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CHAPTER XX.

HYDRAULIC MACHINES.

253. *Hydraulic Motors in General*—Hitherto the energy exerted by means of a head of water has been supposed to be wholly employed in overcoming frictional resistances, and in generating the velocity with which the water is delivered at some given point. We now proceed to consider the cases in which only a fraction of the head is required for these purposes; the remainder then becomes a source of energy at the point of delivery by means of which useful work may be done. A machine for utilizing such a source is called an Hydraulic Motor.

Hydraulic energy may exist in three forms, according as it is due to motion, elevation, or pressure. In the first two cases it is inherent in the water itself, being a consequence of its motion or its position as in the case of any other heavy body. In the third it is due to the action of gravity or some other reversible force, sometimes on the water itself, but oftener on other bodies, as, for example, the load on an accumulator ram. The water is then only a transmitter of energy and not directly the source of it. As, however, the energy transmitted is proportional to the weight of water delivered, just as in the two other cases, the water is as before described as possessing energy. The energy per unit of weight is called "head," as sufficiently explained in the preceding chapter, and the "total head" is the sum of the "velocity head," the "actual head," and the "pressure head."

Hydraulic motors are classed according to the mode in which the water operates upon them, which may be either by weight, or by pressure, or by impulse, including in the last term also "reaction."

254. *Weight Machines.*—To utilize a head of water, consisting of an actual elevation (h) above a datum level at which the water can be delivered and disposed of, a machine may be employed in which the direct action of the weight of the water, while falling through the height h, is the principal motive force.

The common overshot water-wheel (Fig. 2, plate III. p. 152) may be taken as a type. Here the driving pair is a simple turning pair, and the driving link is the force of gravity upon the falling water which acts directly on buckets open to the atmosphere. If G be the delivery in gallons per minute, the energy exerted in foot-pounds per minute is

$$E = 10Gh.$$

The head h is here measured from the level of still water in a reservoir which supplies the wheel. If v be the velocity of delivery to the wheel, the portion $v^2/2g$ is converted into energy of motion before reaching the buckets and operates by impulse. In a wheel of this class, therefore, the water does not operate wholly by weight. The speed of the wheel is limited to about 5 feet per second by the centrifugal force on the water, which, if too great, causes it to spill from the buckets. It will be seen hereafter that the velocity of the water should be about double this, so that v is about 10 feet per second, and the part of the fall operating by impulse is therefore about 1·5 feet. The remainder operates by gravitation, but a certain fraction is wasted by spilling from the buckets, and emptying them before reaching the bottom of the fall. More than one half the head operating by impulse is always wasted (Art. 260), and this class of wheels is therefore only suitable for falls exceeding 10 feet. The great diameter of wheel required for very high falls is inconvenient, but examples may be found of wheels 60 feet diameter and more. The efficiency of these wheels under favourable circumstances is ·75, and is generally about ·65.

In "breast wheels" the buckets are replaced by vanes which move in a channel of masonry partially surrounding the wheel. The water is admitted by a moveable sluice through a grating of fixed blades in the upper part of the channel. The channel is thus filled with water, the weight of which rests on the vanes and furnishes the motive force on the wheel. There is a certain amount of leakage between the vanes and the sides of the channel, but this loss is not so great as that by spilling from the buckets of the overshot wheel. The efficiency is found by experience to be as much as ·75. As the diameter of the wheel is greater than the fall a breast wheel can only be employed for moderate falls.

In both these machines the water virtually forms part of the piece on which it acts. This link of the kinematic chain forms one element of the driving pair, while that attached to the earth forms the other. In the overshot wheel the water is contained in open buckets, in the breast wheel it is contained in a closed chamber or channel. A third class of weight machines is referred to farther on under the head of pumps.

255. *Hydraulic Pressure Machines in Steady Motion.*—A water wheel of great diameter is a slow-moving cumbrous machine, and for heads of 100 feet and upwards it is therefore necessary to employ a pressure or an impulse machine. Such machines are also often more convenient for low falls.

In pressure machines the driving link is compressed water, which is forced between the elements of the driving pair by some source of energy which supplies the necessary head. The head is sometimes an actual elevation either natural or artificial: in the docks at Great Grimsby the hydraulic machinery is operated from a tank placed on a tower 200 feet high. It is however difficult to get a considerable pressure in this way, and an apparatus called an Hydraulic Accumulator is therefore generally resorted to. Two forms occur, of which one is shown in Pl. IX. In the first a plunger or ram is forced into a cylinder by heavy weights placed in a plate-iron cage suspended from it and stayed by iron rods. The accumulator is supplied by pumps generally worked by steam, which is the ultimate source of the energy, the accumulator merely serving the purpose of a store of energy which can be drawn on at pleasure. For ordinary hydraulic machinery the pressure is limited to 750 lbs. per square inch from the difficulty of obtaining pipes of sufficient strength and of working slide valves under heavy pressures. In machines for riveting and other special purposes, however, pressures of 1500 lbs. per square inch and upwards are employed. The accumulator then consists of a cylinder B (Fig. 1, Pl. IX., p. 497), loaded with ring weights EE, sliding on a fixed spindle F, divided into two lengths, of which the upper portion is of smaller diameter than the lower.

In either form the accumulator provides a store of compressed water which can be supplied by suitable pipes to any number of machines, placed often at considerable distances. A head of 1700 feet is thus readily obtained, and for special purposes much more: differences of level may therefore be disregarded as of small importance, and the water considered as operating wholly by pressure.

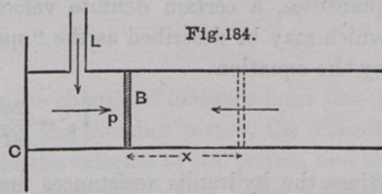


Fig. 184.

The driving pair of the machine forms a chamber of variable size which is alternately enlarged by the pressure of the water, and contracted to expel it. In most cases it is a simple cylinder C and piston B (Fig. 184): the water is admitted by a port from a pipe L, transmitting it from the accumulator at pressure p. Let the piston move through a space x, let A be its area, then

$$\text{Energy exerted} = pAx = p \,.\, X,$$

where X is the volume swept through by the piston. If w be as usual the weight of a cubic foot, wV is the weight of water which enters the cylinder as the piston moves through the distance x, and therefore

$$\text{Energy exerted per lb. of water} = \frac{p}{w} = \text{pressure head in cylinder.}$$

This might have been anticipated from what was said in the last chapter as to the meaning of the term "head," and in fact it is equally true if the driving pair be not a simple piston and cylinder, but of any other kind.

The head in the cylinder is less than that in the accumulator, on account of the friction in the supply pipe and other frictional resistances, and it is on the action of these resistances that the working of the machine depends. Let V be the velocity of the piston in its cylinder, p_0 pressure in accumulator, F the co-efficient of hydraulic resistance *referred to the velocity of the piston* (Art. 248), then, neglecting differences of level, also the heights due to velocities of working and accumulator pistons,

$$\frac{p_0 - p}{w} = F \,.\, \frac{V^2}{2g}.$$

If the machine be moving steadily the pressure p will be equal to the useful resistance which the piston is overcoming, increased by the friction of the piston in its cylinder. Thus p and p_0 will be known quantities, a certain definite velocity V_0 will then be determined, which may be described as the "speed of steady motion": it is given by the equation

$$V_0^2 = \frac{2g}{wF}(p_0 - p).$$

Since the hydraulic resistances may be increased to any extent at pleasure by the turning of a cock, it follows, that the speed of an hydraulic pressure machine can be regulated at pleasure. Further, if the resistance to the movement of the piston be diminished, the speed will increase only by a limited amount, and can, under no circumstances, be greater than is given by

$$\overline{V}_0^2 = p_0 \frac{2g}{wF},$$

which can be regulated as before. The surplus energy is here absorbed by the frictional resistances, and an hydraulic pressure machine therefore possesses the very important, and, for many purposes, valuable characteristic that *it contains within it its own brakes.*

256. *Hydraulic Pressure Machines in Unsteady Motion.*—Although the speed of a pressure engine cannot exceed a certain limit, which is easily found, yet it does not follow that that limit will ever be reached. When the engine starts, the piston and the water in the pipes have to be set in motion, the force required to do this is so much subtracted from that available to overcome resistances. A considerable time therefore elapses before a condition approaching steady motion can be obtained.

In Fig. 170, p. 460, water is supposed flowing through a pipe with a velocity u. Two pistons at a distance x enclose water between them, as in Art. 242, then the difference of pressure $p_1 - p_2$ in the case of steady motion is simply balanced by the surface friction, but in unsteady motion is partially employed in accelerating the flow of the water. Neglecting friction the acceleration g' will be given by the formula

$$(p_1 - p_2)\, A = W \,.\, \frac{g'}{g},$$

where A is the sectional area of the pipe and W is the weight of the water between the pistons. Replacing W by $Ax \, . \, w$, as in the preceding article,

$$\frac{p_1 - p_2}{w} = x \, . \, \frac{g'}{g},$$

which gives a simple formula for the change of pressure head due to inertia. Now if nA be the area of the working piston, the velocity of the water in the pipe is n times the velocity of the piston, and the accelerations are necessarily in the same ratio; and hence it follows that the difference of pressure head between cylinder and accumulator due to an acceleration g' of the piston is for a length of pipe l

$$\frac{p_1 - p_2}{w} = nl \, . \, \frac{g'}{g}.$$

The inertia of the piston itself requires a certain pressure to accelerate it. Let q_0 be the "pressure equivalent to that weight" found, as in Art. 109, page 235, then the pressure due to inertia is

$$q = q_0 \, . \, \frac{g'}{g};$$

then, dividing by w,

$$\frac{p_1 - p_2}{w} + \frac{q}{w} = \left(nl + \frac{q_0}{w} \right) \frac{g'}{g} = \lambda \frac{g'}{g},$$

where λ is a certain length. This may be described as the "length of working cylinder equivalent to the inertia of the moving parts," and may always be readily calculated for any given engine. (See Appendix.) The pressure in feet of water necessary to overcome inertia will then always be given by the simple formula

$$\text{Pressure due to inertia} = \lambda \frac{g'}{g}.$$

It will now be seen that the weight of water in the pipes and cylinders is so much added to the weight of the piston, that in the pipes being multiplied by the ratio of areas of cylinder and pipe. A water-pressure engine is therefore a machine with very heavy moving parts, a circumstance which greatly limits its speed irrespectively of frictional resistances. The smaller the pipes the heavier the parts virtually are, and this must be considered as well as friction (p. 461) in fixing their diameter.

It will be advisable to consider a particular case more in detail. Suppose, as is sometimes the case in practice, that a water-pressure engine is employed to turn a crank, and let us suppose that the crank shaft rotates nearly uniformly as in Ch. IX., then the difference between the pressure in the accumulator and that transmitted to the crank pin may be represented graphically thus :—

Let V be the velocity of the crank pin and let the stroke be $2a$ or AB in the diagram (Fig. 185). Set up

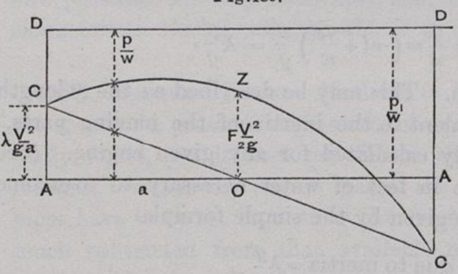
$$CA = \lambda \cdot \frac{V^2}{ga},$$

and draw the sloping line COC. Then, as in Art. 109, already cited, the ordinate of that line represents the pressure necessary to overcome the inertia of the piston and the water connected with it. Again, set up

$$OZ = F\frac{V^2}{2g} = CA \cdot \frac{Fa}{2\lambda},$$

and on the oblique base COC draw the parabola CZC, then (comp. Arts. 20, 109) the ordinate of this parabola will represent the pressure necessary to overcome the hydraulic resistances at every point. If then the horizontal line DD be drawn at a height representing the pressure in the accumulator, the intercept between that line and the parabola will represent the pressure transmitted to the crank pin at each point of the stroke. The slope of CC and the height of the parabola increase rapidly with the speed, which must never be great enough to cause the parabola to touch DD, otherwise a violent shock will occur. The same effect will be produced by any falling off in the useful resistance: the angular acceleration of the crank shaft then raises the central part of the line CC and with it the line of frictional resistances. It should be observed that the curve of frictional resistances may also be taken to represent the kinetic energy of the piston, both these quantities being proportional to the square of the velocity of the piston. It is therefore the graphical integral of the curve of acceleration (Ch. IX.).

Fig.185.



The simple example here given will serve as an illustration of the great variations of pressure which occur in water-pressure engines and their consequent liability to shocks. For which reason escape valves or air chambers must be provided to relieve the pressure when it becomes excessive. Unless the resistance be very uniform an additional accumulator is required as near as possible to the machine.

257. *Examples of Hydraulic Pressure Machines.*—Water-pressure engines form a large and interesting class of hydraulic motors of which a few examples will now be given.

(1) In direct-acting lifts a weight is raised by the direct action of fluid pressure on a ram the stroke of which is equal to the height lifted. The weight here rests on a cage or platform fixed to the upper end of the ram and sliding in guides. The water is frequently supplied from a tank at a moderate elevation, so that the pressure head diminishes as the lift rises, This is a very convenient arrangement for the purpose, as it supplies an additional pressure at the bottom of the stroke where it is required to overcome inertia at starting, and a diminished pressure at the top where the lift requires to be stopped. The useful resistance is here constant and the pressure head would be represented by the ordinates of a sloping line. A diagram of speed and acceleration may be constructed by a process similar to that given in the last article.

(2) A direct-acting lift necessarily occupies a great space, and the stroke of the working cylinder is therefore often multiplied by the use of blocks and tackle as shown in Fig. 2, Plate IX. The cylinder may be placed in any convenient position, and the chain passes from the blocks over fixed pullies to the cage which is suspended from it. The friction of the pullies is here considerable, and there is a liability to breakage; but for convenience the arrangement is one which is frequently employed.

(3) In hydraulic cranes the working cylinder is sometimes placed below and sometimes occupies the crane post which is tubular. The stroke is multiplied by tackle as in the previous case, the chain passing through the crane post and over fixed pullies to the extremity of the jib. An example is shown in Fig. 2, Plate IX., p. 497.

(4) A water-pressure engine may be employed to turn a crank. Three working cylinders inclined at 120° are frequently used as shown in Fig. 1, Pl. X., p. 497. They are single-acting and drive the

same crank as in the small steam engines of the same type employed where great speed is required. The water is admitted to the outer ends of the cylinders, so that the piston rods are always in compression.

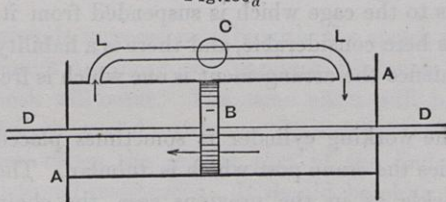
(5) The hydraulic mechanism applied to work heavy guns on board ship consists of a cylinder in which works a piston attached to a rod, the sectional area of which is one-half that of the cylinder. If water be admitted at both ends of the cylinder the piston moves outwards, but if to the inner end only, it moves inwards. The motive force in either case is the same, being due to the difference of areas. This apparatus serves also as a brake of the kind described in the next article. For details and illustrations the reader is referred to the *Gunnery Manual.*

258. *Hydraulic Brakes.*—It has been sufficiently explained that hydraulic resistances absorb an amount of energy which varies as the square of the speed. A hydraulic machine therefore may be employed as a brake, and it is in this way that large amounts of surplus energy are most easily disposed of. Moreover, by its use the speed of any machine to which it is applied is readily controlled.

An hydraulic brake is constructed by interposing a mass of fluid between the elements of a pair so that any motion of the pair causes a breaking-up of the fluid with a corresponding resistance.

A common case is that of a sliding pair consisting of a piston and cylinder filled with water or oil, which passes from one side of the piston to the other whenever the piston moves. Two examples of this apparatus are shown in skeleton in Figs. 186*a*, 186*b*. In the first (Fig. 186*a*) the piston rod *DD* projects through both cylinder covers, and communication is made between the two ends of the cylinder by a pipe *LL* provided with a cock *C*, which can be closed at pleasure. At *D* the rod is attached to the piston rod of a steam cylinder employed to obtain the very considerable force necessary to work the starting and reversing gear of large marine engines. The resistance of this brake is zero when the piston begins to move, but

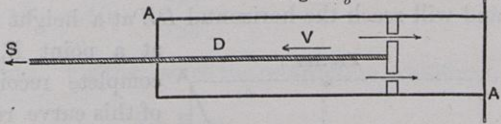
Fig.186$_a$



increases as the square of the speed, and thus effectually prevents it from moving too rapidly. The maximum speed is controlled by turning the cock. For a detailed description of this gear the reader is referred to a treatise on the *Marine Engine*, by Mr. Sennett.

In the second (Fig. 186*b*) the water passes from one end of the cylinder through orifices in the piston itself. This is the common "compresser" or Service Buffer.* The piston rod in

Fig. 186$_b$



this case passes out at one end only of the working cylinder, and is attached to the gun, the recoil of which is to be checked. The theory of this apparatus is of some interest, and will now be briefly considered.

Let n be the ratio of the area of the piston to the *effective* area of the orifices, then the loss of head must be

$$\frac{p_1 - p_2}{w} = (n-1)^2 \frac{V^2}{2g},$$

where V is the speed of piston and p_1, p_2 are the pressures on the two sides of the piston. Hence the pull

$$S = wA(n-1)^2 \frac{V^2}{2g}$$

on the piston rod is necessary to overcome the hydraulic resistance at this speed. The gun is gradually brought to rest by this resistance, aided by the friction of the slide.

At the instant of firing a certain amount of kinetic energy is generated in the gun, given by the formula

$$\text{Energy of Recoil} = k \cdot \frac{WV_0^2}{2g}, \quad \text{(Art. 135, p. 279)}$$

where V_0 is the initial velocity of recoil, and k is a co-efficient (about 1·2) introduced on account of the inertia of the powder gases. As the gun recoils its velocity diminishes, and if P_0 be the friction of the slide the retarding force will be

$$S + P_0 = wA(n-1)^2 \frac{V^2}{2g} + P_0.$$

The maximum value of S will be found by writing V_0 for V, and may be denoted by S_0.

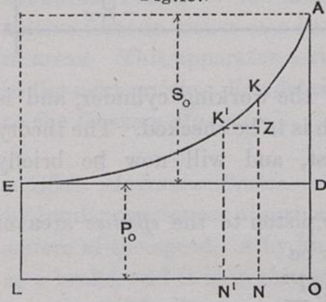
* *Manual of Gunnery for Her Majesty's Fleet*, p. 68.

To represent this graphically, in Fig. 187 draw a curve in which the ordinate KN at any point N represents the retarding force after the gun has recoiled through the space ON from the point O, at which the action of the powder pressure ceases, and the gun has its maximum velocity V_0. This curve will start from a point A such that

$$OA = S_0 + P_0,$$

and will reach the horizontal DE at a height P_0 above the base line at a point E, such that OL is the complete recoil. The area $OAEL$ of this curve represents the energy of recoil which has all been absorbed by the frictional resistance of the slide and the hydraulic resistance of the compressor. Further, the area $KNN'K'$ between two ordinates will represent the diminution of energy as the gun recoils through the space NN' between them, a circumstance

Fig. 187.



which enables us to construct the curve, for if VV' be the velocities of the recoiling gun at NN' respectively,

$$\text{Area } KNN'K' = k.\frac{W(V^2 - V'^2)}{2g}.$$

But if SS' be the corresponding values of S,

$$KZ = S - S' = wA(n-1)^2\frac{V^2 - V'^2}{2g};$$

and if the ordinates be taken near together the area in question will be nearly $KN \,.\, NN'$. We have therefore, by division,

$$\frac{KZ}{KN} = NN' \,.\, \frac{wA(n-1)^2}{kW}.$$

That is, if a number of equidistant ordinates be drawn near together the ratio of consecutive ordinates is constant. The curve may be roughly traced from this property; it is identical with the curve already drawn in Art. 123, p. 262, except that it is a linear instead of a polar curve.

The mean resistance to recoil is given by the equation

$$(\bar{S} + P_0)l = \text{Energy of Recoil},$$

where l is the distance traversed. It would, of course, be advantageous to have a uniform resistance to recoil, because the maximum pressure in the compressor would be diminished and less strain thrown on the gear. This is the object of the various modified forms of the compressor, in which the orifices are not of constant area, but become smaller as the recoil proceeds. In order that the resistance may be constant we must have

$$\bar{S} = wA(n-1)^2 \frac{V^2}{2g},$$

so that $(n-1)V$ is constant. Further, since the retardation is uniform,

$$V^2 = 2g \cdot \frac{\bar{S} + P_0}{W} x,$$

where x is the distance from the end of the recoil. It appears therefore that the orifices should vary in such a way that $(n-1)^2 x$ should be constant. Descriptions of two forms of compressor, with varying orifices, will be found in the *Gunnery Manual.*

Instead of a sliding pair we may employ a turning pair. This is the common "fan" or "fly" brake used to control the speed and absorb the surplus energy of the striking movement of a clock, or in other similar cases. A friction dynamometer (p. 290) was designed by the late Mr. Froude for the purpose of measuring the power of large marine engines, in which the ordinary block or strap surrounding a shaft or drum is replaced by a casing in which a wheel works. Vanes attached to the wheel and the fixed casing thoroughly break up a stream of water passing through the casing. Any amount of energy may thus be absorbed without occasioning any considerable rise of temperature. Siemens' combined brake and regulator has been mentioned already (p. 288).

259. *Transmission of Energy by Hydraulic Pressure.*—Energy may be distributed from a central source, and transmitted to considerable distances with economy by hydraulic pressure. The delivery in gallons per minute of a pipe d'' diameter is

$$G = 27 \sqrt{\frac{h'}{l}} \cdot d^{\frac{5}{2}} \qquad \text{(Art. 243.)}$$

Assume now that the pipe supplies an hydraulic machine at a distance of l feet from an accumulator in which h is the head. Further, suppose that n per cent. is lost by friction of the pipe, then the power transmitted in foot-lbs. per minute is

$$10Gh = 270\ h\sqrt{\frac{nh}{100l}} \cdot d^{\frac{5}{2}},$$

and the distance to which N horse power can be transmitted with a loss of n per cent. is in feet

$$l = \frac{h^3 d^5 n}{1,500,000 N^2} \qquad \text{(nearly)}.$$

With the usual pressure in accumulators of 750 lbs. per square inch, or 1700 feet of water, this gives the simple approximate formula

$$l = 3300\,\frac{d^5 n}{N^2}.$$

Thus for example, 100 horse power may be transmitted by a 5″ pipe to a distance of 4 miles, or 10 horse power by a 1′ pipe to a distance of 220 yards, with a loss by friction not exceeding 20 per cent. The diameter of pipe is limited by considerations of strength and cost.

The power of a motor supplied by a given pipe does not increase indefinitely as its speed increases, but is greatest when one-third of the head is lost by friction.* The maximum possible power is therefore given by the formula

$$H.P. = 220\sqrt{\frac{d^5}{l}} \qquad \text{(approximately)}.$$

This is of course two-thirds the value of N in the preceding formula.

260. *Pumps.*—If the direction of motion of an hydraulic motor be reversed by the action of sufficient external force applied to drive it, while, at the same time, the direction of the issuing water is reversed so as to supply the machine at the point from which it originally proceeded, we obtain a machine which raises water instead of utilizing a head of water. Every hydraulic machine therefore may be employed to raise water as well as to do work, and most of them actually occur in this form; they are then called PUMPS, though in some cases this name would not be used in practice. Much of what has been said about motors applies equally well to pumps: the principal difference lies in the fact that the useful resistance which the pump overcomes is always reversible, whereas in the motor this is

* This result was pointed out to the writer by Mr. Hearson. It appears to be little known.

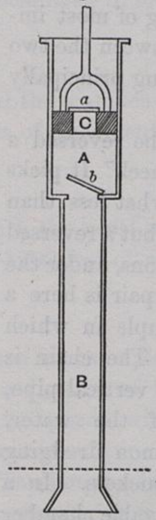
not necessarily the case. The principles of action and the classification of hydraulic machines are, in the main, the same in both cases. Some points omitted while considering motors as being of most importance in pumps, and certain differences of action between the two will now be briefly noticed. Certain machines occurring principally as pumps will be mentioned.

(1) If the direction of motion of an overshot wheel be reversed a machine is obtained which is known as a "Chinese Wheel." It picks up water in its buckets and raises it to a height somewhat less than the diameter of the wheel. This machine is little used, but a reversed breast wheel is frequently employed in drainage operations, under the name of a "scoop," or "flash" wheel. The working pair is here a turning pair, but in the chain pump we find an example in which one of its elements is a chain passing over pulleys. The chain is endless and is provided with flat plates fitting into a vertical pipe, the lower end of which is below the surface of the water, and through which the water is raised. In the common dredging machine the closed channel (p. 484) is replaced by buckets. In a third class of weight machines the water occupies a movable chamber and forms with it a kinematic pair with only one solid element, while it forms, with the link attached to the earth, a working pair which has also but one solid element. The Archimedian screw, and certain varieties of "scoop" wheel, in which the water enters the scoop at the circumference of the wheel and is delivered at the centre, are examples of this kind.

(2) The most common forms of pumps are the "lift" or "force" pumps, which consist of a chamber which expands to admit the water to be lifted and contracts in the act of lifting; they are therefore pressure machines like those considered in Art. 255-6, but reversed. The name "pump" originally applied to these machines alone.

Fig. 188 shows a common lift pump. A is a cylinder at a certain height h_1 above the water to be raised, B is a piston working in the cylinder by the action of which the water is lifted. The piston has orifices in it which permit the water to pass through. The orifices are closed by a valve, as is also the opening at the bottom of the cylinder. These valves are simple "flaps" which open on hinges to permit the water to pass upwards, but close the passage to motion in the opposite direction, thus acting as a ratchet (p. 171). Assuming the piston at the bottom of its stroke, at rest close to the bottom of the cylinder, let it be supposed to rise; the valve a will rise and allow air to pass if any. After several strokes the air will be nearly exhausted, and if h_1 be not too great the empty space will be filled with water raised from the tank by atmospheric pressure. Thus the water will pass into the cylinder closely following the piston. At the top of the stroke the piston commences to descend, a closes and b opens, allowing the water to pass above the piston. This water is now raised by the piston to any required height. In force pumps the process is the same, but the water passes out through an orifice in the bottom of the cylinder instead of through the piston; the raising of the water above the level of the cylinder is done in the down stroke instead of the up.

Fig.188.



The difference between this action and that of a pressure motor lies mainly in the valves, which here open and close automatically by the action of the water, instead of by external agency. Further, the pump wholly or partly works by *suction*, a method by no means peculiar to pumps, for it also occurs in motors, but not so frequently. The height of the water barometer is 34 feet, but the height to which a pump will work by suction is not so great. When the piston is at the bottom of its stroke there must, for safety, always be a certain clearance space below. This space always contains air, the pressure of which diminishes as the piston rises, but cannot be reduced to zero. Further, a certain pressure is required to overcome the weight and friction of the valve before it opens. At least 3 feet of the lift is absorbed in this way, and generally considerably more. To obtain a high vacuum for scientific purposes, air pumps are specially designed to meet these difficulties. Also, leakage must be allowed for and the diminution on account of friction and inertia, which will be considerable if the speed be too great or the pipes too small, as will be understood on reference to Arts. 255-6, all of which applies to pumps as much as to motors. It is hardly necessary to observe that power is neither gained nor lost by the use of suction; it simply enables the working cylinder to be placed above the water to be lifted, an arrangement which is in most cases convenient. The limit in practice is about 25 feet.

PLATE IX.

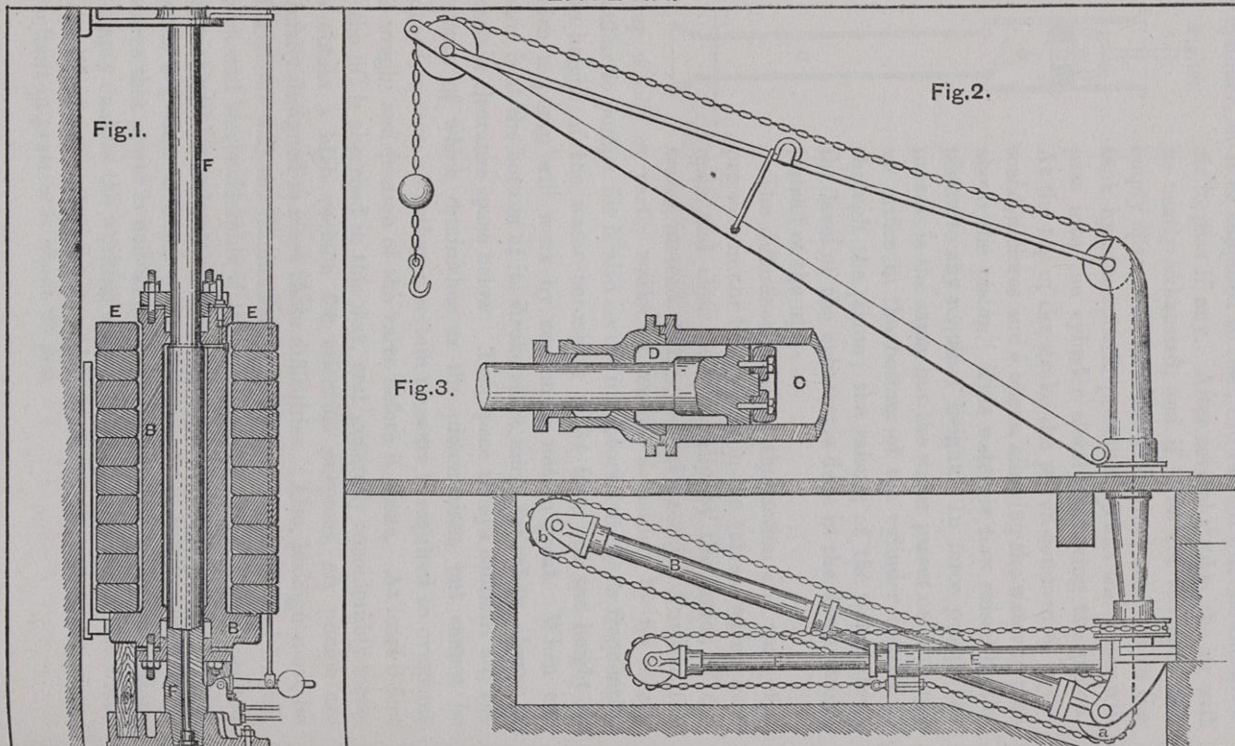


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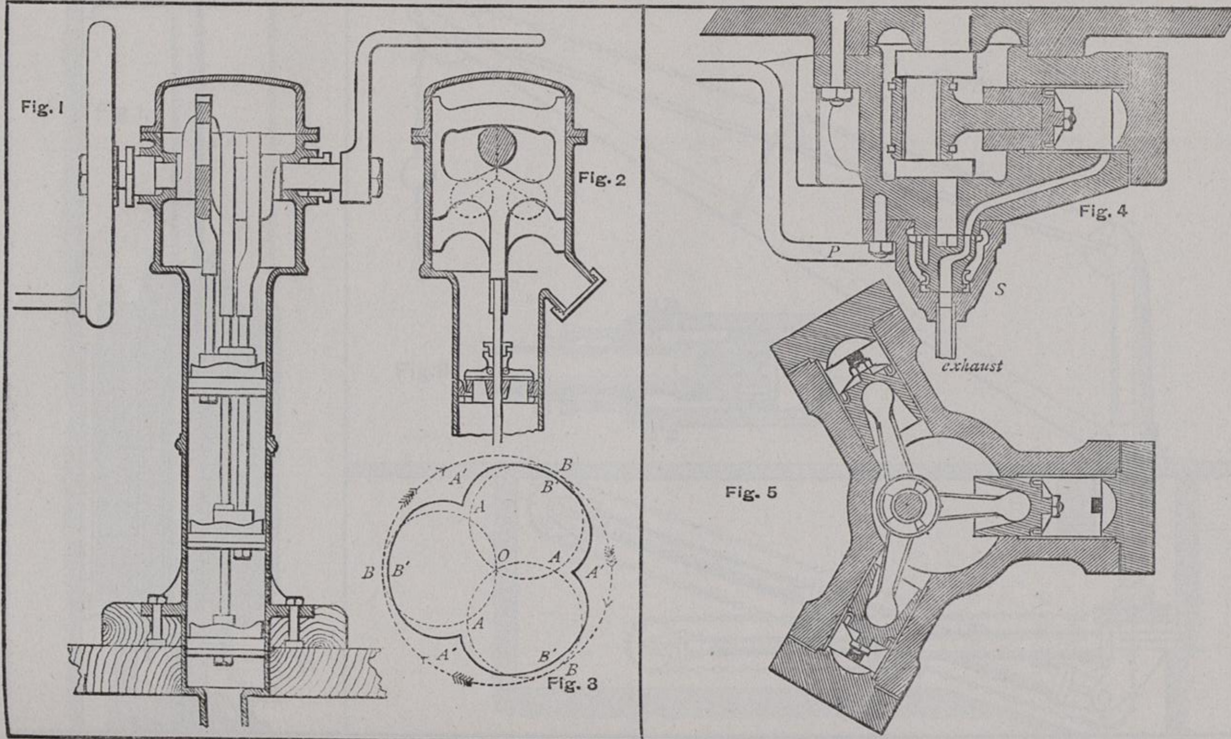
PLATE X.



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Pumps are commonly, but not always, single-acting; they are worked by the direct action of a reciprocating piece, or by means of a rotating crank. In the first case, when independent, a piston acted on by steam or water pressure is attached to a prolongation of the pump plunger: a crank and fly-wheel is often added, as in Fig. 4, Plate II., p. 121, to control the motion and define the stroke. When driven by the crank three working cylinders, placed side by side with a three-throw crank, are commonly used, in order to equalize the delivery, and so to avoid the shocks due to changes of velocity. An air-chamber, forming a species of accumulator, may also be used with the same object. An arrangement of pumps, as applied by Messrs. Donkin & Co. to raise water from a well 200 feet deep and force it to a height of 143 feet above the engine-house, may be mentioned as an example. A set of lift pumps at the bottom of the well worked by "spear" rods from the surface, are combined with a set of force pumps in the engine house itself. The speed of these pumps is about 80 feet per minute, and they deliver about 600 gallons per minute. Pumps almost always have a certain "slip," that is, they deliver less water than corresponds to the piston displacement and number of strokes: in this example the slip was 12 per cent. The efficiency of the pumps and mechanism of the engine was found to be 66 per cent. by careful experiments.*

In raising water from great depths in mines, force pumps at the bottom of the mine are used, worked by heavy "spear" rods from a beam engine at the surface. The weight of the rod supplies the motive force during the downward stroke of the pump; while the engine, which is single-acting, raises the rods again during the downward stroke of the steam piston.

DESCRIPTION OF PLATES IX. AND X.

In order further to illustrate the action of water-pressure machines Plates IX. and X. have been drawn.

Fig. 1, Plate IX., shows the differential accumulator described on page 494.

In Fig. 2 is represented a hydraulic crane, designed by Sir W. Armstrong, for lifting weights of 2 to 3 tons. In it the hydraulic power is applied to rotate the crane as well as to lift the weight.

In order to effect the lift the high-pressure water from the accumulator is admitted to the cylinder A, and forces out the plunger B. There are two pulleys at a and two at b. One end of the chain is secured to the cylinder A, it is led round b, then round

* *Minutes of Proceedings of the Institution of Civil Engineers*, vol. 66.

a, again round b, then under the second pulley at a up through the hollow crane post on to the weight as shown. The effect of this arrangement is that any movement of the plunger B is at the hook multiplied four times.

If B is simply a plunger working in a stuffing box, then the expenditure of energy is always the same whatever weight is being lifted, and the amount must be equal to that which corresponds to lifting the maximum possible weight.

This is an objection which is common to all such machines. The surplus energy is expended in overcoming frictional resistances (p. 486). To mitigate this evil, in cranes of high power the plunger has a piston end, which fits a bored cylinder, and is provided with a cup leather, as shown in Fig. 3. The sectional area of the plunger is about one-half that of the cylinder. If a light weight is to be lifted, water is admitted to both sides of the piston, and the difference of the pressures, equal to what would be exerted on a simple plunger, is available for effecting the lift. When it is required to lift a heavy weight water is admitted to the side C only of the piston, the annular space D being put in communication with the atmosphere. Thus the full pressure due to the area of the piston is exerted with the corresponding expenditure of water.

For the purpose of rotating the crane a pair of cylinders, E, are provided, of which one only is shown in the figure. The thrusting out of the plunger F of one of them by the pressure of the water causes the other to be drawn in by means of a chain which passes around a recessed pulley secured to the crane post.

In Plate X., Figs 1 and 2 show the construction of Downton's Pump, so much used on board ship. In the barrel work three buckets with flap valves, as shown in Fig. 2. The rods to which the upper and second buckets are attached are necessarily out of centre. The rods to the lower buckets pass through deep stuffing boxes in the buckets above, and thus the buckets are maintained from canting seriously. The movement of the buckets is effected by a three-throw crank, the crank pins, which are not round, being set at 120° apart. These pins fit and work in a curved slot in the bucket rod heads. Assuming the admission of no air but water only from below, the discharge of the pump will at each instant equal the displacement of the fastest upward moving bucket. Accordingly the rate of discharge may be represented by a curve, as in Fig. 3. If the slot in the rod head were straight and the pin round, then, the crank moving uniformly, in direction shown, the velocity of discharge would be represented by the radii from O to the dotted curve $BABABA$, which is made up of parts of three circles, the position of the radius being that of either of the three cranks. The effect of the curved slot is to diminish the maximum and increase the minimum discharge, as shown by the full curve $B'A'B'A'B'A'$.

Figs. 4 and 5 of this Plate are sections of the hydraulic engine referred to on page 489, employed to rotate a capstan. It need only be further added that a single rotating valve V suffices for admission and exhaust of all three cylinders. The high-pressure water is supplied by the pipe P to the passage S surrounding the valve and exhausted from the cylinders through the central passage.

EXAMPLES.

1. In estimating the power of a fall of water it is sometimes assumed that 12 cubic feet per second will give 1 H.P. for each foot of fall: what efficiency does this suppose in the motor? *Ans.* ·72.

2. An accumulator ram is 9 inches diameter, and 21 feet stroke: find the store of energy in foot-lbs. when the ram is at the top of its stroke, and is loaded till the pressure is 750 lbs. per square inch? *Ans.* 958,000 foot-lbs.

3. In a differential accumulator the diameters of the spindle are 7 inches and 5 inches; the stroke is 10 feet: find the store of energy when full, and loaded to 2,000 lbs. per square inch. *Ans.* 377,000 foot-lbs.

4. A direct-acting lift has a ram 9 inches diameter, and works under a *constant* head of 73 feet, of which 13 per cent. is required by ram friction and friction of mechanism. The supply pipe is 100 feet long and 4 inches diameter. Find the speed of steady motion when raising a load of 1,350 lbs., and also the load it would raise at double that speed?

Ans. Speed = 2 feet per second.
Load = 150 lbs.

5. In the last question, if a valve in the supply pipe is partially closed so as to increase the co-efficient of resistance by $5\frac{1}{2}$, what would the speed be?

6. Eight cwt. of ore is to be raised from a mine at the rate of 900 feet per minute by a water-pressure engine, which has four single-acting cylinders, 6 inches diameter, 18 inches stroke, making 60 revolutions per minute. Find the diameter of a supply pipe 230 feet long, for a head 230 feet, not including friction of mechanism. *Ans.* Diameter = 4 inches.

7. Water is flowing through a pipe 20 feet long with a velocity of 10 feet per second. If the flow be stopped in one-tenth of a second, find the intensity of the pressure produced, assuming the retardation during stoppage uniform. *Ans.* 62 feet of water.

8. If λ be the length equivalent to the inertia of a water-pressure engine, F the co-efficient of hydraulic resistance, both reduced to the ram, v_0 the speed of steady motion: find the velocity of ram, after moving from rest through a space x against a constant useful resistance. Also find the time occupied.

$$Ans.\ v^2 = v_0^2\left(1 - \epsilon^{-\frac{Fx}{\lambda}}\right); \quad t = \frac{\lambda}{F \cdot v_0} \log_\epsilon \frac{v_0 + v}{v_0 - v}.$$

9. An hydraulic motor is driven from an accumulator, the pressure in which is 750 lbs. per square inch, by means of a supply pipe 900 feet long, 4 inches diameter; what would be the maximum power theoretically attainable, and what would be the velocity in the pipe at that power? Find approximately the efficiency of transmission at half power. *Ans.* H.P. = 240; $v = 22$; efficiency = ·96 nearly.

10. A gun recoils with a maximum velocity of 10 feet per second. The area of the orifices in the compressor, after allowing for contraction, may be taken as one-twentieth the area of the piston: find the initial pressure in the compressor in feet of liquid. *Ans.* 621.

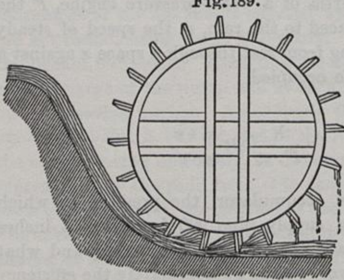
11. In the last question assume weight of gun 12 tons; friction of slide 3 tons; diameter of compressor 6 inches; fluid in compressor water: find the recoil. *Ans.* 4 feet $2\frac{1}{2}$ inches.

12. In the last question find the mean resistance to recoil. Compare the maximum and mean resistances each exclusive of friction of slide. *Ans.* Total mean resistance = 4·4 tons. Ratio = 2·5.

261. *Impulse and Reaction Machines in General.*—The source of energy may be a current of water or the head may be too small to obtain any considerable pressure, and it is then necessary to have some means of utilizing the energy of water in its kinetic form. A machine for this purpose operates by changing the motion of the water and utilizing the force to which the change gives rise. If the water strikes a moving piece and is reduced to rest relatively to it, the machine works by "impulse," and if it be discharged from a moving piece, by "reaction." There is no difference in principle between these modes of working, and both may occur in the same machine. In either case the motive force arises from the mutual action between the water and the piece which changes their relative motion. Machines of this class are also employed for high falls when the low speed of pressure machines renders their use inconvenient or impossible. The water is then allowed to attain a velocity equivalent to a considerable portion of the head immediately before entering the machine, so that its energy is, in the first instance, wholly or partially converted into the kinetic form.

The simplest machine of this kind is the common undershot wheel, consisting of a wheel (Fig. 189) provided with vanes against which the water impinges directly. Let the velocity of periphery of the wheel be V, then the water after striking the vanes is carried along with them at this velocity. If, then, the original velocity of the water be v, the diminution of velocity due to the action of the vanes will be $v - V$. Let W be the weight of water acted on per second, then the impulse on the wheel must be

Fig. 189.



$$P = \frac{W(v-V)}{g},$$

but if A be the sectional area of the stream,

$$W = Avw,$$

this being the weight of water per second which comes in contact with all the vanes taken together.

$$\therefore \quad P = \frac{w}{g} Av\,(v - V).$$

The power of the wheel is PV foot-lbs. per second, and the energy of the stream is $Wv^2/2g$, therefore

$$\text{Efficiency} = \frac{2V\,(v - V)}{v^2}.$$

This is greatest when $V = \frac{1}{2}v$ and its value is then ·5, showing that the wheel works to best advantage when the speed of periphery is one-half that of the stream, but that the efficiency is low, never exceeding ·5.

Such wheels may be seen working a mill floating in a large river, or in other similar circumstances, but they are cumbrous and, allowing for various losses, not included in the preceding investigation; their efficiency is not more than 30 per cent. In the early days of hydraulic machines, they were often used for the sake of simplicity or, as in the example shown in the figure, from a want of comprehension of their principle.* In mountain countries, where unlimited power is available, they are still found. The water is then conducted by an artificial channel to the wheel, which sometimes revolves in a horizontal plane. When of small diameter their efficiency is still further diminished.

In overshot wheels and other machines operating chiefly by weight the head corresponding to the velocity of delivery is partly utilized by impulse, and the speed of the wheel is determined by this consideration. In all cases of direct impulse, if h is that part of the head operating by impulse, the speed of maximum efficiency is

$$V = \tfrac{1}{2}\sqrt{2gh} = 4\sqrt{h},$$

or in practice somewhat less, and at that speed at least half that head is wasted.

The great waste of energy in this process is due partly to the velocity V with which the water moves onward with the wheel, and partly to breaking-up during impulse. It is in fact easy to see that

* See Fairbairn's *Millwork and Machinery*, from which this figure is taken, vol. I., p. 149.

one-fourth the head is wasted by each of these causes. To avoid it, the water must be received by the moving piece against which it impinges without any sudden change of direction, and must be discharged at the lowest possible velocity, effects which may be produced by a suitably-shaped vane curved so as to deflect the water gradually and guide it in a proper direction. The principle on which such a vane is designed may be explained by the annexed diagram. In Fig. 190, AB is a vane, moving with velocity V in a given direction, against which a jet strikes. Drawing a diagram of velocities, let Oa represent v, the velocity of the jet, and let OO' represent V. Then as before (p. 477) $O'a$ represents the velocity of the jet relatively to the vane, and, in order that the water may impinge without shock, the tangent to the vane at A must be parallel to $O'a$. The vane is now curved so as gradually to deflect the water, in doing which there is a mutual action between the jet and the vane which produces the motive force which drives the wheel. If the water leave the vane at B, its velocity relatively to the vane is represented by $O'b$ drawn parallel to the vane at B, and somewhat less than $O'a$ in magnitude, to allow for friction, unless the water be enclosed in a passage, when it will bear some given proportion to $O'a$.

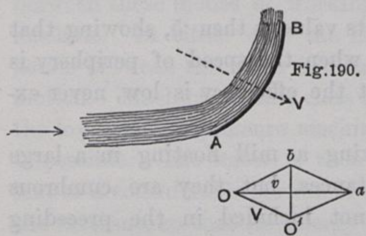


Fig. 190.

The absolute velocity with which the water moves at B is now represented by Ob, and this may be arranged to deliver the water in a convenient direction with a velocity just sufficient to clear the wheel and no more. The efficiency may then theoretically be unity, and, practically, after allowing for losses, may be increased to ·65 or ·7. Vanes of this kind were applied to water wheels by Poncelet. The wheel, in this case, revolves in a vertical plane, and the water, on impinging at A, ascends to B: it then descends under the action of gravity, and is discharged at the same point A at which it entered, so that $O'b$ is approximately equal and opposite to $O'a$.

In all impulse and reaction machines there is a speed of maximum efficiency which, as in the simple case first considered, is given by the formula

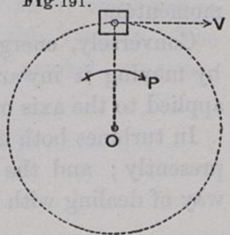
$$V = k\sqrt{2gh},$$

where k is a fraction depending on the type of machine.

262. *Angular Impulse and Momentum.*—The most important of these machines are those in which the change of motion produced in the water is a motion of rotation, and it is needful to consider that form of the principle of momentum which is applicable to such cases.

In Fig. 191, W is a weight describing a circle round O with velocity V; then the product of its momentum by the radius r is called the "moment of momentum" of the weight about O. If O represent an axis to which W is attached rigidly, we may imagine it turning under the action of a force P at a radius R. The moment of P multiplied by the time during which it acts is called the "moment of impulse."

Fig. 191.



During the action of P the weight will move quicker and quicker, and the motion is governed by the principle expressed by the equation

Moment of Impulse = Change of Moment of Momentum.

If L be the moment of P, then, taking the time as one second,

$$L = \text{Change of Moment of Momentum per second.}$$

This equation is true, not only for a single weight and a single force, but also for any number of weights and any number of forces. As in other forms of the principle of momentum it is also true, notwithstanding any mutual actions or any relative movements of the weights or particles considered. Further, any radial motions which the particles possess may be left out of account, for they do not influence the moment of momentum. A particular case is when $L = 0$, then the moment of momentum remains constant, a principle known as the Conservation of Moment of Momentum. The terms "moment of momentum" and "moment of impulse" are often replaced by "angular momentum," "angular impulse."

A weight rotating about an axis is capable of exerting energy in two ways. First, it may move away from the axis of rotation, overcoming by its centrifugal force a radial resistance which it just overbalances. Secondly, it may overcome a resistance to rotation in the shaft to which it is attached. In either case the work done will be represented by a diminution in the kinetic energy of the weight.

If the shaft be free, the diminution of kinetic energy must be equal to the work done by the centrifugal force, and it may be proved in this way, that if V be the velocity of rotation of the weight, r the radius,

$$Vr = \text{constant},$$

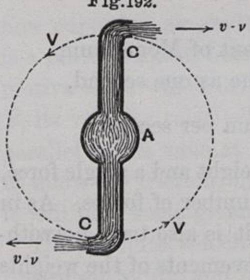
an equation equivalent to the conservation of the momentum of momentum.

Conversely, energy may be applied to a rotating weight either by moving it inwards against its centrifugal force, or by a couple applied to the axis of rotation.

In turbines both modes of action occur together as we shall see presently; and the principle of momentum is the most convenient way of dealing with the question.

263. *Reaction Wheels.*—Fig. 192 shows a reaction wheel in its simplest form. CAC is a horizontal tube communicating with a vertical tubular axis to which it is fixed, and with which it rotates. Water descends through the vertical tube, and issues through orifices at the extremities of the horizontal tube so placed that the direction of motion of the water is tangential to the circle described by the orifices. The efflux is in opposite directions from the two orifices, and a reaction is produced in each arm which furnishes a motive force. There are two methods of investigating the action of this machine which are both instructive. Frictional resistances are, in the first instance, neglected.

Fig. 192.



(1) Let the orifices be closed, and let the machine revolve so that the speed of the orifices in their circular path of radius r is V. Centrifugal action produces a pressure in excess of the head h existing when the arms are at rest, the magnitude of which is $V^2/2g$, and this pressure is so much addition to the head, which now becomes

$$H = h + \frac{V^2}{2g}.$$

This quantity H may also be considered as the head "relative to the moving orifices" estimated as in Art. 239

When the orifices are opened, the water issues with velocity v given by

$$v^2 = 2gH = V^2 + 2gh;$$

thus the water issues with a velocity greater than V, and after leaving the machine has the velocity $v - V$ relatively to the earth. The energy exerted per lb. of water is h, and this is partly employed in generating the kinetic energy corresponding to this velocity. The remainder does useful work by turning the wheel against some useful resistance, so that we have per lb. of water

$$\text{Useful Work} = h - \frac{(v-V)^2}{2g} = \frac{V(v-V)}{g},$$

and, dividing by h,

$$\text{Efficiency} = \frac{V(v-V)}{gh} = \frac{2V}{v+V}.$$

(2) A second method is to employ the principle of the equality of angular impulse and angular momentum already given in Art. 262. Originally the water descends the vertical tube without possessing any rotatory motion, but after leaving the machine it has the velocity $v - V$; its angular momentum is therefore for each lb. of water

$$\text{Angular Momentum} = \frac{(v-V)}{g} \,.\, r.$$

Now according to the principle the angular momentum generated per second is also the angular reaction on the wheel which, when multiplied by V/r, the angular velocity of the wheel, gives us the useful work done per second. Performing this operation, and dividing by the weight of water used per second, we get per lb. of water

$$\text{Useful Work} = \frac{V(v-V)}{g}.$$

This is the result already obtained, and the solution may now be completed by adding the kinetic energy on exit.

From the result it appears that the proportion which the waste work bears to the useful work is $v - V : 2V$, which diminishes indefinitely as v approaches V; but in this case the velocities become very great, since $v^2 - V^2$ is always equal to $2gh$. The frictional resistances then become very great, so that in the actual machine there is always a speed of maximum efficiency which may be investigated as follows :—

Let F be the co-efficient of hydraulic resistances referred to the orifices, then

$$(1+F)\frac{v^2}{2g}=H=h+\frac{V^2}{2g}.$$

The useful work remains as before, and therefore

$$\text{Efficiency}=\frac{2V(v-V)}{v^2-V^2+F\,.\,v^2},$$

a fraction which can readily be shown to be a maximum when

$$v-V=V\sqrt{\frac{F}{1+F}},\ \text{or}\ v=V\left\{1+\sqrt{\frac{F}{1+F}}\right\},$$

which value of v, when substituted in the preceding equation, will give the value of V in terms of h for maximum efficiency. The existence of a speed of maximum efficiency is well known by experience with these machines. In general it is found to be about that due to the head, so that

$$V^2=2gh,$$

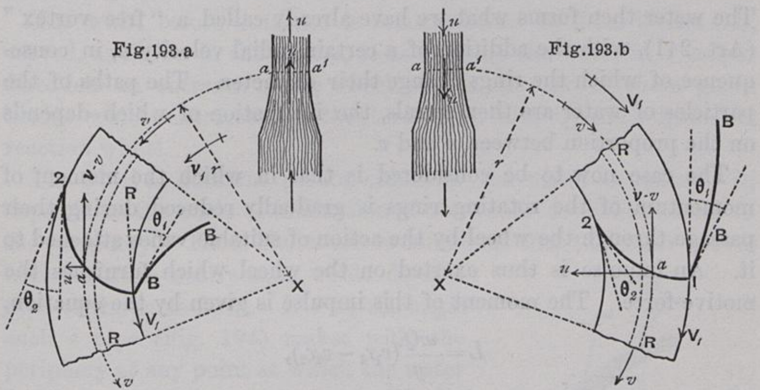
a value which corresponds to $F=\cdot 125$, and gives an efficiency of $\cdot 67$. This is about the actual efficiency of these machines under favourable circumstances; of the whole waste of energy two-thirds, that is two-ninths of the whole head, is spent in overcoming frictional resistances, and the remaining one-third, or one-ninth the whole head, in the kinetic energy of delivery.

The reaction wheel in its crudest form is a very old machine known as "Barker's Mill." It has been employed to some extent in practice as an hydraulic motor, the water being admitted below and the arms curved in the form of a spiral. These modifications do not in any way affect the principle of the machine, but the frictional resistances may probably be diminished.

264. *Turbine Motors.*—A reaction wheel is defective in principle, because the water after delivery has a rotatory velocity in consequence of which we have seen a large part of the head is wasted. To avoid this, it is necessary to employ a machine in which some rotatory velocity is given to the water before entrance in order that it may be possible to discharge it with no velocity except

that which is absolutely required to pass it through the machine. Such a machine is called in general a TURBINE, and it is described as "outward flow," "inward flow," or "parallel flow," according as the water during its passage through the machine diverges from, converges to, or moves parallel to, the axis of rotation.

Fig. 193a shows in plan and section part of an annular casing forming a wheel revolving about an axis XX through which water is



flowing, entering at the centre and spreading outwards. The water leaves the wheel at the outer circumference. Fig. 193b is similar, but the flow is inward instead of outward.

If we consider a section aa made by a concentric cylinder of length y and radius r, the flow will be

$$Q = u \cdot 2\pi r y,$$

where u is the radial velocity or, as we may call it, the "velocity of flow." The area of the section $(2\pi r y)$ may conveniently be called the "area of flow." The value of Q is everywhere the same, and therefore ury must be constant. It is generally desirable to make u constant or nearly so, and then the form of the casing is such that ry is constant. Whether this be so or not, the value of u can always be calculated at any radius for a given wheel with a given delivery.

The water which at any given instant is at a given distance r from the axis may be considered as forming a ring RR, which rotates while at the same time it expands or contracts according as the flow is outward or inward. The velocity of the periphery of this ring may be described as the "velocity of whirl," and if it be called v, the moment of momentum of a ring, the weight of which is W, is

$$M = \frac{W}{g} . vr.$$

If the wheel has no action on the water, this quantity cannot be altered, and we must then have

$$vr = \text{Constant}.$$

The water then forms what we have already called a "free vortex" (Art. 241), with the addition of a certain radial velocity u, in consequence of which the rings change their diameter. The paths of the particles of water are then spirals, the inclination of which depends on the proportion between u and v.

The case now to be considered is that in which the moment of momentum of the rotating rings is gradually reduced during their passage through the wheel by the action of suitable vanes attached to it. An impulse is thus exerted on the wheel which furnishes the motive force. The moment of this impulse is given by the equation,

$$L = \frac{wQ}{g}(v_1 r_1 - v_2 r_2),$$

where wQ is the weight of all the rings passing through the machine in a second, and the suffixes 1, 2 refer to entrance and exit respectively, as indicated in the figures for the two cases of outward flow and inward flow. The turbine works to best advantage when the water is discharged without any whirl, that is when $v_2 = 0$, and putting aside friction the only loss then is that due to the velocity of flow u, which may be made small by making the wheel of sufficient breadth at the circumference where the water is discharged.

In practice there are of course always frictional resistances, but, for given velocities, the impulse on the wheel is not altered by them, so that the moment of impulse is always given by the above equation. Suppose, now, h the *effective* head found from the actual head by deducting (1) the height due to the velocity of delivery, (2) the friction of the supply pipe and passages in the wheel, (3) the loss (if any) by shock on entering the wheel; then

$$\text{Work done per second} = wQh.$$

But, if V_1 be the speed of periphery of the wheel at the radius r_1 where the water enters, V_1/r_1 is the angular velocity of the wheel, and $L \, . \, V_1/r_1$ is the work done per second. We have then for the case where there is no whirl at exit

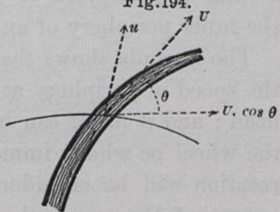
$$V_1 v_1 = gh.$$

The effective head h in this formula includes (1) a part equivalent to the useful work, and (2) a part equivalent to the frictional resistances to the rotation of the wheel, such as friction of bearings and friction of the water surrounding the wheel (if any) on its external surface. This last item is often described as "disc friction."

The whirl before entrance is communicated by fixed blades BB, curved, as shown in the figures, so as to guide the water in a proper direction on entrance to the wheel. It is the use of these guide blades which characterizes the turbine as distinguished from the reaction wheel.

The whirl at different points, either in the wheel or outside it, depends on the angle of inclination of the vanes or guide blades to the periphery. These blades are so numerous that the water moves between them nearly as it would do in a pipe of the same form. If θ be the angle such a pipe (Fig. 194) makes with the periphery at any point at which the water is flowing through it with velocity U, the radial and tangential components of that velocity will be $U. \sin \theta$ and $U. \cos \theta$. The first of these is always the velocity of flow u, whether the pipe be fixed or whether it be attached to the revolving wheel. In the fixed pipe the second is the velocity of whirl which we may call v', and we have for motion before entrance

Fig. 194.



$$v' = u \, . \cot \theta.$$

In the moving pipe, however, it is the velocity of whirl relatively to the revolving wheel, and this is $V - v$, therefore

$$V - v = u \, . \cot \theta.$$

Suppose the vanes of the wheel are radial at the circumference where the water enters. In order that the water may have no velocity of whirl relatively to the wheel on entrance, and that the water may enter without shock, we must then have $v' = V_1$, that is, the value of θ for the fixed guide blade at entrance should be given by

$$\tan \theta_1 = \frac{u}{V_1}.$$

Further, the water should be discharged without whirl, that is, v should be zero at the circumference where the water leaves the wheel, hence

$$\tan \theta_2 = \frac{u}{V_2}.$$

The inclination of the fixed blades at entrance, and of the vanes at entrance and exit is thus determined. At intermediate points it would be desirable that it should so vary that vr should diminish uniformly from entrance to exit in order that the action of all parts of the vane upon the water may be the same. This condition would completely determine the form of the vane, but, in practice, any "fair" form would be a sufficient approximation.

Supposing the vanes thus designed $v_1 = V_1$, and the speed of periphery of the wheel at the circumference where the water enters is then given by the simple formula

$$V_1 = \sqrt{gh},$$

a value which applies to the outer periphery of an inward-flow and the inner periphery of an outward-flow turbine (see Appendix).

The formula shows that the turbine works to best advantage when the speed of periphery at entrance is that due to *half* the *effective* head: and it never can be advantageous to run it quicker. But, if the wheel be wholly immersed in water, the frictional resistance to rotation will be considerable, and as that resistance varies as the square of the speed the wheel may be run slower without much reduction of efficiency, or, it may be, even with an increase.

Many forms of outward-flow turbines exist, of which the best known was invented by Fourneyron, and is commonly known by his name. The inward-flow or vortex turbine was invented by Prof. James Thomson. For descriptions and illustrations of these machines the reader is referred to the treatises cited at the end of this chapter.

265. *Turbine Pumps.*—Impulse and reaction machines are always reversible, and every motor may therefore be converted into a pump by reversing the direction of motion of the machine and of the water passing through it. If, for example, in the reaction-wheel of Fig. 192 we imagine the wheel to turn in the opposite direction with velocity V, while by suitable means the water is

caused to move in the opposite direction with velocity $v - V$, so as to enter the orifices with velocity v, it will flow through the arms to the centre and be delivered up the central pipe. The only difference will be that the lift of the pump will not be so great as the fall in the motor on account of frictional resistances. So, any turbine motor is at once converted into a turbine pump by reversing the direction of its motion and supplying it with water moving with a proper velocity. An inward-flow motor is thus converted into an outward-flow pump, and conversely.

No inward-flow pump appears as yet to have been constructed, though it has occasionally been proposed. The "centrifugal" pump so common in practice is, of course, always an outward-flow machine.

The earliest idea for a centrifugal pump was to employ an inverted Barker's mill, consisting of a central pipe dipping into water connected with rotating arms placed at the level at which water is to be delivered. This machine, which must be carefully distinguished from the true reversed Barker's Mill mentioned above, operates by suction. Its efficiency, which may be investigated as in Art. 263, is very considerable (Ex. 4, p. 520), but there are obvious practical inconveniences which prevent its use in ordinary cases. The actual centrifugal pump is a reversed inward-flow turbine.

All that was said about motors in the last article applies equally well to pumps, and the same formula

$$Vv = gh$$

applies, V being the speed of rotation of the wheel, now usually called the "fan" and v that of the water, both reckoned at the outer periphery where the water issues. The quantity h is now the *gross* lift found by adding to the actual lift, the head corresponding to the velocity of delivery, the friction of the ascending main, the friction of the suction pipe and passages through the wheel into the main, and the losses by shock at entrance and exit.

A pump, however, works under different conditions from a motor, and corresponding differences are necessary in its design. The energy of a fall can by proper arrangements be readily converted, wholly or partially, into the kinetic form without any serious loss by frictional resistances, and the water can, therefore, be delivered to the wheel with a great velocity of whirl to be afterwards reduced by the action of the wheel to zero. When such a motor is reversed, the water enters without any velocity of whirl, and leaves with a velocity, the moment of momentum corresponding to which represents the couple by which the wheel is driven. To carry out the reversal exactly, this velocity ought to be reduced to as small an amount as possible in the act of lifting. Now the reduction of a velocity without loss of head is by no means easy to accomplish, and (see Appendix) always requires some special arrangement.

In Thomson's inward-flow turbine, when reversed, the water is discharged with a velocity of whirl which is equal to the speed of periphery V, and given by the formula

$$V=\sqrt{gh}.$$

The corresponding kinetic energy represents at least half the power required to drive the pump, and if it be wasted, as was the case in some of the earlier centrifugal pumps constructed with radial vanes, the efficiency is necessarily less than ·5, and in practice will be at most ·3. To avoid this loss, the wheel must revolve in a large "vortex chamber," at least double the diameter of the wheel from the outer circumference of which the ascending main proceeds. The water before entering the main forms a free vortex, and its velocity is reduced one-half as it spreads radially from the wheel; three-fourths the kinetic energy is thus converted into the pressure form. The speed of periphery in pumps of this class is that due to *half* the *gross* lift. Assuming their efficiency as ·65, the gross lift is found by an addition of 50 per cent. to the actual lift.

Many examples of vortex-chamber pumps exist, but they are comparatively rare, probably because the machine is more cumbrous; in practice a different method of reducing the velocity of discharge is generally employed. Instead of the vanes being radial at the outer periphery, they are curved back so as to cut it at an angle θ, given by the formula (p. 509)

$$V-v=u\,.\cot\theta,$$

the velocity of whirl is thus reduced from V to kV, where k is a fraction, and the speed is then

$$V=\sqrt{g\frac{h}{k}}.$$

If the efficiency be supposed ·65, and the velocity be reduced in this way to one-half its original value, this gives about $10\sqrt{H}$ for the speed where H is the actual lift. The greater speed is a cause of increased friction as compared with the vortex-chamber arrangement, but on the other hand the friction of the vortex is by no means inconsiderable, and this is so much subtracted from the useful work done.

The centrifugal pump in this form was introduced by Mr. Appold in 1851, and is commonly known by his name.

Another important point in which the pump differs from the motor is in the guidance of the water outside the wheel. In the motor there are four or more fixed blades which guide the water to the wheel; but in the pump the outer surface of the chamber surrounding the wheel forms a single spiral guide blade. The whole of the water discharged from the wheel rotates in the same direction, and in order that the discharge may be uniform at all points of the circumference the sectional area of this chamber should increase uniformly from zero at one side of the ascending main to a maximum value at the other side. In some of the earlier designs of centrifugal pumps it was supposed that some of the water would rotate one way and some the other, but in fact all the discharged water rotates with the wheel, and the passage should be so designed as to permit this, the area corresponding to the proposed velocity of whirl. There are, however, examples in which the water is discharged in all directions into an annular casing, and guided by spiral blades parallel to the axis of rotation. (See a paper by Mr. Thomson, *Min. Proc. Inst. C.E.*, vol. 32.)

Centrifugal pumps work to best advantage only at the particular lift for which they are designed When employed for variable lifts, as is constantly the case in practice, their efficiency is much reduced and does not exceed ·5. It is often much less.

Few centrifugal pumps utilize more than a small fraction of the energy of motion possessed by the water at exit from the wheel, and an investigation of their efficiency on the supposition that this energy is wholly wasted is therefore of considerable interest.

Let h_0 be the actual lift, and let all frictional losses except that specified be neglected; then, if u be the velocity of flow, and v the velocity of whirl at exit, the loss of head is $(u^2+v^2)/2g$, and the gross lift is

$$h = h_0 + \frac{u^2 + v^2}{2g}.$$

Substituting this value of h in the formula for V, and replacing u by its value $(V-v)\tan\theta$, we obtain

$$Vv - gh = gh_0 + \frac{v^2 + (V-v)^2\tan^2\theta}{2}.$$

Adding $\frac{1}{2}V^2$ to each side, and re-arranging the terms,

$$\tfrac{1}{2}V^2 = gh_0 + \tfrac{1}{2}(V-v)^2 . \sec^2\theta,$$

a formula from which we find

$$\text{Efficiency} = \frac{h_0}{h} = \frac{V^2 - (V-v)^2\sec^2\theta}{2Vv}$$

$$= \sec^2\theta\left(1 - \tfrac{1}{2}\left(\frac{v}{V} + \frac{V}{v}\sin^2\theta\right)\right).$$

This result shows that the efficiency is greatest when

$$v = V . \sin\theta;$$

and on substitution we find

$$\text{Maximum efficiency} = \sec^2\theta(1 - \sin\theta) = \frac{1}{1+\sin\theta}.$$

The speed of maximum efficiency is found from the equation

$$\tfrac{1}{2}V_0^2 = gh_0 + \tfrac{1}{2}V_0^2\sec^2\theta . (1 - \sin\theta)^2,$$

which gives

$$V_0^2 = (1 + \operatorname{cosec}\theta)gh_0.$$

The proper velocity of flow is

$$u_0 = V_0 . \tan\theta(1 - \sin\theta),$$

and the area of flow through the periphery of the wheel should be made to give this velocity with the intended delivery.

At any other speed V the velocity of flow will be given by

$$u^2 = (V^2 - 2gh_0)\sin^2\theta,$$

and the efficiency may be found by the preceding formula.

In the best examples of centrifugal pumps working at a suitable lift, it is probable that enough of the energy of motion on exit from the wheel is utilized to provide for the various frictional resistances neglected in this investigation. But in ordinary cases this will not be true, and the efficiency is much reduced. The theory, taking into account all the resistances, is too intricate to enter on here.

When a centrifugal pump is started the fan is filled with water which, in the first instance, rotates as a solid mass with the fan. If the radius of the inner periphery be m times that of the outer where m is a fraction, it will not commence to deliver water till the speed reaches the value

$$V^2(1 - m^2) = 2gh_0.$$

But when once started, the speed may be reduced below this value without stopping the delivery, provided that some of the energy of motion on exit from the wheel is utilized. This has been observed to occur in practice, and it will serve as a test of efficiency.

266. *Limitation of Diameter of Wheel.*—For a given fall in a motor or lift in a pump the diameter of wheel in a turbine is limited, because the frictional resistances increase rapidly with the diameter.

Let u be the velocity of flow, d the diameter, b the inside breadth of wheel at exit; then the delivery in cubic feet per second is

$$Q = ub\pi d.$$

Now, if the breadth b be too small as compared with the diameter, the surface friction of the passages through the wheel will be too great, as in the case of a pipe the diameter of which is too small for the intended delivery. Thus b is proportional to d: also, we have seen that u is proportional to V_1, that is to $\sqrt{h}$, and it follows therefore, by substitution for b and u, that

$$Q = Cd^2\sqrt{h},$$

where C is a co-efficient.

If the wheel be wholly immersed in the water the surface friction (Ex. 8, p. 521) is relatively increased by increasing the diameter. On investigating how great the diameter may be without too great a loss we arrive at the same formula.

Where it is of importance to have as large a diameter as possible to reduce the number of revolutions per minute, the diameter of wheel in a pump or a turbine is therefore found by the formula

$$d = \sqrt{\frac{G}{c\sqrt{h}}}.$$

If G be the delivery in gallons per minute, h the actual fall in feet, d the external diameter also in feet, the value of c for an outward-flow turbine is about 200. In a centrifugal pump the value is probably not very different, h being now the actual lift, but it varies in different types of pump.

Centrifugal pumps cannot generally be employed for high lifts, partly because it becomes increasingly difficult to utilize the energy of motion on exit from the wheel, and partly on account of disc friction. The fan rotates more than twice as fast as the wheel of a turbine, and the disc friction is consequently more than four times as great.

267. *Impulse Wheels.*—The formula

$$V_1^2 = gh,$$

which gives the speed of a turbine wheel in terms of the effective head, also gives the velocity of whirl at entrance, and therefore shows that, of the whole head employed in driving the wheel and producing the velocity of flow, one-half operates by impulse. The remainder operates by pressure, and this class of turbines is therefore not simple impulse, but impulse-pressure machines. It is necessary therefore that the wheel should revolve in a casing, and that the passages should be always completely filled with water. The diameter of wheel is then limited as explained in the last article, and for a small supply of water and a high fall the number of revolutions per minute becomes abnormally great. This consideration and the necessity of adaptation to a variable supply of water render it often advisable to resort to a machine in which the passages are actually or virtually open to the atmosphere. The whole of the energy of the fall is then converted into the kinetic form before reaching the wheel, and consequently operates wholly by impulse.

A wheel of this kind approaches closely in principle to the Poncelet water wheel mentioned in Art. 261, but is often still described as a "turbine," because the water is guided by fixed blades before reaching the wheel. A common example is the Girard turbine. The flow of the water is here parallel to the axis of the wheel, spiral guide blades are ranged round the circumference of a cylinder like the threads of a screw in order to give the necessary whirl to the water before entrance. The wheel is provided with a similar set of spiral vanes curved in the opposite direction, which reduce it to rest as it passes through. In the French *roue à poire* the wheel is conical, the water enters at the circumference, and, guided by spiral vanes, descends to the apex where it is discharged.

268. *Propellers.*—The subject of propellers is outside the limits of simple hydraulics for reasons already indicated when considering the resistance of ships (Art. 249). Nevertheless they may be regarded as hydraulic machines, and their connection with the machines already referred to forms a proper subject for consideration.

Every propeller operates by means of the mutual action between it and the water on which it acts consequent on a change of velocity which it produces in the water of the sea; it is therefore an impulse and reaction machine applied so as to produce a propelling force which drives the vessel through the water. Since the resistance of the vessel is directly astern, the change of motion produced is sternward so far as it is of any utility for the purpose. Some forms of propeller, as, for example, the screw, give lateral motions to the water, but the energy thus employed is wasted.

(1) The simplest form of propeller is that in which the water is drawn into the vessel through orifices of proper size, and projected by means of a centrifugal pump through two orifices in the side of the vessel so placed that the water issues directly astern. The reaction of the issuing jet furnishes a propelling force on the vessel. The problem here is just the same as that of the simple reaction wheel already considered in Art. 263: the fact that the orifices move in straight lines instead of in a circle making no difference in the propelling reaction. Hence, if R be the resistance of the vessel, v the velocity of discharge, V the velocity of the vessel,

$$R = \frac{wQ}{g}(v - V).$$

The engine power is employed in the first instance in creating a head h, but, supposing this known, the question is unaltered, and therefore neglecting frictional resistances,

$$\text{Efficiency} = \frac{2V}{v + V} = \frac{1}{1 + e}.$$

If we call the counter-efficiency $1 + e$, and if we further suppose A to be the *joint* area of the orifices, and $K \,.\, \Delta^{\frac{2}{3}} V^2$ the resistance of the ship (Art. 249), we shall find by substitution that for all speeds

$$K \,.\, \Delta^{\frac{2}{3}} = \frac{wA}{g}(1 + 2e)\ 2e,$$

a formula which shows that the efficiency is increased by increasing the size of the orifices, and enables us to calculate the size for a given efficiency.

In every propeller, in the absence of frictional resistances and of any disturbance due to the passage of the vessel through the water, *the efficiency of a propeller is greater the greater the quantity of water on which it operates.* In the case of the jet propeller which we are now considering, this general conclusion is modified by the effect of frictional resistances. If we make the same supposition as to these as in the reaction wheel, the efficiency will be found, as before, to be greatest for a certain value of v/V, and this will correspond to a certain definite area of orifice. The circumstances being somewhat different, a different way of expressing the frictional resistances would probably more closely represent the facts, but the general conclusion must be the same. It is of little use to consider this more closely, as the disturbance of the water passing near the vessel, produced by drawing a large quantity of water through the orifices of entry, must modify in an unknown way the resistance of the ship. Hence the best magnitude and position of the orifices of entry and efflux can only be found by careful experiments. Such experiments have not hitherto been carried out, but it may be considered probable that the jet propeller cannot compete with other forms of propeller so far as efficiency is concerned. If it is ever introduced, it will be for the sake of facility in manœuvring or some other similar reason.

(2) If the action of paddles be observed, two streams of water are seen proceeding from the floats which play the part of the jets in a jet propeller. The most efficient kind are those in which the floats turn about axes on which they are mounted in such a way as to enter and leave the water without any shock. The streams are then simple jets of area not very different from that of the floats, and are driven back with a velocity which is about the same as that of the floats themselves. If, then, v be the speed of the paddles as calculated from their diameter and revolutions, V the speed of the ship, the propelling reaction is given by the same formula as for jets, namely,

$$R = \frac{w}{g} Q.(v - V),$$

the velocities being in feet per second, and Q the quantity acted on in cubic feet per second. The energy exerted per second is, however, now Rv, and therefore

$$\text{Efficiency} = \frac{V}{v}.$$

For given values of v and V the efficiency is less than that of the simple jet when the frictional resistances are left out of account. The reason of this is that the value just found includes the loss due to breaking up as the paddles strike the water and drive it upwards in a mass of foam before it settles down to the nearly undisturbed motion of the streams. In smooth water, when the paddles are not too deeply immersed, the efficiency however of paddles is far greater than that of the jet, because the area of orifice which can practically be employed is so limited in the jet and the frictional losses in the pump and passages so great.

(3) In rough water the efficiency of paddles is much reduced, and this is also the case when the immersion varies in consequence of alterations in the displacement of the vessel due to consumption of fuel on a voyage or other causes. In sea-going vessels, therefore, the paddles are replaced by a screw propeller.

In this case the action of the propeller is much more complex: the water has a rotatory velocity communicated to it as well as a sternward velocity, and these velocities are different for each portion of the screw blade. Further, the water in which the screw works has very complex motions due to the action of the sides of the ship upon it, a circumstance which affects the resistance of the ship as well as the action of the propeller. For these reasons the screw is not so efficient, other things being the same, as well constructed paddles. On the other hand the quantity of water acted on is large, and the action is not greatly influenced by the circumstances just mentioned as reducing the efficiency of paddles.

On comparing a screw propeller with the machines already considered, it will be perceived that it is a parallel-flow impulse wheel reversed, with two important modifications. First, the fixed guide blades are omitted. It is true that it has been proposed to employ such blades in order to avoid the loss occasioned by the transverse velocity given to the water which is useless for propelling purposes, but the gain of efficiency, if any, is very small. Secondly, the number of blades in the wheel is large; whereas in a screw propeller there are often only two, and in all cases they occupy much less than half the whole circumference.

Both these modifications are due to the same cause, and arise from the fact that in the propeller the changes of velocity produced by the action of the blades are small compared with the whole velocity of the water through the propeller, whereas in the wheel they are large. Hence the frictional resistances in the propeller are disproportionately great and render it necessary to reduce the surface exposed to the water as much as possible.

This reasoning also shows that the frictional losses in the centrifugal pump and passages of a jet propeller must be great. Such a pump should receive the water at the velocity of the ship, and gradually increase its velocity to that attained on efflux from the orifices. The frictional resistances will depend on the whole velocity, but the propelling reaction on the difference of velocities. Or if the alternative be adopted of checking the motion of the water on entry, and afterwards giving it the whole velocity of efflux, the reduction of velocity will be difficult to accomplish without considerable loss, and the propelling reaction will not be greater than before. All unnecessary changes of velocity are a cause of loss.

EXAMPLES.

1. In a reaction wheel the speed of maximum efficiency is that due to the head. In what ratio must the resistance be diminished to work at four-thirds this speed and what will then be the efficiency? Obtain similar results when the speed is diminished to three-fourths its original amount.

Ans. Efficiency = ·63 or ·64.
Ratio = ·84 or 1·14.

2. Water is delivered to an outward-flow turbine, at a radius of 2 feet, with a velocity of whirl of 20 feet per second, and issues from it in the reverse direction at a radius of 4 feet, with a velocity of 10 feet per second. The speed of periphery at entrance is 20 feet per second, find the head equivalent to the work done in driving the wheel. *Ans.* 24·22 feet.

3. In a Fourneyron turbine the internal diameter of the wheel is $9\frac{1}{2}$ inches, and the outside diameter 14 inches. The effective head (p. 509) is estimated at 270 feet: find the number of revolutions per minute. *Ans.* 2200.

NOTE.—These data are about the same as those of a turbine erected at St. Blasien in the Black Forest.

4. An inverted Barker's mill (p. 511) is used as a centrifugal pump. If the coefficient of hydraulic resistances referred to the orifices be ·125, show that the speed of maximum efficiency is that due to twice the lift, and find the maximum efficiency. *Ans.* Maximum efficiency = ·75.

5. A centrifugal pump delivers 1500 gallons per minute. Fan 16 inches diameter. Lift 25 feet. Inclination of vanes at outer periphery to the tangent 30°. Find the breadth at the outer periphery that the velocity of whirl may be reduced one-half, and also the revolutions per minute, assuming the *gross* lift $1\frac{1}{2}$ times the actual lift. *Ans.* Breadth = $\frac{3}{4}$ inch. Revolutions = 700.

6. In the last question find the proper sectional area of the chamber surrounding the fan (p. 513) for the proposed delivery and lift. Also examine the working of the pump at a lift of 15 feet. *Ans.* 24 sq. inches.

7. A jet of water, moving with a given velocity, strikes a plane perpendicularly. Find how much of the energy of the jet is utilized in driving the plane with given

speed. Determine the speed of the plane for maximum efficiency, and the value of the maximum efficiency. *Ans.* Speed of maximum efficiency = one-third that of jet. Maximum efficiency $= \frac{8}{27}$.

8. Assuming the ordinary laws of friction between a fluid and a surface, and supposing that any motion of the fluid due to friction does not affect the question: find the moment of friction (L), and the loss of work per second (U), when a disc of radius a rotates with speed of periphery V.

$$Ans.\ L = f \, . \, \frac{2\pi}{5} \, . \, a^3 V^2 \, ; \quad U = f \, . \, \frac{2\pi}{5} \, . \, a^2 \, . \, V^3.$$

9. If the rotating disc in question 8 be surrounded by a free vortex of double its diameter, show that the loss by friction of the vortex on the flat sides of the vortex chamber is $2\frac{1}{2}$ times the loss by friction of the disc.

10. The resistance of the *Waterwitch* at 8 knots is 5,500 lbs., the orifices of her jet propeller are each 18 inches by 24 inches, what must be the delivery of her centrifugal pump in gallons per minute to propel her at this speed, and what will be the efficiency, neglecting frictional resistances.

Ans. Velocity of efflux = 29·3 per second.
Delivery = 66,000 gallons per minute.
Efficiency = ·63.

11. In the last question find the H.P. required for propulsion, assuming the efficiency of the pump and engines ·4. *Ans.* 525.

12. If the jet propeller of the *Waterwitch* be replaced by feathering paddles, what will be the area of the stream driven back for a slip of 25 per cent. Find the efficiency and the water acted on in gallons per minute.

Ans. Joint area of streams = 34 square feet.
Efficiency = ·75.
Delivery = 236,000 gallons per minute.

REFERENCES.

The subject of hydraulic machines is very extensive, and it is impossible within the limits of a single chapter to do more than give a general idea of their working. For descriptive details and illustrations the reader is referred, amongst other works, to

GLYNN. *Power of Water.* Weales' Series.
FAIRBAIRN. *Millwork and Machinery.* Longman.
COLYER. *Water-Pressure Machinery.* Spon.
BARROW. *Hydraulic Manual.* Printed by authority of the Lords Commissioners of the Admiralty.

The theory of hydraulic machines is further developed in Professor Unwin's work on Hydraulics, already cited on page 481, and the various special treatises therein mentioned, also in Rankine's treatise on the *Steam Engine and other Prime Movers.*