CHAPTER XXI.

PNEUMATICS AND THERMODYNAMICS.

269. *Preliminary Remarks.*—An elastic fluid under pressure is a source of energy which, like a head of water in hydraulics (p. 445), may be employed in doing work of various kinds by a machine, or simply in transferring the fluid from one place to another. In hydraulics we commence with the case of simple transfer, but the density of gases is so low that, unless the differences of pressure considered are very small, the inertia and frictional resistances of the fluid employed in a pneumatic machine have little influence: it is the elastic force, which is the principal thing to be considered. In studying pneumatics, therefore, we commence with machines working under considerable differences of pressure and then pass on to consider the flow of gases through pipes and orifices together with those machines in which the inertia and frictional resistances of the fluid cannot be neglected.

270. *Expansive Energy.*—The special characteristic of an elastic fluid is its power of indefinite expansion as the external pressure is diminished. While expanding, it exerts energy of which the fluid itself is, in the first instance, the source, whereas the energy exerted by an incompressible fluid is transmitted from some other source. Expansive energy is utilized by enclosing the fluid in a chamber which alternately expands and contracts; the common case being that of a cylinder and piston.

Fig. 195 represents in skeleton a cylinder and piston enclosing a mass of expanding fluid. Taking a base line aa' to represent the stroke, set up ordinates to represent the total pressure S on the piston in each position; a curve $1Q2$ drawn through the extremities of these ordinates is the Expansion Curve. Reasoning as in Art. 90, p. 195, the area of this curve represents the energy exerted as the piston moves from the position 1, where the expansion com-

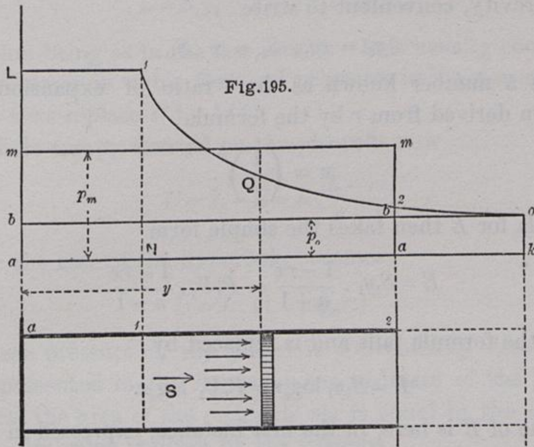


Fig. 195.

mences, to the position 2, where it terminates. One common case was considered in the article cited, namely, that in which the expansion curve is a common hyperbola. This is included in the more general supposition,

$$Sy^n = S_1y_1^n = S_2y_2^n,$$

where y is the distance of the piston from the end a of its stroke, and n is an index which, for the particular case of the hyperbola, is unity. Most cases common in practice may be dealt with by ascribing a proper value to n; for air it ranges between 1 and 1·4, and for steam it is roughly approximately unity. The suffixes indicate the points at which the expansion commences and terminates.

If, now, E be the energy exerted during expansion,

$$E = \int_{y_2}^{y_1} Sdy = S_1y_1^n \int_{y_1}^{y_2} y^{-n}dy = S_1y_1^n.\frac{y_2^{1-n} - y_1^{1-n}}{1-n}.$$

This formula may be written in the simpler form

$$E = \frac{S_1y_1 - S_2y_2}{n-1},$$

in applying which, the terminal pressure S_2 is supposed to have been previously found from the equation

$$S_2 = S_1 \cdot \left(\frac{y_1}{y_2}\right)^n.$$

It is, for brevity, convenient to write

$$y_2 = ry_1;\ S_2 = x \,.\, S_1,$$

where r is a number known as the "ratio of expansion," and x is a fraction derived from r by the formula

$$x = \left(\frac{1}{r}\right)^n.$$

The formula for E then takes the simple form

$$E = S_1 y_1 \,.\, \frac{1 - rx}{n - 1} = P_1 V_1 \,.\, \frac{1 - rx}{n - 1}.$$

If $n = 1$ the formula fails and is replaced by

$$E = S_1 y_1 \log_\varepsilon r = P_1 V_1 \log_\varepsilon r.$$

The value of E is here, in the first instance, expressed in terms of the total pressure on the piston, but as in Art. 255, p. 485, we may replace S by PA, and Ay by V, so that $S_1 y_1$ is replaced by $P_1 V_1$. In "rotatory" engines and pumps the expanding chamber is not a simple cylinder and piston, but is formed from a turning pair. Or, more generally, the chamber pair may be formed from any two links of a kinematic chain which it may be convenient to select for the purpose. In its last form the formula is applicable in every case. If the expansion curve be not given in the form supposed, the value of E is determined graphically by measuring the area of the curve, in doing which, when the chamber is not a simple cylinder, the base of the diagram must represent the volume swept out by the chamber pair, and the ordinates the pressures per unit of area.

271. *Transmitted Energy.*—The energy exerted by an elastic fluid consists not merely of that derived from the expansive power of the fluid pressing against the piston, but also of that which is transmitted in the same way as would be the case if it were incompressible. The fluid is supplied from a reservoir, which may either be an accumulator in which it is stored by the action of pumps, or a vessel in which, by the action of heat, it is generated or its elasticity increased.

In any case, so long as the cylinder remains in communication with the reservoir the fluid enters at nearly constant pressure, and energy is exerted on the piston just as in the water-pressure engine. During this period of admission the energy exerted is

$$L = S_1 y_1 = P_1 V_1 = 144 p_1 V_1,$$

the notation being as in the last article. It is usually convenient to express volumes in cubic feet and pressures in lbs. per square inch. We must thus replace P by $144p$.

The whole energy exerted on the piston is now

$$U = L + E = L \,.\, \frac{n - rx}{n - 1},$$

which for the case of the hyperbola becomes

$$U = L \,.\, (1 + \log_\epsilon r.)$$

The mean pressure on the piston is conveniently denoted by p_m, and is represented in the figure by the ordinate of the line mm so drawn that the area of the rectangle ma is equal to the area of the diagram. Its value is given by the formulæ

$$p_m = \frac{p_1}{r} \,.\, \frac{n - rx}{n - 1}\,;\; p_m = p_1 \,.\, \frac{1 + \log_\epsilon r}{r}.$$

A reservoir filled with an elastic fluid at high pressure is an accumulator, the absolute amount of energy stored in which is the expansive energy or the total energy according as the pressure is not, or is, maintained by the addition of fresh fluid in place of that discharged, the expansion being supposed indefinite in either case. With the law of expansion already supposed, when n is greater than unity, rx vanishes when the expansion curve is prolonged indefinitely. The total absolute energy is then

$$U_1 = P_1 V_1 \,.\, \frac{n}{n - 1},$$

where V_1 is the whole volume of fluid considered.

When n is not greater than unity, U_1 is infinite. In practice, however, there is always a "back" pressure P_0 on the working piston, or, more generally, on the sides of the chamber in which the fluid is enclosed. In overcoming this, the work $P_0 r V_1$ is done, and nothing is gained by prolonging the expansion beyond the point 0 at which the terminal pressure P_2 has fallen to P_0. The corresponding ratio r_0 is given by the formula

$$\log r_0 = \frac{1}{n}\log\left(\frac{1}{x_0}\right) = \frac{\log P_1 - \log P}{n}.$$

The available energy is found by writing $r = r_0$, $x = x_0$ in the value of U, and subtracting $P_0 r_0 V_1 = P_1 V_1 r_0 x_0$. The result is

$$\frac{n}{n-1} \cdot P_1 V_1 (1 - r_0 x_0) = U_1 - U_0,$$

being the difference of the values of U when the expansion commences at 1 and at 0. It is always finite, and is graphically represented by the area $L10bb$.

In the transmission and storage of energy by elastic fluids this quantity plays the same part as the "pressure-head" in hydraulics, to which indeed it reduces if n be supposed very great, r_0 unity, and V_1 the volume of a lb. of water. It is the energy of a given quantity of fluid due to a given difference of pressure. The term "head" may be used for this when the quantity considered is a mass of 1 lb. When $n = 1$

$$U_1 - U_0 = P_1 V_1 \log_\epsilon r_0.$$

When the reservoir is not kept full the only available energy is the expansive energy, less the work done in overcoming P_0 through the volume $V_0 - V_1$. This is graphically represented by the curvilinear triangle $N01$, and is most conveniently given by the formula

$$A.E = U_1 - U_0 - (P_1 - P_0) V_1.$$

272. *Cycle of Mechanical Operations in a Pneumatic Motor—Mechanical Efficiency.*—Motors operating by the pressure of an elastic fluid may be described generally as Pneumatic Motors. They are either supplied from an accumulator as in hydraulic motors of the same class, or they may be heat-engines serving as the means by which heat energy is utilized. In either case the mechanism of the motor is the same, and consists of a chamber which expands to admit the fluid and contracts to discharge it, with a proper kinematic chain for utilizing the motion of the chamber pair.

In water-pressure engines the contraction to expel the water from the chamber is not considered, because all pressures are reckoned above the atmosphere, and the pressure in the accumulator is so great that small differences of pressure may be disregarded. With elastic fluids it is commonly different: the "exhaust" of the chamber must be taken into account.

Returning to Fig. 195, suppose that the piston has reached the end of its stroke, the cylinder is then filled with fluid of a certain pressure p_2 which may be supposed known. Let now a valve be opened allowing the cylinder to communicate with the atmosphere, or with a reservoir containing fluid at a lower pressure p_0. The fluid in the cylinder then rushes out into the reservoir, and the pressure in the cylinder speedily subsides to p_0; the fluid expands in this process, but its expansive energy is wasted in producing useless motions in the air which afterwards subside by friction. After subsidence let the piston be moved back by an external force applied to it which supplies the energy necessary to overcome the "back" pressure p_0. The fluid is discharged from the chamber, and so long as the communication with the exhaust reservoir is open the pressure remains constantly p_0. We represent this on the diagram by drawing a horizontal line bb, the ordinate of which is p_0. The work done in overcoming back pressure is $144p_0V_2$ and is represented on the diagram by the rectangle ba; this is so much subtracted from the energy exerted by the motor.

Thus the volume of the chamber goes through a cycle of changes alternately expanding and contracting. During expansion energy is exerted, the corresponding mean pressure p_m is the "mean forward pressure." During contraction work is done, and the corresponding mean pressure is the "mean back pressure." The difference between the two is the "mean effective pressure" which measures the useful work done, as shown by the equation

$$\text{Useful work} = (p_m - p_0)144V_2,$$

and is graphically represented by the area of the closed figure $L12bb$.

In most cases the moveable element of the chamber pair divides the chamber into two parts, one of which expands while the other contracts, and conversely: the motor is then described as "double acting." The force acting on the moving piece is then the difference between the forward pressure in one chamber and the back pressure in the other, and when the stress on the parts of the machine is to be considered this is the effective pressure upon which the stress depends (p. 240). For all other purposes, however, the back pressure is to be taken as just explained.

If the pressures p_1, p_0 in the supply and exhaust reservoirs be given, and also the form of the expansion curve, the only waste of energy in this process arises from incomplete expansion. Imagine the expansion curve prolonged to the point o where it meets the back pressure line, and suppose the stroke lengthened so as to reach this point, then additional work would be done by the fluid which would be represented graphically by the area of the curvilinear triangle $2ob$. This area represents energy lost by unbalanced expansion, and to avoid it the expansion must be "complete," that is, the fluid must be allowed to expand till its pressure has fallen to p_0, the pressure in the exhaust reservoir, a condition seldom fulfilled in practice, because the loss by friction and other causes becomes disproportionately great. Leaving this out of account, a pneumatic motor is capable of exerting only a certain maximum amount of energy, quite irrespectively of the nature of its mechanism, but dependent only on the pressures between which it works and the nature and treatment of the fluid. A motor which reaches this maximum power may be described as *mechanically* perfect, and the ratio of the actual useful work done to the theoretical maximum may be described as the MECHANICAL EFFICIENCY of the motor.

In practice the mechanical efficiency is diminished not only by incomplete expansion, but also by a portion of the fluid being retained in the "clearance" space of the chamber after the exhaust is completed. This, however, is a detail which cannot be considered here. The theoretical maximum is clearly the same as the store of energy in the fluid used, already found in the last Article, and denoted by $U_1 - U_0$. The consumption of fluid (neglecting clearance) is one cylinder full, at the terminal pressure, in each stroke.

273. *Pneumatic Pumps.*—A pneumatic like an hydraulic motor may be reversed by applying power to drive it in the reverse direction, and the machine thus obtained is a Pump which takes fluid at a low pressure and compresses it into a reservoir at high pressure.

The cycle in the pump is the same as the cycle in a motor, but the operations take place in reverse order. As the chamber expands fluid is drawn in from the low-pressure reservoir and energy is exerted on the piston by the original "back" pressure: as the chamber contracts the fluid is compressed till it reaches the pressure p, when a valve opens and admits it to the high-pressure reservoir. There is, however, this important difference, namely, that the process of unbalanced expansion in the motor cannot be reversed; and therefore, if the pump is to operate on the same weight of fluid, the volume of the working cylinder must be enlarged so that the expansion curve may start from o. If this be supposed, the compression curve will, for the same fluid treated in the same way, be identical with the expansion curve of the motor. If there were no unbalanced expansion the motor would be exactly reversible, and the condition of a motor being mechanically perfect may therefore be described by saying that it must be mechanically reversible. The difference of working of the valves in pumps and motors has already been referred to in Art. 260.

Air pumps are employed either to compress atmospheric air to a high pressure or to exhaust a chamber. In the second case the atmosphere is the high pressure reservoir into which the exhaust air is compressed. In the old "atmospheric" engine the steam was employed merely to produce a vacuum, the motive force being the pressure of the atmosphere: this machine is therefore a reversed air pump.

In all pneumatic motors a pump is required to replace the fluid in the supply reservoir. Unless the motor be a heat engine this pump must be driven by external agency, and the whole process is one of distribution, transmission, and storage of energy, as in water-pressure engines. In Whitehead torpedoes indeed, and in some other similar cases, the reservoir is not kept full: but the motor then works with constantly diminishing power till the store of energy is exhausted. In condensing steam engines an air pump is employed to exhaust the condenser.

The work done in pumping is found by the formulæ of the last article. Examples will be found at the end of this chapter.

274. *Indicator Diagrams.*—The pressure existing in the chamber of a pneumatic machine may be graphically exhibited by means of an instrument called an Indicator. In steam engines especially its use is indispensable to enable the engineer to study the action of the steam.

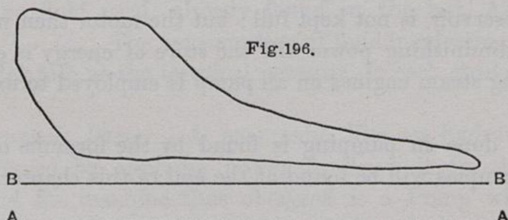
Figs. 1 and 2, Plate XII., show an indicator in elevation and section. *S* is a drum revolving in a vertical axis, *A* is a cylinder communicating with the steam cylinder, the pressure in which is to be measured. *P* is a pencil connected by linkwork with a small piston *H* so as to move with it up or down in a vertical line. The piston is pressed down by a spring which measures the pressure, while the drum, by means of a cord passing over pulleys and connected with the steam piston, revolves through arcs exactly proportional to the spaces traversed by it. A card is folded round the drum, and as the engine moves a curve is traced by the pencil upon it which shows the pressure at each point of the stroke. In practice many precautions are necessary to secure accuracy in the diagram; the more so the higher the speed, because the friction and inertia of the parts of the indicator, together with unequal stretching of the cord and inaccuracy in the reducing motion connecting the drum with the steam piston, may give rise to serious errors. To diminish the effect of inertia the stroke of the indicator piston is made short and multiplied by linkwork.

In the example shown (Crosby's patent) the spring applied to the drum to keep the cord tight has a tension which increases as the drum rotates from rest. This increase compensates for the inertia of the drum, and is said to give a more nearly uniform tension of the cord.

Fig. 196 shows an indicator diagram taken in this way from the high-pressure cylinder of a compound engine.

BB is the atmospheric line drawn on the card by the indicator

Fig.196.



pencil when the cylinder communicates with the atmosphere. *AA* is the vacuum line laid down on the diagram at a distance below *BB*, which represents the pressure of the atmosphere, as found by the barometer, reckoned on the scale of pressures. Then on the

PLATE XI.

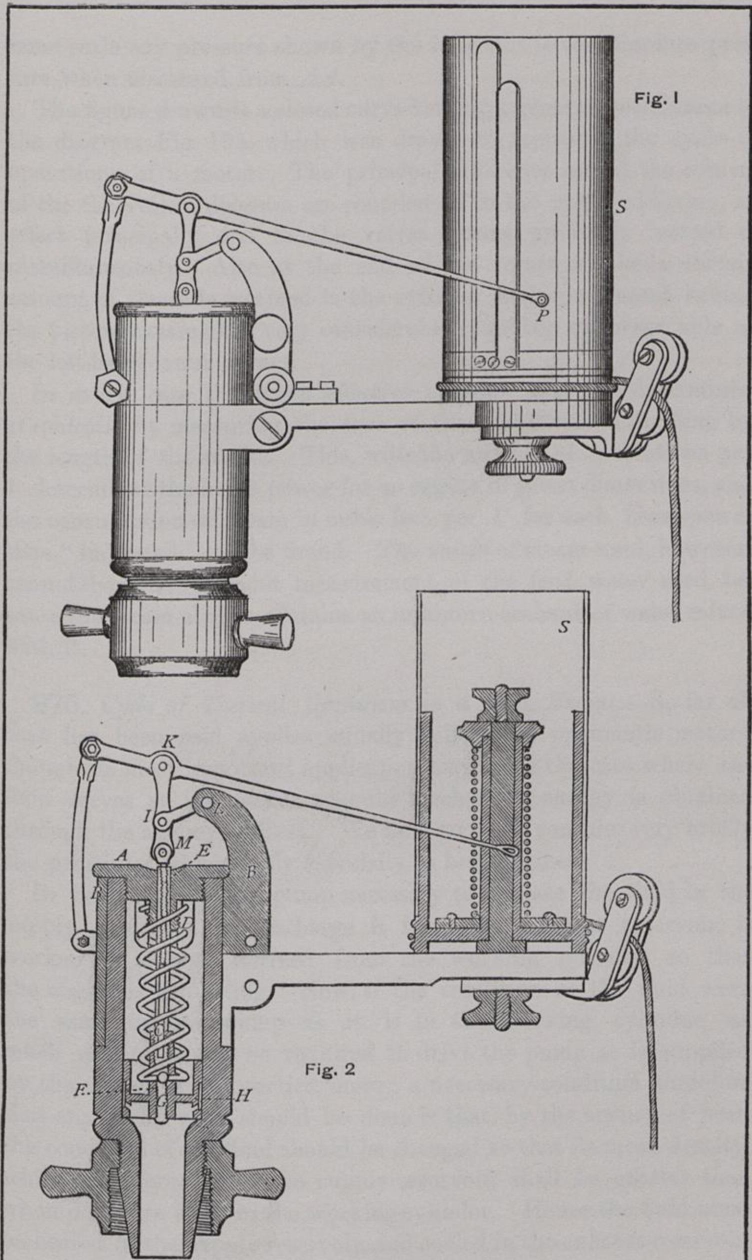


Fig. 1

Fig. 2

To face page 530.

same scale any pressure shown by the indicator is the absolute pressure when measured from *AA*.

The figure drawn is a closed curve bearing a general resemblance to the diagram Fig. 195, which was drawn to represent the cycle of operations of a motor. The principal difference is that the corners of the theoretical diagram are rounded off in the actual diagram, an effect principally due to the valves closing gradually instead of instantaneously. Also at the end of the return stroke a certain amount of steam is retained in the cylinder and compressed behind the piston, causing the very considerable rounding off observable at the left-hand lower corner.

In every case the mean effective pressure may be determined graphically by measuring the area of the diagram and dividing by the length of the stroke. This, with the number of revolutions per 1′ determines the horse power for an engine of given dimensions, and the consumption of steam in cubic feet per 1′ for each horse-power thus "indicated" can be found. The *weight* of steam used, however, cannot be found without measurement of the feed water used, because the steam always contains an unknown amount of water mixed with it.

275. *Cycle of Thermal Operations in a Heat Engine.*—So far all that has been said applies equally well to all pneumatic motors, though its most important application may be to the case where the fluid serves as the means whereby mechanical energy is obtained through the agency of Heat. We now go on to consider very briefly the principles which apply especially to heat engines.

In heat engines the pump necessary to replace the fluid in the supply reservoir, or discharge it from the exhaust reservoir, is worked by energy derived from the working cylinder, so that the engine is self-acting. Now, if the condition of the fluid were the same in the pump as it is in the working cylinder, as much energy would be required to drive the pump as is supplied by the motor, or, in practice, more; a necessary condition therefore that any useful work should be done is that, by the agency of heat, the condition of the fluid should be changed so that its mean density, while being forced into the supply reservoir, shall be greater than when doing its work in the working cylinder. Hence the fluid must be heated in the supply reservoir, and cooled in the exhaust reservoir, and therefore, in every heat engine, in addition to the cycle of mechanical operations there is a cycle of thermal operations consisting of an alternate addition and subtraction of heat; the heat in question being supplied by a body of high temperature and abstracted by a body of low temperature.

In non-condensing steam engines the pump is the feed pump which supplies the boiler with the fluid in the state of water; in the boiler heat is supplied which converts it into steam of density many hundred times less than that of water. The pump is, in this case, very minute, and requires a trifling amount of energy to work it. In condensing engines we have, in addition, the air pump.

In air engines the compressing pump is generally a conspicuous part of the apparatus and requires a large fraction of the power of the motor to drive it; because the changes of density due to the alternate heating and cooling are comparatively small.

276. *Mechanical Equivalent of Heat.*—Heat and mechanical energy are mutually convertible: a unit of heat corresponding to a certain definite amount of mechanical energy which is called the "MECHANICAL EQUIVALENT" of heat. In British units the thermal unit is equivalent to 772 ft.-lbs.

The statement here made is the First Law of the science of Thermodynamics, and it shows that quantities of heat may be expressed in units of work, and, conversely, quantities of work in units of heat. In dealing with questions relating to heat and work, a common unit of measurement must be selected. In most cases the thermal unit is adopted, and quantities of work reduced to such units by division by 772.

In heat engines the cycle of thermal operations consists of an alternate addition of heat (Q), and subtraction of heat (R), so that, if kU be the useful work,

$$kU = Q - R,$$

that is, the work is done at the expense of an equivalent amount of heat which disappears during the action of the engine. In steam engines this has been tested experimentally by measuring the heat supplied in the boiler and the heat discharged from the condenser. The difference should be, and in fact is found to be, the thermal equivalent of the work done by the engine. The ratio kU/Q is called the "absolute," or sometimes, for reasons we shall see presently, the "apparent" efficiency of the engine. It is always a small fraction: in the best steam engines, for example, it does not exceed ·13, losses connected with the furnace and boiler not being included. (Comp. Art. 278.) The quantity U is here the theoretical maximum (p. 528) for a pneumatic motor working between the given limits of pressure, and k is the "mechanical" efficiency.

277. *Mechanical Value of Heat. Thermal Efficiency.*—In stating the first law of thermodynamics nothing is said about the temperature at which the heat is used. In other words, the mechanical equivalent of heat is just the same whether the temperature be low or high. Yet common experience tells us that the value of heat for mechanical purposes depends very much on this circumstance. The heat discharged from the condenser of a condensing steam engine, or with the exhaust steam of a non-condensing engine, is of little value for the purposes of the engine. So obvious is this fact that the first attempts at connecting the work done by a heat engine with the heat supplied to it may be partly described as attempts to show that temperature, not quantity, was equivalent to energy, heat being supposed as indestructible as matter.

It is now known, however, that difference of temperature is not in itself energy, but merely an indispensable condition that heat may be capable of being converted into work. The power of a heat-engine depends on difference of temperature being greater, the greater that difference is; but in all cases only a fraction of the heat supplied is converted into mechanical energy.

In the converse operation of converting mechanical energy into heat it is possible, by employing it in overcoming frictional resistances, to obtain an amount of heat equal to the energy employed, but such processes are always irreversible. The only way of converting heat into work is by means of a heat engine in which the rejection of heat at low temperature is as essential as the supply of heat at high temperature.

Difference of temperature is wasted if heat be allowed to pass from a hot body to a cold one without the agency of steam, air, or some other body, the density of which is changed by its action. When once wasted it cannot be recovered, a fact of common experience which is expressed by stating the Second Law of Thermodynamics.

Heat cannot pass from a cold body to a hot one by a purely self-acting process.

By a "self-acting" process in this statement is meant any process of the nature of a perpetual motion which is independent of any external agency. By the employment of mechanical energy drawn from external bodies, heat may be made to pass from a cold body to a hot one, the amount of energy required being greater the greater the difference of temperature. And the method recently introduced of raising steam, without the use of a furnace, by means of heat derived from the exhaust steam condensed in a solution of caustic soda, shows that energy derived from chemical affinity may serve the purpose. But, if no energy is employed, no heat will pass.

Difference of temperature must therefore be carefully utilized, and since the smallest difference of temperature is sufficient to cause heat to pass from a source into the air or steam which exerts energy, it at once follows that the process of conversion of heat into work will be most efficient if all the heat be supplied while the fluid has the temperature of the source of heat, and all the heat rejected while it has the temperature of the body which subtracts heat. These are the conditions of maximum efficiency, and if they are satisfied it is possible to show that a mechanically perfect motor (p. 528) supplied with heat Q will exert the energy

$$U = Q \cdot \frac{t_1 - t_2}{t_1 + 461},$$

t_1, t_2 being the temperatures Fahrenheit of addition and subtraction of heat. This is true whatever be the nature of the heat engine employed for the purpose, and no more heat can be converted into work under any circumstances. A heat engine which satisfies these conditions may be described as "thermally perfect."

If two bodies be at the same temperature heat may be made to flow in either direction from one to another, the actual direction being determined by a difference which may be made as small as we please: that is, the process is *reversible.* Hence the conditions of maximum thermal efficiency may also be described by saying that the cycle of thermal operations must be "thermally reversible." And the condition that an engine may be both mechanically and thermally perfect may be completely described by stating that the engine is reversible.

Whichever way we adopt of stating the result it follows at once that a unit of heat has a certain definite MECHANICAL VALUE given by the equation

$$M = 772 \cdot \frac{t_1 - t_2}{t_1 + 461}.$$

where t_1, t_2 are the temperatures between which it can be used.

278. *Thermal Efficiency.*—If an engine be mechanically perfect the work done per unit of heat will be simply the mechanical value, if the conditions of maximum efficiency are satisfied. In general, however, some of the heat will be supplied at a lower temperature than the source of heat, and some will be abstracted at a higher temperature than that of the refrigerator. When this is the case difference of temperature is wasted and there is a corresponding loss of thermal efficiency. If the temperature is known at which the air or steam is, while it is being supplied with a certain quantity of heat, or while a certain quantity of heat is being abstracted, the mechanical value of that heat can be found corresponding to that temperature. This quantity represents the work actually done since the engine is supposed mechanically perfect, and the same calculation being made for all the heat supplied or abstracted, the total actual work will be known. Dividing this by the total quantity of heat the actual work (U) per unit of heat will be known. The ratio

$$e = \frac{U}{M}$$

may be described as the "THERMAL EFFICIENCY" of the engine.

In practice the engine will not be either mechanically or thermally perfect: its efficiency will then be the product (ek) of the mechanical efficiency and the thermal efficiency. The efficiency thus calculated is estimated relatively to an engine which is mechanically and thermally perfect, and may be described as the "relative" or "true" efficiency, as distinguished from the "absolute" or "apparent" efficiency defined in a former article.

To estimate the efficiency of a heat engine without any reference to the temperatures between which the heat can be used is very misleading. The true efficiency of the best condensing steam engines is about 50 per cent., instead of 13 per cent. as it appears to be merely from the quantity of heat used. In comparing engines of different kinds, however, the same limits of temperature should be employed. (See Appendix.)

279. *Compound Engines.*—The working fluid may be discharged from one contracting chamber into a second which simultaneously expands. In many cases an intermediate reservoir is employed, which receives the fluid from the first chamber and supplies it to the second; the two chambers are then virtually separate, and form two distinct motors, the power of which can be separately calculated. The sum of the two is the power of the compound motor; it is necessarily the same as if the fluid had been used with the same expansion curve between the same extreme pressures in a single chamber; except that the frictional resistance of the passages between the chambers and the intermediate reservoir represents a certain loss of energy in the compound motor which does not occur in the simple one. When there is no intermediate reservoir there is no distinct period of admission or expansion in the low-pressure chamber, but the power may still be determined graphically for each chamber, and the results added.

In every case the energy of the fluid is the same, and cannot be affected by the mechanism employed to utilize it, unless its density or elasticity be altered by contact with the sides of the chamber in which it is enclosed. In steam engines, however, the action of the sides of the cylinder has great influence by condensing steam as it enters the cylinder. The liquefied steam is re-evaporated towards the end of the stroke as the temperature of the steam falls, but the process is nevertheless a very wasteful one. The action is greater the greater the degree of expansion employed, because the range of temperature is greater, and the gain by expansion is thus in great measure neutralized or even converted into a loss. By employing two cylinders instead of one the expansion is divided into two parts each of moderate amount, and liquefaction may be in great measure avoided. Compound engines are therefore being used more and more wherever economy of fuel is a consideration, and in marine practice have almost superseded the simple engine.

The principal losses in steam engines are (1) a mechanical loss due to incomplete expansion, and (2) a thermal loss due to liquefaction. One of these cannot be diminished without increasing the other; but considerable economy may be effected by the use of a "steam jacket," by the employment of superheated steam, and by compounding.

280. *Internal Energy. Internal Work.*—The distinction between internal work and external work was pointed out in Art. 92, p. 199, and the corresponding distinction between internal and external energy of motion in Art. 134, p. 278. These distinctions are principally important in fluids, because the extreme mobility of their parts renders internal motions, of great magnitude, of common occurrence. We have already seen in Chap. XIX. how energy is dissipated by the internal action of liquids; in gases the same dissipation occurs, and is even more important.

In liquids the absorption of energy is completely irreversible, but in gases it is not so. We may have internal energy as well as internal work: the greater part of the expansive energy of a gas being due to internal actions.

The state of an elastic fluid is completely known when its pressure and volume are known, but these quantities are capable of any variation we please within wide limits, provided only that we have unlimited power of adding or subtracting heat. If, however, a third quantity, the temperature, be considered, it will be found that the three are always connected together by a certain equation depending on the nature of the fluid, so that when any two are given the third is known. For example, in the so-called "permanent" gases such as dry air, the equation is very approximately

$$PV = c \, . \, T,$$

where T is the temperature reckoned from the "absolute" zero, a point 461° F. below the ordinary zero of Fahrenheit's scale, and c is a constant which for pressures (P) in lbs. per square foot and volumes (V) in cubic feet per lb. has, for dry air, the value 53·2. The "state" of the fluid is completely known if any two of these three quantities are known, but not otherwise.

To produce a given change of *state* a certain definite amount of work must be done in overcoming molecular resistances; this is the internal work, and is the same under all circumstances. But in gaseous fluids, the molecular forces being reversible, may tend to give rise to the change of state, and then we have internal energy instead of internal work. Taking the first case: if the change be at constant volume this internal work will be the total work done; but in general the volume changes, and in consequence external work is done, the amount of which depends not merely on the change of state, but also on the way in which that change is carried out. The total amount of work is the sum of the internal and external work: it is done by the agency of heat energy supplied from without, so that we write

Heat Expended = Internal Work + External Work,

the three quantities being expressed in common units.

For the application of this equation to questions relating to the formation of steam the reader is referred to a treatise on the Steam Engine by the present writer. We have now to consider the flow of gases through pipes and orifices, for which purpose the equation is written

Expansive Energy = Internal Energy + Heat Supply,

or, in other words, of the whole expansive energy of the fluid, a part is derived from internal molecular forces, and apart from heat supplied from without.

If no heat is supplied from without the expansive energy is equal to the internal energy: this case is called "adiabatic" expansion, obtained by writing $n = 1{\cdot}4$ in the formulæ of Art. 270. More generally, it is shown in treatises on thermodynamics that the internal work done in changing the temperature of a lb. of air from T_1 to T_2 is $I_2 - I_1$, where

$$I = K_v \, . \, T = 2{\cdot}5\ PV,$$

K_v being the specific heat at constant volume, which is $2{\cdot}5\ c$. Hence the equation becomes for a heat supply Q

$$E = 2{\cdot}5\ c\ (P_2V_2 - P_1V_1) + Q.$$

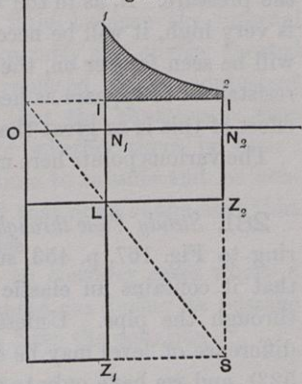
This is the fundamental equation from which all cases may be derived.

If heat be supplied to a permanent gas at a uniform rate as the temperature falls, it may be shown that the law of expansion is $PV^n =$ constant, as supposed in Art. 270, and this is generally permissible with sufficient approximation. The expansion index n then depends upon the proportion which Q bears to E. If $Q = E$ the expansion is hyperbolic, and the whole of the expansive energy is derived from heat supplied from without.* The manner in which the expansive

* For further explanation of the statements here made the reader is referred to Cotterill's *Steam Engine*, chapter IV.

energy (E) depends on the heat supply (Q) is well seen by the annexed diagram (Fig. 197). Let, as before, the ordinates of the point 1 represent the pressure and volume before expansion and those of the point 2 after expansion, 1, 2 being the expansion curve. Set downwards N_1Z_1, N_2Z_2 each equal to $2\frac{1}{2}$ the corresponding pressure ordinates, and complete the rectangles OZ_1, OZ_2. Then complete the rectangle Z_1Z_2, and draw the diagonal SL to meet the vertical through O. Finally through the intersection draw II horizontally: then the rectangle IN_2 will be found to be the difference of the rectangles OZ_1, OZ_2, and therefore represents the internal energy exerted during expansion. Thus the area $12II$ (shaded in the figure) represents the heat supply: which will depend not only on the points 1, 2, that is, on the change of state of the air, but also on the form of the expansion curve, that is, on the way in which the change takes place.

Fig.197



When the pressure of air is changed the changes of temperature are generally so great that, unless the process be very rapid, they are accompanied by a flow of heat to or from external bodies. The amount of heat thus abstracted from, or supplied to the air, is unknown, and the index n cannot, therefore, be found accurately. An approximate value is assumed according to circumstances in each special case.

When air is compressed into a reservoir for the purpose of storing energy, its temperature cannot remain permanently above that of surrounding bodies, and the process will be most economical when the temperature is kept as low as possible by surrounding the pump with cold water. We then assume $n = 1$, but the result of the calculation will generally be too small. The heat to be abstracted is the thermal equivalent of the absolute amount of work done in compression alone. The energy exerted in pumping is *greater* than this by the amount of energy transmitted by the air (if any) which leaves the reservoir, and *less* by the energy exerted by the back pressure which here serves as a source of energy. (See Art. 271, p. 525.)

When air expands from a reservoir in which it is stored at a moderate pressure it receives heat from without, but the amount is uncertain and cannot be relied on. We therefore assume $n = 1{\cdot}4$, though the result of the calculation will be too small. The ratio of results for $n = 1{\cdot}4$ and $n = 1$ will be a measure, though a crude and imperfect one, of the efficiency of the process of storage and transmission of energy by compressed air. The efficiency is less the higher the pressure. If, as in the Whitehead torpedo, the pressure employed is very high, it will be necessary to reduce it by wire-drawing. As will be seen farther on, the mechanical energy dissipated by internal resistances re-appears as heat which maintains the temperature. The effect of this is so great that the process is (probably) economical.

The various points here mentioned are illustrated by Ex. 1-4, p. 552.

281. *Steady Flow through a Pipe. Conservation of Energy.*—Referring to Fig. 167, p. 453, suppose that the reservoir is closed, and that it contains an elastic fluid at high pressure which is flowing through the pipe. Unless the change of pressure be very small, difference of level may be disregarded as relatively unimportant (p. 522), and we have only to consider differences of pressure, while, on the other hand, we must now remember that, when the pressure changes, energy is exerted by expansion as well as by transmission. The energy transmitted from the reservoir *to* any point where the pressure is P and volume V is $P_0 V_0$, where the suffix indicates the state of the fluid in the reservoir. Of this the amount PV is transmitted through the point, and the difference $P_0 V_0 - PV$ together with the expansive energy E is employed in generating the kinetic energy which the gas possesses in consequence of the velocity u with which it is rushing through the pipe at the point considered. Thus, if the motion be steady,

$$\frac{u^2}{2g} = 3{\cdot}5\,(P_0 V_0 - PV) + Q,$$

where Q is the heat (if any) supplied during the passage from the reservoir to the point. If no heat be supplied,

$$\frac{u^2}{2g} + 3{\cdot}5\,PV = \text{Constant},$$

an equation which may also be written

$$\frac{u^2}{2g} + K_p . T = \text{Constant},$$

where K_p is the specific heat at constant pressure. If we have to do with any elastic fluid other than a permanent gas, $3{\cdot}5\ PV$ must be replaced by $I+PV$, where I is the internal energy, and if the question be such that the elevation of the point considered has any sensible influence, the term z must be added as in the corresponding case of an incompressible fluid.

The equation for a compressible fluid, however, is much more general than that for an incompressible fluid, because the internal energy is taken into account, and consequently any energy exerted in overcoming frictional resistances is replaced by an equivalent amount of heat generated. It follows that the equation is true whether there be frictional resistances or whether there be none : provided that the internal motions have time to subside and be converted into heat by friction, and provided that none of the heat thus generated is transmitted to external bodies.

It sometimes happens that we have to consider cases where a quantity of heat Q is supplied to a permanent gas during its passage from a point 1 to a point 2, we shall then have the equation

$$Q = K_p\ (T_2 - T_1) + \frac{u_2^2 - u_1^2}{2g},$$

an equation which is true, however great the variations of pressure or temperature are, and whether or not there are frictional resistances.

282. *Velocity of Efflux of a Gas from an Orifice.*—The most important applications of the equation for the steady flow of a gas are to the discharge of air or steam from an orifice and to the flow of air through long pipes.

In the first case the frictional resistances are small and are consequently neglected. It will be desirable to give a method of treating the question which is independent of the general equation.

In Fig. 198a $o12k$ represents the expansion curve for a small portion of the gas as it rushes out of the reservoir A (Fig. 198b) in which it is confined through a small orifice into the atmosphere. The jet contracts at issue to a contracted section kk, nearly as in the case where the fluid is incompressible, and then, in general, expands again in some such way as is shown in the figure. The velocity through the contracted section may be denoted by u, and the pressure there by P. The area of the contracted section is connected with the area of the orifice by the equation

$$A = kA_0,$$

as on page 449, k being a co-efficient.

Each small portion of the fluid expands from the state represented by the point o on the diagram to that represented by K:

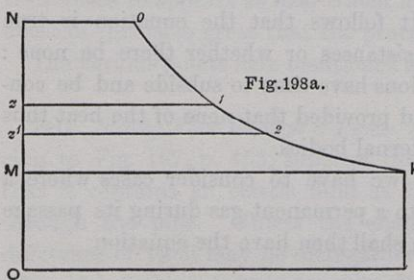


Fig.198a.

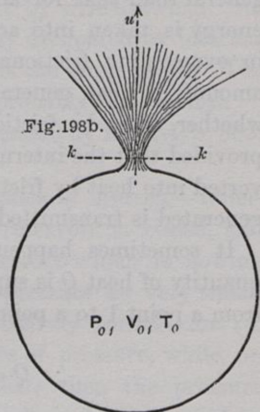


Fig.198b.

in some intermediate state it will be represented by a point 1 on the expansion curve, and immediately after by 2, a point near to 1. Let u_1, u_2 be the corresponding velocities, then

$$\frac{u_2^2 - u_1^2}{2g} = \frac{P_1 - P_2}{w} = V.\delta P,$$

where w is the mean density and V the mean specific volume represented graphically by the mean of $z1$, $z'2$. Hence $V.\delta P$ is represented by the area of the strip cut off by these ordinates. Dividing the whole area into strips, the area of each strip represents the corresponding change in $u^2/2g$, so that the total area represents the final value of this quantity. We have then

$$\frac{u}{2g} = \text{Area } No\kappa M = \int_P^{P_0} VdP = h.$$

The quantity h thus found and graphically represented is the "head" due to difference of pressure, as fully explained in Art. 271.

Assuming the expansion curve $PV^n =$ Constant, as before,

$$\frac{u^2}{2g} = \frac{n}{n-1}(P_0V_0 - PV) = \frac{nc}{n-1}(T_0 - T).$$

Now, if the expansion be adiabatic $n = 1{\cdot}4$, and $nc/(n-1)$ is equal to K_p, so that the result might have been written down at once from the general equation of the preceding article.

Employing the notation of Art. 270, but replacing the suffix 1 by the suffix 0, the velocity of efflux is given by the formula

$$\frac{u^2}{2g} = \frac{n}{n-1}.P_0V_0\,(1 - rx).$$

283. *Discharge from an Orifice.*—The weight of gas discharged per second from an orifice of contracted area A is now found from the formula

$$W = \frac{Au}{V},$$

where V is the specific volume of the gas at the instant of passing through the contracted section, and therefore supposing A unity the weight per unit of area is given by

$$W^2 = 2g\,.\,\frac{n}{n-1}\,.\,\frac{P_0V_0}{V}\,(1 - rx).$$

For V we now write rV_0 and finally obtain

$$W^2 = 2g\,.\,\frac{n}{n-1}\,.\,\frac{P_0}{V_0}\,.\,\frac{1 - rx}{r^2}.$$

In applying this formula x must be supposed known and r calculated from it by the equation on p. 524.

It will be found on examination that as x diminishes from unity W increases to a maximum value and then diminishes again to zero. That is, if the pressure in the throat of the jet at the contracted section be diminished the discharge does not increase indefinitely, but reaches a maximum and then decreases. On substitution for r in terms of x it will be seen that for a given pressure (P_0) in the reservoir W is greatest when $x^{\frac{2}{n}} - x^{\frac{n+1}{n}}$ is greatest.

This will be found to be the case when

$$x = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}.$$

The expansion is adiabatic, and the values of n with the resulting values of x for maximum discharge are shown in the annexed table.

NATURE OF GAS.	VALUE OF n.	VALUE OF x.
Dry Air, ...	1·408	·527
Superheated Steam, ...	1·3	·546
Dry Saturated Steam, ...	1·135	·577
Moist Steam, ...	1·1	·582
	1	$\frac{1}{\sqrt{\epsilon}} = \cdot 6$

The discharge is therefore a maximum when the external pressure is from ·5 to ·6 the pressure in the reservoir. For dry air it will be found on substitution that the maximum discharge per second per unit of contracted area is

$$W_m = \frac{3\cdot 9\, P_0}{\sqrt{P_0 V_0}} = \frac{P_0}{1\cdot 87\,\sqrt{T_0}},$$

and for dry steam

$$W_m = \frac{3\cdot 6\, P_0}{\sqrt{P_0 V_0}}.$$

The pressure P_0 was originally supposed expressed in lbs. per square foot, but it may now be taken as lbs. per square inch in the numerator of these fractions, in which case W_m will be the discharge per square inch.

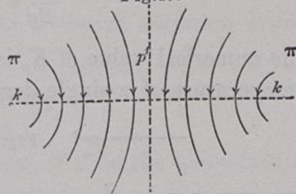
The diminution of the discharge on diminution of the external pressure below the limit just now given, is an anomaly which had always been considered as requiring explanation, and M. St. Venant had already suggested that it could not actually occur. In 1866 Mr. R. D. Napier showed by experiment that the weight of steam of given pressure discharged from an orifice really is independent of the pressure of the medium into which the efflux takes place; and in 1872 Mr. Wilson confirmed this result by experiments on the reaction of steam issuing from an orifice.*

The explanation lies in the fact that the pressure in the centre of the contracted jet is not the same as that of the surrounding medium.

* *Discharge of Fluids*, by R. D. Napier. Spon, 1866. *Annual of the Royal School of Naval Architecture for* 1874.

The jet after passing the contracted section suddenly expands, and the sudden change of direction of the fluid particles gives rise to centrifugal forces which cause the pressure to increase on passing from the surface of the jet to the interior on the principle explained on page 455. This will be better understood by reference to the annexed figure (Fig. 199) which shows a longitudinal section of the jet at the point where the contraction of transverse section is greatest. The particles describe curves the radius of curvature of which increases from a small minimum value at the surface k to an infinite value at the centre. The pressure p, increases from that of the medium (π) at k to a maximum p' at the centre, the increase being very rapid at first and afterwards more gradual. The problem is therefore far more complicated than we have supposed, each small portion of the jet having its own pressure and (consequently) its own velocity and density.

Fig. 199.



The results of experiment however suggest that an approximate solution may be obtained by the assumption of a mean pressure in the throat of the jet, with a corresponding mean velocity; this mean pressure being that which gives maximum discharge in every case in which that quantity is greater than π. At lower pressures it is to be assumed equal to π.

Adopting this hypothesis we see that whenever steam is discharged from a boiler the pressure in which is greater than, about, 25 lbs. per square inch absolute, or 10 lbs. above the atmosphere, the formula given above for maximum discharge is to be used. If we assume the mean value 252 for $\sqrt{P_0V_0}$ this gives $p_1/70$ for the weight discharged from an orifice per square inch of effective area per second, the pressure p_1 being the absolute pressure in the boiler expressed in lbs. per square inch. Contraction and friction must be allowed for by use of a co-efficient of discharge, the value of which however is more variable than that of the corresponding co-efficient for an incompressible fluid. Little is certainly known on this point.

283. *Flow of Gases through Pipes.*—Returning to the general equation, we have now to examine the case where air or steam flows through a pipe of considerable length. As in the case of water, the frictional resistances are then so great that most of the head is taken up in overcoming them. The velocities of the fluid are therefore not excessive, and the value of $u^2/2g$ varies comparatively little.

Now in the equation

$$\frac{u^2}{2g} + K_p \, . \, T = \text{Constant}$$

the numerical value of K_p is about 184, and therefore a variation of temperature of a single degree will correspond to a great change in $u^2/2g$; it may therefore be assumed that the temperature remains very approximately constant, provided only that the difference of pressure at the two ends of the pipe is not too great compared with its length.

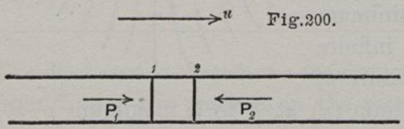


Fig. 200.

In Fig. 200 suppose 1, 2 to be two sections of the pipe at a distance Δx so small that, in estimating the friction, the velocity may be taken at its mean value u; then the force required to overcome friction is

$$R = f \, . \, s \, . \, \Delta x \, . \, u^2,$$

where s is the perimeter and f the co-efficient of friction, defined as on page 460. Replacing this by a new co-efficient f' as in the passage cited,

$$R = f' \, \frac{w}{2g} \, . \, s \, \Delta x \, u^2,$$

in which equation w means the weight of unit-volume of the gas. Now, it was pointed out on page 473 that surface friction was a kind of eddy resistance, and that in the case of water it was proportional to the density. This leads us to suppose that in fluids of varying density, not f but f' is a constant quantity. Replacing w by its equivalent $1/V$, we obtain

$$R = f \, . \, \frac{s\Delta x}{V} \, . \, \frac{u^2}{2g}.$$

We now apply the principle of momentum which will be expressed by the equation

$$(P_1 - P_2)A = W \, . \, A \frac{u_2 - u_1}{g} + R,$$

where W is the weight of gas flowing through the pipe per unit of area per second, and the suffixes refer to the two sections in question, the area of which is A. Now, the motion through the pipe being steady, W is the same throughout, so that

$$\frac{u}{V} = W = \text{Constant}.$$

By substitution for W and writing H for $u^2/2g$, an equation is obtained in the differential form

$$-V \,.\, \delta P = \delta H + f \,.\, \frac{\delta x}{m} \,.\, H,$$

m being the hydraulic mean depth. Now, if T be the temperature, which, as remarked above, is sensibly constant,

$$P = \frac{cT}{V} = W \,.\, \frac{cT}{u}; \therefore \delta P = -W \,.\, \frac{cT}{u^2} \,.\, \delta u.$$

Substitute again for W and u, we then find

$$-V \,.\, \delta P = \frac{1}{2} cT \,.\, \frac{\delta P}{H}.$$

On substitution, the differential equation becomes integrable by dividing by H, and we obtain

$$\frac{1}{2} cT \left\{ \frac{1}{H_0} - \frac{1}{H} \right\} = \log_\epsilon \frac{H}{H_0} + f \,.\, \frac{l}{m},$$

where l is the length of the pipe, and H_0, H the values of $u^2/2g$ at entrance and exit respectively. In application of this equation the term containing the logarithm is small as compared with the rest, and may generally be omitted; also

$$\frac{H}{H_0} = \frac{p_0^2}{p^2},$$

a ratio which is known if the pressures are the data of the question.

The value of the co-efficient is found by experience to be nearly the same as for water, that is, about ·007. In the case of steam cT is to be replaced by the nearly constant product PV, which is to be taken from a table for this quantity so as to obtain a mean value according to the pressure considered.

The equation just found must not be applied to cases in which the difference of pressure is too great compared with the length of the pipe. The friction is then not great enough to prevent the velocity from becoming excessive; the temperature then sensibly falls, instead of remaining constant as supposed in the calculation. An equation can be found which takes account of the fall of temperature when necessary, but in such cases as commonly occur in practice, the supposition of constant temperature is amply sufficiently approximate. When the difference of pressure is small the equation will be found to reduce to the hydraulic formula for flow in a pipe. This case will be considered presently.

The head is given by the formula

$$h = PV \cdot \log_\epsilon \frac{p_0}{p} = PV \log_\epsilon r,$$

and the power expended in forcing the air through is Wh or $PAu \cdot \log_\epsilon r$ ft. lbs. per 1″.

284. *Flow of Gases under Small Differences of Pressure.*—When the differences of pressure are small and no heat is added or subtracted, a gas flows in the same way as a liquid of the same mean density. In the case of air the mean specific volume is found from the equation

$$V = \frac{cT}{P} = \frac{T}{40},$$

the units being feet and pounds, the mean pressure that of the atmosphere, and the temperature measured on Fahrenheit's scale from the absolute zero. At 59° this gives $V = 13$ cubic feet, but the actual volume will vary slightly from variations in the mean pressure.

The small differences of pressure with which we have now to do are commonly measured by a syphon gauge in inches of water. One inch of water is equivalent to a pressure of 5·2 lbs. per sq. ft.

If now ΔP be the difference of pressure in lbs. per sq. ft., i the corresponding number of inches of water, the head due to it will be, as in Art. 283,

$$h = V \cdot \Delta P = \frac{T}{7 \cdot 7} \cdot i.$$

The velocity due to this head, or, what is the same thing, the volume discharged per sq. ft. of *effective* area per second in the absence of frictional resistances, is in cubic feet

$$u = \sqrt{2gh} = 2 \cdot 89 \sqrt{Ti},$$

and the weight-discharge in pounds per second

$$W=\frac{u}{V}=115{\cdot}6\sqrt{\frac{i}{T}}.$$

At 59° one inch of water gives a head of 67·5 feet and a discharge of 65·9 cubic feet, or 5·07 lbs. per second; but at 539° the head is 130 feet and the discharge 91·3 cubic feet, or 3·67 lbs., results which show that the effect of a given difference of pressure is entirely different according to the temperature of the flowing air. This is a point which must always be borne in mind in applying hydraulic formulæ to the flow of gases. Frictional resistances are taken into account by the employment of a co-efficient as in hydraulics, and as elsewhere explained, there is reason to believe that the values of these co-efficients are the same, except so far as they may be dependent directly or indirectly on the co-efficient of contraction (p. 463). Co-efficients of contraction are more variable in air than in water, but their average value does not differ widely in the two cases, and may provisionally be assumed the same.

In particular, it is well established that the formula for the discharge of a pipe in cubic feet per second (p. 462),

$$Q=k\sqrt{\frac{h'}{l}}\cdot d^{\frac{5}{2}},$$

applies to air as well as to water with the same value of the co-efficient k, that is (d in feet) about 40. The head h' is calculated, as just explained, according to the temperature of the air, for a given difference of pressure.

It is sometimes necessary to consider the flow of some gas other than atmospheric air. In approximately permanent gases this is easily done if we know the density of the gas. For example, the density of common coal gas is about ·43, air being unity. The value of c in the formula $PV=cT$ is then proportionately increased, but in other respects the formulæ are unaltered, the index of the adiabatic curve and the constants 2·5, 3·5 which depend on it remaining unaltered. The formula for small differences of pressure may also be employed for the non-permanent gases, such as steam, with a corresponding modification.

Pneumatic machines in which the variation of pressure is small are analogous to hydraulic machines, and most of what was said in the last chapter is applicable to them. The common fan, for example, is a centrifugal pump, the lift of which is the difference of pressure reckoned in feet of air, that is, at ordinary temperatures, about 67 feet for each inch of water. The speed of periphery is $\sqrt{gh}$ in feet per second, where h is the lift increased, as explained in the case of the pump, on account of frictional resistances and the curving-back of the vanes.

Fans are employed to produce a current of air for the purpose of ventilating a mine, ship, or structure of any kind. In mines they are often 30 feet diameter or more. The pressure required is here small and the speed moderate. They are also used to produce a forced draught in torpedo boats, or the blast of a smithy fire. The pressure is then 5 inches of water and more, corresponding to a lift of 300 feet and upwards. The speed of periphery is consequently excessive, and for the comparatively great pressures required for a foundry cupola or a blast furnace, it is necessary to resort to some other form of blowing machine. The speed of periphery might perhaps be reduced by the use of fixed guide blades to give the air a rotatory motion before entering the fan.

285. *Varying Temperature. Chimney Draught.*—If the temperature of the flowing air is varied by the addition or subtraction of heat, its density will be altered during the flow, and it is then necessary to know the mean density, in order that we may be able to calculate the "head" due to a given difference of pressure as measured by the water gauge.

In Fig. 201 OX, OY are axes of reference parallel to which ordinates are drawn as usual to represent volumes and pressures. A given difference of pressure ΔP is represented by the difference ab of a pair of ordinates. The original volume of the air is represented by $a1$. Suppose now that in flowing through a passage of any description the air is heated, as for example in passing through a furnace, the volume increases greatly while the pressure falls slightly; this will be represented by the curve 12, terminating at a point 2. The form of the curve will depend on the law of heating, and will be very different according to the state of the fire; if the bars of the grate

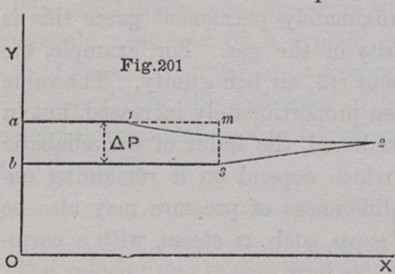


Fig.201

be blocked by clinker and the surface of the fire be free from special obstruction, most of the frictional resistance and corresponding fall of pressure will occur before the air is heated, and the curve will slope rapidly near 1 and slowly afterwards; while, conversely, if the fire be covered with fresh fuel and the grate bars clear, the reverse may be true. After being heated let the air pass through a boiler tube, by which heat is abstracted, till it reaches the chimney: the volume then diminishes greatly while the pressure falls slightly, as shown by the curve 23, terminating at a point 3, such that $b3$ represents the volume of the air in the chimney. The form of the curve 23 will depend on the law according to which the tube abstracts heat. The area of the whole figure $a123b$ represents the "head" due to the whole difference of pressure ΔP, and it will now be obvious that this head will vary according to circumstances which cannot be precisely known. Thus the mean density cannot be found except by empirical formulæ derived by direct experience, and consequently applicable only to the special cases for which they have been determined. It has hitherto been most usually assumed in the case of a furnace and boiler that the mean density was that of the air in the chimney, which amounts to supposing that the forms of the curves 12, 23 are such that the area of the rectangle $a3$ is equal to the area of the whole figure. This is the supposition employed by Rankine, and in many cases it appears to lead to correct results.

In every case of the flow of heated air it must be carefully considered what the mean density will probably be. Its value can often be foreseen without difficulty. It is only in the case of long passages, where the air suffers great frictional resistance while being heated or cooled, that it is uncertain what value to adopt.

The draught which draws air through a fire may be produced artificially or by the action of a chimney. In the latter case there is a difference of pressure within and without the chimney at its base due to the difference of weight of a column of air of the height of the chimney at the temperature of the chimney and at that of the atmosphere. Radiation causes the temperature of the air to be less in the upper part of the chimney and so diminishes the draught, and frictional resistances have the same effect. If these be disregarded the draught in inches of water will be

$$i = 7{\cdot}7\left\{\frac{1}{T_0} - \frac{1}{T}\right\}l,$$

where T_0 is the temperature of the atmosphere, T that of the chimney, while l is the height of the chimney in feet. The temperatures are here reckoned from the absolute zero.

If, for example, the temperature of the chimney be 539° F., and that of the atmosphere 59° F., the height of chimney required for a draught of 1 inch of water will be about 141 feet, or in practice more on account of friction and radiation.

The effect of this draught in drawing air through a furnace or through passages of any kind will vary according to the circumstances which have just been explained.

EXAMPLES.

1. Find the store of energy in the reservoir of a Whitehead torpedo. Capacity 5 cubic feet. Pressure 70 atmospheres.

Ans. If $n=1$ 2,420,000 ft.-lbs., or 3,130 thermal units.
$n=1·4$ 1,092,000 ,, or 1,414 ,,
Ratio of results = ·45.

2. In the last question the air is supplied to the torpedo engines by a reducing valve, so that the pressure in the supply chamber remains constantly at 13 atmo spheres: find the available energy.

Ans. If $n=1$ 1,900,000 ft.-lbs.
$n=1·4$ 1,346,000 ,,

NOTE.—The difference between these results and the preceding is the effect of wire-drawing (resistance of valve). The efficiency of the process may be taken as 1346/2420 or ·56. The supply chamber is supposed small.

3. Air is stored in a reservoir the pressure in which is maintained always nearly at 10 atmospheres: find the store of energy per cubic foot of air supplied from the reservoir.

Ans. If $n=1$ 48,700 ft.-lbs.
$n=1·4$ 35,700 ,,
Ratio = ·733.

4. A chamber of 100 cubic feet capacity is exhausted to one-tenth of an atmosphere: find the work done, assuming $n=1$.

Here if the chamber be imagined to contract, compressing the air still remaining in it, the energy exerted will be due to the pressure of the atmosphere, and the difference between this and the work done in compression will be available for other purposes. In exhausting this is reversed. *Ans.* 142,000 ft.-lbs.

5. Find the mechanical efficiency of an engine so far as due to incomplete expansion (ratio r): assuming the expansion hyperbolic.

Ans. If R be the ratio of complete expansion,

$$\text{Efficiency} = \frac{1 + \log_\varepsilon r - \dfrac{r}{R}}{\log_\varepsilon R}.$$

6. In the last question obtain numerical results for a condensing engine, taking the back pressure at 2 lbs. and boiler pressure 60 lbs.

Ans. Ratio of expansion,	1	2	5	10,
Efficiency, - -	·284	·48	·72	·87.

7. Find the comparative mechanical efficiencies in a condensing and a non-condensing engine. Back pressure in condensing engine 2, in non-condensing 16. Boiler pressure 60 and 100. Ratio of expansion 5 in both cases.

The engines must here be supposed to have the same lower limit of pressure of 2 lbs.; and the result for the non-condensing engine includes the loss by the actual back pressure being 16 lbs. *Ans.* ·72, ·46.

8. Find the loss by wire-drawing between two cylinders from one constant pressure of 60 lbs. to another constant pressure of 40 lbs. Expansion hyperbolic. *Ans.* ·405 PV.

9. One vessel contains A lbs. of fluid at a given pressure P_A, and a second B lbs. of the same fluid at a lower pressure P_B. A communication is opened between the vessels, and fluid rushes from A to B: find the loss of energy.

The loss here is the difference between the energy exerted by A lbs. expanding from V_A to V, and the work done in compressing B lbs. from V_B to V: where V_A, V_B are the specific volumes of the fluid in A and B, and V that of the fluid after equilibrium has been attained, found from the formula

$$V=\frac{AV_A+BV_B}{A+B}.$$

Hence the loss is very approximately

$$\text{Loss}=\frac{AB}{A+B}\cdot\frac{(V_B-V_A)(P_A-P_B)}{2}.$$

10. In a compound engine the receiver is half the volume of the high-pressure cylinder, and at release the pressure in the cylinder is 25 lbs. per square inch, while that in the receiver is 15 lbs. per square inch: find the loss of work per lb. of steam. Obtain the results also when the receiver is double instead of one half the cylinder.

Ans. Case I., 1638 ft.-lbs.
Case II., 3873 ,,

11. In a condensing engine find the mean effective pressure and the consumption of steam in cubic feet per I.H.P. per minute at the boiler pressure: being given, back pressure 3, boiler pressure 60 lbs. per square inch (absolute), ratio of expansion 5.

Ans. Mean effective pressure = 28·33 lbs. per square inch.
Consumption of steam = 1·62 cubic feet per minute.

12. If the volume of 1 lb. of dry steam at the boiler pressure be taken in the preceding question as 7 cubic feet and the liquefaction in admission 20 per cent.: find the weight of steam consumed in lbs. per I.H.P. per hour. *Ans.* 17·5.

13. Find the mechanical value of a unit of heat, the limits of temperature being 600° and 60°; 300° and 100°; 400° and 212°.

Ans. 393, 203, 169 ft.-lbs.

14. The limits of temperature in a heat engine are 350° and 600°; find the thermal efficiency when two-thirds of the whole heat supplied is used between 300° and 100°, one-sixth between 200° and 100°, and one-sixth between 250° and 100°. *Ans.* ·705.

15. In question 6, on account of a gradual increase in the liquefaction the thermal efficiency at the several ratios of expansion mentioned is assumed as ·9, ·85, ·7, ·5: find the true efficiency. *Ans.* ·256, ·408, ·504, ·435.

16. In a compound engine the pressure of admission is 100 lbs. per square inch, the steam is cut off at one-third in the high-pressure cylinder, the ratio of cylinders is $2\frac{1}{2}$: the back pressure is 3 lbs. per square inch, the large cylinder 40 inches diameter, and the speed of piston 400 feet per second. Find the H.P., neglecting wire-drawing and sudden expansion. *Ans.* 567.

17. In the last question suppose that the engine has a very large intermediate reservoir, and that the cut-off in the low-pressure cylinder is ·5; find the pressure in the reservoir, neglecting wire-drawing, also the loss per cent. by sudden expansion at exhaust from the high-pressure cylinder, and the percentage of power developed in the two cylinders.

Obtain the results also for a cut-off of one-third in the low-pressure cylinder.

Ans.

	Cut off $\frac{1}{2}$.	Cut off $\frac{1}{3}$.
Pressure in reservoir, ...	26·7	40
Loss by sudden expansion per cent., ...	·8	·7
Percentage of power in high-pressure cylinder ...	46·5	32·4
,, ,, low-pressure ,, ...	52·6	67·6

18. Compare the efficiencies of the simple and compound engine, assuming the liquefaction the same at the best ratio of expansion, which is 5 in the simple engine and 7 in the compound engine, while in the latter 5 per cent. of the work is lost by wire-drawing between the cylinders. Back pressure and boiler pressure in both cases 3 lbs. and 84 lbs. respectively.

Ans. Gain by compounding $2\frac{1}{2}$ per cent.

19. In question 16, instead of supposing the whole expansion represented by a single hyperbolic curve, assume that at the end of the stroke in the high-pressure cylinder the steam is dry, while at the end of the stroke in the low-pressure cylinder the steam contains 10 per cent. water. Obtain the required result for the cut-off ·5 and find the weight of steam used (exclusive of jacket steam) in lbs. per I.H.P. per hour. Also obtain the results when the steam at the end of the stroke in the high-pressure cylinder contains 30 per cent. water, all other data remaining the same.

Ans.

	Case I.	Case II.
Pressure in reservoir, ...	22·5	14·9
Percentage of power in high-pressure cylinder,	55	37·5
,, ,, low-pressure ,,	55	62·5
Lbs. of steam per I.H.P. per hour, ...	13	16·5

NOTE.—The results of this question may be readily obtained by use of tables of the properties of steam. They show clearly the great influence on the working of a compound engine of the relative liquefaction in the cylinders. If liquefaction be permitted in the high-pressure cylinder the compound engine loses its advantage.

20. Air at a pressure of 1000 lbs. per sq. inch and a temperature of 539° expands to 6 times its volume without gain or loss of heat; find the pressure and temperature at the end of the expansion. *Ans.* $p = 81$, $t = 27°$.

21. In the last question suppose the air at the end of the expansion to have a pressure equal to $1\frac{1}{2}$ times that given by the adiabatic law, and heat to be supplied at a uniform ratio as the temperature falls; find the index of the expansion curve and

the work done during expansion. Compare the heat supplied with the work done and find the specific heat.

Ans. $n = 1\cdot174$. Specific heat $= 1\cdot3$.
Work done $= 1,134,000$ ft.-lbs. Ratio $= \cdot435$.

22. Air is contained in a vessel at a pressure of 25 lbs per sq. inch and temperature 70°. What will be the velocity with which the air issues into the atmosphere (pressure 15 lbs. per sq. inch)? Also find the discharge and the head.

Ans. $h = 13,420$: $u = 930$ ft. per second.
$W = 34\cdot26$ lbs. per sq. inch of orifice per minute.

23. In the last question find the initial pressure corresponding to maximum discharge for all external pressures less than that of the atmosphere. Find this discharge. *Ans.* Pressure $= 28\cdot5$ lbs. per sq. inch.
Discharge $= 39\frac{1}{2}$ lbs per sq. inch per minute.

24. What weight of steam will be discharged per minute from an orifice 2 inches diameter, the absolute boiler pressure being 120 lbs. per sq. inch. Coefficient of discharge ·7. *Ans.* 227 lbs.

25. Air flows through a pipe 6 inches diameter and 4000 feet long; the initial pressure is 20 and the final pressure 15 lbs. per sq. inch; temperature 70°; find the velocities and the discharge. $4f = \cdot03$.

Ans. Velocity at entrance = 39 feet per second.
,, exit = 52 feet ,,
Discharge = 4 lbs. ,,

26. In the last question find the loss of head and the H.P. required to keep up the flow. *Ans.* $h' = 8124$ feet. H.P. $= 59$.

27. Steam at 50 lbs. rushes through a pipe 3 inches diameter and 100 feet long with velocity at entrance of 100 feet per second; find the loss of pressure. $4f = \cdot03$.

Ans. 1·6.

REFERENCES.

For descriptive details and illustrations of the mechanism of steam engines the reader is referred amongst other works to

THURSTON. *History of the Growth of the Steam Engine.* International Scientific Series. Kegan Paul.

SENNETT. *The Marine Steam Engine.* Longman.

RIGG. *Practical Treatise on the Steam Engine.* Spon.