wegen ist in Fig. 1 der Maßstab des Magnetfeldes zehnmal so groß gewählt worden wie in den beiden anderen Figuren.

Bei diesem und bei allen anderen diesbezüglichen Versuchen zeigte sich, daß die Angaben der verwendeten Askania-Drehwaagen durch die verschiedenartigsten äußeren Magnetfelder in keiner meßbaren Weise beeinflußt wurden. Selbst Magnetfeldänderungen, die 60mal stärker waren als das Erdmagnetfeld, hatten bei den verwendeten Instrumenten keinen störenden Einfluß auf die Meßergebnisse. Die bisher erhaltenen Resultate über die Abhängigkeit der Schwerkraft vom Zwischenmedium sind also nicht durch störende magnetische Kräfte beeinflußt.

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On the Determination of the Lunar Atmospheric Tide*)

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New methods are devised for eliminating the shar diurnal variation, and a large part of the accidental variations, when determining the lunas atmospheric tide from hourly values of the barometric pressure. The theory of the method, and its practical application, are described.

1. The changes of barometric pressure at any station contain two parts periodic in the solar and lunar days respectively, and may also contain a part periodic in the year. The remainder is not periodic, but is mainly associated with variations of sunshine, cloudiness, and other weather conditions, and with the passage of cyclones and anticyclones.

*) For references the following may be consulted: S. Chapman and M. Hardman, Mem. Roy. Meteor. Soc. **2**, Nr. 19, p. 153 (1928), and J. Bartels, Veröff. d. Pr. Met. Inst. Nr. 346, Abh. Bd. 8, Nr. 9, 1927.

Let the deviation of the pressure at any station, from its mean value over a long period, be denoted by p or $p(t)$, a function of the time t. This can be represented as the sum of four functions

$$p(t) = p_s(t) + p_l(t) + p_y(t) + p_a(t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

respectively representing the solar diurnal variation, the lunar diurnal variation, the yearly variation, and the remaining "accidental" variation. The functions p_s and p_l vary with the season; p_s also varies with the weather, particularly its first harmonic component — the second harmonic is nearly constant throughout the year, and its seasonal variation is small; p_l varies also with the moon's distance. These variations of p_s and p_l can be represented by combinations of terms periodic in intervals differing slightly from the mean solar or lunar day, but it is on the whole more convenient to treat them as variations of the amplitude and phase of a function of fixed period.

When the mean diurnal variation of p is determined at a particular season or epoch in the year, p_y produces a small non-cyclic term in the variation, different at different seasons. It is convenient to combine p_s and p_y into a single function p_S, represented, in a given calendar month, by the mean diurnal inequality including the non-cyclic variation due to p_y. This inequality will be supposed determined, for the given calendar month, from a number of years and not from one only. Thus

$$p_S = p_s + p_y, \quad p = p_S + p_a + p_l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The variation p_S will be supposed constant throughout each calendar month, though differing slightly from one month to another. Actually p_S varies continuously throughout the year, but the slight change relative to the mean of the month will be ignored, or regarded as included in p_a.

The object of this paper is to discuss the best means of determining p_l from a series of hourly values of the barometric pressure.

2. The range of variation (R_s) of p_S is in general fifteen or more times as great as that (R_l) of p_l at the same station. Hence, and also because of the nearly equal lengths of the periods of p_S and p_l, it is imperative to eliminate p_S from the whole variation $p(t)$, when determining p_l. If all days over a sufficiently long period are used together, the hourly values of p being rearranged and summed according to lunar time, p_S is thereby averaged out; but this will not be so, in general, when only a limited selection of days in a given interval is considered, such as the days corresponding to a particular lunar distance, or the barometrically "quiet" days, that is, the days of small pressure range. In such cases p_S must be eliminated by some specific method, either before the re-tabulation of the original hourly data according to lunar time, or in some other way.

J. Bartels has shown that on quiet days p_S is not the same as on the average of all days; hence when groups of quiet days are dealt with, the p_S appropriate to such days should be used in the elimination.

3. Both p_S and p_l diminish, in general, with increasing latitude, whereas p_a, on the contrary, increases. On both accounts, p_l is most easily determined in low

latitudes, while above about 50° latitude it is difficult to disentangle p_l from p_S and p_a. The few existing determinations in such latitudes have been made by confining the computations to quiet days, of range about 0.1 inch (or 2.5 mm) of mercury.

A simple calculation indicates that though, on the average, for the "high" latitude stations considered hitherto, this excludes about two days out of every three, the remaining quiet days give a more reliable determination of p_l than is obtainable, by the methods hitherto used, from all days. For if $\bar{p}_a$ is the average numerical value of p_a, the influence of p_a on the mean value of p at any particular solar or lunar hour, when observations at this hour, on n different days, are averaged, is given by $\overline{p_a}/\sqrt{n}$, when n is a large number. By using all days during a given period, instead of only the quiet days, n is increased three fold, and $\overline{p_a}/\sqrt{n}$ is reduced in the ratio $1/\sqrt{3}$, but the $\overline{p_a}$ for all days is more than $\sqrt{3}$ times its value for quiet days, so that the inclusion of the unquiet days, with all the additional labor which it involves, only decreases the accuracy of the determination of p_l.

4. These considerations also indicate that, in order to obtain equally good determinations of p_S and p_l, in which the accidental error bears an equal proportion, in each case, to the range R_S and R_l of p_S or p_l, $(R_S/R_l)^2$ times as many days must be used to determine p_l as are required for p_S; since R_S/R_l is fifteen or more, $(R_S/R_l)^2$ is not less than 225. In latitude 50° or more, many days must be taken in order to determine p_S, and the number required to determine p_l would be enormous, and almost impracticable, if not reduced by confining the work to quiet days.

5. The object of this paper is to describe certain new processes of computation, which it is believed are improvements on the existing methods for the determination of p_l. One relates to the elimination of p_S, while the other is concerned with the elimination of the accidental variation p_a; it is hoped that the second process will render unquiet days as useful for the determination of p_l as the quiet days were in the past determinations by the ordinary method.

6. The two processes will first be discussed in relation to a particular set of determinations of p_l on which the writer is engaged, but the principles involved can easily be applied to cases in which other modes of tabulation of the original hourly data are adopted.

The determinations referred to are for Honolulu, Kimberley, and several Canadian stations. In this work the published hourly data were used in a modified form, after a plan due to J. Bartels, which is very convenient in such computations as these; instead of using the hourly values themselves, the hour-to-hour differences Δ were tabulated on the computation sheets. The published hourly values were given to the nearest 0.01 inch, and Δ rarely exceeded $\pm$ 0.09 inch, and could therefore be represented by a single positive or negative digit, taking 0.01 inch as the unit.

These hour-to-hour differences were tabulated in rows of 25, indicating the hourly changes of p over a period of 25 hours, consisting of a complete solar day together with an hour of the next day; 25 hours were taken because this is approximately the duration of the lunar day. The series of differences, which may be denoted by $\Delta_0, \Delta_1, \ldots \Delta_{24}$, represented the differences for the solar hours $2^h - 1^h$, $3^h - 2^h, \ldots 26^h - 25^h$.

The entries were checked by a method likewise due to J. Bartels. On each daily row the differences between the final and initial values of the pressure for three 8-hour intervals were noted, i. e., for the intervals $9^h - 1^h$, $17^h - 9^h$, $25^h - 17^h$; let these be denoted D, E, F. They should of course equal the sums of the three successive groups of eight Δ's for the day, i. e.

$$D = \sum_{r=0}^{7} \Delta_r, \quad E = \sum_{r=8}^{15} \Delta_r, \quad F = \sum_{r=16}^{23} \Delta_r \quad \ldots\ldots \quad (3)$$

and this check was applied to the entries Δ_0 to Δ_{23}. The value of Δ_{24} was simply copied from the first entry (Δ_0) for the following day.

The Greenwich lunar transit time for the 25-hour day represented by each row of Δ's was entered on the same row. In entering these times from the Nautical Almanac, the minutes were omitted, so that $1^h\,13^m$ or $1^h\,50^m$ would both be entered as 1. Thus the transit times 1^h, as entered on the sheets, would correspond to an average solar time $1^1/_2{}^h$, i. e., to the middle of the interval to which Δ_0 relates. The times, as entered, ranged*) from 1^h to 25^h.

Each daily row of 25 Δ's was also classified according to the distance of the moon on that day; four groups were considered, denoted by the letters A, N, P, R; the group A comprised days of apogee and the two days before and after; likewise the group P comprised the five days centred at each perigee. The groups N and R comprised the days between the A and P days, or the P and A days, when the moon was respectively nearing or receding from the earth.

The data for a single calendar month in one year were entered on each sheet.

7. "Vertical" sums of the Δ's, that is, sums by columns, were formed on each sheet; if divided by the number of days (i. e. of rows) involved, these would give the mean values of $\Delta_0, \Delta_2, \ldots \Delta_{24}$ for the month. Such sums were combined from all the sheets for the same calendar month in several years; let the sequence of sums thus obtained be denoted by $S_0, S_1, \ldots S_{24}$, so that, if $\mathfrak{N}$ is the total number of days involved, these divided by $\mathfrak{N}$ give the mean values of $\Delta_0, \Delta_2, \ldots \Delta_{24}$, which represent (in the form of hour-to-hour differences) the mean solar diurnal variation p_S, including any non-cyclic variation due to p_y. Apart from accidental error, which should be much reduced in the mean Δ's, S_0 and S_{24} should be equal. The mean non-cyclic variation, given by $(S_0 + \ldots + S_{23})/\mathfrak{N}$, represents the variation in 24 hours, at the given season of the year, due to p_y.

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*) These times $1^1/_2{}^h$ to $25^1/_2{}^h$ are those which occur within the interval $26^h - 1^h$ represented on the daily rows. If this interval had been taken as $25^h - 0^h$, the transit times entered would range from 0 to 24, really corresponding to $0^1/_2{}^h$ to $24^1/_2{}^h$; this would have been slightly preferable in connection with the mathematical notation used in the following discussion, though not in any sense affecting the results of the work.

The sums S_0 to S_{23} were checked by forming sums also of the D's, E's and F's; let the sums be denoted by D_S, E_S, F_S. Clearly

$$D_S = \sum_{r=0}^{7} S_r, \quad E_S = \sum_{r=8}^{15} S_r, \quad F_S = \sum_{r=16}^{23} S_r \quad \ldots \ldots \quad (4)$$

For later use (cf. §§ 15—18) a further sequence of numbers S_r^c was formed, defined as follows:

$$\left.\begin{array}{lll} r = 0 \text{ to } r = 7 \text{ and } r = 24 & & S_r^c = S_r - \frac{1}{8} D_S \\ r = 8 \text{ to } r = 15 & & S_r^c = S_r - \frac{1}{8} E_S \\ r = 16 \text{ to } r = 23 & & S_r^c = S_r - \frac{1}{8} F_S \end{array}\right\} \quad \ldots \quad (5)$$

These numbers were written down to the nearest unit, that is, fractions occurring in $\frac{1}{8}(D_S, E_S, F_S)$ were ignored.

8. Sloping sums of the Δ's were made in such a way as to combine the Δ's corresponding to a particular lunar hour, just as the sums S_r refer to a particular solar hour. Such sums were made not only for all the days in the same calendar month in several years, but also separately for the groups of days A, N, P, R. The sloping sums were made with the aid of templates sliding in a frame, so constructed and used as to indicate the Δ's which refer to the same lunar hour; in different rows these occur in different columns. In the rows for days of transit time 1^h, as entered, the difference Δ_r refers to the hour centred at the lunar hour r; in the rows for days of transit time n^h, Δ_r refers to the hour centred at the lunar hour $r - n + 1$. The sloping sums are equivalent to vertical sums on a sheet containing the daily rows of Δ's written in the same cyclic order as on the original sheets, but beginning with the entry Δ_{n-1} for days of transit time n^h; this implies a transposition of the first $n - 1$ entries, on the rows for such days, to the end of the sequence.

The combined sloping sum, for the days of a particular group (say group A) for the same calendar month in several years, may be called a lunar sum. Let the sequence of sums be denoted by L_r^A. ($r = 0$ to $r = 24$), the A (or N, P, R) indicating the group of days concerned; they are sums of hour-to-hour differences for hours centred at the lunar times 0^h, 1^h, ..., 24^h.

Let the number of days of transit time n^h involved in such a sum associated with group A be denoted by N_{n-1}^A, and let $\mathfrak{N}^A$ be the total number of days involved, given by

$$\mathfrak{N}^A = N_0^A + \cdots + N_{24}^A \quad \ldots \ldots \ldots \ldots \quad (6)$$

Clearly (cf. § 7)

$$\mathfrak{N} = \mathfrak{N}^P + \mathfrak{N}^N + \mathfrak{N}^A + \mathfrak{N}^R \quad \ldots \ldots \ldots \ldots \quad (7)$$

The lunar sums for the groups A, N, P, R were checked by comparing their combined sum with the lunar sum for the whole series of days.

9. In connection with the process for eliminating part of the influence of p_a on the determination of p_l, in a manner to be described later (§§ 15—18), the

following further tabulation was made relative to the 𝔑 days referred to in §§ 7, 8.

The numbers D, E and F entered in each daily row (§ 6), together with a further number, denoted by G, which was simply the D for the following day, were recopied on a sheet which had 100 columns, divided into 25 sets of four; each set was headed by one of the numbers 1 to 25, and in each set the four columns were headed D, E, F, G. The numbers D, E, F, G for the days of transit time n^h were entered in the set of columns headed n^h; inks of four different colours were used in the tabulations on this sheet, each colour distinguishing the days belonging to one of the groups A, N, P, R. The numbers in each colour in each column were then summed; let the four sums in the column D of the set headed n^h be denoted by D_{n-1}^A, D_{n-1}^N, D_{n-1}^P, D_{n-1}^R, and the total sum by D_{n-1}, i. e.,

$$D_{n-1} = D_{n-1}^A + D_{n-1}^N + D_{n-1}^P + D_{n-1}^R; \quad \ldots \ldots \ldots \quad (8)$$

similarly for the columns headed E, F, G.

The sequences D_r, E_r, F_r were rewritten in three successive rows (1, 2, 3) of 25 columns, 0 to 24, the sequences being shifted backwards through 4, 12 and 20 places respectively, i. e., transposed so as to begin with D_3, E_{11} and F_{19}. The sums by columns were formed, in row 4; they may be denoted by H_r. In a further row (5) the sum of the numbers in columns r, $r+1$ of row 4 was entered in each column r, i. e., the sums $H_r + H_{r+1}$. This sequence of numbers was then multiplied by $k \equiv \sin 8\,\varphi / \sin 2\,\varphi$, where $\varphi = 2\,\pi/25 = 14.4^0$; thus $k = 0.9048/0.4818 = 1.878$; this new sequence was entered in row 6. In row 7 the sequence G_r, shifted one place forward, so as to read G_{24}, G_0, ..., G_{23} (i. e. G_{r-1}) was written. The sum of the numbers in each column of rows 6 and 7 was written in row 8; let this sequence be denoted by J_0, J_1, ..., J_{24}.

Similar sequences J_r^A, J_r^N, J_r^P, J_r^R were formed for the groups of days A, N, P, R. Evidently, for each value of r,

$$J_r = J_r^A + J_r^N + J_r^P + J_r^R. \quad \ldots \ldots \ldots \ldots \quad (9)$$

The formation and use of these sequences for the partial elimination of the influence of p_a on the determination of p_l constitutes the main point of this paper.

10. The sequences L_r, S_r^c, J_r, N_r, for the various groups of days A, N, P, R, and for these groups combined, were analyzed to determine the second harmonic component. Only this component was calculated, because previous investigations have shown that there is no appreciable component of frequency other than 2 in the lunar day; tidal theory indicates that there must be a diurnal component depending on the moon's declination, but it is so small as to have escaped detection as yet; it may in time be determined, from a large amount of material at a suitable station, perhaps with the aid of the refinements of computation here introduced. The present discussion will refer only to the second component, but the same principles can be applied to the determination of any other harmonic.

The sequences analyzed, such as L_r, will be regarded as sequences of values of functions $f(\theta)$, with the period 2π, for the values $\theta = r\varphi$, where $\varphi = 2\pi/25 = 14.4^0$. Let the component of frequency σ be denoted (for L_r) by

$$L_{(\sigma)} \sin(\sigma\theta + l_\sigma) \quad \ldots \ldots \ldots \ldots \ldots \quad (10)$$

with a similar notation for other sequences. Then by the theory of harmonic analysis, $L_{(\sigma)}$ and l_σ may be derived from the formulae

$$L_{(\sigma)} \exp(i l_\sigma) = \frac{2i}{25} \sum_{r=0}^{24} L_r \exp(-i r \sigma \varphi) \quad \ldots \ldots \quad (11)$$

or

$$L_{(\sigma)} \exp(-i l_\sigma) = -\frac{2i}{25} \sum_{r=0}^{24} L_r \exp(i r \sigma \varphi) \quad \ldots \ldots \quad (12)$$

by separating the real and imaginary parts; thus

$$\left. \begin{aligned} L_{(\sigma)} \cos l_\sigma &= \frac{2}{25} \sum_{r=0}^{24} L_r \sin r\sigma\varphi \\ L_{(\sigma)} \sin l_\sigma &= \frac{2}{25} \sum_{r=0}^{24} L_r \cos r\sigma\varphi \end{aligned} \right\} \quad \ldots \ldots \ldots \quad (13)$$

Since we are here mainly concerned with the case $\sigma = 2$, the suffix σ may be omitted except when it is not 2. Moreover, in the further formulae of this paper involving summations with respect to r, the limits will be indicated only when they are not from 0 to 24; when no limits are indicated, it is to be understood that the summation is over the whole range (0 to 24) of r.

It is convenient to write

$$L_A = L \cos l, \qquad L_B = L \sin l \quad \ldots \ldots \ldots \ldots \quad (14)$$

so that

$$L \sin(2\theta + l) = L_A \sin 2\theta + L_B \cos 2\theta \quad \ldots \ldots \ldots \quad (15)$$

similarly for the other sequences analyzed.

In connection with a given set of data (referring to a particular month or seasonal set of months), the sequences to be analyzed for the second component are S_r^c and five sets of sequences L_r, J_r, N_r, for the groups A, N, P, R and the whole set of days: in all, this is $1 + 5 . 3$ or 16. The analysis leads to the harmonic constants S^c, s^c; L, l; J, j; N, n; the last six appear five times, with the upper affix A, N, P or R, or with no upper affix.

11. The lunar sums L_r would represent p_l if p_S and p_a were fully averaged out, but in general this is not the case, and p_S and p_a have to be eliminated from the lunar sums by specially devised means.

It may be questioned whether p_S should be eliminated from the data month by month, using for each month the p_S derived from that month, or whether the mean p_S from the same calendar month in a series of years should be used for all those months. The choice is perhaps not of great importance, since only a small fraction of p_S is likely to remain in any lunar sum. The second method is here used, and is perhaps preferable because p_S as determined from one month only may be appreciably in error owing to p_a, which is much more completely eliminated when p_S is determined from several months: the reason might not be a valid one if p_S were really different, in the same calendar month, in different years, but this is not the case to any great extent, at least as regards its second component, with which we are principally concerned.

In many previous investigations, by the writer and others, p_S has been removed at the outset by subtracting the monthly mean value at each solar hour from the values for that hour on each day, the differences being the basis of the subsequent work. The mean hourly values thus subtracted may be derived from a single month, or may be the means from the same calendar month in a series of years: the former has generally been the plan adopted.

When J. Bartels' method of using hour-to-hour differences as the basis of the subsequent computations is adopted, as here, p_S is not removed at the outset, and must be eliminated at a later stage.

One method of doing this in connection with the lunar sum for each set of days, e. g., for the total set of $\mathfrak{N}$ days referred to in § 7, is to form an auxiliary sheet on which, in 25 rows, the sequences of hour-to-hour differences representing $N_0\, p_S$, $N_1\, p_S, \ldots, N_{24}\, p_S$ are entered, the sequence in the n^{th} row being transposed so as to begin with its n^{th} term. Thus the sequence in the n^{th} row will be

$$(N_{n-1}/\mathfrak{N})\,\{S_n,\ S_{n+1},\ \cdots,\ S_{24},\ S_0,\ \cdots,\ S_{n-1}\}. \qquad (16)$$

Sums by columns then give the amounts by which p_S affects the lunar sums. The difference between the lunar sequence and this sequence will represent p_l freed from p_S, but usually containing a residue of p_a.

This method has been used by J. Bartels, and the same plan in principle had been followed by the writer in some previous investigations, in which the original hourly values of p, without removal of p_S, were taken as the basis of computation.

12. The elimination of p_S, so far as it affects the second harmonic in the lunar sums, can be achieved more simply in the following way.

The second harmonic in the sequence of sums at the foot of the auxiliary sheet is the sum of the second harmonics in each of the 25 rows. In the first row this harmonic is $(N_0/\mathfrak{N})S \sin(2\,\theta + s)$, in the notation of § 10; in row r, owing to the transposition of the first $r-1$ terms after the remaining terms, the harmonic is $(N_r/\mathfrak{N})S \sin(2\,\theta + s + 2\,r\varphi)$. Thus the second harmonic in the total is

$$\left.\begin{aligned} &(S/\mathfrak{N}) \sum N_r \sin(2\,\theta + s + 2\,r\,\varphi) \\ &= (S/\mathfrak{N})\,\{\sin(2\,\theta + s) \sum N_r \cos 2\,r\,\varphi + \cos(2\,\theta + s) \sum N_r \sin 2\,r\,\varphi\} \\ &= \tfrac{25}{2}\,(S/\mathfrak{N})\,\{N \sin n \sin(2\,\theta + s) + N \cos n \cos(2\,\theta + s)\} \\ &= \tfrac{25}{2}\,(S\,N/\mathfrak{N}) \cos(2\,\theta + s - n) \end{aligned}\right\} \quad (17)$$

by (13).

Thus the part of the second harmonic in L which is due to p_S can be determined in simple terms by analyzing the sequence N_r, once the sequence S_r (which is the same for the four groups of days A, N, P, R and the whole set) has been analyzed.

This is substantially the method here adopted for the removal of p_S, but instead of dealing with the sequence S_r, the modified sequence S_r^c (§ 7) is considered; this is only a part of p_S, and this part affects the second harmonic term of the lunar sequence L_r by the amount

$$\left.\begin{aligned} &\frac{25}{2}\frac{S^c N}{\mathfrak{N}}\cos(2\,\theta + s^c - n) \\ &= \frac{25}{2}\frac{S^c N}{\mathfrak{N}}\{\sin(n - s^c)\sin 2\,\theta + \cos(n - s^c)\cos 2\,\theta\} \\ &\equiv S_A^c \sin 2\,\theta + S_B^c \cos 2\,\theta \end{aligned}\right\} \quad \ldots \quad (18)$$

where the equation gives the definition of the numbers S_A^c, S_B^c; similarly (with the superfix A, N, P or R attached to $\mathfrak{N}$, N and n) for the lunar sums $L_r^A, \ldots, L_r^R$.

The reason for this modification is bound up with the method here introduced to eliminate a large part of the accidental variation p_a. This will be described in §§ 15—18.

13. Imagine that chords of the graph of $p(t)$ are drawn joining successive points at 1^h, 9^h and 17^h on each solar day. These chords form a rectilinear graph which defines a function $p^-(t)$ — the "rectilinear" part of the barometer variation, in 8-hour intervals. Let

$$p^c(t) = p(t) - p^-(t) \quad \ldots \ldots \ldots \ldots \ldots \quad (19)$$

so that $p^c(t)$ is the function — the "curved" part of the barometric variation, in 8-hour intervals — which is left over if $p^-(t)$ be subtracted from $p(t)$.

The graphs of the component functions of $p(t)$, that is, of $p_S(t)$, $p_l(t)$, $p_a(t)$ may be similarly analyzed into "rectilinear" and "curved" parts, p_S^-, p_l^-, p_a^- and p_S^c, p_l^c, p_a^c, relating to the same 8-hour intervals. Evidently

$$p^-(t) = p_S^-(t) + p_l^-(t) + p_a^-(t) \quad \ldots \ldots \ldots \ldots \quad (20)$$

$$p^c(t) = p_S^c(t) + p_l^c(t) + p^c(t) \quad \ldots \ldots \ldots \ldots \quad (21)$$

The hour-to-hour differences associated with the function $p^-(t)$, when written out in a sequence of 25 terms as for $p(t)$, are evidently (cf. § 6) equal to

$$\tfrac{1}{8}\,D \text{ (eight times)}, \ \tfrac{1}{8}\,E \text{ (eight times)}: \ \tfrac{1}{8}\,F \text{ (eight times)}, \ \tfrac{1}{8}\,G. \quad \ldots \quad (22)$$

For the mean $p_S(t)$, from the $\mathfrak{N}$ days of a set of data (cf. § 7) they are $1/(8\,\mathfrak{N})$ multiplied by D_S, E_S, F_S (each eight times in succession) and G_S, while for $p_S^c(t)$ they are $S_r^c/\mathfrak{N}$. Thus the expression (18) represents the part of the second harmonic in the sequence L_r due to the component function $p_S^c(t)$.

14. Of the functions p, p^-, p^c; p_S, p_S^-, p_S^c; p_l, p_l^-, p_l^c; p_a, p_a^-, p_a^c; the first three are directly known from the observations ,while the vertical sums (§ 7) determine p_S,

and therefore also p_S^-, p_S^c, at least within limits of accidental error which are small compared with p_S itself. The last six functions are not known, but the relation between p_l, p_l^- and p_l^c can be determined owing to the known (purely lunar semidiurnal) character of this variation.

Suppose that

$$p_l = C \sin (2\,\theta + c) \quad \ldots\ldots\ldots\ldots \quad (23)$$

where θ denotes lunar time, measured from lunar transit, and reckoned in angle at the rate $2\,\pi$ per lunar day; C and c, which depend on the lunar distance, may be considered constant in each group of days A, N, P, R, with values indicated by C^A, $c^A, \ldots$, while C, c refer to the mean values. Then to the term L_r of the lunar sum, derived from $\mathfrak{N}$ days, p_l contributes the amount

$$\left.\begin{aligned} &\mathfrak{N}\, C\,[\sin\{(2\,r+1)\,\varphi + c\} - \sin\{(2\,r-1)\,\varphi + c\}] \\ &= 2\,\mathfrak{N}\, C \sin\varphi \cos(2\,r\,\varphi + c) \end{aligned}\right\} \quad \ldots\ldots \quad (24)$$

being $\mathfrak{N}$ times the difference between p_l at the lunar hours $r + \frac{1}{2}$ and $r - \frac{1}{2}$, at which 2θ has the values $(2r+1)\,\varphi$ and $(2r-1)\,\varphi$: thus the true lunar (semidiurnal) component of the sequence L_r is

$$\left.\begin{aligned} &2\,\mathfrak{N} C \sin\varphi \cos(2\,\theta + c) \\ &= 2\,\mathfrak{N} C \sin\varphi \sin(2\,\theta + c + \tfrac{1}{2}\,\pi) \\ &= C' \sin(2\,\theta + c') \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (25)$$

where

$$C' = 2\,\mathfrak{N} C \sin\varphi, \qquad c' = c + \tfrac{1}{2}\,\pi \quad \ldots\ldots\ldots \quad (26)$$

It will be shown in § 19 that the separate contributions of p^- and p^c to L_r are, very approximately, respectively

$$\left.\begin{aligned} &0.1959\ C' \sin(2\,\theta + c' + 5.235^0) \\ &0.8052\ C' \sin(2\,\theta + c' - 1.27^0) \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (27)$$

In latitudes 50^0 or over, p_a is responsible for most of the variation p, and it has a range of three inches or more. Its changes are usually gradual, the rate often not varying much within eight hours. Consequently the rectilinear part of p_a is nearly equal to p_a itself, while the curved residue, p_a^c, is much smaller. The new method here introduced is to eliminate p_a^-, leaving only the smaller part of p_a, that is, p^c, to be eliminated by the averaging resulting from the combination of many days' data. Though the method is most useful for stations in moderate or high latitudes, it may be employed with advantage also for stations in low latitudes.

15. Since p_l is unknown, the distinction between p_l and p_a cannot be made except á posteriori; the following is the plan adopted for the removal of p_a^-.

The lunar sums L_r are made up of contributions from the six parts of p, namely,

$$p_S^-,\ p_l^-,\ p_a^-,\ p_S^c,\ p_l^c,\ p_a^c \quad \ldots\ldots\ldots\ldots \quad (28)$$

The contributions made by the first three parts (which together equal p^-), and also the contribution by the fourth part, p_S^c, are removed, leaving, as the residue, only the contributions made by p_l^c and p_a^c; it is assumed that the latter is rendered inappreciable by averaging, so that the residue of the lunar sums is composed solely of p_l^c. Owing to the known relation (cf. 25, 28) between p_l^c and p_l, the latter can be inferred from the former.

The removal of the contributions made by p_S^-, p_l^-, p_a^-, that is, by p^-, is made in a single stage, for convenience; the process is described in §§ 16—18. The inclusion of a part of p_l itself, namely p_l^-, in this elimination, is inevitable because it is at this stage impossible to separate p_l^- and p_a^-; but the fraction of p_l removed is only small, as is shown by comparison of (25) and (27).

It is unnecessary to remove the whole of the contributions made to L_r by p^- and p_S^c, as we are only interested in the second harmonic component of the lunar sums. Hence the analysis is confined to the removal of the second harmonic in the contributions of p^- and p_S^c. The second harmonic in the contribution by p_S^c has already been found, namely (18).

16. The contribution by p^- to the lunar sums L_r consists of the sum of the contributions from each of the daily rows involved. These appear on the computation sheets as the sequence of hour-to-hour differences (22), i. e.,

$$\tfrac{1}{8} D \text{ (eight times)}, \ \tfrac{1}{8} E \text{ (eight times)}, \ \tfrac{1}{8} F \text{ (eight times)}, \ \tfrac{1}{8} G.$$

But in the lunar sums this sequence is transposed (§ 8), so that if the day is of transit time n^h, the n^{th} term comes first. The combined contribution from the days of the same transit time n^h can be considered as a whole, and is represented by the sequence

$$\tfrac{1}{8} D_{n-1} \text{ (eight times)}, \ \tfrac{1}{8} E_{n-1} \text{ (eight times)}, \ \tfrac{1}{8} F_{n-1} \text{ (eight times)}, \ \tfrac{1}{8} G_{n-1} \quad (29)$$

(where $D_{n-1}, \ldots$ are as defined in § 9), transposed as stated.

17. Consider the second harmonic component of a sequence of the form

$$D \text{ (eight times)}, \ E \text{ (eight times)}, \ F \text{ (eight times)}, \ G \quad \ldots\ldots (30)$$

such as (29). Let it be denoted by $A \sin (2\,\theta + a)$. Then, by § 10,

$$\left.\begin{aligned} A\,e^{i a} &= \frac{2\,i}{25}\left\{D \sum_{r=0}^{7} e^{-2 i r \varphi} + E \sum_{r=8}^{15} e^{-2 i r \varphi} + F \sum_{r=16}^{23} e^{-2 i r \varphi} + G e^{-48 i \varphi}\right\} \\ &= \frac{2\,i}{25}\left\{(D + E e^{-16 i \varphi} + F e^{-32 i \varphi}) \sum_{r=0}^{7} e^{-2 i r \varphi} + G\,e^{2 i \varphi}\right\} \end{aligned}\right\} (30)$$

since $50\,\varphi = 4\,\pi$.

Now

$$\sum_{r=0}^{7} e^{-2 i r \varphi} = \frac{1 - e^{-16 i \varphi}}{1 - e^{-2 i \varphi}} = \frac{\sin 8\,\varphi}{\sin \varphi}\,e^{-7 i \varphi} \quad \ldots\ldots (31)$$

as appears on putting $\alpha = -\varphi$, $\alpha = -8\varphi$ in

$$1 - e^{2i\alpha} = -2ie^{i\alpha}\sin\alpha \quad \ldots\ldots\ldots\ldots (32)$$

Hence

$$Ae^{ia} = \frac{2i}{25}\left\{\frac{\sin 8\varphi}{\sin\varphi}(De^{-7i\varphi} + Ee^{-23i\varphi} + Fe^{-39i\varphi}) + Ge^{2i\varphi}\right\}. \quad (33)$$

If the sequence (30) is transposed so as to begin with the n^{th} term, the second harmonic component is changed to $A\sin\{2\theta + a + 2(n-1)\varphi\}$, or $A\sin(2\theta + a_{n-1})$, where

$$a_{n-1} = a + 2(n-1)\varphi \quad \ldots\ldots\ldots\ldots (34)$$

and therefore, by (33),

$$Ae^{ia_{n-1}} = Ae^{ia + 2(n-1)i\varphi} = \frac{2i}{25}\left\{\frac{\sin 8\varphi}{\sin\varphi}e^{2(n-1)i\varphi}\right.$$

$$\left. + (De^{-7i\varphi} + Ee^{-23i\varphi} + Fe^{-39i\varphi}) + Ge^{2ni\varphi}\right\} \quad (35)$$

18. Let the contribution made by p^- to the second harmonic of the lunar sums, through the days of transit time n^h, be denoted by $A_n \sin(2\theta + a_n)$. It is obtained (cf. 26) by substituting the values $\frac{1}{8}D_{n-1}$, $\frac{1}{8}E_{n-1}$, $\frac{1}{8}F_{n-1}$, $\frac{1}{8}G_{n-1}$, for D, E, F, G in (35). The whole contribution, which will be denoted by

$$\prod \sin(2\theta + \overline{\omega}) \quad \ldots\ldots\ldots\ldots (36)$$

is obtained by summing the resulting expression for the values of n from 1 to 25, or of $r = n - 1$ from 0 to 24. Hence

$$\prod \sin(2\theta + \overline{\omega}) = \sum_{n=0}^{24} A_n \sin(2\theta + a_n) \quad \ldots\ldots (37)$$

whence it follows that

$$\left.\begin{aligned}
\prod e^{i\overline{\omega}} &= \sum A_r e^{ia_r} \\
&= \frac{1}{8}\,\frac{2i}{25}\left\{\frac{\sin 8\varphi}{\sin\varphi}\sum e^{2ri\varphi}(D_r e^{-7i\varphi} + E_r e^{-23i\varphi} + F_r e^{-39i\varphi})\right. \\
&\quad \left. + \sum G_r e^{(2r+2)i\varphi}\right\} \\
&= \frac{i}{100}\left\{\frac{\sin 8\varphi}{\sin\varphi}e^{-i\varphi}\sum (D_{r+3} + E_{r+11} + F_{r+19})e^{2ir\varphi}\right. \\
&\quad \left. + \sum G_{r-1}e^{2ir\varphi}\right\} \\
&= \frac{i}{100}\left\{\frac{\sin 8\varphi}{\sin\varphi}e^{-i\varphi}\sum H_r e^{2ir\varphi} + \sum G_{r-1}e^{2ir\varphi}\right\}
\end{aligned}\right\} \quad (38)$$

where

$$H_r = D_{r+3} + E_{r+11} + F_{r+19} \quad \ldots\ldots\ldots (39)$$

These are the numbers H_n already defined in § 9. The sequences D, E, F are here regarded as periodic, so that an increase or decrease of the suffix by 25 leaves any term unchanged.

Since

$$1 + e^{-2i\varphi} = 2\cos\varphi\, e^{-i\varphi} \quad \ldots\ldots\ldots\ldots (40)$$

it follows that

$$\left.\begin{aligned} \frac{\sin 8\varphi}{\sin\varphi} e^{-i\varphi} \sum H_r e^{2ir\varphi} &= \frac{\sin 8\varphi}{\sin 2\varphi} \sum H_r (1 + e^{-2i\varphi}) e^{2ri\varphi} \\ &= \frac{\sin 8\varphi}{\sin 2\varphi} \sum (H_r + H_{r+1}) e^{2ri\varphi} \end{aligned}\right\} \quad (41)$$

Hence, remembering the definition of J_r in § 9, it appears that

$$\prod e^{i\bar{\omega}} = \frac{i}{100} \sum J_r e^{2ri\varphi} \quad \ldots\ldots\ldots\ldots (42)$$

But by analogy with (12) it follows that, denoting the second harmonic of the sequence J_r by $J\sin(2\theta + j)$

$$J e^{-ij} = -\frac{2i}{25} \sum J_r e^{2ir\varphi} \quad \ldots\ldots\ldots\ldots (43)$$

Hence

$$\prod e^{i\bar{\omega}} = -\tfrac{1}{8} J e^{-ij} \quad \ldots\ldots\ldots\ldots (44)$$

so that

$$\prod_A = \prod \cos\bar{\omega} = -\tfrac{1}{8} J\cos j = -\tfrac{1}{8} J_A \quad \ldots\ldots (45)$$

$$\prod_B = \prod \sin\bar{\omega} = \tfrac{1}{8} J\sin j = \tfrac{1}{8} J_B \quad \ldots\ldots\ldots\ldots (46)$$

in accordance with the notation (14).

Thus the analysis of the sequence J_r of § 9 suffices to indicate the second harmonic of the contribution made by p^- to L_r.

19. It remains to determine the contribution made by p_l^- to the second harmonic component of L_r; this contribution has been eliminated in p^-, and this undesired elimination must be allowed for.

On the monthly sheets of hour-to-hour differences Δ the value Δ_r, on a day of transit time n^h, corresponds (§ 8) to an interval centred at the lunar hour $r - n + 1$ ($\pm$ 25). The part of this Δ_r which is due to p_l is (§ 14)

$$\left.\begin{aligned} C\sin[\{2(r-n+1)+1\}\varphi + c] - C\sin[\{2(r-n+1)-1\}\varphi + c] \\ = 2C\sin\varphi\cos\{2(r-n+1)\varphi + c\} \end{aligned}\right\} \quad (47)$$

which is the real part of

$$2C\sin\varphi \exp i\{2(r-n+1)\varphi + c\} \quad \ldots\ldots\ldots (48)$$

Magnitudes D_l, E_l, F_l, G_l analogous, with respect to p_l, to D, E, F, G with respect to p (§ 6) may be obtained as follows. For D_l the value is the real part of

$$\left.\begin{aligned} &2\,C\sin\varphi \sum_{r=0}^{7} \exp i\,\{2\,(r-n+1)\,\varphi + c\} \\ &= 2\,C\sin\varphi \exp i\,\{-2\,(n-1)\,\varphi + c\} \sum_{r=0}^{7} \exp 2\,i\,r\,\varphi \end{aligned}\right\} \quad \ldots (49)$$

By using (31), with the sign of φ changed, (49) becomes

$$2\,C\sin 8\,\varphi \exp i\,\{-(2\,n-9)\,\varphi + c\} \quad \ldots\ldots\ldots (50)$$

so that

$$D_l = 2\,C\sin 8\,\varphi \cos\{(2\,n-9)\,\varphi - c\} \quad \ldots\ldots\ldots (51)$$

Similarly

$$E_l = 2\,C\sin 8\,\varphi \cos\{(2\,n-25)\,\varphi - c\} \quad \ldots\ldots\ldots (52)$$

$$F_l = 2\,C\sin 8\,\varphi \cos\{(2\,n-41)\,\varphi - c\}\,; \quad \ldots\ldots\ldots (53)$$

all these equations refer to days of transit time n^h.

The quantity G_l is the D_l for the next day, which is obtained from (51) by changing n to $n+1$, i. e.,

$$G_l = 2\,C\sin 8\,\varphi \cos\{(2\,n-7)\,\varphi - c\} \quad \ldots\ldots\ldots (54)$$

since the time of lunar transit is one hour later each day, according to our treatment of the lunar day as being of duration 25 solar hours. Actually it is $24^h\,50^m$, so that in one out of about six days the next day's transit time (as entered) is the same as for the day itself. The error caused in our work by ignoring the difference between 25^h and the length of the lunar day will be considered in § 24.

Let the second harmonic in the sequence $\frac{1}{8}\,D_l$ (eight times), $\frac{1}{8}\,E_l$ (eight times), $\frac{1}{8}\,F_l$ (eight times), $\frac{1}{8}\,G_l$, for a day of transit time n, be denoted by $K_{n-1}\sin(2\,\theta + k_{n-1})$. Then, substituting $\frac{1}{8}\,D_l, \ldots, \frac{1}{8}\,G_l$ for $D, \ldots, G$ in (33), we have

$$\left.\begin{aligned} K_{n-1}\exp i\,k_{n-1} = \frac{i\,C\sin 8\,\varphi}{50}\Big[&\frac{\sin 8\,\varphi}{\sin\varphi}\,\big\{e^{-7i\varphi}\cos\big(\overline{2\,n-9}\,\varphi - c\big) \\ &+ e^{-23i\varphi}\cos\big(\overline{2\,n-25}\,\varphi - c\big) + e^{-39i\varphi}\cos\big(\overline{2\,n-41}\,\varphi - c\big)\big\} \\ &+ e^{2i\varphi}\cos\big(\overline{2\,n-7}\,\varphi - c\big)\Big] \end{aligned}\right\} \quad (55)$$

In L_r this second harmonic appears as $K_{n-1}\sin\{2\,\theta + k_{n-1} + 2\,(n-1)\,\varphi\}$, owing to the transposition of the first $n-1$ terms of the sequence for the days of transit time n^h; moreover it occurs N_{n-1} times, this being the number of such days. If the second harmonic of the whole contribution of p_l^- to L_r be denoted by $K\sin(2\,\theta + k)$, we have

$$K\sin(2\,\theta + k) = \sum N_r K_r \sin(2\,\theta + k_r + 2\,r\,\varphi) \quad \ldots\ldots (56)$$

so that

$$\left.\begin{aligned} K e^{ik} &= \frac{i\,C \sin 8\,\varphi}{50} \sum N_r \left[\frac{\sin 8\,\varphi}{\sin \varphi} \left\{ e^{i\,(2\,r-7)\,\varphi} \cos\left(\overline{2\,r - 7}\,\varphi - c\right) \right.\right. \\ &\quad + e^{i\,(2\,r-23)\,\varphi} \cos\left(\overline{2\,r - 23}\,\varphi - c\right) \\ &\quad \left.\left. + e^{i\,(2\,r-39)\,\varphi} \cos\left(\overline{2\,r - 39}\,\varphi - c\right)\right\} + e^{i\,(2\,r+2)\,\varphi} \cos\left(\overline{2\,r - 5}\,\varphi - c\right)\right] \\ &= \frac{i\,C \sin 8\,\varphi}{50} \sum \left[\frac{\sin 8\,\varphi}{\sin \varphi} \left(N_{r+3} + N_{r+11} + N_{r+19}\right) \right. \\ &\quad \left. + N_{r+2}\, e^{7\,i\,\varphi} \right] e^{i\,(2\,r-1)\,\varphi} \cos\left(\overline{2\,r - 1}\,\varphi - c\right) \end{aligned}\right\} \quad (57)$$

By using (40) with the sign of φ changed, (57) may be rewritten as

$$\left.\begin{aligned} &(K/C)\, e^{i\,(k-c)} \\ &= \frac{i \sin 8\,\varphi}{100} \sum \left[\frac{\sin 8\,\varphi}{\sin \varphi} \left(N_{r+3} + N_{r+11} + N_{r+19}\right) \right. \\ &\quad \left. + N_{r+2}\, e^{7\,i\,\varphi} \right] \left\{1 + e^{2\,i\,\{(2\,r-1)\,\varphi - c\}}\right\} = \frac{i\,\Re \sin 8\,\varphi}{100} \left(3\,\frac{\sin 8\,\varphi}{\sin \varphi} + e^{7\,i\,\varphi}\right) \\ &\quad + \frac{i \sin 8\,\varphi}{100}\, e^{-2\,i\,(\varphi + c)} \sum \left[\frac{\sin 8\,\varphi}{\sin \varphi} \left(N_{r+3} + N_{r+11} + N_{r+19}\right) \right. \\ &\quad \left. + N_{r+2}\, e^{7\,i\,\varphi} \right] e^{4\,r\,i\,\varphi} \end{aligned}\right\} \quad (58)$$

Let the fourth harmonic component of the sequence N_r be denoted [by analogy with (10)] by

$$\tfrac{1}{25}\,\Re\, N_{(4)} \sin\left(2\,\theta + n_4\right); \quad \ldots \ldots \quad (59)$$

$N_{(4)}$ is likely to be a very small fraction. Then, by (12),

$$\Re\, N_{(4)}\, e^{-i\,n_4} = -2\,i \sum N_r\, e^{4\,r\,i\,\varphi} \quad \ldots \ldots \quad (60)$$

so that

$$\sum N_{r+m}\, e^{4\,r\,i\,\varphi} = \frac{i}{2}\, \Re\, N_{(4)}\, e^{-i\,n_4 - 4\,m\,i\,\varphi} \quad \ldots \ldots \quad (61)$$

Hence (58) may be written

$$\left.\begin{aligned} &(K/2\,\Re\, C \sin \varphi)\, e^{i\,(k-c)} = \frac{i \sin 8\,\varphi}{200 \sin \varphi} \left(3\,\frac{\sin 8\,\varphi}{\sin \varphi} + e^{7\,i\,\varphi}\right) \\ &- \frac{1}{400}\,\frac{\sin 8\,\varphi}{\sin \varphi}\, N_{(4)}\, e^{-i\,n_4 - 2\,i\,(\varphi + c)} \left[\frac{\sin 8\,\varphi}{\sin \varphi} \left(e^{-12\,i\,\varphi} + e^{-44\,i\,\varphi} + e^{-76\,i\,\varphi}\right) \right. \\ &\left. + e^{-i\,\varphi} \right] = \frac{i \sin 8\,\varphi}{200 \sin \varphi} \left(3\,\frac{\sin 8\,\varphi}{\sin \varphi} + e^{7\,i\,\varphi}\right) \\ &- \frac{1}{400}\, N_{(4)}\, \frac{\sin 8\,\varphi}{\sin \varphi} \exp\left\{-i\,(n_4 + 2\,c - 4\,\varphi)\right\} \left[\frac{\sin 8\,\varphi}{\sin \varphi} \left(1 + 2 \cos 7\,\varphi\right) + e^{-7\,i\,\varphi} \right] \end{aligned}\right\} \quad (62)$$

By means of a few numerical calculations this can be reduced to the form

$$\left.\begin{aligned} (K e^{ik})/(2\,\Re\, C\, i \sin \varphi\, e^{ic}) &= 0.1959 \exp i\,\chi \\ &\quad + 0.0210\, N_{(4)} \exp\left\{-i\,(n_4 + 2\,c - \lambda)\right\} \end{aligned}\right\} \quad (63)$$

where

$$\chi = 5.235^0, \ \lambda = 122.41^0 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (64)$$

Usually $N_{(4)}$, which is the ratio of the semi-amplitude of the fourth harmonic in the sequence N_r, to the mean N_r, will be a small fraction, less than 0.1; if the N_r's are all equal, $N_{(4)}$ is, of course, zero; its value is easily determined in any actual case. If it does not exceed 0.1, the term containing it may be neglected, as being only 1 per cent or less of the whole expression (63), and affecting the final determination of p_l by not more than $\frac{1}{5}$ per cent. If, however, in any particular instance, $N_{(4)}$ exceeds 0.1, a correction to the final value of p_l can readily be made on this account. Hence in place of (63) we may use the approximate form

$$(K e^{ik})/(3\,\mathfrak{N}\,C i \sin\varphi\, e^{ic}) = 0.1959\, e^{i\chi} \quad . \ . \ . \ . \ . \ . \ . \ . \ (65)$$

By (25), p_l contributes to the lunar sum L_r the second harmonic $2\,\mathfrak{N} C \sin\varphi \sin(2\,\theta + c + \frac{1}{2}\pi)$, of which the amplitude and phase may be represented by $2\,\mathfrak{N} C \sin\varphi\, e^{i\,(c + {}^1/_2\,\pi)}$ or $2\,\mathfrak{N} C i \sin\varphi\, e^{ic}$, just as $K e^{ik}$ represents the amplitude and phase of $K \sin(2\,\theta + k)$, the contribution of p_l^- to L_r. The equation (65), neglecting the term in $N_{(4)}$, may be interpreted as indicating that the second harmonics in L_r due to p_l^- and to p_l have amplitudes which are in the ratio 0.1959, while in phase the former is 5.235° in advance of the latter. Thus the process used to eliminate p_a from the lunar sums also removes one-fifth of p_l; this fraction is too small to outweigh the great advantage gained in the reduced accidental error in the determination of p_l.

The second harmonic component of p_l, due to p_l^c, which remains in the lunar sums after p^- and p_s^c have been eliminated, is clearly (if we neglect $N_{(4)}$)

$$\left.\begin{aligned} C'\{\sin(2\,\theta + c') - 0.1959 \sin(2\,\theta + c' + 5.235^0)\} \\ = 0.8052\, C' \sin(2\,\theta + c' - 1.27^0) \end{aligned}\right\} \quad . \ . \ . \ . \ (66)$$

20. The complete calculation of p_l or $C \sin(2\,\theta + c)$ can now be summarized, with respect to the complete lunar sum L_r for a given calendar month or season; the procedure for the sum derived from any other group, such as the A, N, P, R groups, is of course precisely similar.

Associated with the sequence L_r there are the sequences N_r, the number of days of each transit time involved in it: S_r^c, representing p_s^c (§ 7); and J_r, formed as described in § 9. The second harmonic in each of these four sequences is to be formed and expressed as in (10) and (15), i. e., by the numbers

$$L,\ l;\ N,\ n;\ S^c,\ s^c;\ J,\ j \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (67)$$

$$L_A,\, L_B;\quad N_A,\, N_B;\quad S_A^C,\, S_B^C;\quad J_A,\, J_B \quad . \ . \ . \ . \ . \ . \ . \ (68)$$

The part of $L \sin(2\,\theta + l)$ due to p_s^c is given by (18); the numbers

$$S_A^C = \frac{25}{2}\,\frac{S^c N}{\mathfrak{N}} \sin(n - s^c),\ S_B^C = \frac{25}{2}\,\frac{S^c N}{\mathfrak{N}} \cos(n - s^c) \quad . \ . \ . \ (69)$$

can be calculated, since $\mathfrak{N} = \sum N_r$.

The part of $L \sin(2\theta + l)$ due to p^- is given by (36), being

$$-\tfrac{1}{8} J_A \sin 2\theta + \tfrac{1}{8} J_B \cos 2\theta \quad \text{. . . . . (70)}$$

The residue of $L \sin(2\theta + l)$ is therefore

$$(L_A - S_A^C + \tfrac{1}{8} J_A) \sin 2\theta + (L_B - S_B^C - \tfrac{1}{8} J_B) \cos 2\theta \quad \text{. . . . (71)}$$

Let this be expressed as

$$Q \sin(2\theta + q) \quad \text{. . . . . (72)}$$

so that

$$Q \cos q = L_A - S_A^C + \tfrac{1}{8} J_A, \quad Q \sin q = L_B - S_B^C - \tfrac{1}{8} J_B \quad \text{. . . (73)}$$

Then (72) represents $p_l^c + p_a^c$, and, apart from any residue of the latter which is not averaged out, we have, by comparison of (72) with (66),

$$Q = 0.8052\, C', \qquad q = c' - 1.27^0 \quad \text{. . . . . (74)}$$

Hence, by (26),

$$C = C'/(2\,\Re \sin\varphi) = Q/(1.6104\,\Re \sin\varphi) \quad \text{. . . . . (75)}$$

$$c = c' - \tfrac{1}{2}\pi = q - 88.73^0 \quad \text{. . . . . (76)}$$

This approximation has been derived on the hypothesis that $N_{(4)}$ is too small to affect the calculation.

It is necessary to verify that $N_{(4)}$, the ratio of the semi-amplitude of the fourth harmonic in the sequence N_r, to the mean value of N_r, is not more than 0.1; it is sufficient to do this by treating N_r as a sequence of 24 terms, neglecting N_{24}, and adding together the four quarter-sequences $N_0, \ldots, N_5$; $N_6, \ldots, N_{11}$; $N_{12}, \ldots, N_{17}$; $N_{18}, \ldots, N_{23}$; the half range of this combined sequence of 6 terms, divided by the mean of the six terms, is in general a good approximation to $N_{(4)}$, which will usually be less than 0.1. If it is 0.1 or more, it is necessary to determine the fourth harmonic of the sequence N_r, so obtaining $N_{(4)}$ and n_4 (§ 19). The correction to be made to the determination of C and c (§ 20) on this account will be very small, and it is scarcely necessary to indicate how it is to be made; the first approximation to c, as obtained in (20), can be used in making the correction.

21. The same plan for reducing the influence of p_a upon computations of the lunar atmospheric tide may be adapted to other modes of tabulation of the data. One of the modes most commonly used consists of the tabulation of the hourly data, after removal of p_s for each individual month, in lunar daily rows of 26 consecutive entries, the first of which (in column 0), in any row, is for the solar hour nearest to the time of lunar transit, while the remaining entries are for the succeeding hours in order. In this mode of tabulation the entries are taken from two successive solar days, in proportions depending on the lunar-transit time: and there is no transposition of earlier hours after later.

Since p_s has already been removed from the data, these refer to $p_l + p_a$. Let this be denoted by P. Let P^- be defined, in relation to P, as, in § 13, p^- was in relation to p, except that the points where $P^- = P$ will be taken as 0^h, 8^h, 16^h, 24^h, instead of 1^h, 9^h, 17^h. The entries for these hours on the lunar daily row for transit time n^h (where n is taken to vary from 1 to 24), occur in the columns $8 - c_n$, $16 - c_n$, $24 - c_n$, and sometimes $32 - c_n$: where

$$c_n \equiv n \pmod 8), \text{ and } 1 \leqslant c_n \leqslant 8 \quad \ldots \ldots \ldots \quad (77)$$

Let these entries be underlined on each row; in connection with each row, we consider underlined entries e_0, e_1, e_2, e_3, e_4 as follows: e_1, e_2, e_3 are the underlined entries in one or other of the columns 1 to 8, 9 to 16, 17 to 24 respectively, while e_0 and e_4 are the entries for the hours preceding that of e_1, and succeeding that of e_3, by 8 hours: thus e_0 occurs in column 0 or in the preceding row, while e_4 may occur in column 25 or in the following row. Then the 8-hour linear changes of P^- which affect the row concerned are the differences between these five underlined entries, i. e.,

$$a = e_1 - e_0, \; b = e_2 - e_1, \; c = e_3 - e_2, \; d = e_4 - e_3 \quad \ldots \quad (78)$$

These changes may be positive or negative; they will usually be expressible in numbers of one digit only. Let them all be entered (in red ink, say) in the column $8 - c_n$ of the row concerned; let the number of the column be added as suffix to a, b, c, d. The suffix (m, say) will thus be as follows for rows of transit time n:

$n =$	1	2	3	4	5	6	7	8
$n =$	9	10	11	12	13	14	15	16
$n =$	17	18	19	20	21	22	23	24
$m =$	1	2	3	4	5	6	7	0

Thus to each value of m there correspond three corresponding transit times (n_m, say).

In a given set of lunar daily rows, for any years, season, or lunar distance, let the number of rows for which a, b, c, d are entered in column m be denoted by N_m, and let the sums of the numbers a, b, c, d in this column be denoted by A_m, B_m, C_m, D_m. Let $\mathfrak{N}$ be the whole number of rows, so that

$$\mathfrak{N} = \sum_{m=0}^{7} N_m \quad \ldots \ldots \ldots \quad (79)$$

Also let

$$A = \sum_{m=0}^{7} A_m, \quad D = \sum_{m=0}^{7} D_m \quad \ldots \ldots \ldots \quad (80)$$

Let σ, σ_m ($m = 0$ to 7) denote the sequences (each of 26 terms) of sums of the entries in columns 0 to 25, for the whole $\mathfrak{N}$ days, and for the eight sets of N_m days separately. Let σ^-, σ_m^- denote the parts of these sequences which are due to P^-; then, apart from a constant additive to all the terms, and which is immaterial for our purpose, the sequence σ_m^- can be defined most simply by the 25 successive differences between its terms, as follows:

$$\left.\begin{array}{c} \frac{1}{8} A_m, \frac{1}{8} A_m, \ldots; \quad \frac{1}{8} B_m, \frac{1}{8} B_m, \ldots; \quad \frac{1}{8} C_m, \frac{1}{8} C_m, \ldots; \quad \frac{1}{8} D_m, \ldots; \\ \text{where } \frac{1}{8} A_m \text{ occurs } 8 - m \text{ times, } \frac{1}{8} B_m \text{ and } \frac{1}{8} C_m \text{ occur 8 times each,} \\ \text{and } \frac{1}{8} D_m \text{ occurs } m + 1 \text{ times} \end{array}\right\} \quad (81)$$

If the semidiurnal component of σ_m^-, corrected for the non-cyclic variation, is $P_m^- \sin(2\theta + p_m^-)$, that of the sequence (81) is (cf. 23 and 25)

$$2 P_m^- \sin\varphi \sin(2\theta + p_m^- + \varphi + \tfrac{1}{2}\pi).$$

The semidiurnal component of the whole contribution of P^- to σ^- will be denoted by $\Pi^- \sin(2\theta + \varpi^-)$, and satisfies the equations

$$\Pi^- \sin(2\theta + \varpi^-) = \sum_{m=0}^{7} P_m^- \sin(2\theta + p_m^-) \quad \ldots\ldots (82)$$

By § 10,

$$\left. \begin{aligned} 2 P_m^- \sin\varphi \exp i(p_m^- + \varphi + \tfrac{1}{2}\pi) = \frac{2i}{25}\Bigg[\sum_{r=0}^{7-m} \tfrac{1}{8} A_m e^{-2ir\varphi} \\ + \sum_{r=8-m}^{15-m} \tfrac{1}{8} B_m e^{-2ir\varphi} + \sum_{r=16-m}^{23-m} \tfrac{1}{8} C_m e^{-2ir\varphi} + \sum_{r=24-m}^{24} \tfrac{1}{8} D_m e^{-2ir\varphi}\Bigg] \end{aligned} \right\} (83)$$

so that

$$\left. \begin{aligned} 200 P_m^- e^{i\varphi} \sin\varphi \exp(i p_m^-)(1 - e^{-2i\varphi}) = A_m \{1 - e^{-2i\varphi(8-m)}\} \\ + \{B_m e^{-2i\varphi(8-m)} + C_m e^{-2i\varphi(16-m)}\}(1 - e^{-16i\varphi}) \\ + D_m e^{-2i\varphi(24-m)}\{1 - e^{-2i\varphi(m+1)}\} \end{aligned} \right\} (84)$$

Summing with respect to m, we have, by (82),

$$\left. \begin{aligned} 400 i \Pi^- \sin^2\varphi \exp(i\varpi^-) = (A - D) + e^{-16i\varphi} \sum_{m=0}^{7} (B_m - A_m) e^{2mi\varphi} \\ + e^{-32i\varphi} \sum_{m=0}^{7} (C_m - B_m) e^{2mi\varphi} + e^{-48i\varphi} \sum_{m=0}^{7} (D_m - C_m) e^{2mi\varphi} \end{aligned} \right\} (85)$$

Let us define a new sequence of 25 terms as follows:

$$\left. \begin{aligned} U_0 &= A - D \\ U_{r+1} &= D_r - C_r \quad (r = 0 \text{ to } 7) \\ U_{r+9} &= C_r - B_r \quad (r = 0 \text{ to } 7) \\ U_{r+17} &= B_r - A_r \quad (r = 0 \text{ to } 7) \end{aligned} \right\} \quad \ldots\ldots\ldots (86)$$

Then (85) may be written

$$\left. \begin{aligned} &400 i \Pi^- \sin^2\varphi \exp(i\varpi^-) \\ &= U_0 + \sum_{m=0}^{7} [U_{m+1} e^{2(m+1)i\varphi} + U_{m+9} e^{2(m+9)i\varphi} + U_{m+17} e^{2(m+17)i\varphi}] \\ &= \sum_{r=0}^{24} U_r e^{2ri\varphi} = -\frac{25}{2i} U e^{-iu} \end{aligned} \right\} (87)$$

by (12), where $U \sin(2\theta + u)$ denotes the semidiurnal component of the sequence U_r.

Consequently,

$$\prod^{-} \exp(i\,\overline{\omega}^{-}) = \frac{U}{32 \sin^2 \varphi} e^{-iu} \quad \dots \dots \dots \dots (88)$$

or

$$\prod^{-} = \frac{U}{32 \sin^2 \varphi} \quad \dots \dots \dots \dots (89)$$

$$\overline{\omega}^{-} = -u \quad \dots \dots \dots \dots (90)$$

Thus the analysis of the single sequence U_r gives the semidiurnal component of the contribution of P^{-} to the lunar sequence σ.

22. It remains to calculate the part of the sequence σ^{-} which is due to p_l^{-}. The true lunar diurnal variation being $C \sin(2\,\theta + c)$, as in § 19, it is clear that, on the days of transit time n_m, the lunar hours corresponding to the underlined entries e_0, e_1, e_2, e_3, e_4 are

$$-m, \ -m+8, \ -m+16, \ -m+24, \ -m+32 \quad \dots \dots (91)$$

hence it follows that p_l contributes to a_m a part a'_m given by

$$\left.\begin{aligned} a'_m &= C\,[\sin\{2\,(8-m)\,\varphi + c\} - \sin\{-2\,m\,\varphi + c\}] \\ &= 2\,C \sin 8\,\varphi \cos\{2\,(4-m)\,\varphi + c\} \end{aligned}\right\} \quad \dots (92)$$

and likewise parts b'_m, c'_m, d'_m, to b_m, c_m, d_m, as follows:

$$\left.\begin{aligned} b'_m &= 2\,C \sin 8\,\varphi \cos\{2\,(12-m)\,\varphi + c\} \\ c'_m &= 2\,C \sin 8\,\varphi \cos\{2\,(20-m)\,\varphi + c\} \\ d'_m &= 2\,C \sin 8\,\varphi \cos\{2\,(28-m)\,\varphi + c\} \end{aligned}\right\} \quad \dots \dots (93)$$

Further, let

$$A'_m = N_m\,a'_m, \quad B'_m = N_m\,b'_m, \quad C'_m = N_m\,c'_m, \quad D'_m = N_m\,d'_m \quad \dots (94)$$

Then the contribution of p_l^{-} to σ^{-} depends on A'_m, B'_m, C'_m, D'_m just as that of P^{-} depends on A_m, B_m, C_m, D_m. In particular, if the semidiurnal component of the contribution of p_l^{-} to σ^{-} be denoted by $\Pi_l^{-} \sin(2\,\theta + \overline{\omega}_l^{-})$, and if $V \sin(2\,\theta + v)$ is the semidiurnal component of the sequence V_r defined, relative to A'_m, ..., as U is defined relative to A_m, ..., we have

$$\prod_l^{-} = \frac{V}{32 \sin^2 \varphi}, \quad \overline{\omega}_l^{-} = -v \quad \dots \dots \dots (95)$$

Now

$$\left.\begin{aligned} V_0 = A' - D' &= 2\,C \sin 8\,\varphi \sum_{m=0}^{7} N_m [\cos\{2\,(4-m)\,\varphi + c\} \\ &\qquad\qquad - \cos\{2\,(28-m)\,\varphi + c\}] \\ &= 4\,C \sin 8\,\varphi \sin 24\,\varphi \sum_{m=0}^{7} N_m \sin\{2\,(16-m)\,\varphi + c\} \\ &= -4\,C \sin \varphi \sin 8\,\varphi\,\{N_0 \sin(7\,\varphi + c) + N_1 \sin(5\,\varphi + c) \\ &\quad + N_2 \sin(3\,\varphi + c) + \cdots + N_7 \sin(-7\,\varphi + c)\} \end{aligned}\right\} \quad (96)$$

since $25\,\varphi = 2\,\pi$.

Let us write

$$\left.\begin{array}{lll} N_0+N_7=\mathfrak{N}\left(\tfrac{1}{4}+\nu_4\right), & N_1+N_6=\mathfrak{N}\left(\tfrac{1}{4}+\nu_3\right), & N_2+N_5=\mathfrak{N}\left(\tfrac{1}{4}+\nu_2\right) \\ & & N_3+N_4=\mathfrak{N}\left(\tfrac{1}{4}+\nu_1\right) \\ N_0-N_7=\nu_{-4}\,\mathfrak{N}, & N_1-N_6=\nu_{-3}\,\mathfrak{N}, & N_2-N_5=\nu_{-2}\,\mathfrak{N} \\ & & N_3-N_4=\nu_{-1}\,\mathfrak{N} \end{array}\right\} \quad (97)$$

Then the numbers ν are likely to be small fractions. Also let

$$\left.\begin{aligned} \alpha_1 &= \nu_1 \cos\varphi + \nu_2 \cos 3\varphi + \nu_3 \cos 5\varphi + \nu_4 \cos 7\varphi \\ &= 0.969\,\nu_1 + 0.729\,\nu_2 + 0.309\,\nu_3 - 0.187\,\nu_4 \end{aligned}\right\} \quad \ldots\ldots (98)$$

$$\left.\begin{aligned} \alpha_{-1} &= \nu_{-1} \sin\varphi + \nu_{-2} \sin 3\varphi + \nu_{-3} \sin 5\varphi + \nu_{-4} \sin 7\varphi \\ &= 0.249\,\nu_{-1} + 0.685\,\nu_{-2} + 0.951\,\nu_{-3} + 0.982\,\nu_{-4} \end{aligned}\right\} \quad \ldots (99)$$

Then since

$$\sin\varphi\,(\cos\varphi + \cos 3\varphi + \cos 5\varphi + \cos 7\varphi) = \tfrac{1}{2}\sin 8\varphi \quad \ldots (100)$$

(96) may be written

$$V_0 = -NC\sin^2 8\varphi \left\{\frac{1}{2}\sin c + 4\,\frac{\sin\varphi}{\sin 8\varphi}\,(\alpha_1 \sin c + \alpha_{-1}\cos c)\right\} \quad \ldots (101)$$

Further

$$\left.\begin{aligned} Ve^{-iv} &= -\frac{2i}{25}\Big[V_0 + \sum_{m=0}^{7}\big\{(D'_m - C'_m)\,e^{2i(m+1)\varphi} \\ &\qquad + (C'_m - B'_m)\,e^{2i(m+9)\varphi} + (B'_m - A'_m)\,e^{2i(m+17)\varphi}\big\}\Big] \\ &= -\frac{2i}{25}\Big[V_0 - 4C\sin^2 8\varphi \sum_{m=0}^{7} N_m \big\{e^{2i(m+1)\varphi}\sin\{2(24-m)\varphi + c\} \\ &\qquad + e^{2i(m+9)\varphi}\sin\{2(16-m)\varphi + c\} + e^{2i(m+17)\varphi}\sin\{2(8-m)\varphi + c\}\big\}\Big] \\ &= -\frac{2i}{25}\Big[V_0 - 4Ce^{ic}\sin^2 8\varphi \sum_{m=0}^{7} N_m \big\{e^{-2i(24-m)\varphi - ic}\sin\{2(24-m)+c\} \\ &\qquad + e^{-2i(16-m)\varphi - ic}\sin\{2(16-m)\varphi + c\} + e^{-2i(8-m)\varphi - ic}\sin\{2(8-m)\varphi + c\}\big\}\Big] \\ &= -\frac{2i}{25}\Big[V_0 + 2iCe^{ic}\sin^2 8\varphi \sum_{m=0}^{7} N_m \big\{3 - e^{-4i(24-m)\varphi - 2ic} \\ &\qquad - e^{-4i(16-m)\varphi - 2ic} - e^{-4i(8-m)\varphi - 2ic}\big\}\Big] \\ &= -\frac{2i}{25}\Big[V_0 + 6i\mathfrak{N}\,Ce^{ic}\sin^2 8\varphi - 2iCe^{-ic}\sin^2 8\varphi \Big(\sum_0^7 N_m e^{4mi\varphi}\Big)\big(e^{-96i\varphi} \\ &\qquad + e^{-64i\varphi} + e^{-32i\varphi}\big)\Big] \\ &= \tfrac{12}{25}\mathfrak{N}\,Ce^{ic}\sin^2 8\varphi \Big[1 + \tfrac{1}{12}ie^{-ic}\sin c + \tfrac{2}{3}ie^{-ic}\frac{\sin\varphi}{\sin 8\varphi}(\alpha_1\sin c + \alpha_{-1}\cos c) \\ &\qquad - \tfrac{1}{3}e^{-64i\varphi - 2ic}(1 + 2\cos 32\varphi)\sum_{m=0}^{7}(N_m/\mathfrak{N})\,e^{4mi\varphi}\Big] \end{aligned}\right\} \quad (102)$$

Let

$$\left.\begin{aligned}\alpha_2 &\equiv \nu_1 \cos 2\varphi + \nu_2 \cos 6\varphi + \nu_3 \cos 10\varphi + \nu_4 \cos 14\varphi \\ &= 0.876\,\nu_1 + 0.063\,\nu_2 - 0.809\,\nu_3 - 0.930\,\nu_4\end{aligned}\right\} \quad (103)$$

$$\left.\begin{aligned}\alpha_{-2} &= \nu_{-1} \sin 2\varphi + \nu_{-2} \sin 6\varphi + \nu_{-3} \sin 10\varphi + \nu_{-4} \sin 14\varphi \\ &= 0.482\,\nu_{-1} + 0.998\,\nu_{-2} + 0.588\,\nu_{-3} - 0.368\,\nu_{-4}\end{aligned}\right\} \quad (104)$$

Then

$$\left.\begin{aligned}\sum_{m=0}^{7} {}^{\bullet}N_m e^{4mi\varphi} &= \Re\, e^{14i\varphi}\left\{\frac{1}{4}(\cos 2\varphi + \cos 6\varphi + \cos 10\varphi + \cos 14\varphi)\right. \\ &\left. + \alpha_2 - i\alpha_{-2}\right\} = \Re\, e^{14i\varphi}\left\{\frac{1}{8}\frac{\sin 16\varphi}{\sin 2\varphi} + \alpha_2 - i\alpha_{-2}\right\}\end{aligned}\right\} \quad (105)$$

on replacing φ by 2φ in (100). Also

$$1 + 2\cos 32\varphi = -\frac{\sin 2\varphi}{\sin 16\varphi} \quad \ldots\ldots\ldots \quad (106)$$

in virtue of $25\varphi = \pi$. Hence

$$\left.\begin{aligned}Ve^{-iv} &= \frac{12}{25}\Re\, Ce^{ic}\sin^2 8\varphi\left[1 + \frac{1}{24}(1 - e^{-2ic}) + \frac{1}{24}e^{-2ic}\right. \\ &\left. + \frac{2}{3}ie^{-ic}\frac{\sin\varphi}{\sin 8\varphi}(\alpha_1 \sin c + \alpha_{-1}\cos c) + \frac{1}{3}e^{-2ic}\frac{\sin 2\varphi}{\sin 16\varphi}(\alpha_2 - i\alpha_{-2})\right] \\ &= \frac{1}{2}\Re\, Ce^{ic}\sin^2 8\varphi\left[1 + \frac{48}{75}ie^{-ic}\frac{\sin\varphi}{\sin 8\varphi}(\alpha_1 \sin c + \alpha_{-1}\cos c)\right. \\ &\left. + \frac{8}{25}e^{-2ic}\frac{\sin 2\varphi}{\sin 16\varphi}(\alpha_2 - i\alpha_{-2})\right]\end{aligned}\right\} \quad (107)$$

Thus when, as will usually be the case, the terms involving α_1, α_{-1}, α_2, α_{-2}, can be neglected, this reduces to

$$Ve^{-iv} = \tfrac{1}{2}\Re\, Ce^{ic}\sin^2 8\varphi \ldots\ldots\ldots\ldots \quad (108)$$

so that

$$\Pi_l^- = \frac{1}{64}\Re\, C\frac{\sin^2 8\varphi}{\sin^2\varphi} = 0.206\,84\,\Re\, C \quad \ldots\ldots \quad (109)$$

$$\overline{\omega}_l^- = -v = c \quad \ldots\ldots\ldots\ldots \quad (110)$$

by (95). In this case, therefore, if the transit times are nearly uniformly distributed throughout the group of lunar days considered, the elimination of P^- from the data removes just over one-fifth of p_l, without change of phase. The remaining fraction of p_l is 0.79316, and the reciprocal of this is 1.2608.

Thus, if $L \sin(2\theta + l)$ is the semidiurnal component of the sequence of lunar sums, corrected only for the non-cyclic variation, and if

$$L \sin(2\theta + l) - \Pi^- \sin(2\theta + \overline{\omega}^-) \equiv L' \sin(2\theta + l') \quad \ldots \quad (111)$$

the corresponding determination of C, c will be

$$C = 1.2608\,L'/\Re, \quad c = l' \quad \ldots\ldots\ldots\ldots \quad (112)$$

23. The preceding method of eliminating a large part of p_a from p in the determination of p_l is to some extent arbitrary as regards the choice of the interval of 8 hours in the definition of p^-; it is of interest to consider whether any other interval whould be more suitable for the purpose in view.

It seems desirable to take the solar day as the basis of definition of p^- in the determination of p_l, because (a) p^- is thereby rendered less dependent on p_l itself, and also (b) the systematic elimination of p^- is most conveniently made in close relation to the tables of original data, which are arranged according to solar time. Hence the interval on which p^- is based must be a submultiple of the solar day; the solar day itself would not suffice — it would practically only eliminate the noncyclic variation during the lunar day, which is part of the existing practice. The smaller submultiples of the solar day, i. ., 2^h, 3^h, 4^h, are too small; the corresponding p^- would contain far too large a proportion of p_l. Thus the only intervals which really need consideration are 12^h, 8^h and 6^h; the choice of the first would have the advantage that p^- would contain an extremely small fraction of p_l, while the p^- corresponding to 6^h would contain a fraction of p_l considerably more than the one-fifth corresponding to 8^h. On the other hand the residual part (p_a^c) of p_a ist greatest for the 12^h interval, and least for that of 6^h; the mean numerical value of p_a^c is likely to vary as the square of the length of the interval, i. e., as 144 : 64 : 36. The best choice is not clear, but that of 8^h, here made, is probably better than that of 12^h, because the fraction of p_l removed with p^- is small, while the residual p_a^c will be less than half that corresponding to the interval of 12^h for p^-; the choice of 6^h would reduce p_a^c still further, but at the cost of removing too large a fraction of p_l itself.

24. It is of interest to consider how far the determination of the lunar atmospheric tide is affected by the use of an interval of M' solar hours (where M' is an integer, here taken as 25) to represent the lunar day, which is actually of variable duration, but of average length M solar hours, where $M = 24.83$ ($24^h\ 50^m$).

First consider the case when hour-to-hour differences form the basis of the lunar tabulations, as in §§ 6—20. For the days of transit time n^h, Δ_r refers (§ 8) to a time $r - n + 1$ solar hours after transit, if $r > n - 1$, or to $n - r - 1$ solar hours before transit, if $r < n - 1$; in the latter case, since p_l is periodic in the time M hours, the time is equivalent to $M + r - n + 1$ solar hours after transit.

Since M' solar hours are taken to represent the lunar day, r ranges from 0 to $M' - 1$. We suppose that n ranges from 1 to M', as in the tabulations described in §§ 6—20 (the conclusion arrived at is not affected if n is taken to range, like r, from 0 to $M' - 1$). In forming the lunar sums the daily rows of Δ's are, in effect, divided into two parts, of which the first is transposed so as to follow the second; the result is that the values of Δ_n from days of different transit time n^h, and for which, for a particular integral value of m (where $0 \leqslant m \leqslant M' - 1$),

$$r \equiv m + n - 1 \pmod{M'} \quad \text{. . . . . (113)}$$

are all grouped together and added. Since $0 \leqq r \leqq M' - 1$, it is clear that according to (113) r will equal $m + n - 1$ on the days for which n lies in the range 1 to $M' - m$; in all these cases, $M' - m$ in number, the true interval after transit, corresponding to Δ_r, is exactly m solar hours. When n lies in the range $M' - m + 1$ to M', r will equal $m + n - 1 - M'$; in these cases, m in number, the true time to which Δ_r corresponds is $m - M'$ solar hours before transit, which is equivalent, so far as concerns p_l, to the time $m - M' + M$ solar hours after transit. Since $M' \neq M$, the entries Δ_r thus combined together do not all refer to precisely the same lunar epoch, except when $m = 0$; they correspond either to m or $m + M - M'$ solar hours after transit, and they are treated, in the subsequent harmonic analysis, as if they referred to an epoch after lunar transit equal to a fraction m/M' of a lunar day, that is, an epoch of $m\,M/M'$ solar hours after transit. This time is intermediate between the two actual epochs of the Δ_r's.

The mean epoch, in solar hours after lunar transit, corresponding to the term L_m in the sequence of lunar sums, is clearly

$$\left\{ m \sum_{n=1}^{M'-m} N_{n-1} + (m + M - M') \sum_{n=M'-m+1}^{M'} N_{n-1} \right\} \div \mathfrak{N},$$

which is intermediate between m and $m + M - M'$. If the values of N_{n-1} are the same for each value of n, and therefore equal to $\mathfrak{N}/M'$, the expression reduces to

$$\frac{1}{M'} \{ m (M' - m) + (m + M - M') m \}$$

or $m\,M/M'$, which is the epoch assumed for L_m in the harmonic analysis. If the values of N_{n-1} are nearly all alike, this result will be nearly but not quite exact.

The differences between $m\,M/M'$ and m or $m + M - M'$ (the two values corresponding to the Δ_r's combined in L_m) are

$$- m \frac{M' - M}{M'} \quad \text{and} \quad (M' - m) \frac{M' - M}{M'};$$

these vary with m, being most unequal when m is 1 or 24, and most nearly equal when m is 12 or 13. The fact that the Δ_r's do not exactly correspond to the epoch assumed for L_m in the harmonic analysis renders the deduced amplitude of p_l slightly too small, but by a quite negligible amount if M' is 25 or even 24. A further slight reduction arises from the spread of the transit times, taken to occur at the exact hour n, over the interval $n - \frac{1}{2}^h$ to $n + \frac{1}{2}^h$.

25. When M' is taken, as above, to be 25, the midnight values of the pressure are always used twice over, that is, as 0^h on one day and 24^h on the preceding day. There is no valid physical reason why they should thus have double weight, as compared with the values at other hours, in determining p_l, and the fact that they do have double weight is a slight blemish on the method of using 25 values. It is probably of no real disadvantage, however, and some theoretical objection seems likely to arise in connection with any method of deriving p_l from data referring to solar hours. For example, though each solar hourly value is used once

only if M' is taken as 24 instead of 25, there is the new difficulty that on certain days none of the 24 hourly values will be within half an hour of lunar transit (the nearest transits occur before the first value, i. e., on the preceding day, and after the last value on the next day); according to the previous plan we should omit these days altogether, which would be as objectionable, theoretically, as giving the midnight values double weight when $M' = 25$. Actually it would be appropriate to treat half these days as days of transit time 1^h, and the others as days of transit time 24^h, though this would not be quite correct. It would add slightly to the errors in the times of the Δ_r's which contribute to L_m; these times, when $M' = 24$, will in any case diverge rather more from $m\ M/M'$ than when $M' = 25$. The errors will, however, only very slightly affect the determination of p_l, and the use of $M' = 24$ certainly has some advantages; (a) it reduces the tabulations and summations by $\frac{1}{25}$ th, or 4 per cent.; (b) it simplifies the removal of p_a (since only D, E, F, and not also G — cf. §§ 9, 13 — would be required to represent p_a on each day); and, finally, the harmonic analysis is simpler for sequences of 24 values than for those of 25.

26. When hour-to-hour differences are taken as the basis of tabulation, the difference between M and M' affects the formula (24), where it is assumed that the interval between the hours over which the difference is taken is $\frac{1}{25}$ th of a lunar day, i. e., M/M' solar hour, whereas actually it is one solar hour. If $M' = 25$, the actual interval is too long in the ratio M/M', so that $\sin\varphi$ in the formula (24) should be $\sin(M'\varphi/M)$. This renders the resulting determination of p_l, by this method, too great in the ratio

$$\sin\left(\frac{25}{24.83}\ 14.4^0\right)\Big/\sin 14.4^0$$

which is 1.0068. The correction necessary on this account is therefore about — 0.7 per cent., which is in itself scarcely worth making, in view of the accidental errors attaching to the determination of p_l.

27. In conclusion, it may be remarked that, owing to the recent extension in the capacity of the cards used in the Hollerith automatic punching, sorting and adding machines, from 45 to 80 columns, the Hollerith system of calculation can now conveniently be applied to the determination of lunar periods in geophysical phenomena. Systematic plans are being devised for the application of the Hollerith machines to this purpose; the details of the work of removal of p_S and p_a^- from the data are altered in many respects, though the principles and essentials of the preceding analysis remain valid.