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Niedersächsische Staats- und Universitätsbibliothek Göttingen
Georg-August-Universität Göttingen
Platz der Göttinger Sieben 1
37073 Göttingen
Germany
Email: gdz@sub.uni-goettingen.de

Man wird annehmen müssen, daß in der Tiefe Uran-Radiumerze vorkommen, wenn auch vielleicht nur in kleineren Mengen. Der hohe CO_2-Gehalt der Wässer hat sicher eine große Bedeutung für die Aufschließung und Verlagerung des Urans. Eine endgültige Klärung muß weiteren Arbeiten überlassen bleiben.

Herrn Generaldirektor Hayer, Radiumbad Brambach, danke ich für sein freundliches Entgegenkommen bei der Durchführung der Arbeit.

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Freiberg i. S., Radiuminstitut der Bergakademie, im Oktober 1934.

The Hayford-Bowie Table and the Definition of Perfect Isostasy

By **Walter D. Lambert,** U. S. Coast and Geodetic Survey, Washington, D. C.
(with 2 figures)

Es wird darauf aufmerksam gemacht, daß Hayford bei der Berechnung seiner Tafeln für die isostatische Reduktion der Schwerebeobachtungen ausnahmslos die exakte Gleichheit der kompensierenden Massen mit den ihnen entsprechenden topographischen Massen angenommen hat; die gegenteilige Meinung Bullards ist daher unbegründet. Die von Bullard vorgeschlagene Definition der Isostasie als Druckgleichheit ist vernünftig, aber für manche Zwecke weniger bequem als eine auf die Massengleichheit begründete Definition. Auch muß man bei den von Bullard gegebenen Formeln für die Schwerkraft unter der Erdoberfläche auf die Anziehung der oberen Teile der Erdkruste Rücksicht nehmen.

Mr. Bullard's*) note raises two distinct questions: (1) On what basis did Hayford compute his tables for the isostatic reduction of gravity? (2) On what basis should they be computed, perfect isostasy being assumed, or in other words, what is the precise mathematical formulation of perfect isostasy?

As to the first question, it is not surprising that Hayford's rather intricate computations, which are equivalent to a double integration by mechanical quadrature, should be rather hard to follow and should therefore have given rise to misapprehension; this has happened before. It may be stated quite definitely

*) E. C. Bullard: Zeitschr. f. Geophys. **10**, 318 (1934).

that Hayford did take the mass of the compensation equal to the mass of the corresponding topography, even in the distant zones. This matter is the subject of an explanatory footnote*) so long that it need not be reproduced in full. The tables of Cassinis based on the Hayford zones (not the tables of Cassinis referred to in the footnote) and recently published in preliminary form have been used to check Hayford's tables in Special Publication No. 10 and they confirm the fact that the masses of topography and compensation were taken exactly equal by Hayford even for distant zones. Mr. Bullard's objection on this score is therefore unfounded.

The footnote admits an error of about two percent in the resultant, which is quite a different matter from an error of two percent in the compensation. for the latter error, as Mr. Bullard notes, would falsify the resultant completely. The conclusion that there is the error of about two percent in the resultant is derived from a careful reading of Special Publication No. 10, p. 24 and following. It will be seen that the computation there explained has the effect of distributing equal elements of the compensating mass among the different equally spaced levels, 1, 2, 3, etc., (Fig. 1) and disregards the fact that at these levels the zone has the smaller area at the greater depth. For uniform density of compensation the amount of mass of compensation distributed to each level should be proportional to the area of the zone at this level, that is, to the radius of the level. The effect of Hayford's method of computation is to increase slightly the density of compensation at the lower levels and this is roughly equivalent to absolutely uniform density of compensation with a slightly greater depth. This error is small, less than two percent, as stated in the footnote of Special Publication No. 40.

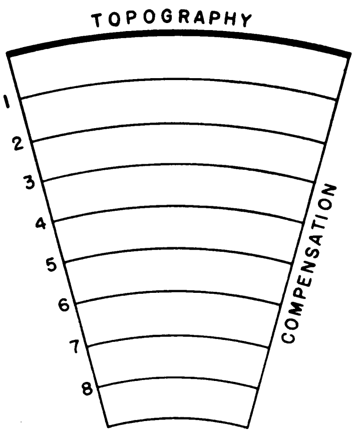
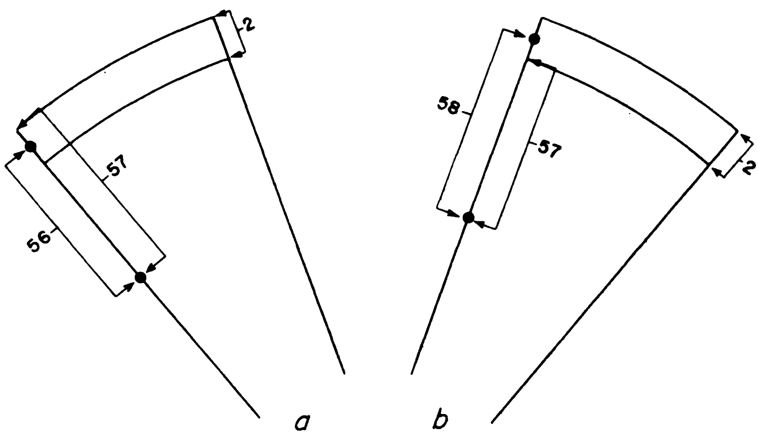


Fig. 1
Hayford's distribution of the mass of compensation—equal amounts to each level

A comparison of Hayford's figures with those obtained from Cassinis's new tables did not bring out this two percent very clearly, because the quantities concerned were small and because the assumption made by Hayford that the resultant in the distant zones is strictly proportional to the thickness of the topography, whether land or water, introduces errors greater than two percent but still not serious. The reason why the assumed proportionality is not exact may be seen by a simple example. The resultant effect is roughly proportional to the

*) W. Bowie: Coast and Geodetic Survey Special Publication Nr. **40**. Investigations of Gravity and Isostasy. p. 98. Washington 1917.

mass of the topography and to the depth through which it is displaced, which may be taken equal to half the depth of compensation, or 57 kilometers. A thickness of topography equal to two kilometers of land is displaced $57 - 1/2 \, . \, 2 = 56$ kilometers (Fig. 2a). The minus sign appears because Hayford always starts the compensation at the surface of the lithosphere, whether for a land compartment or a water compartment. A water compartment two kilometers in depth undergoes a displacement (Fig. 2b) of $57 + 1/2 \, . \, 2 = 58$ kilometers, so that the effects are not in the ratio of the masses involved, which in Hayford's table is taken as 0.615, but in the ratio $0.615 \, . \, 58/56$. The two-percent correction is therefore obscured by the lack of strict proportionality.



(a) Land compartment
(b) Water compartment

Fig. 2. Displacement of mass for Hayford isostatic compensation

In section 3 of Mr. Bullard's note he states the condition for complete isostasy by equating two integrals. This is the condition of numerical equality of pressure treated as if it were hydrostatic and exerted on the surface of compensation by the topography and compensation respectively; Mr. Bullard, however, does not use the word „pressure". In basing the concept of isostasy on equality of pressure rather than on equality of mass Mr. Bullard is almost certainly getting a little closer to ideas of the original proponents of the theory of isostasy. His formulas, however, are in need of correction and it is worth while mentioning some of the practical advantages of adopting equality of mass rather than equality of pressure as the working definition for isostatic computations.

Mr. Bullard assumes that gravity below the surface of the earth is inversely proportional to the square of the distance from the center. If r is the radius of the earth, g_0 the gravity at depth x, then this assumption, if the square of x/r is neglected, gives for value of gravity, g, in terms of surface gravity, g_0,

$$g = g_0 \left(1 + \frac{2x}{r}\right) \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad (1)$$

But the effect of the matter above the depth x must be considered. If we attribute to this a density δ and call δ_m the mean density of the earth, we have instead of (1)

$$g = g_0 \left[1 + \left(2 - \frac{\delta}{3\delta_m}\right) \frac{x}{r}\right] \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad (2)$$

Since $\delta/\delta_m = 1/2$, approximately, the increase in gravity with depth according to (2) is only about one-fourth as great as would be given by (1), which corresponds to Mr. Bullard's assumption.

There are, however, certain disadvantages in basing the concept of isostasy on hydrostatic pressure in the crust, as is done by Mr. Bullard and others. We know very well that the pressure is not hydrostatic in the crust, even at considerable depths. It savors of fiction, or at least of arbitrary convention, to assume hydrostatic pressure with in a mass of rock well above the general level, or indeed above any level other than that of the lowest ocean deeps, for if the pressure were hydrostatic, the higher masses would run down and flow over the lower ones.

The chief advantage of assuming equality of mass of the topography and its compensation rather than equality of pressure on the surface of compensation is, however, that of simplicity. If we assume equality of mass, we can reckon our loads from any convenient level, in particular from sea level. If we do not assume equality of mass, and equality of pressure requires inequality of mass, because of variation of gravity and non-parallelism of the verticals, then we must reckon our loads from such a level that the average load over the globe is zero, or make an appropriate correction. This level, called by the writer 'mean load level'*), lies about 1400 meters below mean sea level.

If we expand the apparent irregularities (ΔM) of mass as shown by the topography in a series of spherical harmonics of various orders, Y_0, Y_1 etc., we have**)

$$\Delta M = Y_0 + Y_1 + Y_2 \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad (3)$$

If ΔM is reckoned from mean load level or some equivalent process is used, the term in Y_0 is zero. Unless Y_0 is zero, we add to or subtract from the total mass of the earth when we make an isostatic reduction on any other basis than that of equality of mass; this is undesirable in reductions intended to determine the figure of the earth. The presence of a term in Y_0 is inconvenient not only in the computation of the direct effect of the disturbing masses but also in the computation of their indirect effect in deforming the geoid. Unless the disturbing

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*) W. D. Lambert: Bulletin Géodésique, Nr. **26**, p. 91, 1930.

) Such an expansion has been made by A. Prey, Abhandlungen der k. Gesellsch. d. Wiss. zu Göttingen **11, 1, 1922.

potential $\varDelta V$ is based on an expansion of type (3) in which Y_0 is zero, we can not use the convenient equation

$$H = \frac{\varDelta V}{g} \quad \ldots \ldots \ldots \ldots \ldots \quad (4)$$

where H is the warping of the geoid due to $\varDelta M$ and to the resulting $\varDelta V$.

If we attempt to adhere strictly to the idea of equality of pressure, we must remember that since pressure depends on gravity, we must take account not only of the normal change of gravity with depth, as in (2), but also of anomalies in gravity due to anomalies in the distribution of mass. The attempt to introduce this latter consideration is responsible for much of the extreme complication of the law of density of compensation suggested by Love*), which, however, was entirely justified for Love's particular purpose, but is impractical in ordinary isostatic reductions.

For these reasons the writer suggests that the definition of isostasy as equality of mass be generally adopted as an arbitrary conventional definition in the computation of isostatic reductions. Even more desirable, however, than the adoption of any particular definition would be the general adoption by agreement of some one definition. The agreement could not and should not exclude the use of other definitions by individual investigators for their own purposes. The argument just given has been conceived mainly with reference to the determination of the figure of the earth but it seems to the writer that the definition of isostasy as equality of mass should be sufficient for other purposes, at least in the present state of knowledge of stresses in the earth. There are, however, other uses for gravity than the determination of the figure of the earth, such as the study of local crustal structure. In any argument about methods of reducing gravity observations it is well to bear in mind the words of Helmert**): „Im allgemeinen wird man wohl von vornherein zugeben müssen, dass verschiedene Verwendungsarten von g auch verschiedene Reductionen auf ein einheitliches Niveau bedingen, dass also die Aufgabe der Reduction nicht nur eine einzige Lösung hat. Wie immer aber auch reduciert wird, so müssen die reducierten g doch mit einer hinreichenden Annäherung den Charakter von Differentialquotienten des Schwerkraftpotentials W eines bestimmten Massensystems nach der Höhe besitzen, also ein $\partial W : \partial h$ sein, genommen für das einheitliche Niveau: das Potentialniveau $W = \text{const.}$“

*) A. E. H. Love: Some Problems of Geodynamics. p. 9. Cambridge, England, 1911.

**) F. R. Helmert: Sitzungsber. d. kgl. Preuß. Akad. d. Wiss. 1902, S. 844.