ckA\sqrt{2gh},$$

where $A_0$, $A$ are the contracted and actual areas of the orifice, and $k$

is their ratio which is a fraction called the Co-efficient of Contraction. The discharge therefore depends on the product of the two co-efficients $c$ and $k$, which may be replaced by

$$C = ck,$$

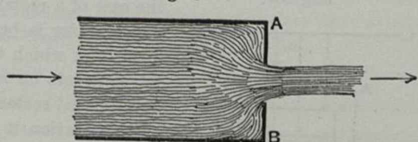
a quantity called the Co-efficient of Discharge.

The value of $C$ can also be determined by direct measurement of the discharge, an observation which can be made with much greater accuracy than those of contraction and velocity on which it depends. In the present case it is found to be ·62, agreeing well with the product ·97 × ·64 of the values given above.

With other forms of orifice the same co-efficients are used, but their numerical values are quite different. In the figure two cases are represented: on the right side of the vessel the water issues through a short pipe the entrance to which from the vessel is square-edged; on the left a similar pipe is employed but it projects inwards instead of outwards. When the pipe projects outwards the water is found to issue in a jet the full diameter of the pipe, that is, $k$ is unity; while, on the other hand, the velocity is much diminished, the value of $c$ being only ·815. When it projects inwards the jet contracts greatly, the value of $k$ being ·5 while the velocity is about the same as in a simple orifice. Thus $C$ instead of being ·62 is ·815 and ·5 in the two cases. The causes of these remarkable differences will be seen hereafter, the results are only given here to illustrate the meaning of the co-efficients under consideration.

**235.** *Incomplete Contraction.*—The contraction of the issuing jet depends on the average angle at which the moving particles converge towards the orifice before reaching it, and this is the reason why it is so great in the case of a short pipe projecting inwards. If the circumstances be such that the convergence is small the contraction diminishes. Fig. 165 shows a pipe of some size through an orifice in the flat end $AB$ of which water is being forced, issuing into the atmosphere. The co-efficient $k$ is found to depend on the proportion which the area of the original orifice $A$ bears to that of the pipe $S$, because the smaller $S$ is, the less is

Fig.165.



the angle of the convergence. This has been expressed by an empirical formula due to Rankine which may be written

$$\frac{1}{k}=\sqrt{2{\cdot}618-1{\cdot}618\frac{A^2}{S^2}},$$

which will be found to give $k=\cdot618$ when $S$ is infinite, as is nearly the case for a simple orifice as explained above, while for smaller values $k$ increases, becoming unity as it should when $S=A$.

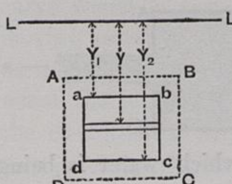
In a similar way if an orifice be near a corner of the vessel the contraction will be diminished. In these cases the contraction is usually described as "incomplete."

**236.** *Discharge from Large Orifices in a Vertical Plane.*—When the orifices are large, compared with the head and the vessel from which the discharge takes place, the question is more complicated.

If the plane of the orifice be vertical the velocities of the several parts of the fluid are not the same as is the case, so far as can be judged by the eye, when the orifice is small. On the contrary the velocity of that part of the stream which issues from the lower part of the orifice is visibly greater than that proceeding from the upper part. Hence it follows that the centre of gravity of the fluid issuing in a given time, to which the head is measured, is not on the same level as the centre of gravity of the contracted section, but lies below it. The corresponding point on the section may be described as the Centre of Flow. Also the internal energy of motion of the jet is of sensible magnitude and cannot be neglected.

By supposing that each part flows independently of the rest the discharge can be found for an orifice of any shape. For example, take the case of a rectangular orifice $ABCD$ (Fig. 166) from which water is being discharged from a reservoir, the level from which the head is measured being $LL$. The jet contracts on efflux, and the contracted section may be supposed rectangular. The position and dimensions of this section it will be necessary to suppose known by experiment; let its breadth be $b$, and let its upper and lower sides be at depths $Y_1$, $Y_2$ below $LL$. Divide the area into strips, and consider any one at depth $y$, then the velocity will be given by the formula (neglecting friction),

Fig.166.




$$v^2=2gy.$$

The quantity discharged per second will be given by

$$Q = \int_{Y_1}^{Y_2} bv \, . \, dy = b\sqrt{2g}\int_{Y_1}^{Y_2} \sqrt{y} \, . \, dy,$$

which by integration gives

$$Q = \tfrac{2}{3}b \, . \sqrt{2g} \, . \, (Y_2^{\frac{3}{2}} - Y_1^{\frac{3}{2}}),$$

which determines the discharge.

The energy of motion of the water discharged per second will be

$$U = w\int_{Y_1}^{Y_2} bv \, . \, \frac{v^2}{2g} \, . \, dy = bw\sqrt{2g}\int_{Y_1}^{Y_2} y^{\frac{3}{2}} . dy,$$

which by integration gives

$$U = \tfrac{2}{5}wb\sqrt{2g} \, . \, (Y_2^{\frac{5}{2}} - Y_1^{\frac{5}{2}}).$$

By dividing $U$ by $wQ$ we get the depth of the centre of gravity of the fluid discharged per second below $LL$, that is to say, the head $h$ is given by the formula

$$h = \tfrac{3}{5} \, . \, \frac{Y_2^{\frac{5}{2}} - Y_1^{\frac{5}{2}}}{Y_2^{\frac{3}{2}} - Y_1^{\frac{3}{2}}}.$$

The velocity of delivery is

$$V = \frac{Q}{A} = \sqrt{2g} \, . \, \tfrac{2}{3} \, . \, \frac{Y_1^{\frac{3}{2}} - Y_2^{\frac{3}{2}}}{Y_2 \; - Y_1},$$

and the energy of translation on delivery is

$$U_0 = \frac{WQV^2}{2g},$$

a quantity less than the whole energy $WQh$ by the energy due to internal motions. A common method of treating the question is to measure the head to the centre of the section and then employ the formula

$$V^2 = 2gh$$

with a proper co-efficient of discharge. This method is not exact, for it underestimates both the head and the energy of motion of the water; but its errors partially compensate one another, and its results are made to agree approximately with experiment by the employment of a variable co-efficient. To apply the exact formulæ it is necessary to know the dimensions and position of the contracted section for which the existing experimental data are insufficient. For further particulars on this subject, the reader is referred to Professor Unwin's work cited on page 481.

Again, if the dimensions of the orifice be not small compared with the surface of the water in the vessel from which the discharge takes place, this surface will sink with a velocity $V$ which is of sensible magnitude. If the area of the surface be $S$ and that of the contracted section $A_0$, the discharge will be

$$Q = A_0 v = SV,$$

an equation which determines $V$. The water will now have a velocity $V$ before descending through the height $h$, and the equation of energy is therefore

$$v^2 - V^2 = 2gh,$$

This may be written if we please

$$\frac{v^2}{2g} = h + \frac{V^2}{2g},$$

showing that in addition to the actual head $h$ we must consider the *virtual* head $V^2/2g$ due to the initial velocity of the water. In many hydraulic questions it is inconvenient or impossible to measure the head from still water. It is then measured from some point where the water is approaching the orifice with a velocity $V$ determined by observation. The actual head $h$ must then be increased by the height due to this velocity.

**237.** *Head relatively to Moving Orifices.*—The passages through which the water is moving may be attached to a ship, locomotive, or other moving structure, in which case the velocity must be reckoned relatively to the structure, and the height due to the velocity must be reckoned as part of the head. If for example in the bow of a vessel moving through the water with velocity $V$ an orifice be opened at the surface level, the water will enter through it, and if unacted on will move within the vessel with velocity $V$ and will possess relatively to the vessel the energy $V^2/2g$ per unit of weight. If it be acted on during entrance by the head due to any difference of level or pressure, so that its velocity is changed from $V$ to $v$, the corresponding change of energy will measure the work which is done, and therefore the equation $v^2 - V^2 = 2gh$ applies as before.

**238.** *Steady Flow through Pipes. Conservation of Energy.*—Fig. 167 represents a vessel of water discharging through a large pipe, the section of which varies according to any law. If the pipe "runs full," that is, if it be always completely filled with water, the discharge is

$$Q = A_1u_1 = A_2u_2,$$

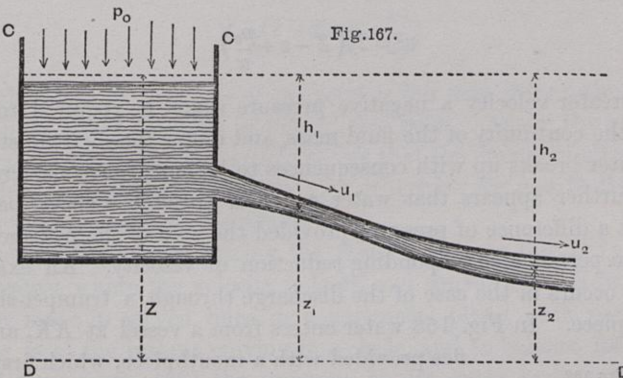
where $u_1$, $u_2$ are the velocities through two sections the areas of which are $A_1$, $A_2$. Hence the velocity is always inversely as the sectional area, and in an ordinary pipe in which the section is uniform must be the same throughout. Let the pressures be $p_1$, $p_2$, and the actual head, that is to say, the depths below the water surface $CC$, $h_1$, $h_2$, then it appears from Art. 234 that

$$\frac{u_1^2}{2g} = h_1 + \frac{p_0 - p_1}{w}; \ \frac{u_2^2}{2g} = h_2 + \frac{p_0 - p_2}{w},$$

where $p_0$ is the pressure on the surface $CC$.

Take now some convenient line $DD$ at a depth $Z$ below the water

Fig.167.




surface $CC$, and let $z_1$, $z_2$ be the elevation of the section above this datum level so that

$$z_1 + h_1 = Z = z_2 + h_2,$$

then the above equations may be written

$$\frac{u_1^2}{2g} + \frac{p_1}{w} + z_1 = Z + \frac{p_0}{w} = \frac{u_2^2}{2g} + \frac{p_2}{w} + z_2.$$

This result shows that if $u$, $p$, $z$ be the velocity, pressure, and elevation for any section of the pipe,

$$\frac{u^2}{2g} + \frac{p}{w} + z = \text{Constant}.$$

Each of the terms of this equation represents a particular kind of energy: the first is energy of motion, the third energy of position, the second is energy due to pressure, the origin of which will be further explained in the next chapter. The equation therefore shows that the total energy of the water remains constant as it traverses the pipe, and is accordingly the algebraical expression of the Principle of the Conservation of Energy. It supposes that no energy is lost by frictional resistances, and that any change in the internal motions of the particles amongst themselves may be disregarded. The word "head," the origin of which we have already seen, is frequently employed for the energy per unit of weight. (See Appendix).

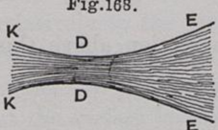
An important consequence of this principle is that where the sectional area of the pipe is least, and consequently the velocity greatest, there the pressure is least. Hence it follows that the velocity cannot exceed a certain limiting value $u$, found by putting $p=0$. At an elevation $z$ above datum level

$$u^2 = 2g\left(Z - z + \frac{p_0}{w}\right).$$

At a greater velocity a negative pressure would be required to preserve the continuity of the fluid mass, and under these circumstances the water breaks up with consequences to be hereafter considered.

It further appears that water can flow through a closed passage against a difference of pressure, provided the area of the passage vary so as to permit a corresponding reduction of velocity. An example of this occurs in the case of the discharge through a trumpet-shaped mouthpiece. In Fig. 168 water enters from a vessel at $KK$, an orifice provided with a mouthpiece, which first contracts to $DD$, and then expands to $EE$ where the jet enters the atmosphere. The pressure at $EE$ is that of the atmosphere, and therefore at $DD$ is less than that of the atmosphere, that is, less than it would be if the trumpet were cut off at the neck. Hence the discharge is increased by the addition of the expanded portion. If the water issued into a vacuum the jet would not expand to fill the wide mouth of the trumpet, which would not in that case have any influence on the discharge. The increased discharge and partial vacuum at $DD$ have been verified by experiment.

Fig.168.




**239.** *Distribution of Energy in an Undisturbed Stream. Vortex Motion.*—If the reservoir in the last article be imagined to supply a stream running in a channel of any size either closed or open, that stream, if undisturbed by any of the causes mentioned hereafter, may be supposed made up of an indefinite number of elementary streams, each of which moves as it would do in a closed pipe, as just described, without in any way intermingling with the rest. The forms of these ideal pipes depend solely on the form of the channel in which the stream is confined. The equation

$$\frac{u^2}{2g} + \frac{p}{w} + z = Z + \frac{p_0}{w}$$

applies to the motion in every pipe, and from it we may draw two important conclusions. In the first place, it may be written in the form

$$\frac{p - p_0}{w} = Z - z - \frac{u^2}{2g};$$

and therefore, *the pressure at any point is less than if the water were at rest by the height due to the velocity at that point.* Again, the equation interpreted as in the last article shows that the energy of all parts of the fluid is the same, or, as we may otherwise express it, *the energy of the fluid is uniformly distributed.*

From either way of stating the result it appears that the pressure is greatest where the velocity is least, and conversely. Now, if the water move in curved lines in a horizontal plane, each particle of water is at the instant moving in a circle, and to balance its centrifugal force (Art. 132) the pressure on its outer surface must be greater than that on its inner. It follows therefore that, if a channel is curved so as to alter the direction of the stream, the pressure increases as we go from the inner side of the channel to the outer; while, on the other hand, the velocity is greatest at the inner side and least at the outer. The change is the greater the sharper the bend, for the centrifugal force is greater. In open channels the change at the surface where the pressure is constant is in elevation instead of in pressure.

The magnitude of the change can be calculated in certain cases (see Appendix), of which we can only here consider one which is of special importance. If the particles of water describe circles about a common vertical axis, the elementary streams will form uniform rings, the centrifugal force of which can be calculated as in Art. 145, pp. 293-4. The resultant force on the half ring is—employing the notation of the article cited—given by

$$P = w \,.\, 2A \,.\, \frac{V^2}{g}.$$

This is balanced by an excess pressure on the outer surface of the half ring, and if that excess be $\Delta p$ the corresponding resultant force is $\Delta p \cdot 2r$, as shown on p. 305. Equating this to $P$

$$\Delta p = \frac{w}{g} \cdot \frac{A}{r} \cdot V^2.$$

The ring is supposed of breadth unity, and for $A$ we may write the thickness of the ring, which may be called $\Delta r$. Dividing by this, and proceeding to the limit

$$\frac{dp}{dr} = \frac{w}{g} \cdot \frac{V^2}{r},$$

an equation from which the pressure can be found if the law of velocity be given. If the fluid rotated about the axis like a solid mass, $V$ would vary as $r$; but the case now to be examined is that in which $V$ varies inversely as $r$, as expressed by the equation

$$Vr = \text{Constant} = k.$$

Substitute and integrate, then replacing $k$ by $Vr$, it will be found that

$$\frac{p}{w} + \frac{V^2}{2g} = \frac{p_0}{w} + \frac{V_0^2}{2g},$$

where the suffix refers to a given point where the pressure is $p_0$ and the velocity $V_0$. This result shows that the energy is uniformly distributed, and we infer that if the direction of a moving current is changed so that the particles of water describe concentric circles, the velocity varies inversely as the distance from the centre.

A mass of rotating fluid is called a "vortex," and in the case just considered the vortex is described as "free," because the motion is that which is naturally produced (comp. Art. 261). A free vortex is necessarily hollow, for to hold the water together a negative pressure would be required near the axis of rotation, but the hollow may be filled up by water moving according to a different law.

**240.** *Viscosity.*—When the motion of a mass of water is free from sudden changes of direction, loss of energy takes place only through the direct action of viscosity, a property of fluids which it will now be necessary briefly to consider. In Fig. 154, p. 409, a block of plastic material is represented, and it was explained that to produce change of form a certain difference of pressure was necessary, depending on the hardness of the material. In a fluid a similar difference of pressure is necessary to produce a change of form *at a given rate*, and the magnitude of the difference is proportionate to the rate. If $u$ be the rate at which the height of the block is diminishing and the breadth increasing, each reckoned per unit of dimension,

$$p = 2cu,$$

where $c$ is a co-efficient called the "co-efficient of viscosity." Or to express the same thing differently, if $\omega$ be the *rate* at which a small rectangular portion of the fluid is distorting, as in Fig. 140, p. 357, $q$ the corresponding distorting stress,

$$q = c \cdot \omega.$$

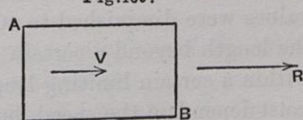
Hence, when a fluid moves, any change of form requires an amount of work to be done which is proportionate to the speed at which the change takes place. In a free vortex the rate of distortion is equal to the angular velocity of the particles round the axis, and varies inversely as the square of the distance; the changes of shape are therefore very rapid near the centre, and energy is consequently dissipated much more rapidly than in the stream from which the vortex is produced.

In the case of water the viscosity is so small that such changes of form as occur in an undisturbed stream are not rapid enough to absorb any large amount of energy. For example, in the discharge from orifices in a thin plate the loss of head is only 5 or 6 per cent. It is only when the water is disturbed by the neighbourhood of a rough surface over which it moves, or otherwise, that large quantities of energy are dissipated and frictional resistances of great magnitude produced.

**241.** *Surface Friction in General.*—We now proceed to study experimentally some of the more important causes of frictional resistance.

Fig. 169 shows a thin flat plate $AB$ with sharp edges completely immersed in the water. The plate is moving edgeways through the water with velocity $V$, then a certain resistance $R$ is experienced which must be overcome by an external force. This resistance consists in a tangential action between the plate and the

Fig.169.



water, and so far is analogous to the friction between solid surfaces but it follows quite different laws, which may be stated as follows:—

(1) The friction is independent of the pressure on the plate.

(2) It varies as the area of the surface in contact with the water.

(3) It varies as the square of the velocity.

These laws are expressed by the formula

$$R = fSV^2,$$

where $f$ is a co-efficient which, as in the friction of solid surfaces, is described as the "co-efficient of friction." The value of this co-efficient depends on the degree of smoothness of the plate. Thus, for example, in some experiments, to be described presently, on thin boards moving through water it was found that the co-efficient was ·004 for a clean varnished surface, and ·009 for a surface resembling medium sand paper, the units being pounds, feet, and seconds.

There are certain limitations to the truth of these laws, as in the case of solid surfaces. In the first place, if the velocity be below a certain limit the water adheres to the surface, and its velocity relatively to the surface is some continuous function of the distance from the surface so that the stream does not break up. This will be further referred to hereafter; for the present it is sufficient to say that the resistance then follows an entirely different law, varying nearly as the velocity instead of the (velocity)$^2$. The limiting velocity, however, at which this is sensibly the case is so low that in most practical applications the effect may be disregarded. In the second place, it is supposed that the water glides over all parts of the surface, with the same velocity; but if the surface be any considerable length the friction of the front portion of the surface on the water furnishes a force which drags the water forward along with the surface and so diminishes the velocity with which it moves over the rear portion. The friction is thus diminished, and in large surfaces very considerably diminished. Thus Mr. Froude experimenting on a surface 4 feet long, moving at 10 feet per second, found the value of $f$ given above, but when the length was 20 feet and upwards, those values were diminished to ·0025 and ·005 respectively. Increasing the length beyond a certain amount produces very little change, and within a certain limiting length the effect is insensible. These limits must depend on the speed, but no exact observations have been made on this point. The power of the speed to which the friction is proportional has, however, been found to be diminished on long smooth surfaces, as shown below. The skin friction of vessels on which, as we shall see hereafter, the resistance chiefly depends at low speeds, is much diminished by the effect of length.

Experiments on surface friction were made by Colonel Beaufoy. They formed part of an elaborate series of experiments on the resistance of bodies moving through water, carried out during many years in the Greenland Dock, Deptford. Beaufoy employed the formula

$$R=f\,.\,SV^n$$

to represent his results, and for the index $n$ obtained the values 1·66, 1·71, 1·9 in three series of experiments. The standard experiments on the subject are however due to the late Mr. Froude: they were made on boards $\frac{3}{16}$ inch thick, 19 inches deep, towed edgeways through the water. The boards were coated with various substances so as to form the surface to be experimented on.

The following table gives a general statement of Froude's results. In all the experiments in this table, the boards had a fine cutwater and a fine stern end or run, so that the resistance was entirely due to the surface. The table gives the resistances per square foot in pounds, at the standard speed of 600 feet per minute, and the power of the speed to which the friction is proportional, so that the resistance at other speeds is easily calculated.

| Nature of Surface. | Length of Surface, or Distance from Cutwater, in Feet. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2 Feet. | | | 8 Feet. | | | 20 Feet. | | | 50 Feet. | | |
| | A | B | C | A | B | C | A | B | C | A | B | C |
| Varnish, ... ... | 2·00 | ·41 | ·390 | 1·85 | ·325 | ·264 | 1·85 | ·278 | ·240 | 1·83 | ·250 | ·226 |
| Paraffin, ... ... | 1·95 | ·38 | ·370 | 1·94 | ·314 | ·260 | 1·93 | ·271 | ·237 | ... | ... | ... |
| Tinfoil, ... ... | 2·16 | ·30 | ·295 | 1·99 | ·278 | ·263 | 1·90 | ·262 | ·244 | 1·83 | ·246 | ·232 |
| Calico, ... ... | 1·93 | ·87 | ·725 | 1·92 | ·626 | ·504 | 1·89 | ·531 | ·447 | 1·87 | ·474 | ·423 |
| Fine Sand, ... | 2·00 | ·81 | ·690 | 2·00 | ·583 | ·450 | 2·00 | ·480 | ·384 | 2·06 | ·405 | ·337 |
| Medium Sand, | 2·00 | ·90 | ·730 | 2·00 | ·625 | ·488 | 2·00 | ·534 | ·465 | 2·00 | ·488 | ·456 |
| Coarse Sand, ... | 2·00 | 1·10 | ·880 | 2·00 | ·714 | ·520 | 2·00 | ·588 | ·490 | ... | ... | ... |

Columns A give the power of the speed to which the resistance is approximately proportional.

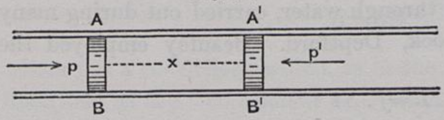
Columns B give the mean resistance per square foot of the whole surface of a board of the lengths stated in the table.

Columns C give the resistance in pounds of a square foot of surface at the distance sternward from the cutwater stated in the heading.

**242.** *Surface Friction of Pipes.*—When water moves through a pipe the friction of the internal surface causes a great resistance to the flow.

Fig. 170 shows a pipe of uniform transverse section (not necessarily circular) provided with two pistons, $AB$, $A'B'$, at a distance $x$, enclosing between them a mass of water. The pistons and included water move forward together with velocity $v$ under the action of a force $R$, required on account of the friction of the pistons and of the water on the pipe. Omitting piston friction the force $R$ will be given by

Fig.170.




$$R = f \, . \, S \, . \, v^2 = f \, . \, sx \, v^2,$$

where $S$ is the wetted surface and $s$ the perimeter.

If we imagine the pipe full of water moving through it with velocity $v$, the force $R$ is supplied by the difference of the pressures $p$, $p'$ on the pistons, and, therefore, if $A$ be the sectional area

$$p - p' = f \, . \, \frac{s}{A} \, . \, xv^2.$$

The quantity $A/s$ may be replaced by $m$ and is described as the "hydraulic mean depth" of the pipe, a term derived from the case of an open channel to be considered hereafter. In the ordinary case of a cylindrical pipe $m = \frac{1}{4}d$. Further, we may reduce the pressures to feet of water by dividing by $w$, and thus obtain for the difference of pressure $h'$

$$h' = f \, . \, \frac{x}{m} \, . \, \frac{v^2}{w} = f' \, . \, \frac{x}{m} \, . \, \frac{v^2}{2g},$$

where $f'$ is a co-efficient connected with $f$ by the equation

$$f = f' \, . \, \frac{w}{2g}.$$

The value of $w$, the weight of a cubic foot of water, differs so little from $2g$ that it is unnecessary, for our present purpose (Art. 283), to distinguish between $f$ and $f'$, especially as the value of $f$ is always determined by special experiment on pipes.

This formula for the head necessary to overcome surface friction is continually in use. The formula gives directly the head necessary for a length $x$ of the pipe, when the water, by being enclosed between pistons, is constrained to move over the surface with a given velocity: when the pistons are removed and the water flows freely it represents the facts very imperfectly. The central parts of the stream move quicker than the parts in immediate contact with the pipe, and besides, though the circumstances are different, we cannot be sure that the velocity over the internal surface is not affected in the same way as in the case of a moving surface. The value of $f$ has therefore to be obtained by special experiment, and the result of such experiments are by no means always in accordance with each other. It is found, however, that $f$ lies between the limits ·005 and ·01 according to the condition of the internal surface, and partly also on the diameter and velocity, the value being greater in small pipes than large ones, and at low velocities than high ones. For the present we assume ·0075 as roughly representing the facts when there is no special cause for increased resistance. For a pipe of circular section, length $l$, we have therefore

$$h' = 4f \cdot \frac{l}{d} \cdot \frac{v^2}{2g},$$

where for $4f$ we commonly assume the value ·03.

243. *Discharge of Pipes.*—The velocity $v$ is the actual velocity with which the water moves, so that $v^2/2g$ is the energy of motion of each pound of the water. The loss of energy by friction is the same as that of raising the water through a height $h'$, and is therefore equal to the energy of motion when

$$\frac{l}{d} = \frac{1}{4f} = 33 \text{ nearly,}$$

that is, a length of pipe equal to 33 diameters absorbs an amount of energy equivalent to the whole energy of motion of the water. In pipes of any length, therefore, the effect of friction is very great, so much so that the size of a pipe is principally fixed by the loss of head which can be permitted. It is easily seen that to deliver water with a given velocity the loss varies inversely as the diameter, and that to deliver a given quantity it varies inversely as the fifth power of the diameter; thus, the smallest permissible diameter is fixed almost entirely by the value of $h'$, which may be supposed already known.

The quantity discharged per second is given us by the formula

$$Q = Av = \frac{\pi}{4} d^2 v,$$

and on substitution this becomes

$$Q = \frac{\pi}{4} \sqrt{\frac{g}{2f}} \cdot \sqrt{\frac{h'}{l}} \cdot d^{\frac{5}{2}}.$$

All dimensions are here in feet and $Q$ is in cubic feet per second. If we require gallons per minute for a diameter of $d$ inches, the formula will be

$$G = C \cdot \sqrt{\frac{h'}{l}} \cdot d;$$

where $C$ is a constant the value of which, for $4f = \cdot 03$, is 30, but which is often taken somewhat less (say 27) to allow for contingencies.

**244.** *Open Channels.*—Returning to Fig. 170, suppose the pipe, instead of being horizontal, is laid at an angle $\theta$ (see Fig. 171 next page), so that the difference of level of the two ends is $y = l \cdot \sin \theta$, then the difference of pressure-head is

$$\frac{p - p'}{w} = f \cdot \frac{1}{m} \cdot \frac{v^2}{2g} - y,$$

and therefore may be made zero if the slope of the pipe be

$$\sin \theta = f \cdot \frac{1}{m} \cdot \frac{v^2}{2g} = \frac{h'}{l}.$$

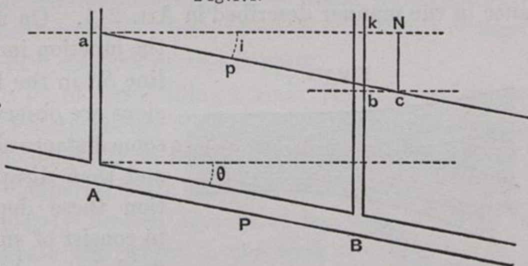
But if the pressure be constant we may remove the upper surface of the pipe and thus obtain the case of an open channel. The quantity $m$ is now the sectional area of the channel divided by the wetted perimeter, and is therefore the actual depth in a very broad shallow channel, but in other cases less in a ratio dependent on the form of section. As before stated it is described as the "hydraulic mean depth" of the channel.

We can now find the velocity and discharge of a stream of given dimensions and fall, provided that we know the value of $f$, or conversely the size of channel for a given discharge and fall. The value of $f$, however, varies for the same reasons as in pipes which indeed apply with still greater force, so that the limits of variation are wider. The average value does not differ very widely from ·0075, already adopted for pipes; but to obtain results of even moderate accuracy a special study of the experiments on the subject is necessary, which will not be attempted in this treatise.

**245.** *Virtual Slope of a Pipe.*—If the pipe be laid at any other angle the pressure will not be constant, and the mode in which it varies is best seen by a graphical construction.

Suppose small vertical pipes $Aa$, $Bb$ to be placed at the end of $A$, $B$ of the pipe we are considering (Fig. 171), then (if they enter the water square, without being bent towards the direction of motion) the water will rise in them to a level representing the pressure in feet of water at these points. If there were no friction the level would be the same in both, and the difference ($bk$ in the figure) therefore represents the loss by friction. Now draw a horizontal line through $b$, and take $c$ on it, so that $ac = AB = l$, then the angle $caN$ is given by the equation

Fig. 171.




$$\sin i = \frac{h'}{l},$$

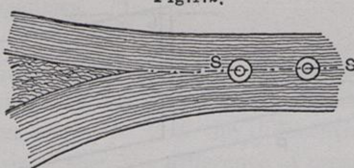
and is therefore the slope of a channel of the same length and hydraulic mean depth which would give the same discharge. This angle is therefore called the VIRTUAL SLOPE of the pipe. At any point $P$ in the pipe, the water would rise to the level of the corresponding point $p$ in the virtual channel, found by taking $ap = AP$. The construction would of course fail if $h'$ were equal to, or greater than $l$, but this case does not occur in practice; on the contrary, in pipes as in channels the angle $i$ is nearly always small. The virtual slope is frequently one of the data of the question. The line $ac$ is variously described as the "pressure line," "line of virtual slope," or "hydraulic gradient."

The pipe need not be straight; it may be curved or be laid in sections at different slopes, there will still be a continuous hydraulic gradient, provided the diameter be the same throughout; but if the sections be of different diameters each section will have its own slope. In practice care must be taken that the pipe does not rise above its hydraulic gradient, for otherwise there will be a partial vacuum: the pipe then acts as a syphon, which is liable to fail on account of leakage and the presence of air in the water.

**246.** *Loss of Energy by Eddies and by Broken Water.*—We now proceed to consider other causes of frictional resistance.

In Fig. 172 two streams of water, moving with different velocities, converge towards each other and unite into one. Each stream, so far as can be judged by the eye, moves originally without disturbance in the manner described in Art. 241. On union, however, near the junction indicated by the dotted line *SS* in the figure, small depressions are observed, which move for some distance along with the stream, and then disappear. On examination these depressions are found to consist of small portions of the fluid in a state of rotation, the speed of rotation being greatest at the centre and gradually dying away towards the circumference. A motion of this kind was called a "vortex" in Art. 241, and in the present case is also described as an "eddy"; it is independent of the general motion of the stream, and its energy is therefore of the internal kind. The disappearance of the eddies thus formed is due to viscosity, the effect of which is much greater in the eddy than in the stream as already explained. After the eddies have disappeared the two streams are found to have become a single one, moving with a velocity intermediate between those of the streams which form it, but possessing less energy. Theoretically there is nothing to prevent two streams of a perfect fluid from moving side by side with different velocities, but such a motion is always unstable, and will not long continue without the formation of eddies by a sudden change of direction (Art. 239) in small portions of the fluid which separate from the rest. The instability is greater the more nearly perfect the fluid is. When-

Fig.172.



ever then water in motion intermingles with water at rest, or moving with a different velocity, internal motions of a complex kind are produced, representing a considerable amount of energy of the internal kind which is virtually lost even before its final dissipation by fluid friction.

Again, in order that a mass of water may form a continuous whole, sufficient pressure must exist on the bounding surface to prevent the pressure at any point within the mass from becoming zero, as explained in Art. 240. If this condition is not satisfied the water breaks up more or less completely, and the result is a confused mass with complex internal motions rapidly disappearing as before by fluid friction. When waves break on a beach, or when paddles strike the water and drive it upwards in a mass of foam, the process takes place on a large scale before our eyes; but the same thing occurs in most cases where the velocity of a mass of water is suddenly changed, and of this we will now consider some examples.

Fig. 173*a* shows a jet of water filling a tank. Here the water pouring in possesses the kinetic energy $Wv^2/2g$ due to the original velocity of the water, and the height from which it falls into the tank. If it be of some size as compared with the tank the water will be completely broken up; if it be small it will penetrate the water in the tank without much apparent disturbance at the surface: in either case the result is a mass of water at rest as a whole, so that its energy is all of the internal kind. If the jet be shut off the water rapidly settles down to rest, the whole energy is then dissipated by fluid friction.

Fig. 173$_a$

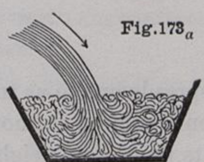


Fig. 173*b* shows a bucket moving horizontally, bottom foremost, with velocity $V$, while a horizontal jet moving with greater velocity strikes it centrally: the bucket is then filled with broken water which pours out under the action of gravity. In water-wheels a series of buckets are filled in succession, and the broken water carried on with the wheel. Here if the bucket were at rest the loss of energy would be, as before, $Wv^2/2g$; but as it is

Fig. 173$_b$

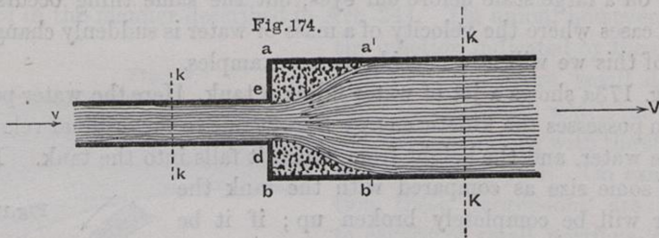
moving with velocity $V$, the striking velocity on which the breaking depends will be $v - V$, and the loss of energy is

$$U = W\frac{(v - V)^2}{2g},$$

where $W$ is the weight of water acted on in the time considered. Both these cases may be treated as examples of the collision of two bodies considered on page 280, one of the bodies being indefinitely great. The energy of collision is employed in breaking up the water. It is represented in the first instance by internal motions, and subsequently dissipated by fluid friction.

Fig. 174 represents a pipe which is suddenly enlarged from the diameter $cd$ to the diameter $ab$. The water is moving through the small part of the pipe with velocity $v$, and, on passing through $cd$

Fig.174.




spreads out so as to fill the larger part. At some distance from the enlargement it moves in a continuous mass with velocity $V$, but in its immediate neighbourhood we have broken water, as in the case of the bucket, from which it only differs in the enclosure of the water in a casing. The loss of energy per unit of weight may be expected to be the same (Art. 252) as before, and is therefore

$$h' = \frac{(v - V)^2}{2g},$$

a formula which gives us the "loss of head." If the sectional areas of the two parts of the pipe be $A$, $a$ the discharge is

$$Q = AV = av,$$

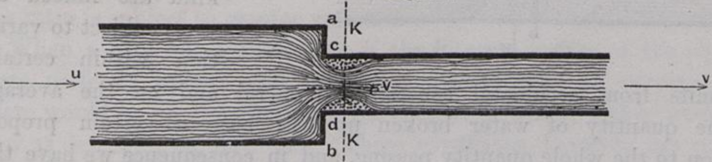
so that if $m$ be the ratio of areas,

$$h' = (m - 1)^2\frac{V^2}{2g} = \left(1 - \frac{1}{m}\right)^2\frac{v^2}{2g}.$$

The co-efficient of resistance is therefore $(m - 1)^2$ or $(1 - 1/m)^2$, according as the velocity to which it is referred is that in the large pipe or that in the small one.

Instead of the water moving from a small pipe into a large one, we may have the converse case of a suddenly contracted pipe as in Fig. 175. The loss here is due to precisely the same cause, namely a sudden enlargement, which is produced as follows. In the figure the stream of water moving with velocity $u$ contracts on passing through

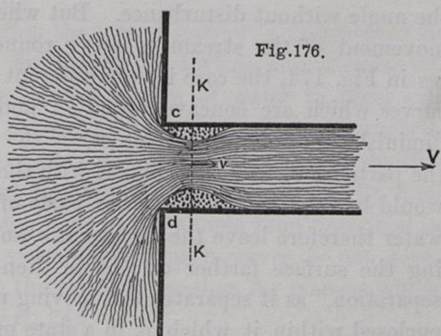
Fig. 175.




$cd$ nearly as it would if the small part of the pipe were removed, as in Fig. 165, p. 449, until it reaches a contracted section $KK$, and is then moving with a velocity $v$ which is greater than $u$ in the ratio of the area of the large pipe to the *contracted* area $KK$. The loss of head in this part of the process is not large. After passing $KK$, however, an expansion takes place to the area of the small pipe, and this is accompanied by breaking up, the space between the contracted jet and the pipe being filled up with broken water.

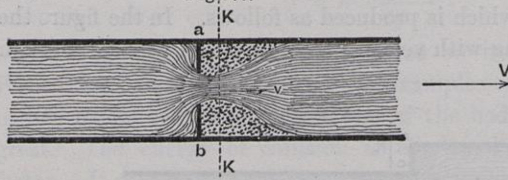
In Fig. 176 we have the extreme case, in which the large pipe is a vessel of any size. We thus obtain the case of a pipe with square edged entrance which has already been referred to in Art. 236. Another modification is that of a diaphragm in a pipe, as in Fig. 177. The small pipe is here larger than the orifice through which the water enters, and in the figure we have simply a single pipe divided into parts by a diaphragm with an orifice in the centre. The stream of water, after passing the contracted section $KK$, expands to fill the pipe. In cocks when partially closed, a loss of head of the same kind occurs, which may be increased to any extent by closing the cock further.

Fig. 176.




In all these cases the loss of head may be calculated approximately by means of the formula for a sudden enlargement, but the ratio of enlargement is not known exactly, on account of the uncertainty of the value of the co-efficient of contraction to be assumed. Losses of head of this kind are indeed always subject to variation within certain limits from accidental causes; in general and on the average the quantity of water broken up will bear a certain proportion to the whole quantity passing, and in consequence we have the general law of hydraulic resistance stated on page 447, but the ratio may vary from time to time, and cannot be stated with precise accuracy. The causes of this uncertainty will be clearly understood on considering somewhat more closely the manner in which the loss takes place.

Fig. 177.




In Figs. 175, 177 two plane surfaces at right angles meet at $a$, forming an internal angle, through which water is flowing. The particles of water there describe curves which are all convex towards $a$, and in conformity with the general principle explained in Art. 239, the pressure must increase and the velocity diminish on going towards $a$. The water then moves slowly and quietly round the angle without disturbance. But when compelled by the general movement of the stream to move round an external angle such as *kea* in Fig. 174, the case is very different; the particles then describe curves which are concave round $e$; and consequently the pressure diminishes in going towards $e$, while the velocity increases. To hold the particles of water in contact with the surface, an infinite pressure would be required in the other parts of the fluid. The particles of water therefore leave the surface at $e$, and describe a path $ea'$, regaining the surface farther on; $ea'$ is then described as a "surface of separation," as it separates the moving mass of water from a portion enclosed within it which is in a state of violent disturbance. Such are the surfaces shown in Figs. 171-178. It is not, however, to be supposed that these surfaces are sharply defined, and that they permanently separate different masses of water. On the contrary, no such equilibrium is possible; the surfaces are continually fluctuating, and a constant interchange takes place between the so-called "dead" water and the stream. In this intermingling eddies are produced nearly as in the comparatively simple case of two streams given on page 464. The process is always essentially the same, and consists in sudden changes of direction being communicated to parts of the stream which become detached from the rest.

**247.** *Bends in a Pipe. Surface Friction.*—In some other cases the process of breaking up by which energy is lost is less obvious, and the ratio is subject to greater variations.

When a pipe has a bend in it, if the internal surface of the pipe were perfectly smooth and free from discontinuity of curvature, there would be no disturbance of the current of water, which would flow as described in Art. 241. These conditions, however, are not satisfied by actual bends in pipes, and there is always a loss of head due to them in addition to the loss by surface friction. This loss can only be determined by experiment, but it is easy to conjecture that the loss will be proportional to the angle through which the pipe is bent, and that it will be greater the quicker the bend, that is, the smaller the radius of the bend is as compared with the diameter of the pipe. The extreme case of a bend is a knee, but the loss is not in this case proportional to the angle of the knee, but follows a complex law. For details respecting bends and knees the reader is referred to the treatises cited at the end of this chapter, but some common examples are given in the table (p. 470).

In the case of surface friction the loss of energy is represented in the first instance by eddies formed at the surface and thrown off. In almost all practical cases of the motion of water in pipes and channels, even when to all outward appearance quite undisturbed, the fluid is in fact in a state of eddy motion throughout, and dissipation of energy at every point is going on much more rapidly than would be the case if the motion were of the simple kind described in Art. 241. The quantity of water broken up, however, is not generally in a fixed proportion to the quantity passing, for reasons which are sufficiently indicated in Art. 242.

**248.** *Summation of Losses of Head.*—The total loss of energy due to a number of hydraulic resistances of various kinds is found by adding together the losses of head due to each cause taken separately. The velocity of the water past each obstacle will not generally be the same for all, and it is then necessary to select some one velocity from which all the rest can be found by multiplication by a suitable factor for each obstacle. If $n$ be this multiplier the loss of head will be

$$h' = \Sigma F n^2 \frac{V^2}{2g},$$

where $V$ is the velocity selected for reference. The value of $V$ is then found for motion under a given head $H$ by the formula

$$(1 + \Sigma F n^2)\frac{V^2}{2g} = H.$$

The various values of $F$ already given are collected with some additions in the annexed table :—

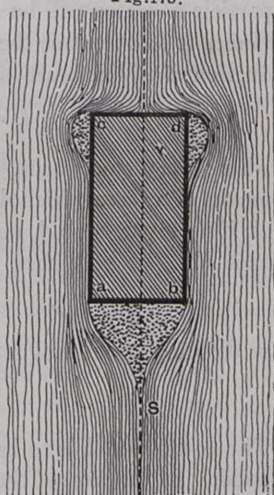
CO-EFFICIENTS OF HYDRAULIC RESISTANCE.

| NATURE OF OBSTACLE. | VALUE OF $F$. | REMARKS. |
|---|---|---|
| Orifice in a Thin Plate. | ·06 | |
| Square-edged Entrance of a Pipe. | ·5 | |
| Sudden Enlargement of a Pipe in the ratio $m : 1$. | $(m-1)^2$ | Referred to Velocity through large part of Pipe. |
| Bend at Right Angles in a Pipe. | ·14 | Radius of Bend = 3 × Diameter of Pipe. |
| Quick Bend at Right Angles. | ·3 | Radius of Bend = Diameter of Pipe. |
| Common Cock partially closed. | ·75, 5·5, 31 | Handle turned through 15°, 30°, 45° from position when fully open. |
| Surface Friction of a Pipe the length of which is $n$ times the diameter. | $4 f.n.$ | For a clean Iron Pipe $d$ inches diameter, according to Darcy, $4f = \cdot 02\left(1 + \frac{1}{d}\right)$ |
| Knee in a Pipe at Right Angles. | Unity | In Bends the co-efficient is proportional to the Angle of the Bend, but in Knees the law is much more complex. |

**249.** *Resistance of deeply Immersed Bodies. Ships at Low Speeds.*—The subject of the resistance of ships is outside the limits of this treatise, for the ship moves on the surface of water, exposed to the atmosphere, on which waves are produced; whereas in the branch of mechanics now under consideration, the water is supposed to move within fixed boundaries. A certain part of the subject, however, may properly be considered as belonging to Hydraulics. If a body be deeply immersed in a fluid, that part of the fluid alone which is in its immediate neighbourhood will be affected by its motion, and the question is not essentially different from the cases already considered of the movement of water in pipes and channels.

Fig. 178 shows a parallelopiped *abcd* moving through water in the direction of its length, the face *cd* being foremost. To an observer whose eye travels along with the body the water will appear to move past the solid in a stream of indefinite extent. At some distance away the action of the solid is insensible, but it becomes increasingly great as the solid is approached, and is greatest for that part of the water which moves in immediate contact with it. At *c* and *d* eddies are formed in passing round the corners exactly as is the case at the same points in Figs. 175, 176—the stream in fact is suddenly contracted in the same way as in passing from a large pipe to a small one, the diminution of area in this case being the transverse section of the solid. After this the water moves in actual contact with the solid until it reaches the corners *ab*, when it describes the curves *aS*, *bS*, meeting in *S*, after which it forms a continuous stream as before. The two curves enclose between them a mass of eddying water exactly similar to the eddies at *a* and *b* in Fig. 174—the stream, in fact, suddenly expands, just as in passing from a small pipe to a large one, the increase of area being in this case the sectional area of the solid. The eddies thus formed during the passage of the solid through the water absorb energy, which must be supplied by means of an external force, which drags the body through the water. The eddies

Fig. 178.



at $cd$ represent an increased pressure on the front face $cd$ of the solid, while those at $aS$, $bS$ diminish that at the rear. This kind of resistance to the movement of a body through water is called Eddy Resistance, and may be almost entirely avoided by employing "fair" forms, that is, by avoiding all discontinuity of curvature in the solid itself, and in the junction of its surface with the direction of motion.

A general formula for eddy resistance is derived thus. As already stated the water suffers no sensible disturbance at a certain distance from the solid. If then we imagine a certain plane area $A$ attached transversely to the solid, and moving with it, all the water affected by the solid will pass through this plane, and its quantity will be

$$Q = AV,$$

where $V$ is the velocity. In similar solids this area must be proportioned to the sectional area $S$ of the solid, so that we write $A = cS$, where $c$ is a constant depending on the form. Of this water a certain fraction will be disturbed by eddies, and the velocity of each particle of water will be some fraction of the velocity of the solid. Hence it follows that the energy $U$ generated per second in the production of eddies must be

$$U = c'wQ \cdot \frac{V^2}{2g} = cc'wS \cdot \frac{V^3}{2g},$$

where $c'$ is a co-efficient. Now this amount of energy is generated by means of a force which drags the solid through the water, at the rate of $V$ feet per second, notwithstanding an equal and opposite resistance $R$. We have then

$$RV = cc'wS \cdot \frac{V^3}{2g},$$

or dividing by $V$, and replacing $cc'$ by a single constant $k$,

$$R = k.wS \cdot \frac{V^2}{2g}.$$

The co-efficient $k$ is to be determined by experiment for each form of solid. In the case of the parallelopiped shown in the figure, the value of $k$ depends little on the length, unless it be so short that the eddies at the corners $cd$ coalesce with those in the rear of the solid, and it then becomes the same as that of a plate moved flatwise. Further it is nearly the same, if the transverse section be circular instead of square, and does not greatly differ from unity. For the flat plate it is greater and may be taken as 1·25. It must be remarked, however, that resistance of this kind is very irregular, and may vary considerably in the course of the same experiment. Different results are therefore obtained by different experimentalists. By some authorities much larger values are given. The same remarks apply to the case of a sphere for which the value may be taken as about ·4.

In all cases the value of $k$ is independent of the units employed. It is also to a great extent independent of the kind of fluid, being approximately the same for example in air as in water; but this would not hold good for fluids of very different viscosity; nor is it true for high speeds in air, because the compressibility of the air affects the question. The same remarks apply to the co-efficient ($F$) of hydraulic resistance employed above. It has been found that co-efficients of surface friction are greater in salt water than in fresh in the ratio of the densities of these fluids, as we might anticipate, since surface friction is a kind of eddy resistance.

In well-formed ships the eddy resistance should not be more than 10 per cent. of the total resistance at low speeds, and is frequently less; the principal cause of resistance here is surface friction, which is given by the formula stated in Art. 241. The surface to be considered is the wetted surface, which can be found by direct measurement. It is convenient, however, to have a formula which gives the resistance in terms of the displacement ($\Delta$) of the vessel. If $R$ be the resistance, $V$ the speed, the formula will be

$$R = K \,.\, \Delta^{\frac{2}{3}} V^2,$$

where $K$ is a co-efficient which for speed in knots per hour, displacement in tons, and resistance in lbs., may be taken from ·55 to ·85 according to the type of vessel, if the bottom be in good condition. The speed, however, must not exceed a certain limit on account of the wave resistance, which increases at a much more rapid rate. At low speeds the value of this resistance is small, and it may approximately be considered as compensating for the somewhat slower rate at which the surface friction increases (Art. 241), but at high speeds it becomes a principal part of the resistance, and has to be separately considered. The speed of a wave is proportional to the square root of its length, and the magnitude of the resistance in similar ships depends on the proportion between the length $L$ of the ship, and the length of waves which travel at the same speed. The limit in question is therefore given by the equation

$$V_0 = K' \,.\, \sqrt{L}$$

where for lengths in feet and speeds in knots per hour the co-efficient $K'$ may be taken from ·6 to ·7. (See Appendix.)

Not only is the speed limited to which the resistance formula given above applies, but it must be further remarked that it supposes that the vessel is towed by an external force. If the vessel be propelled by steam power on board, the effective resistance is much greater, because the action of the propeller has (probably always) the effect of increasing the resistance. In screw propulsion this augmentation is very great, being at the rate of 20 to 40 per cent. ; the larger value is of common occurrence. In reckoning the engine power required, the resistance must be taken at its augmented value, and the formula of Art. 128, p. 269, employed for the efficiency of the mechanism, which is much less at low speeds than at high speeds, as the formula shows.

**250.** *Direct Impulse and Reaction.*—The generalized form of the second and third laws of motion, described as the Principle of Momentum in Chapter XI. of this work, may be employed with great advantage when the motion of water in large masses is under consideration, because the total momentum of a fluid mass depends solely on the motion of the centre of gravity (p. 277), and not on the very intricate motions of the parts of the fluid amongst themselves. Further, the energy dissipated by frictional resistances is accounted for by these internal motions, or by the mutual actions of the fluid particles, and the total momentum is therefore independent of these resistances. Hence it follows that results may be obtained which are true notwithstanding any frictional resistances, and in some cases the loss of energy by them may be determined *a priori*. Also the pressures on fixed surfaces may be found which do no work, and to which therefore the principle of work does not directly apply.

Fig.179.

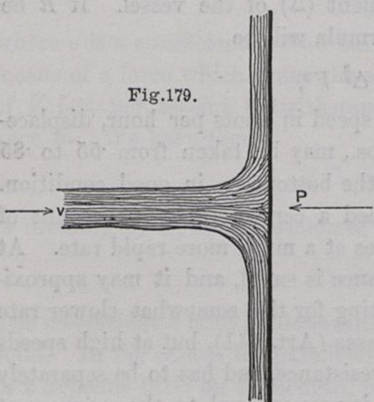


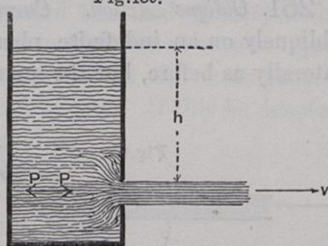

Fig. 179 shows a jet of water striking perpendicularly a fixed plane of infinite extent, and exerting on it a pressure $P$. The magnitude of this pressure is found by considering that the plane exerts an equal and opposite pressure on the water, which changes its velocity. The water, originally moving with velocity $v$, spreads out laterally; and any motion which it possesses is parallel to the plane. In time $t$ the impulse is $Pt$, and the change of momentum is $Mvt$, where $M$ is the mass of water delivered per second. Equating these we have

$$P = Mv = \frac{W}{g} . v,$$

where $W$ is the weight of water delivered per second.

If the plane be smooth, and gravity be neglected, the motion of the water will be continuous; but if it be rough to any extent, so that breaking-up occurs, the result will still be correct, provided only the roughness be symmetrical about the axis of the jet. And the action of gravity parallel to the plane does not affect the question.

Fig. 180.




In Fig. 180 we have the converse case of water issuing from a vessel with a lateral orifice. Here the water, which originally was at rest, issues with velocity $v$, and the momentum generated in time $t$ is $Mvt$. To produce this momentum a corresponding impulse is required, which is derived from the resultant horizontal pressure $P$ of the sides of the vessel upon the water. We have as before

$$P = Mv = \frac{Wv}{g}.$$

A pressure equal and opposite to $P$ is exerted by the water on the vessel: this is described as the "reaction" of the water; and, if the vessel is to remain at rest, must be balanced by an external force supplied by the supports on which it rests.

A remarkable connection exists between the change of pressure on the sides of the vessel consequent on the motion and the co-efficients of contraction and resistance.

First, suppose the water at rest, the orifice being closed, then the value of $P$ is zero, and the pressure on the area of the orifice is $w . A . h$, the notation being as in Art. 236. When the orifice is opened the pressure on that side is diminished, first, by the quantity $w \,.\, A \,.\, h$; secondly, by an unknown diminution $S$ due to the motion of the water (p. 455) over the surface near the orifice. Now

$$P = S + w \,.\, A \,.\, h = \frac{wA_0 v'^2}{g} = 2wA_0 (h - h'),$$

the notation still being as in the article cited. Replacing $A_0$ by $kA$ we obtain

$$S = wA \left\{2k (h - h') - h\right\} = wAh \left(\frac{2k}{1 + F} - 1\right).$$

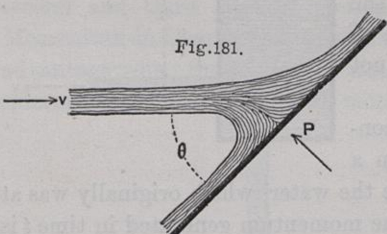
Since $S$ is always positive the least value of $k$ is

$$k = \frac{1 + F}{2}.$$

If there be no frictional resistances $k = \cdot 5$, and this is the smallest value $k$ can have under any circumstances. For a small pipe projecting inwards as in Fig. 164, p. 448, these conditions are approximately realized, the water being at rest over the whole internal surface of the vessel.

**251.** *Oblique Action. Curved Surfaces.*—When a jet impinges obliquely on an indefinite plane (Fig. 181), the water spreads out laterally as before, but the quantity varies according to the direction.

Fig. 181.



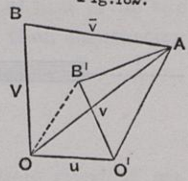

In the absence of friction the velocity of individual particles is the same as that of the jet in whatever direction the water passes. At the same time the velocity of the whole mass of water parallel to the plane cannot be altered by the action of the plane, and is therefore $v \,.\, \cos\theta$, where $\theta$ is the angle the jet makes with the plane. It immediately follows that any small portion of water diverging from $K$ the centre of the jet at an angle $\phi$ with the jet must be balanced by another portion diverging in the direction immediately opposite, and the quantities so diverging must be in the ratio $1 - \cos\phi : 1 + \cos\phi$, being inversely as the changes of velocity parallel to the plane. But if the circumstances be such that breaking-up takes place, the motion of the water parallel to the plane will be undetermined, and in general there will be a tangential action on the plane of the nature of friction.

The normal pressure on the plane is in all cases the same, being given by the formula

$$P = Mv \,.\, \sin\theta = \frac{W}{g} \,.\, v \,.\, \sin\theta.$$

If the surface on which the water impinges be curved it is necessary to know the average direction and magnitude of the velocity with which the water leaves the surface. In the absence of friction, as already noticed, the velocity of the individual particles is unaltered unless the water be enclosed in a pipe so that the pressure can be varied—a case for subsequent consideration; the direction, however, will depend on the way in which the water is guided. In cases which occur in practice it will generally be found either that the whole of the water is guided in some one direction, or that it leaves the surface in all directions symmetrically.

Fig.182.




Taking the first case, suppose the original velocity ($v$) of the water to be represented by $OA$ (Fig. 182), and the final velocity to be diminished to $V$ by friction, and altered in direction so as to be represented by $OB$. Then the change of velocity in the most general sense of the word (p. 275) is represented by $AB$. If this be denoted by $\bar{v}$ the change of momentum per second is

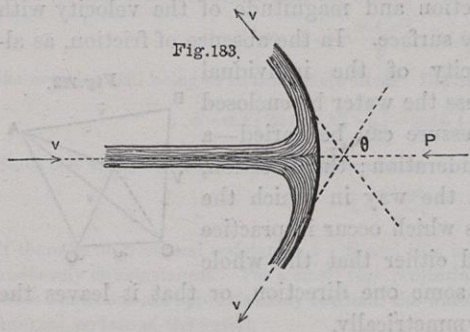
$$P = \frac{W\bar{v}}{g}.$$

The resultant pressure on the surface is parallel to $AB$ and numerically equal to $P$.

In applications to machines the curved surface is frequently a vane which is not fixed, but moves with a given velocity; the pressure can then be found by a simple addition to the diagram. Through $O$ draw $OO'$, representing the velocity ($u$) of the moving surface in direction and magnitude, then $O'A$ represents the velocity with which the water strikes the surface. Considering the vane as fixed, the velocity is now estimated with which the water would leave it, and $O'B'$ drawn to represent it; the change is now $AB'$ instead of $AB$. If the absolute velocity is required with which the water leaves the surface, it may be found simply by joining $OB'$, which will completely represent it: the change of velocity being $AB'$, whether the velocities are absolute or relative to the moving surface.

The cup vane $ACA$ (Fig. 183), against which a small jet of water impinges centrally, may be taken as an example where the water spreads in all directions symmetrically. If $OA$ be tangent to the vane at $A$, making an angle $\theta$ with the centre line of the jet, the water leaves the vane in the direction $OA$ with unaltered velocity (neglecting friction). The resultant pressure $P$ is in the direction of the jet, and the velocity in that direction is altered from $v$ to $v \cos\theta$ in the opposite direction, so that the change of velocity is $v(1+\cos\theta)$. Thus we have

Fig. 183.




$$P = \frac{Wv}{g}(1+\cos\theta).$$

**252.** *Impulse and Reaction of Water in a Closed Passage.*—When the water is moving in a closed passage the resultant pressure to be considered in applying the principle is not merely that on the sides of the passage, but also that on the ideal surfaces which separate the mass of water we are considering from the complete current. In the previous cases the pressure of the atmosphere on the free surface bounding the fluid was the same throughout, and was balanced by an equal pressure of the surface against which it impinges, which is not included in the preceding results. This is now no longer the case.

An important example is that of the sudden enlargement in a pipe already referred to in Art. 246. In Fig. 174, page 466, take ideal sections $KK$, $kk$ of the large and small portions of the pipe, and consider the whole mass of water between them. This mass is acted on (1) by the pressure ($p$) on the transverse section $kk$, (2) by the pressure ($P$) on the transverse section $KK$, and (3) by the pressure of the sides of the pipe. If we resolve in the direction of the length of the pipe, the only part of (3) which we need consider is the pressure ($p'$) on the annular surface $ac$, $bd$, the area of which is $A-a$, and the whole resultant pressure is therefore $PA - pa - p'(A-a)$ in the direction opposite to the motion of the water. Now let $W$ be the weight of water delivered in one second, then in that space of time $W$ passes from the small pipe, where its velocity is $v$, to the large pipe, where it has a velocity $V$, so that if we equate the resultant pressure to diminution of momentum

$$PA - pa - p'(A-a) = \frac{W}{g}(v-V) = \frac{wAV(v-V)}{g},$$

a formula which may be written

$$\frac{P}{w}-\frac{p}{w}=\frac{V(v-V)}{g}+(p'-p)(1-\frac{1}{m}),$$

$m$ being as in Art. 246 the ratio of enlargement. Let now $H$ be the total head in the large pipe and $h$ in the small one, then subtracting $(v^2-V^2)/2g$ from both sides and re-arranging the terms

$$h-H=\frac{(v-V)^2}{2g}+(p-p')(1-\frac{1}{m}).$$

Comparing this result with that obtained in the article cited, it appears that the value of the loss of head there given is a necessary consequence of supposing $p=p'$, but cannot otherwise be correct. That the pressure in the broken water at *ac*, *bd* is nearly equal to the pressure in the small pipe may be considered probable *a priori*, independently of the experimental verification which the formula has received.

## EXAMPLES.

1. The injection orifices of the jet condenser of a marine engine are 5 feet below the surface of the sea, and the vacuum is 27 inches of mercury: with what velocity will the water enter the condenser, supposing three-fourths the head lost by frictional resistances? Also find the co-efficients of velocity and resistance and the effective area of the orifices to deliver 100,000 gallons per hour. *Ans.* Velocity = 23·6′ per second; Area = 27 sq. inches.

2. Water is discharged under a head of 25′ through a short pipe 1″ diameter with square-edged entrance; find the discharge in gallons per minute. *Ans.* $66\frac{1}{3}$.

3. Water issues from an orifice the area of which is ·01 sq. feet in a horizontal direction and strikes a point distant 4′ horizontally and 3′ vertically from the orifices. The head is 2′ and the discharge 25 gallons per min.; find the co-efficients of velocity, resistance, contraction, and discharge. *Ans.* $c=$ ·816, $F=$ ·5, $k=$ ·57, $C=$ ·57.

4. The wetted surface of a vessel is 7,500 sq. feet, find her skin resistance at 8 knots and the H.P. required to propel her, taking the resistance to vary as $V^2$ with a co-efficient of ·004. *Ans.* Resistance = 5,600 lbs., H.P. = 137.

5. The diameter of a screw propeller is 18′, the pitch 18′, and the revolutions 91 per min. Neglecting slip find the H.P. lost by friction per sq. feet of blade at the tips, taking a co-efficient ·008 to include both faces of the blade. *Ans.* Friction = 65 lbs. per sq. feet. H.P. = 10·6.

6. Two pipes of the same length are 3″ and 4″ diameter respectively: compare the losses of head by skin friction (1) when they deliver the same quantity of water, (2) when the velocity is the same. *Ans.* Ratio = 4·21 and 1·33.

7. Water is to be raised to a height of 20′ by a pipe 30′ long 6″ diameter: what is the greatest admissible velocity of the water if not more than 10 per cent. additional power is to be required in consequence of the friction of the pipe? *Ans.* $8\frac{1}{2}$′ per sec.

8. Two reservoirs are connected by a pipe 6″ diameter and three-fourths of a mile long. For the first quarter mile the pipe slopes at 1 in 50, for the second at 1 in 100, while in the third it is level. The head of water over the inlet is 20 feet and that over the outlet 9 feet. Neglecting all loss except that due to surface friction, find the discharge in gallons per min., assuming $f = \cdot 0087$. *Ans.* $v = 3\cdot 43'$ per sec. Discharge = 253 gallons per min.

9. A river is 1000′ wide at the surface of the water, the sides slope at 45°, and the depth is 20′; find the discharge in cubic feet per sec. with a fall of 2′ to the mile, assuming $f = \cdot 0075$. *Ans.* 154,000.

10. A tank of 250 gallons capacity is 50′ above the street. It is connected with the street main, the head in which is 52′ by a service pipe 100′ long: find the diameter of the pipe that the tank may be filled in 20 min. What must the head in the main be to fill the tank in 5 min. with this service pipe? *Ans.* $d = 1\cdot 6''$. Head in main = 82′.

11. Water is discharged by a vessel from a long pipe: show that the discharge is the same for all pipes of the same length with the discharging extremity in the same horizontal line. Draw the hydraulic gradient and examine the case of a syphon.

12. In question 2 suppose the pipe instead of being short to be 25″ long, find the discharge, assuming for surface friction $f = \cdot 01$. *Ans.* 52.

13. A horizontal pipe is reduced in diameter from 3″ to $\frac{1}{2}''$ in the middle, the reduction being very gradual. The pressure head in the pipe is 40′, what would be the greatest velocity with which water could flow through it, all losses of head being neglected? *Ans.* 1·4′ per sec.

14. A pipe 2″ diameter is suddenly enlarged to 3″. If it discharge 100 gallons per min., the water flowing from the small pipe into the large one, find the loss of total head and the gain of pressure head at the sudden enlargement. State the two values of the co-efficient of resistance.

*Ans.* Loss of head $= 8\frac{1}{2}''$. $F = 1\cdot 59$ or $\cdot 31$.
Gain of pressure = 1′ 2″.

15. In the last question suppose the water to move in the reverse direction. Find the loss of head and the change of pressure consequent on the sudden contraction, assuming the co-efficient of contraction to be ·66.

*Ans.* Loss of head $= 7\frac{1}{2}''$.
Diminution of pressure = $2'\ 5\frac{3}{4}''$.

16. A horizontal pipe 30′ long is suddenly enlarged from 2″ to 3″ and then suddenly returns to its original diameter. Length of each section = 10′. Draw the hydraulic gradient when the pipe is discharging 100 gallons per min. into the atmosphere, assuming as coefficient of surface friction $4f = \cdot 03$. Find the total loss of head. *Ans.* Total loss of head = $10'\ 2\frac{1}{2}''$.

17. A pipe contains a diaphragm with an orifice in it the area of which is one-fifth the sectional area of the pipe. Find the co-efficient of resistance of the diaphragm, assuming the contraction on passing through the orifice the same as that on efflux from a vessel through a small orifice in a thin plate. *Ans.* $F = 46$.

18. Find the loss of head in inches due to a bend through 45° of radius 6″ in a pipe 2″ diameter, the velocity of the water being 12′ per sec. *Ans.* ·2″.

19. A plane area moves perpendicularly through water in which it is deeply immersed; find the resistance per sq. feet at a speed of 10 miles per hour. Deduce the pressure of a wind of 20 miles per hour using the same co-efficient. *Ans.* Resistance = 269 lbs. Wind pressure = 1·312 lbs.

20. Compare the resistance of an area moving flatwise through the water with its resistance moving edgewise so far as due to surface friction, the co-efficient for which is ·004. *Ans.* Ratio = 312.

21. In question 1 suppose the ship moving at 10 knots and the orifice of entry so arranged as to cause no additional resistance: find the velocity of delivery. *Ans.* Additional head = 4·42′; velocity = 25′ per sec.

22. Water is supplied by a scoop to a locomotive tender at a height of 7′ above the trough. Assuming half the head lost by frictional resistances, what will be the velocity of delivery when the train is running at 40 miles per hour, and what will be the lowest speed of train at which the operation is possible? *Ans.* 39′ per sec.; $14\frac{1}{2}$ miles per hour.

23. A stream of water delivering 500 gallons per min. at a velocity of 15 feet per sec. strikes an indefinite plane (1) direct, (2) at an angle of 30°; find the pressure on the plane.

24. Employ the principle of momentum to prove the formula on page 455 for the resultant centrifugal force of one-half a rotating ring of fluid.

## REFERENCES.

For further information on subjects connected with the present chapter, the reader is referred to a treatise on Hydraulics by Professor W. C. Unwin, M.I.C.E., forming part of the article Hydro-Mechanics in the edition of the "Encyclopædia Britannica" now (1883) in course of publication.