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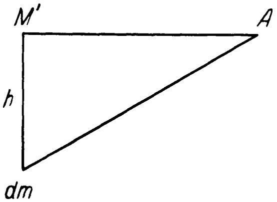
Potsdam, Geodätisches Institut, den 15. März 1929.

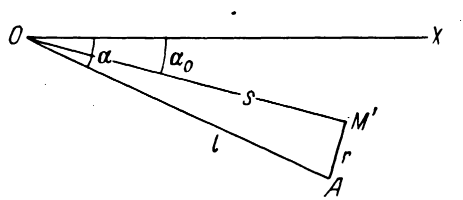
Interrelation between local gravity anomalies and the derivatives of the potential.

By **B. Numerov.** — (With 3 Illustrations.)

Let us consider a portion of the Earth's surface as a level plane where the distribution of gravity, or its deviation Δg from a certain normal law of distribution, is supposed to be known. The anomalies Δg may be naturally explained by the unhomogeneity of structure of the nearest layers and depend on the geological structure of the region. Let the element dm of the disturbing mass lie at a depth h from the Earth's surface and project on the horizontal



Diag. 1.



Diag. 2.

plane at point M' (see diagram 1). The gravity anomaly Δg at point A depending on the element dm will be equal to:

$$\Delta g = \frac{k^2 h\, dm}{(h^2 + r^2)^{3/2}} \quad \ldots\ldots\ldots\ldots (1)$$

where $k^2 = 667 \cdot 10^{-10}$ is the gravitational constant and $r = AM'$.

We shall consider now (diagram 2) in a horizontal plane a system of polar coordinates with the pole at point O and the axis x, directed, e. g., southwards. The coordinates of point A will be denoted by l and α, and those of point M' by s and α_0. Let us further consider the integral:

$$J = \frac{1}{2\pi}\int_0^{2\pi} d\alpha \int_0^{\infty} \Delta g . dl \quad \ldots \ldots \ldots \ldots \quad (2)$$

which we may compute, substituting the anomaly Δg by the expression (1), or

$$\Delta g = \frac{k^2 h dm}{[h^2 + l^2 + s^2 - 2sl\cos(\alpha - \alpha_0)]^{3/2}} \quad \ldots \ldots \quad (1^*)$$

In fact, we can admit that:

$$\left.\begin{aligned} J &= \frac{k^2 h dm}{2\pi}\int_{\alpha_0}^{\alpha_0 + 2\pi} d\alpha \int_0^{\infty} \frac{dl}{[h^2 + l^2 + s^2 - 2sl\cos(\alpha - \alpha_0)]^{3/2}} \\ &= \frac{k^2 h dm}{2\pi}\int_0^{2\pi} d\alpha \int_0^{\infty} \frac{dl}{(h^2 + l^2 + s^2 - 2sl\cos\alpha)^{3/2}} \\ &= \frac{k^2 h dm}{2\pi}\int_0^{2\pi} d\alpha \left[\frac{l - s\cos\alpha}{(h^2 + s^2\sin^2\alpha)(h^2 + s^2 + l^2 - 2sl\cos\alpha)}\right]_0^{\infty} \\ &= \frac{k^2 h dm}{2\pi}\int_0^{2\pi} \frac{d\alpha}{h^2 + s^2\sin^2\alpha} = \frac{k^2 dm}{\sqrt{h^2 + s^2}} = dV \end{aligned}\right\} \quad \ldots (3)$$

By dV at point O is denoted the potential of the elementary mass dm. Summing up with regard to all elements dm of the disturbing mass we arrive to write a general formula, valid for any arbitrary distribution of the disturbing masses, viz:

$$V = \frac{1}{2\pi}\int_0^{2\pi} d\alpha \int_0^{\infty} \Delta g . dl = \frac{1}{2\pi}\int \frac{\Delta g}{l} d\sigma \quad \ldots \ldots \ldots \quad (4)$$

This formula is nothing but a particular case of Stokes*) and Green's formula, in assuming a certain angle ψ in that formula to be small, i. e. in admitting that the globe represents a plane surface.

If we seek to obtain the expression of derivatives of the first and second order of the potential, we shall have to fulfil the integration with regard to the area $d\sigma$, on the basis of formula (4), starting from the outer pole P. However before proceeding to the integration let us rewrite the formula (4) in the form:

$$V = \frac{1}{2\pi}\int \frac{\Delta g - \Delta g_0}{l} d\sigma + V' \quad \ldots \ldots \ldots \ldots \quad (4^*)$$

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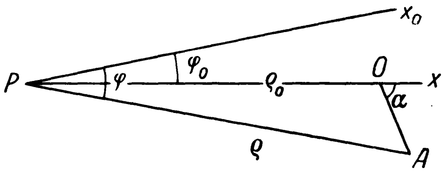
*) Helmert: Die mathematischen und physikalischen Theorien der höheren Geodäsie, S. 251.

where V' is the potential of the flat layer occasioning at all points of the primary plane an anomaly of gravity Δg_0. It is easy to ascertain that all derivatives $\left(\text{except } \frac{\partial V'}{\partial z}\right)$ of the potential V' are equal to zero.



Diag. 3.

Denoting (diagram 3) the coordinates of points O and A with regard to a new pole P by ϱ_0, φ_0 and ϱ, φ we get:

$$\frac{1}{l} = [\varrho^2 + \varrho_0^2 - 2\varrho\varrho_0\cos(\varphi - \varphi_0)]^{-1/2} \quad \ldots \ldots \ldots \quad (5)$$

and formula (4) may be rewritten

$$V = \frac{1}{2\pi}\int(\Delta g - \Delta g_0)[\varrho^2 + x_0^2 + y_0^2 - 2\varrho x_0\cos\varphi - 2\varrho y_0 \sin\varphi]^{-1/2} d\sigma + V' (6)$$

in assuming that $x_0 = \varrho_0 \cos\varphi_0$ and $y_0 = \varrho_0 \sin\varphi_0$ are rectangular coordinates of the point O relatively to pole P.

Differenciating formula (6) with regard to the parameters x and y and supposing further that $x_0 = y_0 = 0$, i. e. in admitting that the pole P coincides with point O, we obtain the following important formulae:

$$\left.\begin{aligned} \frac{\partial V}{\partial x} &= \frac{1}{2\pi}\int_0^{2\pi}\cos\alpha\, d\alpha \int_0^{\infty}(\Delta g - \Delta g_0)\frac{dl}{l}, \\ \frac{\partial V}{\partial y} &= \frac{1}{2\pi}\int_0^{2\pi}\sin\alpha\, d\alpha \int_0^{\infty}(\Delta g - \Delta g_0)\frac{dl}{l}, \end{aligned}\right\} \quad \ldots \quad (7)$$

$$\left.\begin{aligned} \frac{\partial^2 V}{\partial x^2} &= \frac{1}{2\pi}\int_0^{2\pi}(3\cos^2\alpha - 1)\, d\alpha \int_0^{\infty}(\Delta g - \Delta g_0)\frac{dl}{l^2}, \\ \frac{\partial^2 V}{\partial y^2} &= \frac{1}{2\pi}\int_0^{2\pi}(3\sin^2\alpha - 1)\, d\alpha \int_0^{\infty}(\Delta g - \Delta g_0)\frac{dl}{l^2}, \end{aligned}\right\} \quad \ldots \quad (8)$$

$$\left.\begin{aligned} \frac{\partial^2 V}{\partial x^2} - \frac{\partial^2 V}{\partial y^2} &= \frac{3}{2\pi}\int_0^{2\pi}\cos 2\alpha\, d\alpha \int_0^{\infty}(\Delta g - \Delta g_0)\frac{dl}{l^2}, \\ 2\frac{\partial^2 V}{\partial x \partial y} &= \frac{3}{2\pi}\int_0^{2\pi}\sin 2\alpha\, d\alpha \int_0^{\infty}(\Delta g - \Delta g_0)\frac{dl}{l^2}. \end{aligned}\right\} \quad \ldots \quad (9)$$

Finally on the basis of Laplace's formula, i. e. $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$, we have:

$$\frac{\partial^2 V}{\partial z^2} = -\frac{1}{2\pi}\int_0^{2\pi} d\alpha \int_0^{\infty} (\varDelta g - \varDelta g_0)\frac{dl}{l^2} \quad \ldots\ldots\ldots \quad (10)$$

The first formula (7) for $\frac{\partial V}{\partial x}$ has been primarily deduced by F. A. Vening Meinesz*) on the basis of Stokes general formula.

The formulae (7), (10) enable us to compute the potential and its derivatives, if we know the distribution of gravity over a plane, which may be presented by a part of the Earth's surface. Thus, naturally, without taking into consideration the remotest zones and without extending the integration over the whole globe, we may find only the differences between the derivatives of the potential for contiguous points. Of practical value are the first derivatives of the potential of disturbing masses, which show the deflection of the plumb-line U_x and U_y along the axis x and axis y

$$U_x = \frac{1}{g}\frac{\partial V}{\partial x}, \qquad U_y = \frac{1}{g}\frac{\partial V}{\partial y},$$

the second derivatives $\frac{\partial^2 V}{\partial x^2} - \frac{\partial^2 V}{\partial y^2}$ and $2\frac{\partial^2 V}{\partial x \partial y}$ or the so called gradients of curvature, observed by means of a gravitational variometer, and finally $\frac{\partial^2 V}{\partial z^2}$ which cannot be determined with the aid of variometer, but the knowledge of which is extremely important for the interpretation of the gravitational observations.

The possibility of computing the deflection of the plumb-line on basis of gravity anomalies may be of great practical importance especially in remote regions where the topographical surveys are to be based on astronomical observations instead of triangulation. In computing the integrals (7), (8), (9) and (10) some difficulties may arise in integrating in the vicinity of point O when l is small. Let the gravity anomaly near point O be represented by the series:

$$\left.\begin{aligned} \varDelta g - \varDelta g_0 = \left(\frac{\partial g}{\partial x}\right)_0 l\cos\alpha + \left(\frac{\partial g}{\partial y}\right)_0 l\sin\alpha + \frac{1}{2}l^2\cos^2\alpha\left(\frac{\partial^2 g}{\partial x^2}\right)_0 \\ + \frac{1}{2}l^2\sin^2\alpha\left(\frac{\partial^2 g}{\partial y^2}\right)_0 + \frac{1}{2}l^2\sin 2\alpha\left(\frac{\partial^2 V}{\partial x \partial y}\right)_0 + \cdots \end{aligned}\right\} \quad (11)$$

*) F. A. Vening Meinesz: A formula expressing the deflection of the plumb-line in the gravity anomalies and some formulae for the gravity-field and the gravity potential outside the geoide. Amsterdam 1928.

Inserting this formula in (7) and (10) and integrating with regard to l within the limits 0 and l_0 [the radii of convergence of series (11)] and with regard to α from 0 to 2π we find:

$$\left.\begin{array}{ll} \frac{\partial V}{\partial x} = \frac{1}{2}\left(\frac{\partial g}{\partial x}\right)_0 l_0, & \\ \frac{\partial V}{\partial y} = \frac{1}{2}\left(\frac{\partial g}{\partial y}\right)_0 l_0, & \frac{\partial^2 V}{\partial x^2} = \frac{1}{16} l_0 \left[5\left(\frac{\partial^2 g}{\partial x^2}\right)_0 - \left(\frac{\partial^2 g}{\partial y^2}\right)_0\right], \\ \frac{\partial^2 V}{\partial x^2} - \frac{\partial^2 V}{\partial y^2} = \frac{3}{8} l_0 \left[\left(\frac{\partial^2 g}{\partial x^2}\right)_0 - \left(\frac{\partial^2 g}{\partial y^2}\right)_0\right], & \frac{\partial^2 V}{\partial y^2} = \frac{1}{16} l_0 \left[5\left(\frac{\partial^2 g}{\partial y^2}\right)_0 - \left(\frac{\partial^2 g}{\partial x^2}\right)_0\right], \\ 2\frac{\partial^2 V}{\partial x \partial y} = \frac{3}{8} l_0 \left(\frac{\partial^2 g}{\partial x \partial y}\right)_0, & \frac{\partial^2 V}{\partial z^2} = -\frac{1}{4} l_0 \left[\left(\frac{\partial^2 g}{\partial x^2}\right)_0 + \left(\frac{\partial^2 g}{\partial y^2}\right)_0\right]. \end{array}\right\} \quad (12)$$

i. e. in the limits of the circle of radius l_0 the derivatives of the potential are computed according to formulae (12) depending: for the first derivatives of the potential upon the gradient of the variation of gravity, $\left(\frac{\partial g}{\partial x}\right)_0$ and $\left(\frac{\partial g}{\partial y}\right)_0$ and for the second derivatives upon their variations.

The further integration for $l > l_0$ may be fulfilled graphically, namely by means of graphics which are used for the account of the influence of topography on the gravitational observations on the basis of a map with isogams*).

Zur Anisotropie der physikalischen Parameter von Gesteinen speziell der magnetischen Suszeptibilität.

Von **J. Koenigsberger.**

Bei der Bestimmung der magnetischen Suszeptibilität von Gesteinen nach einer für Feldmessungen geeigneten Methode zeigten kristalline Schiefer eine deutliche magnetische Anisotropie, analog ihrer thermischen, elektrischen, elastischen Anisotropie. Man kann die Anisotropie auch ohne die Annahme anisotroper Träger der magnetischen Induktion, wie es z. B. Pyrrhotit ist, schon durch eine bestimmte Anordnung isotroper oder ungeordneter anisotroper Erzkörner erklären.

Die magnetische Suszeptibilität von Gesteinen in magnetischen Feldern, die nicht viel stärker als das Erdfeld sind, kann, wie a. a. O.**) gezeigt wurde, einfach rasch dadurch bestimmt werden, daß man ein Stück des zu untersuchenden Materials mit einer einigermaßen ebenen Fläche in bestimmten kleinen Abstand, z. B. 1.5 cm, von einer an einem Faden aufgehängten Magnetnadel bringt. Man mißt die durch die Anziehung bei paramagnetischen Substanzen oder Abstoßung bei Diamagnetika bewirkte Drehung der Magnetnadel in Skalenteilen. Man

*) B. Numerov: Die topographische Reduktion bei Drehwaagenbeobachtungen. Zeitschr. f. Geophys., Jahrg. 4, Heft 3 (1928).

) Zeitschr. f. Phys. **54, 511 (1929).