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Autor: Hentschius, Joannes Jacobus

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Idem vero rectang. sub AD, DC = qvadrato rectæ DB (per hypoth.);

Ergo qvadratum DE = qvadrato DB (per 1. ax.) ac propterea linea DE = lineæ DB (per 8. ax.)

Porro recta FE = rectæ FB (per 15. def. 1.); in triangulis igitur DEF, DBF, duo latera DE, EF, duobus DB, BF sunt æqvalia & basis ipsorum FD communis, angulus igitur DBF = angulo DEF (per 8. 1.);

Rectus autem est DEF angulus (per 18. 3.),

Ergo & angulus DBF est rectus (per 1. ax.);

Et qvoniam recta FB per circuli centrum transit, ideoqve est circuli semidiameter (per 15. def. 1.), recta DB ab extremitate semidiametri FB ad angulos rectos ducta circulum ABC contingit (per cor. 16. 3.).

Qvod erat demonstr.

EUCLIDIS ELEMENTORUM, LIBER QUARTUS

DEFINITIONES.

1. Figura rectilinea in figura rectilinea inscribi dicitur, qvando unusqvisqve figuræ inscriptæ angulus contingit unumqvodqve latus ejus, in qva inscribitur.
2. Figura similiter circa figuram circumscribi dicitur, qvando unumqvodqve latus circumscri-

ptæ

ptæ contingit unumqvemqve angulum ejus, qvæ inſcribitur.

3. Figura rectilinea in circulo inſcribi dicitur, qvando unusqvisqve inſcriptæ figuræ angulus circuli circumferentiam contingit.

4. Figura rectilinea circa circulum circumſcribi dicitur, qvando unumqvodqve latus circumſcriptæ circuli circumferentiam contingit.

5. Circulus ſimiliter in figura rectilinea inſcribi dicitur, quando Circuli circumferentia unumqvodqve latus ejus, in qva inſcribitur, contingit.

6. Circulus circa figuram rectilineam circumſcribi dicitur, qvando circuli circumferentia unumqvemqve angulum ejus, circa qvam circumſcribitur, contingit.

7. Recta linea in circulo aptari dicitur, quando ejus termini in circuli circumferentia fuerint.

PROP. I. PROBL.

In dato circulo datæ rectæ lineæ, qvæ diametro ejus non major ſit, æqvalem rectam lineam aptare.

Sit datus circulus ABC, data autem recta linea D *non major circuli diametro: oportet in circulo ABC rectæ lineæ* D *æqvalem rectam lineam aptare.*

Conſtructio.

1. Ducatur circuli ABC diameter; Si igitur BC ſit æqvalis ipſi D, factum jam erit qvod proponebatur.

ponebatur. Etenim in circulo ABC aptata est BC rectæ lineæ D æqvalis;

2. Sin autem major est BC qvam D, ponatur ipsi D æqvalis CE (per 3. 1.); deinde centro qvidem C intervallo autem CE describatur circulus AEG (per 3. postul.), & CA jungatur (per 1. post.).

Demonstratio.

Qvoniam punctum C est centrum circuli AEG, erit

CA = CE (per 15. def. lib. 1.

Sed D = CE (per constr.)

Ergo recta D = rectæ CA (per 1. ax.).

In dato igitur circulo ABC datæ rectæ lineæ D, qvæ non major est circuli diametro, æqvalis aptata est CA. *Quod erat faciendum & demonstrandum.*

PROP. II. PROBL.

In dato circulo inscribere triangulum æqviangulum dato triangulo.

Sit datus circulus ABC, datum autem triangulum DEF: oportet in circulo ABC inscribere triangulum triangulo DEF æqviangulum.

Constructio.

1. Ducatur recta linea HAG contingens circulum ABC in puncto A;
2. Ad rectam lineam HG & ad punctum in ea A constituatur angulus HAC = angulo DEF (per 23. 1.);
3. Ad rectam lineam HG & ad punctum in ea A

rurſus conſtituatur angulus GAB = angulo DFE & BC jungatur.

Demonſtratio.

Qvoniam circulum ABC contingit recta GH, a contactu autem ducta eſt AC, erit angulus HAC æqvalis ei, qvi in alterno circuli ſegmento conſiſtit, angulo, videlicet ipſi ABC (per 32. 3.);

Sed angulus HAC = angulo DEF (per conſtruct.);

Ergo & angulus ABC = angulo DEF (per 1. ax.).

Eadem ratione & angulus ACB eſt æqvalis angulo DFE:

Reliqvus igitur angulus BAC, reliqvo angulo EDF æqvalis erit (per 32. 1.)

Ergo triangulum ABC triangulo DEF eſt æqviangulum, & in circulo ABC inſcriptum eſt (per 3. def. 4.).

Quod erat fac. & demonſtr.

PROP. III. PROBL.

Circa datum circulum circumſcribere triangulum, æqviangulum dato triangulo.

Sit datus circulus ABC, datum autem triangulum DEF: oportet circa circulum ABC circumſcribere triangulum æqviangulum triangulo DEF.

Conſtructio.

1. Protrahatur EF ex parte utraqve ad puncta H, G, (per 2. poſt.);
2. Sumatur circuli ABC centrum K (per 1. 3.);

3. Recta

3. Recta linea KB vtcunqve ducatur, constituaturqve ad lineam KB, & ad punctum in ea K angulus BKA = angulo DEG, angulo autem DFH = angulus BKC (per 23. 1.);
4. Per puncta A, B, C, ducantur rectæ lineæ LAM, MBN, NCL, circulum ACB contingentes (per 17. 3.)

Demonstratio.

Qvoniam rectæ LM, MN, NL circulum contingunt in punctis A, B, C; (per construct.) a centro autem K ad puncta A, B, C, ductæ sunt rectæ KA, KB, KC, erunt anguli ad puncta contactûs A, B, C, recti (per 18. 3.);

Porro qvoniam AMBK (qvod in duo triangula dividi potest) anguli qvatuor æqvales sunt qvatuor angulis rectis (per 32. 1.), e qvibus anguli KAM, KBM sunt recti; erunt reliqvi AKB, AMB duobus rectis æqvales;

Sunt autem & DEG & DEF duobus rectis æqvales (per 13. 1.): Anguli igitur AKB, AMB angulis DEG, DEF sunt æqvales, e qvibus AKB ipsi DEG est æqvalis (per construct.): ergo reliqvus AMB reliqvo DEF æqvalis erit (per 3. ax.)

Similiter demonstrabitur angulus LNM ipsi DFE æqvalis: Ergo & reliqvus MLN est æqvalis reliqvo EDF (per 32. 1.).

Est igitur LMN triangulum æqviangulum triangulo DEF, & circa circulum ABC circumscribitur (per 4. def. 4.)

Quod erat faciendum.

PROP. IV. PROBL.

In dato triangulo circulum inſcribere.

Sit datum triangulum ABC : oportet in triangulo ABC circulum inſcribere.

Conſtructio.

1. Secentur anguli ABC, BCA bifariam per rectas CD, BD, productas usqve dum conveniant in puncto D (per 9. 1.),
2. A puncto D ad rectas lineas AB , BC, CA ducantur perpendiculares DE , DG, DH (per 12. 1.)

Demonſtratio.

Qvoniam angulus ABC bifariam ſectus eſt, erit ang. ABD = angulo CBD (per conſtruct.) & porro rectus angulus BED = recto ang. BHD (per ax. 10.); duo igitur triangula DEB, DHB habent duos angulos duobus angulis æqvales & unum latus DB utriqve commune, qvod ſcilicet uni æqvalium angulorum ſubtenditur : Qvare reliqva latera reliqvis lateribus æqvalia habebunt, ſcilicet latus EB = lateri BH, & lat. DE = lat. DH (per 26. 1) Eadem ratione erit etiam DG = DE = DH : Ideoqve centro D, intervallo autem DG vel DE vel etiam DH deſcriptus circulus tranſibit per puncta E, H, G , atqve in his punctis rectas AB, BC, CA continget (per coroll. 16. 3.); propterea etiam circulus in triangulo ABC inſcriptus eſt (per 5. def. 4.).

Qvod erat faciendum.

PROP.

PROP. V. PROBL.

Circa datum triangulum circulum circumſcribere.

Sit datum triangulum ABC: oportet circa datum triangulum ABC circulum circumſcribere.

Conſtructio.

1. Rectæ AB, AC bifariam ſecentur in punctis D, E (per 10. 1.);
2. A punctis D, E, ipſis AB, AC ad rectos angulos ducantur DF, EF (per 11. 1.)

Demonſtratio.

Lineæ DF, EF, ad rectos angulos ductæ, vel intra triangulum ABC, vel in trianguli latere BC, vel extra triangulum ABC convenient in puncto F.

1. *Conveniant DF, EF intra triangulum in puncto F (vide Fig. 1.) & BF, CF, AF jungantur.*

Qvoniam igitur AD eſt æqvalis rectæ DB: bifariam enim ſecta eſt AB (per conſtruct.) recta autem DF utriqve triangulo ADF, BDF communis, & angulus ADF æqvalis angulo BDF (per 10. ax.); erit baſis AF = baſi FB (per 4, 1.): Similiter oſtendetur & CF æqvalis AF: ergo & BF = CF: tres igitur FA, FB, FC inter ſe ſunt æqvales.

Qvare centro F, intervallo autem æqvali uni ipſarum FA, FB, FC, circulus deſcriptus etiam per reliqva puncta tranſibit: atqve erit circulus circa triangulum ABC circumſcriptus (per 6. def. 4.)

2. *DF, EF, conveniant in recta linea BC, in puncto F (ut in 2. Fig.), & AF jungatur:* Similiter demonſtrabimus punctum F centrum eſſe circuli circa triangulum ABC circumſcripti.

3. *DF, EF conveniant extra triangulum ABC rurſus in puncto F (ut in Fig. 3.)*: & jungantur AF, BF, CF;

Qvoniam igitur AD = DB (per conſtr.); communis autem & ad angulos rectos DF; baſis AF baſi BF æqvalis erit (per 4. 1.)

Similiter demonſtrabimus & CF ipſi AF æqvalem eſſe, qvare & BF eſt æqvalis CF, rurſus igitur centro F, intervallo autem æqvali uni ipſarum AF, BF, CF, circulus deſcriptus etiam per reliqva tranſibit puncta; atqve erit circa triangulum ABC circumſcriptus.

Quod erat faciendum.

Corollarium.

Ex his manifeſtum eſt, qvod, ſi centrum circuli intra triangulum ceciderit, angulus BAC exiſtens in ſegmento ſemicirculo majore minor eſt recto: Si autem ceciderit in recta linea BC, angulus in ſemicirculo rectus erit: & ſi extra triangulum ABC, angulus in ſegmento minore ſemicirculo erit major recto. Qvare ſi datum triangulum ſit oxygonium, DF, EF intra triangulum

gulum convenient: Sin in eo ſit angulus rectus BAC in ipſa AC: & ſi ſit major recto, extra ABC.

PROP. VI. PROBL.

In dato circulo qvadratum inſcribere.

Sit datus circulus ABCD: oportet in circulo ABCD quadratum inſcribere.

Conſtructio.

1. Ducantur Circuli ABCD diametri ad rectos angulos inter ſe AC, BD (per 11. 1.);
2. Jungantur AB, BC, CD, DA (per poſt. 1.).

Demonſtratio.

Quoniam E eſt centrum circuli, qvatuor autem anguli ad centrum, E conſtituti, ſcil. AEB, AED, DEC, CEB ſunt recti (per conſtruct.) ideoqve omnes inter ſe æqvales (per 10. ax.); porro rectæ EA, EB, EC, ED ſunt æqvales (per 15. def. 1.):

Triangula igitur BEA, AED, DEC, CEB ſunt inter ſe æqvalia, ac proinde baſes BA, AD, DC, CB ſunt æqvales (per 4. 1.); Qvare qvadrilaterum ABCD eſt æqvilaterum.

Rurſus qvoniam recta BD eſt diameter circuli ABCD; erit BAD ſemicirculus; qvapropter angulus BAD rectus eſt (per 31. 3.); Cum vero eadem ratione demonſtretur reliqvos angulos ADC, DCB, CBA etiam eſſe rectos; rectangulum igitur eſt ABCD qvadrilaterum; oſtenſum autem est æqvilaterum esse : igitur qvadratum est (per 29. def. 1.) & inscriptum est in circulo ABCD (per 3. def. 4.).

Qvod erat faciendum.

PROP. VII. PROBL.

Circa datum circulum quadratum circumscribere.

Sit datus circulus ABCD : oportet circa ABCD circulum qvadratum describere.

Constructio.

(1) Ducantur circuli ABCD duæ diametri AC, BD ad rectos inter se angulos ; & (2) per puncta A, B, C, D, ducantur rectæ FG, GH, HK, KF contingentes circulum ABCD (per 17. 3.).

Demonstratio.

Qvoniam recta FG circulum contingit, a centro autem E ; ad punctum contactûs A ducta est recta EA ; erunt anguli ad A recti (per 18. 3.)

Eadem ratione & anguli ad puncta B, C, D, sunt recti.

Porro qvoniam angulus AEB est rectus, & EBG etiam rectus ; erit (per 28. 1.) GH ipsi AC parallela ; eadem ratione & AC parallela est rectæ FK ; qvare GH & FK inter se sunt parallelæ (per 30. 1.).

Similiter demonstrabitur & utramqve ipsarum GF, HK ipsi BED parallelam esse ; ideoqve GF, HK etiam inter se parallelas.

Paral-

Parallelogramma igitur ſunt GK, GC, AK, FB, BK; ac propterea GF = HK; GH vero = FK (per 34. 1.).

Et qvoniam AC = BD (per 15. def. 1.) ſed & AC qvidem utriqve ipſarum GH, FK eſt æqvalis, BD vero æqvalis utriqve, GF, HK utraqve igitur GH, FK utriqve GF, HK, æqvalis erit. Qvare æqvilaterum eſt FGHK qvadrilaterum. Dico & rectangulum eſſe: qvoniam enim parallelogrammum eſt GBEA atqve angulus AEB eſt rectus & ipſe AGB angulus rectus erit, (per 34. 1.) Similiter demonſtrabimus angulos etiam, qvi ad puncta H, K, F ſunt conſtituti rectos eſſe: Rectangulum igitur eſt qvadrilaterum FG, HK; demonſtratum autem eſt & æqvilaterum: igitur qvadratum eſt; & circumſcriptum præterea eſt circa circulum ABCD.

Qvod erat faciendum.

PROP. VIII. PROBL.

In dato qvadrato circulum inſcribere.

Sit datum qvadratum ABCD: oportet in ABCD qvadrato circulum inſcribere.

Conſtructio.

1. Utraqve ipſarum GH, GF ſecetur bifariam, in punctis, A, B (per 10. 1.)?
2. Per punctum A alterutri ipſarum GH, FK, parallela ducatur AC, per punctum vero B ducatur BD parallela alterutri GF, HK (per 32. 1.).

Dico: circulus centro E intervallo EA deſcriptus qvadrato inſcribetur.

Demonſtratio.

Qvadrati FGHK latus FG = lateri GH; ergo lateris FG dimidium GA æqvatur lateris GH, dimidio GB (per 7. ax.), & qvoniam recta AC eſt parallela rectæ GH, recta autem BD parallela rectæ GF; eſt igitur AGBE parallelogrammum habens oppoſita latera æqvalia.

latus nempe AG = lateri BE }
& lat. GB = lateri AE } (per 34. 1.)

Sed latus AG, & GB ſunt ejusdem magnitudinis (ut ſupra oſtenſum eſt); ergo & BE, AE ſunt æqvales (per 1. ax.);

Eadem ratione demonſtrabitur parallelogramma eſſe BHEC, AEDF, eorumqve oppoſita latera eſſe æqvalia,

latus nempe BH = lateri EC }
& latus AF = lateri ED } (per 34. 1.)

Qvoniam autem BH = GB, & AF = AG (per conſtr.);

erit etiam GB = EC }
& AG = ED } (per 1. ax.).

Sed GB = AG (ut ſupra); ergo & FC = ED.

Et rurſus, qvoniam oſtenſum eſt, iisdem æqvalibus AG, GB lateribus, etiam æqvalia eſſe latera BE, AE, qvatuor igitur latera EC, ED, BE, AE erunt inter ſe æqvalia. Qvare centro E, intervallo EA ſi deſcribitur circulus, per reliqva puncta B, C, D qvoqve tranſibit, & unumqvodqve qvadrati latus in punctis A, D, C, B tanget; datoqve igitur qvadrato inſcriptus erit.

Quod erat faciendum.

PROP.

PROP. IX. PROBL.

Circa datum qvadratum circulum circumſcribere.

Sit datum qvadratum ABCD: Oportet circa qvadratum ABCD circulum circumſcribere.

Conſtructio.

Jungantur AC, BD, qvæ ſe invicem in puncto E ſecent.

Demonſtratio.

Triangulorum ADC, ABC, duo latera ſunt æqvalia, latus ſcil. AD = lateri AB; latus autem AC utriqve eſt commune; & qvoniam baſis BC etiam æqvatur baſi DC, erit angulus BAC = angulo DAC: angulus igitur DAB bifariam ſectus eſt a recta linea AC.

Similiter demonſtrabimus unumqvemqve angulorum ABC, BCD, CDA, bifariam ſecari a rectis lineis AC, BD.

Qvoniam igitur angulus DAB angulo ABC eſt æqvalis; erit etiam anguli DAB dimidium EAB = EBA, dimidio nempe anguli ABC (per 7. ax.); ideoqve æqvalibus hisce angulis oppoſita latera EA, EB inter ſe ſunt æqvalia (per 6. 1.)

Eadem ratione demonſtrabimus & vtramqve rectarum EC, ED utriqve EA, EB, æqvalem eſſe: ergo qvatuor rectæ lineæ EA, EB, EC, ED inter ſe ſunt æqvales.

Centro

Centro igitur E intervallo autem æqvali uni ipſarum EA, EB, EC, ED circulus deſcriptus etiam per reliqva puncta tranſibit, atqve erit circumſcriptus circa quadratum ABCD.

Quod erat faciendum.

PROP. X. PROBL.

Iſoſceles triangulum conſtituere, habens alterutrum angulorum, qvi ſunt ad baſin duplum reliqvi.

Iſoſceles triangulum ABD eſt conſtruendum, cujus anguli ad baſin ABD & BDA ſinguli ſint dupli ejus ad verticem DAB.

Conſtructio.

1. Ponatur recta qvædam linea AB, & ſecetur in puncto C ita, ut rectangulum comprehenſum ſub AB, BC, æqvale ſit qvadrato ex CA (per 11. 2.):
2. Centro A intervallo AB circulus deſcribatur. BDF, (per 3. poſt);
3. In circulo BDF aptetur recta linea BD æqvalis ipſi AC (per 1. 4.);
4. Jungantur DA, DC; & triangulo ACD circumſcribatur circulus ACD (per 5. 4.)

Demonſtratio.

Qvoniam rectangulum ſub AB, BC æqvale eſt qvadrato rectæ AC (per conſtr.), æqvalis autem eſt AC ipſi BD; erit rectangulum ſub AB, BC æqvale qvadrato rectæ BD.

Porro

Porro, qvoniam extra circulum ACD ſumptum eſt punctum B, ab hoc autem puncto cadunt duæ rectæ lineæ BCA, BD, qvarum altera qvidem circulum ſecat, altera vero in eum incidit, & qvia rectangulum ſub AB, BC æqvale eſt qvadrato rectæ BD; recta igitur linea BD circulum ACD in puncto D continget (per 37. 3.);

Rurſus qvoniam BD circulum contingit, & a contactu D ducta eſt recta DC, erit angulus BDC æqvalis ei, qvi in alterno circuli ſegmento conſtituitur, videlicet angulo DAC (per 32. 3);

Cum autem angulus BDC æqvalis ſit ipſi DAC, communis addatur CDA: totus igitur BDA eſt æqvalis duobus angulis CDA, DAC. Sed his ipſis duobus angulis CDA, DAC etiam æqvalis eſt exterior angulus BCD (per 32. 1.): ergo & angulus BDA æqvalis eſt ipſi angulo BCD (per 1. ax.).

Iterum angulus BDA eſt æqvalis angulo DBA (per 5. 1.), nam latus AB æqvale eſt lateri AD (per 15. def. 1.) ergo & DBA ipſi BCD æqvalis erit.

Tres igitur anguli BDA, DBA, BCD inter ſe ſunt æqvales.

Qvoniam vero angulus DBA, vel (qvod idem eſt) angulus DBC æqvalis eſt angulo DCB; erit latus BD æqvale lateri DC (per 6. 1.).

Sed recta BD æqvalis eſt rectæ CA (per conſtruct.): ergo & DC æqvatur rectæ CA: quare & angulus CDA æqvalis eſt angulo CAD (per 5. 1.): anguli igitur CDA, CAD ſimul ſumpti ipſius anguli CAD dupli ſunt.

Eſt

Est autem & angulus BCD æqvalis angulis CDA, CAD simul sumptis: ergo & ang. BCD duplus est ipsius CAD.

Sed angulus BCD est æqvalis alterutri ipsorum BDA, DBA: qvare & alteruter ipsorum BDA, DBA ipsius DAB (vel CAD) est duplus.

Isosceles igitur triangulum ADB constitutum est, habens alterutrum eorum angulorum, qvi sunt ad basin BD duplum reliqvi.

Qvod erat faciend.

PROP. XI. PROBL.

In dato circulo pentagonum æqvilaterum & æqviangulum inscribere.

Sit datus circulus ABCDE: oportet in ABCDE circulo pentagonum æqvilaterum & æqviangulum inscribere.

Constructio.

1. Ponatur triangulum isosceles FGH habens alterutrum angulorum ad G, H duplum anguli, qvi est ad F (per 10. 4.)
2. In circulo dato inscribatur triangulum ACD, triangulo FGH æqviangulum (per 2. 4.);
3. Anguli ad Basin ACD, ADC secentur bifariam rectis CE, DB, occurrentibus circumferentiæ in punctis B, E; (per 9. 1.);
4. Ducantur rectæ AB, BC, DE, EA.

Demonstratio.

1. Qvoniam uterque angulorum ACD, CDA duplus est anguli CAD, & secti sunt bifariam a

rectis

rectis lineis CE, DB (per constr.); qvinqve anguli DAC, ACE, ECD, CDB, BDA sunt inter se æqvales. Æqvales autem anguli æqvalibus circumferentiis insistunt (per 26. 3.); qvinqve igitur circumferentiæ AB, BC, CD, DE, EA æqvales sunt inter se. Sed æqvales circumferentias æqvales rectæ lineæ subtendunt (per 29. 3.); ergo & qvinqve rectæ lineæ AB, BC, CD, DE, EA inter se sunt æqvales: æqvilaterum est igitur ABCDE pentagonum. *Quod primo erat demonstr.*

2. Qvoniam circumferentia AB æqvalis est circumferentiæ DE (ut supra ostens) communis addatur circumf. BCD, tota igitur ABCD circumferentia toti circumferentiæ EDCB est æqvalis.

Circumferentiæ qvidem ABCD insistit angulus AED, circumferentiæ vero EDCB insistit angulus BAE: ergo & BAE angulus est æqvalis angulo AED (per 27. 3.).

Eadem ratione & unusqvisqve angulorum ABC, BCD, CDE alterutri ipsorum BAE, AED est æqvalis: æqviangulum igitur est ABCDE pentagonum. *Quod 2do erat demonstr.*

In dato igitur circulo inscriptum est pentagonum æqvilaterum & æqviangulum.

Quod erat faciendum.

PROP. XII. PROBL.

Circa datum circulum pentagonum æqvilaterum & æqviangulum circumscribere.

Sit datus circulus ABCDE: oportet circa circulum ABCDE pentagonum æqvilaterum & æqviangulum circumscribere.

Con-

Conſtructio.

1. Intelligatur circumferentia tota circuli in qvinqve partes æqvales diviſa per puncta A, B, C, D, E pentagoni circulo inſcripti (per 11. 4.);
2. Per puncta, A, B, C, D, E ducantur rectæ circulum contingentes GH, HI, IK, KL, LG (per 17. 3.);
3. Sumatur circuli centrum F (per 1. 3.);
4. Jungantur FB, FI, FC, FK, FD (per 1 poſt.).

Demonſtratio.

1. Qvoniam recta IK contingit circulum in puncto C, & a centro F ad contactum ducta eſt FC; erit FC ad ipſam IK perpendicularis (per 18. 3.): rectus igitur eſt uterqve angulorum, qvi ſunt ad C.

Eadem ratione & anguli, qvi ad puncta B, D ſunt recti.

Cum autem rectus eſt angulus FCI, erit qvadratum rectæ FI æqvale qvadrato rectæ FC † qvadr. rectæ CI (per 47. 1.).

Eandem ob cauſam qvadrato rectæ FB † qvadr. rectæ BI æqvale eſt qvadratum rectæ FI: qvare qvadratum rectæ FC † qvadrat: rectæ CI æqvalia ſunt qvadrato rectæ FB, † qvadrato rectæ BI (per 1. ax.).

Sed recta FC æqvalis eſt rectæ BF, ideoque qvadratum rectæ FC æqvale qvadrato rectæ FB: qvare qvadratum reliqvum rectæ BI æqvale eſt reliqvo qvadrato rectæ CI (per 3. ax.); æqvalis igitur eſt recta BI ipſi rectæ CI (per 8. ax.).

Qvoniam

Qvoniam vero in duobus triangulis FBI, FCI duæ rectæ FB, BI duabus FC, CI sunt æqvales, Communis autem utriqve FI; erit angulus BFI æqvalis angulo IFC, & angulus BIF æqvalis angulo FIC (per 8. 1.). Duplus igitur est BFC anguli IFC, & angulus BIC duplus ipsius FIC.

Eadem ratione & angulus CFD duplus est anguli CFK, angulus vero CKD duplus anguli CKF.

Et qvoniam circumferentia BC circumferentiæ DC est æqvalis (per constr.) & angulus BFC angulo CFD æqvalis erit (per 27. 3.)

Atqvi angulus BFC duplus est anguli IFC, angulus vero CFD duplus ipsius CFK (ut supra): æqvalis igitur est angulus IFC angulo CFK (per 7. ax.).

Sunt igitur duo triangula FIC, FCK habentia duos angulos duobus angulis æqvales, alterum alteri, & unum latus uni lateri æqvale, qvod ipsis commune est nempe FC, ergo & reliqva latera reliqvis lateribus æqvalis habent, & reliqvum angulum reliqvo angulo æqvalem (per 26. 1.); recta igitur IC est æqvalis rectæ CK, & angulus FIC æqvalis angulo FKC.

Qvoniam autem IC est æqvalis rectæ CK, erit IK ipsius IC dupla.

Eadem ratione & HI ipsius BI dupla ostendetur.

Rursus qvoniam BI ostensa est æqvalis ipsi IC, atqve est IK qvidem dupla rectæ IC, HI vero dupla ipsius BI; erit HI ipsi IK æqvalis (per 6. ax.)

Similiter & unaqvæqve ipsarum GH, GL, LK ostendetur æqvalis alterutri HI, IK: æqvilaterum igitur est GHIKL pentagonum. *Qvod primo erat demonstrandum.*

2. Qvoniam angulus FIC est æqvalis angulo FKC, & ostensus est ipsius qvidem FIC duplus angulus HIK; ipsius vero FKC duplus IKL, erit & HIK angulus angulo IKL æqvalis (per 6. ax.)

Simili ratione ostendetur & unusqvisqve ipsorum IHG, HGL, GLK, alterutri HIK, IKL æqvalis: Qvinqve igitur anguli GHI, HIK, IKL, KLG, LGH inter se sunt æqvales. Ergo æqviangulum est GHIKL pentagonum. *Qvod 2do erat demonstrand.*

Qvare circa circulum ABCDE datum circumscriptum est pentagonum æqvilaterum & æqviangulum. *Qvod erat faciendum.*

PROP. XIII. PROBL.

In dato pentagono æqvilatero & æqviangulo circulum inscribere.

Sit datum pentagonum æqvilaterum & æqviangulum ABCDE: oportet in ABCDE pentagono circulum inscribere.

Constructio.

1. Uterqve angulorum BCD, CDE a rectis CK, DF bifariam secetur (per 9. 1.);
2. A puncto I, in qvo conveniunt inter se CI, DI, ducantur rectæ IB, IA, IE.

Demonstratio.

Qvoniam pentagoni latus BC æqvale est lateri CD (per hypoth.), & latus IC commune, duo igitur triangula IBC, ICD habent duo latera æqvalia, alterum alteri,

alteri, habent vero & angulos BCI, DCI inter æqvalia latera BC, CI & CD, CI comprehenſos æqvales: qvare baſis BI baſi DI eſt æqvalis, & triangulum BIC æqvale triangulo DIC, & reliqvi anguli reliqvis angulis æqvales, qvibus æqvalia latera ſubtenduntur (per 4. 1.): angulus igitur CBI angulo CDI æqvalis erit.

Cum autem angulus CDE anguli CDI eſt duplus (per conſtr.), & angulus qvidem CDE angulo ABC æqvalis, angulus vero CDI angulo CBI æqvalis; erit & CBA angulus duplus anguli CBI; ac propterea angulus ABI angulo IBC æqvalis: angulus igitur ABC bifariam ſecatur a recta linea BI.

Similiter demonſtrabitur & unumqvemqve angulorum BAE, AED a rectis lineis IA, IE bifariam ſecari. Itaqve a puncto I ad rectas lineas AB BC, CD, DE, EA ducantur perpendiculares IF, IG, IH, IL, IK.

Rurſus, qvoniam angulus GCI eſt æqvalis angulo HCI (per conſtr.), & rectus IGC recto IHC æqvalis (per 10 ax): erunt IGC, IHC duo triangula duos angulos duobus angulis æqvales habentia & unum latus uni lateri æqvale, commune ſcilicet IC, qvod utriqve æqvalium angulorum ſubtenditur: ergo & reliqva latera reliqvis lateribus æqvalia habebunt, atqve erit perpendicularis IG perpendiculari IH æqvalis (per 26. 1.)

Similiter oſtendetur & unaqvæqve ipſarum IL, IK, IF, æqvalis alterutri IH, IG, qvinqve igitur rectæ lineæ IF, IG, IH, IL, IK inter ſe ſunt æqvales.

Qvare centro I intervallo autem æqvali uni ipſarum IF, IG, IH, IL, IK circulus deſcriptus etiam per reliqva tranſibit puncta, & rectas lineas AB, BC, CD, DE, EA continget, propterea qvod anguli ad F, G, H, L, K ſunt recti.

In dato igitur pentagono æqvilatero & æqviangulo circulus eſt inſcriptus. *Qvod erat faciendum.*

PROP. XIV. PROBL.

Circa datum pentagonum æqvilaterum & æqviangulum circulum circumſcribere.

Sit datum pentagonum æqvilaterum & æqviangulum ABCDE, oportet circa pentagonum ABCDE circulum circumſcribere.

Conſtructio.

1. Uterqve BCD, CDE angulorum bifariam a rectis lineis CF, DF ſecetur (per 9. 1.);
2. A puncto F, in qvo conveniunt rectæ CF, DF, ad puncta B, A, E, ducantur FB, FA, FE.

Demonſtratio.

Similiter, vt in antecedente prop. 13., demonſtrabitur unumqvemqve angulorum CBA, BAE, AED, a rectis lineis BF, FA, FE bifariam ſecari.

Et qvoniam angulus BCD angulo CDE eſt æqvalis, atque eſt anguli BCD dimidius angulus FCD, anguli vero CDE dimidius CDF; erit FCD angulus æqvalis angulo FDC (per 7. ax.); qvare & latus FC lateri FD eſt æqvale.

Eadem ratione demonſtrabitur unaqvæqve ipſarum FB, FA, FE æqvalis alterutri FC, FD: qvinqve igitur rectæ lineæ FA, FB, FC, FD, FE ſunt inter ſe æqvales. Ergo centro F & intervallo æqvali

æqvali uni ipſarum FA, FB, FC, FD, FE circulus deſcriptus etiam per reliqva tranſibit puncta, eritqve circumſcriptus circa pentagonum æqvilaterum & æqviangulum ABCDE. *Qv. e. faciendum.*

PROP. XV. PROBL.

In dato circulo hexagonum æqvilaterum & æqviangulum inſcribere.

Sit datus circulus ABCDEF: oportet in circulo ABCDEF hexagonum æqvilaterum & æqviangulum inſcribere.

Conſtructio.

1. Sumatur circuli ABCDEF centrum G (per 1. 3.);
2. Ducatur circuli ABCDEF diameter AD; & centro D intervallo autem DG deſcribatur circulus EGCH (per 3 poſt.);
3. A punctis E, C per centrum G ducantur rectæ EB, CF; & jungantur AB, BC, CD, DE, EF, FA: Dico hexagonum ABCDEF æqvilaterum eſſe & æqviangulum.

Demonſtratio.

1. Qvoniam G eſt centrum circuli ABCDEF, D vero eſt centrum circuli EGCH erit Recta GE = GD, & recta DE = GD (per 15. def. 1.), ideoq; recta GE = rectæ DE (per 1. ax.): æqvilaterum igitur eſt GED triangulum treſqve ipſius anguli EGD, GDE, DEG ſunt inter ſe æqvales (per 5. 1.)

Sunt autem & hi tres anguli ſimul ſumpti æqvales duobus angulis rectis (per 32. 1.); unusqvisqve igitur ipſorum trium angulorum EGD, GDE, DEG eſt tertia pars duorum rectorum.

Similiter oſtendetur triangulum GCD eſſe æqvilaterum, ejusqve tres angulos inter ſe eſſe æqvales & unumqvemqve horum angulorum DGC, GCD, CDG eſſe tertiam partem duorum rectorum: qvare duo anguli EGD, DGC ſunt inter ſe æqvales.

Qvoniam recta CG inſiſtens rectæ EB angulos, qvi ſunt deinceps CGE, CGB, duobus rectis æqvales efficit; angulus autem CGE æqvatur angulis EGD, DGC, qvorum unusqvisqve eſt una tertia pars duorum rectorum; reliqvus igitur angulus CGB erit etiam una tertia pars duorum rectorum; qvare anguli EGD, DGC, CGB, ſunt inter ſe æqvales.

Cum vero anguli BGA, AGF, FGE ipſis EGD, DGC, CGB angulis ad verticem ſint oppoſiti & propterea æqvales (per 15. 1.): ſex igitur anguli EGD, DGC, CGB, BGA, AGF, FGE ſunt inter ſe æqvales: & ſex proinde circumferentiæ AB, BC, CD, DE, EF, FA, qvibus iſti æqvales anguli inſiſtunt, inter ſe ſunt æqvales (per 26. 3.).

Qvæ autem circumferentias ipſas æqvales ſubtendunt rectæ lineæ AB, BC, CD, DE, EF, EA, etiam æqvales ſunt (per 29. 3.): qvare æqvilaterum eſt hexagonum ABCDEF. *Qvod primo erat demonſtr.*

2. Qvoniam circumferentia AF æqvalis eſt circumferentiæ ED, communis addatur circumferentia ABCD: tota igitur circumferentia FABCD æqvalis eſt toti circumferentiæ EDCBA (per 2. ax.); & propterea, qvi æqvalibus ipſis circumferentiis inſiſtunt anguli AFE, DEF æqvales ſunt (per 27. 3.).

Similiter

Similiter oſtendentur & reliqvi anguli hexagoni ABCDEF ſigillatim æqvales alterutri ipſorum AFE, DEF: eſt igitur æqviangulum ABCDEF hexagonum. *Quod 2do erat demonſtr.*

In dato igitur circulo inſcriptum eſt hexagonum æqvilaterum & æqviangulum. *Quod erat faciendum*

Corollarium.

Ex hoc manifeſtum eſt hexagoni latus circuli ſemidiametro æqvale eſſe.

Et ſi per puncta A, B, C, D, E, F ducamus contingentes circulum, circa circulum circumſcribetur hexagonum æqvilaterum & æqviangulum, ad modum eorum qvæ de pentagono dicta ſunt. Ad qvorum modum etiam dato hexagono circulum inſcribemus & circumſcribemus.

PROP. XVI. PROBL.

In dato circulo qvindecagonum æqvilaterum & æqviangulum inſcribere.

Sit datus circulus ABCD oportet in circulo ABCD qvindecagonum æqvilaterum & æqviangulum inſcribere.

Conſtructio.

1. Circulo ABCD inſcribatur triangulum æqvilaterum ACD (per 2. 4.);
2. Eidem circulo inſcribatur etiam pentagonum æqvilaterum (per 11. 4.).
3. Circumferentia BC dividatur bifariam in puncto E (per 30. 3.);

Dico utrumqve rectarum BE, EC eſſe latus qvindecagoni circulo inſcribendi.

Demonſtratio.

Si tota circuli circumferentia in qvindecim partes æqvales ſecta concipiatur, trianguli æqvilateri latus AC ab ipſis æqvalibus qvindecim partibus auferet partes qvinqve æqvales;

Pentagoni vero æqvilateri latus AB earundem partium tres partes æqvales auferet, circumferentia igitur AB lateris pentagoni a circumferentia ABC lateris trianguli ablata, relinqvetur circumferentia BC, duas partes decimas qvintas totius circuli circumferentiæ comprehendens: qvare, ipſa BC circumferentia in puncto B bifariam ſecta, erit utraqve rectarum BE, EC una decima qvinta pars totius circumferentiæ ABCDA.

Si igitur dato circulo ABCD in continuum aptentur rectæ lineæ æqvales uni ipſarum BE, EC (per 1. 4.), erit in ipſo inſcriptum qvindecagonum æqvilaterum, & ſimul æqviangulum (per 27. 3.),

Qvod erat faciendum.

Ad modum autem eorum, qvæ dicta ſunt de pentagono, ſi per circuli diviſiones ducamus rectas lineas circulum contingentes, circa ipſum circumſcribetur quindecagonum æqvilaterum & æqviangulum. Et inſuper ad modum eorum, qvæ dicta ſunt de pentagono, dato qvindecagono æqvilatero & æqviangulo circulum inſcribemus & circumſcribemus.