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Notes on Wave Theories for the Propagation of T-, Lg-, Rg-, G-Waves and Microseisms from Storms over Deep Sea

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Summary: Some years ago the author gave wave theories for the propagation of the above mentioned waves. But there the approximations were not completely justified and in the theory for G-waves the earth's model and the analysis were of a different kind from those used in the other theories. In the present notes, the theories have been modified to some extent and substantiated by graphical representations. In the cases of T-, Lg-, and Rg-waves the principal aim has been to demonstrate low period energy concentration. In the case of microseisms from storms over deep sea, this aspect has been examined to find whether they can be present while sub-oceanic Rayleigh Waves are always absent. Lastly, developing a theory similar to the previous four cases, it has been shown that for G-waves there is practically no energy concentration due to the low velocity layer.

Zusammenfassung: Theorien für die Ausbreitung der im Titel genannten Wellen, die der Autor vor Jahren entwickelt hat, werden modifiziert und durch graphische Darstellungen vervollständigt. Die bei der Rechnung seinerzeit benutzten Näherungen waren nicht voll gerechtfertigt. Außerdem lag der Theorie der G-Wellen ein anderes Erdmodell zugrunde als den übrigen Theorien.

Für die T-, Lg- und Rg-Wellen zeigt sich vor allem eine Energiekonzentration bei kurzen Perioden. Unter diesem Aspekt wird auch die durch Stürme über tiefem Ozean erzeugte Mikroseismik untersucht. Es wird der Frage nachgegangen, ob diese Mikroseismik auftreten kann, während sub-ozeanische Rayleigh-Wellen stets fehlen. G-Wellen werden schließlich theoretisch wie die anderen genannten Wellentypen behandelt. In diesem Fall gibt es keine nennenswerte Energiekonzentration, die auf den Einfluß des Geschwindigkeitskanals (low velocity layer) zurückzuführen wäre.

Procedure

Following the notations previously used [BOSE 1963; 1964a, b, c; 1965], we shall represent the displacement potentials φ , ψ or the transverse displacement v in the form

$$\varphi = e^{-i\omega t} \int_{-\infty}^{\infty} \Phi(z, k) e^{ikx} dk$$

where $2\pi/\omega$ is the period and the x -axis is in the direction of wave propagation. Φ is found to contain a denominator $\Delta(\omega, k, \varepsilon)$ where ε is a small factor representing departure from homogeneity. $\Delta = 0$ forms the dispersion equation of the medium

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and corresponding to these phases φ has maximum contribution at long distances—the case in which we are interested. But, we shall take the dispersion equation as $\Delta(\omega, k, \rho) = 0$ showing graphically that the error committed in this way is small. Moreover, the important first mode will only be considered except in section 4.

1. T-Waves

Using the equations (14)-(17) of [BOSE 1964a], it can be shown that at the three typical levels $z = H/4, H/2, 3H/4$

$$\begin{aligned} \Phi\left(\frac{H}{4}, k\right) &= -\frac{e^{-r_1 h}}{\Delta} \sin \frac{3}{4} r_0 H \left[1 + \varepsilon_1 \frac{r_0^2 + k^2}{a^2 - 4r_0^2} \left\{ 1 - \frac{2r_0}{a} \cot \frac{3}{4} r_0 H \right. \right. \\ &\quad \left. \left. \times \left(1 - \frac{\varepsilon_2}{4\varepsilon_1} \frac{a^2 - 4r_0^2}{a^2 - r_0^2} \right) \right\} \right] \\ \Phi\left(\frac{H}{2}, k\right) &= -\frac{e^{-r_1 h}}{\Delta} \sin \frac{r_0 H}{2} \left(1 + 2\varepsilon_1 \frac{r_0^2 + k^2}{a^2 - 4r_0^2} \right) \end{aligned} \quad (1.1)$$

$$\begin{aligned} \Phi\left(\frac{3H}{4}, k\right) &= -\frac{e^{-r_1 h}}{\Delta} \sin \frac{r_0 H}{4} \left[1 + \varepsilon_1 \frac{r_0^2 + k^2}{a^2 - 4r_0^2} \left\{ 1 + \frac{2r_0}{a} \cot \frac{r_0 H}{4} \right. \right. \\ &\quad \left. \left. \times \left(1 + \frac{\varepsilon_2}{4\varepsilon_1} \frac{a^2 - 4r_0^2}{a^2 - r_0^2} \right) \right\} \right] \end{aligned}$$

where

$$\begin{aligned} \Delta &= \frac{\rho}{\rho_1} r_1 \sin r_0 H + r_0 \cos r_0 H \\ &\quad + 2\varepsilon_1 r_0 \frac{r_0^2 + k^2}{a^2 - 4r_0^2} \cos r_0 H - \frac{\varepsilon_2}{2} a \frac{r_0^2 + k^2}{a^2 - r_0^2} \sin r_0 H \end{aligned} \quad (1.2)$$

From the model of the Atlantic Ocean as given in EWING, JARDETZKY and PRESS [1957, pp. 335–337] we take $H = 3.657$ km, $\alpha_0 = 1.509$ km/sec, $\alpha_1 = 1.539$ km/sec, $\varepsilon_1 = 0.02016$, $\varepsilon_2/\varepsilon_1 = 0.951$. The value of ε_1 is obtained from equation (1) of [BOSE 1964a] and from the condition that the amplitude variation of α is 0.03047 km/sec. The value of $\varepsilon_2/\varepsilon_1$ is obtained by making α a minimum at a depth of 0.2438 km.

In Fig. 1 the negative of the ε terms of (1.2) has been plotted for the modified first mode and has been shown by the broken line. Evidently, the approximation in neglecting this deteriorates as the period diminishes. The continuous lines represent the proportional increment in φ due to the ε terms and it is apparent that, (i) at a given level, the proportional concentration of energy increases with decreasing period and (ii) for a given period, the proportional concentration of energy increases

with height. With this we must consider the fact that since $\varphi = 0$ at $z = H$, the absolute concentration will be more towards the sound channel axis.

Similar inference about the higher modes can not be drawn. For, the periods become smaller and the first order approximations in ε break down.

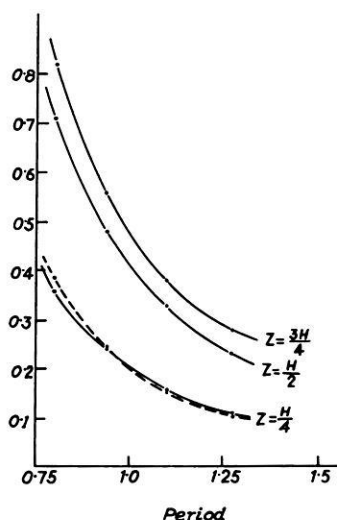


Fig. 1: Proportional increment in $\varphi(z)$ and the error $\Delta(\omega, k, 0) - \Delta$ (dotted line).

2. SOFAR Channel and Microseisms from Storms over Deep Sea

A better expression for the sound velocity than that in [BOSE 1964b] is

$$\alpha = \alpha_0 \left(1 + \frac{\varepsilon}{2} \cos az \right), \quad aH = \frac{3\pi}{2} \quad (2.1)$$

Taking clue from the remarks of PRESS and EWING [1948] that the sea surface being an antinode for vertical motion is the proper place for application of forces, we shall take a periodic source on the sea surface. Thus from equations (1)–(18) of [BOSE 1964b] with (2.1) it can be shown that at the ocean floor $z = 0$ and at the level of the sound channel axis

$$\begin{aligned} \Phi(0, k) &= \frac{P\mu_1 r_0}{\rho\omega^2 k^5 \Delta} [(s_1^2 + k^2)^2 - 4k^2 r_1 s_1] \\ \Phi\left(\frac{2H}{3}, k\right) &= -\frac{P}{\rho\omega^2 k^5 \Delta} \left[r_0 \{4k^2 r_1 s_1 - (s_1^2 + k^2)^2\} \left(1 + 2\varepsilon_1 \frac{r_0^2 + k^2}{a^2 - 4r_0^2} \right) \cos \frac{2}{3} r_0 H \right. \\ &\quad \left. - \frac{\rho\omega^4}{\rho_1 \beta_1^4} r_1 \sin \frac{2}{3} r_0 H \right] \quad (2.2) \end{aligned}$$

where

$$\begin{aligned}
 k^5 \Delta \equiv & \frac{\varrho \omega^4}{\varrho_1 \beta_1^4} r_1 \sin r_0 H - r_0 [4 k^2 r_1 s_1 - (s_1^2 + k^2)^2] \cos r_0 H \\
 & - \varepsilon \frac{r_0^2 + k^2}{a^2 - 4 r_0^2} \left[\frac{\varrho \omega^4}{\varrho_1 \beta_1^4} r_1 \left(\sin r_0 H + \frac{2 r_0}{a} \cos r_0 H \right) \right. \\
 & \left. + r_0 \{4 k^2 r_1 s_1 - (s_1^2 + k^2)^2\} \left(\cos r_0 H + \frac{2 r_0}{a} \sin r_0 H \right) \right] \quad (2.3)
 \end{aligned}$$

For the modified dispersion equation $\Delta(\omega, k, 0) = 0$ the negative of the ε term in (2.3) becomes

$$2 \varepsilon \frac{r_0^2 + k^2}{a^2 - 4 r_0^2} \frac{\varrho}{\varrho_1} \frac{c^4}{\beta_1^4} \sqrt{1 - \frac{c^2}{\alpha_1^2}} \left(\sin r_0 H + \frac{2 r_0}{a} \cos r_0 H \right) \quad (2.4)$$

and the proportional increment in φ at $z = \frac{2}{3} H$ is

$$2 \varepsilon \frac{r_0^2 + k^2}{a^2 - 4 r_0^2} \sin r_0 H \cos \frac{2}{3} r_0 H \operatorname{cosec} \frac{1}{3} r_0 H \quad (2.5)$$

Evidently there is no increment at $z = 0$ due to the sound channel.

To discuss the absence of low period sub-oceanic Rayleigh (S.O.R.) waves, we shall consider their generation from a periodic compressional line source at a depth h below the ocean floor. Thus a corresponding term should be added to (6) of [BOSE 1964b], as in [BOSE 1964a]. The ocean surface, in this case, is free and therefore the boundary conditions will have the same form as (18) of [BOSE 1964b] with $P = 0$. Thus, we get for the proportional increment corresponding to the modified modes

$$\text{at } z = 0, \quad \varepsilon \frac{r_0^2 + k^2}{a^2 - 4 r_0^2} \left(1 + \frac{2 r_0}{a} \tan r_0 H \right) \quad (2.6)$$

$$\text{at } z = \frac{2}{3} H, \quad \varepsilon \frac{r_0^2 + k^2}{a^2 - 4 r_0^2} \left(1 + \frac{2 r_0}{a} \tan r_0 H + 2 \sin r_0 H \cos \frac{2}{3} r_0 H \operatorname{cosec} \frac{1}{3} r_0 H \right) \quad (2.7)$$

We shall compute these expressions for $\varrho_1/\varrho = 2.5$, $\beta_1^2/\alpha_1^2 = 1/3$, $\beta_1^2/\alpha_0^2 = 4$ (granitic bottom) and $\alpha_0 = 1.524$ km/sec with amplitude of variation 0.03047 km/sec giving $\varepsilon = 0.04$ (Atlantic Ocean, cf. Section 1).

In Fig. 2, fifty times (2.4) has been plotted as a broken line, showing the smallness in error. The unbroken lines show that the proportional increment in φ is slightly greater for S.O.R. waves at both the levels. This is true for a superficial source. But according to Longuet-Higgins' theory the source of microseisms extends through the ocean depth, due to which it will be much greater and may even exceed that for the S.O.R. waves. Further, there is an attenuating effect of focal depth on S.O.R. waves, which is entirely absent for microseisms.

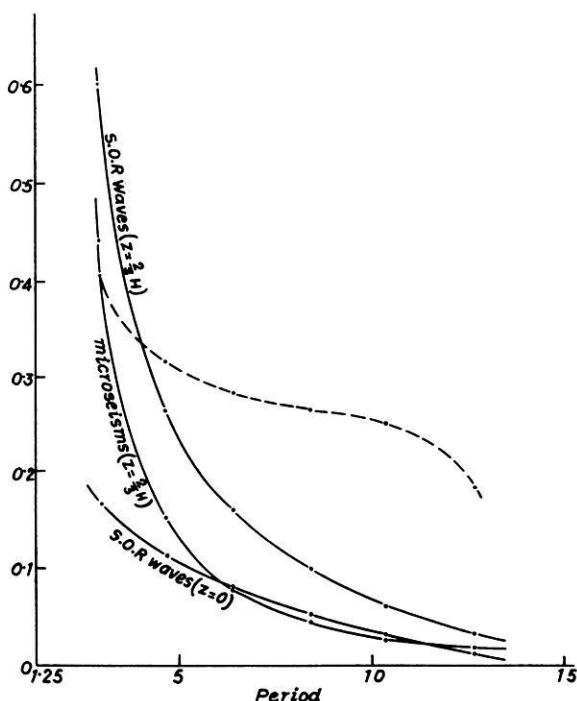


Fig. 2: Proportional increment in $\varphi(z)$ for microseisms and S.O.R. waves and fifty times the error $\Delta(\omega, k, 0) - \Delta$ (dotted line).

Now, the attenuating effect of focal depth is greater on T-Waves than on S.O.R. waves, because $r_1 h$ is greater. But the proportional increment in φ is much greater for T-Waves. Also, the T-Waves are concentrated in the upper part of the ocean only, whereas the S.O.R. waves are distributed throughout its depth. Thus the T-Waves have to cross a smaller length of inclined continental barrier. The last two factors override the first, making T-Waves present and S.O.R. Waves absent on seismograms. Similarly since all the three factors are practically in favour of microseisms, they can be observed from storms over deep sea.

3. Lg-Waves

It is known that the S-velocity in the earth's crust increases at nearly a uniform rate [DORMAN, EWING and OLIVER 1960]. From equations (11)–(14) of [BOSE 1965], it can be proved that at the surface of the earth

$$V(H, k) = \frac{G}{\Delta} \left[\left(1 + 3 \sum \varepsilon_n \frac{s_0^2 + k^2}{n^2 m^2 - 4 s_0^2} \right) \cos s_0 H + \frac{\mu_1 s_1}{\mu_0 s_0} \left(1 - \sum \varepsilon_n + \sum \varepsilon_n \frac{s_0^2 + k^2}{n^2 m^2 - 4 s_0^2} \right) \sin s_0 H \right] \quad (3.1)$$

where

$$\Delta = s_0 \sin s_0 H - \frac{\mu_1}{\mu_0} s_1 \cos s_0 H + \sum \varepsilon_n \frac{\mu_1}{\mu_0} s_1 \cos s_0 H + \sum \varepsilon_n \frac{s_0^2 + k^2}{n^2 m^2 - 4 s_0^2} \left(s_0 \sin s_0 H + \frac{\mu_1}{\mu_0} s_1 \cos s_0 H \right) \quad (3.2)$$

For the modified dispersion equation $\Delta(\omega, k, 0) = 0$, the ε terms in (3.2) become

$$\left(\sum \varepsilon_n + 2 \sum \varepsilon_n \frac{s_0^2 + k^2}{n^2 m^2 - 4 s_0^2} \right) s_0 \sin s_0 H \quad (3.3)$$

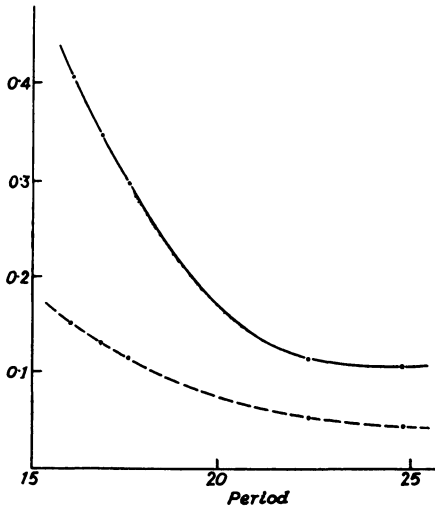


Fig. 3: Proportional increment in $v(H)$ and five times the error $\Delta - \Delta(\omega, k, 0)$ (dotted line).

and the proportional increment in $v(H)$ is

$$4 \sum \varepsilon_n \frac{s_0^2 + k^2}{n^2 m^2 - 4 s_0^2} \cos^2 s_0 H - \sum \varepsilon_n \sin^2 s_0 H \quad (3.4)$$

We take $H = 35$ km, $\mu_1/\mu_0 = 1.8$ [EWING et al. 1957, p. 212] and Jeffreys' values for S-velocity in the granitic, intermediate and upper mantle layers as 3.36, 3.74, 4.36 km/sec respectively. These give $\beta_0 = 3.55$ km/sec, $\beta_1 = 4.36$ km/sec. We shall neglect $\varepsilon_3, \varepsilon_5, \dots$ as they are small. From the above data we get $\varepsilon_1 = 0.1068$.

In Fig. 3 we have considered the period range 16–25 sec, although we are interested in the range 1–6 sec of Lg-Waves. This has to be done because

$$\varepsilon_1 \frac{s_0^2 + k^2}{m^2 - 4 s_0^2}$$

becomes too large for lower periods to invalidate the first order approximations. The broken line representing five times (3.3), shows the smallness in error in the modified dispersion equation. The unbroken line representing (3.4) indicates rapid concentration of energy for diminishing period.

4. Vertical component of Lg- and Rg-Waves

From equations (3) and (8)–(15) of [BOSE 1964c], we get

$$\begin{aligned} u(H) = & -e^{-i\omega t} \frac{2iF}{\mu_0} \int_{-\infty}^{\infty} \frac{k}{A} \left[e^{kH} \left\{ s_0 (s_0^2 - 5k^2) \cos s_0 H + k (k^2 - 5s_0^2) \sin s_0 H \right. \right. \\ & \left. \left. + 4ks_0 (k \cos s_0 H + s_0 \sin s_0 H) \sum \varepsilon_n \frac{s_0 + k^2}{n^2 m^2 - 4s_0^2} \right\} \right. \\ & \left. + e^{-kH} \left\{ s_0 (s_0^2 - 5k^2) \cos s_0 H - k (k^2 - 5s_0^2) \sin s_0 H \right. \right. \\ & \left. \left. + 4ks_0 (k \cos s_0 H - s_0 \sin s_0 H) \sum \varepsilon_n \frac{s_0^2 + k^2}{n^2 m^2 - 4s_0^2} \right\} \right. \\ & \left. + 2s_0 (3k^2 - s_0^2) - 2s_0 (k^2 - s_0^2) \sum \varepsilon_n \frac{s_0^2 + k^2}{n^2 m^2 - 4s_0^2} \right. \\ & \left. + \frac{1}{A} (\text{some first order terms}) \right] e^{ikx} dk \quad (4.1) \end{aligned}$$

$$\begin{aligned} w(H) = & -\frac{2F}{\mu_0} e^{-i\omega t} \int_{-\infty}^{\infty} \frac{k}{A} \left[\left\{ e^{kH} (s_0 \cos s_0 H - k \sin s_0 H) \right. \right. \\ & \left. \left. - e^{-kH} (s_0 \cos s_0 H + k \sin s_0 H) \right\} \left\{ (3k^2 + s_0^2) + 4k^2 \sum \varepsilon_n \frac{s_0^2 + k^2}{n^2 m^2 - 4s_0^2} \right\} \right. \\ & \left. + \frac{1}{A} (\text{some first order terms}) \right] e^{ikx} dk \quad (4.2) \end{aligned}$$

In the low period modes corresponding to the dispersion equation $\Delta = 0$, we can retain terms containing e^{kH} only since kH is large. The dispersion equation from (17) of [BOSE 1964c] thus becomes

$$\tan(kH \sqrt{\xi - 1}) = \sqrt{\xi - 1} \frac{(2 - \xi)^2 + 4}{(2 - \xi)^2 - 4(\xi - 1)} \tag{4.3}$$

where $\xi = c^2/\beta_0^2$. In the first mode, ξ can be less than 1 while in the second mode ξ is always greater than 1 with

$$\pi < s_0 H < \frac{3\pi}{2}$$

We shall neglect the terms containing $1/\Delta^2$ in (4.1), (4.2). For, a similar situation arises in sections 1, 2, 3 if $1/\Delta$ is expanded in powers of the ϵ 's. But the contribution of the ϵ terms in Δ being small, those of $1/\Delta^2(\omega, k, 0)$ will also be so. The proportional increments in $u(H)$ and $w(H)$ are thus found to be

$$\frac{2 - \xi}{\xi - 1} \sum \epsilon_n \frac{s_0^2 + k^2}{n^2 m^2 - 4s_0^2} \tag{4.4}$$

and

$$\frac{4}{2 + \xi} \sum \epsilon_n \frac{s_0^2 + k^2}{n^2 m^2 - 4s_0^2} \tag{4.5}$$

Also the ratio of amplitudes of $u(H)$ and $w(H)$ is

$$|2 - \xi| \left| \frac{\xi - 1 + (2 - \xi) \sum \epsilon_n \frac{s_0^2 + k^2}{n^2 m^2 - 4s_0^2}}{\xi + 2 + 4 \sum \epsilon_n \frac{s_0^2 + k^2}{n^2 m^2 - 4s_0^2}} \right| \tag{4.6}$$

From the model of section 3, we have $H = 35$ km, $\beta(z = H) = 3.36$ km/sec and $\beta(z = 0) = 3.74$ km/sec. Since β decreases linearly with depth, we must have $\beta(z = H/6) = 3.6767$ km/sec. These values give $\beta_0 = 3.55$ km/sec, $\epsilon_1 = 0.08387$ and $\epsilon_3 = 0.02290$.

For the first mode, Fig. 4 shows that there is considerable proportional increment in $w(H)$ which first slowly decreases up to roughly 10 sec and then rapidly increases up to 15.11 sec. In $u(H)$ there is rapid proportional decrement up to 12.39 sec. After this, it can be shown from (4.4) that there is a large amount of proportional increment up to 15.11 sec. This portion has not been plotted out because of breakdown in the first order approximations. Indeed the periods mentioned above also suffer from the same defect. For periods exceeding 15.11 sec ($\xi = 1.172$), (4.4) and (4.5) are negative showing decrement in energy. The broken line represents five times (4.6) and evidently it remains quite small. These waves therefore appear as Rg-Waves.

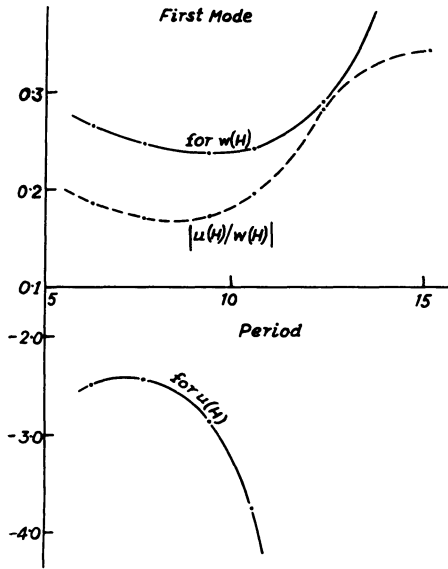


Fig. 4: Proportional increment in $u(H)$ and $w(H)$ and the ratio of their amplitudes (First Mode).

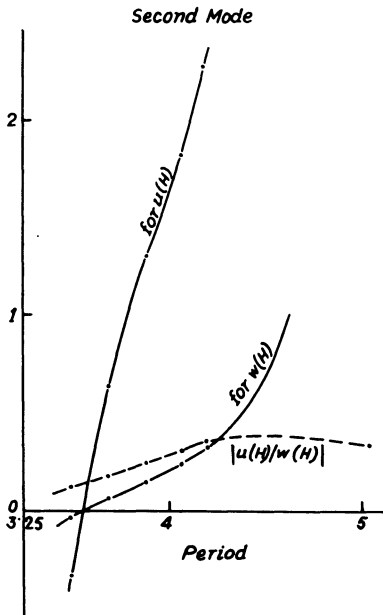


Fig. 5: Proportional increment in $u(H)$ and $w(H)$ and the ratio of their amplitudes (Second Mode).

For the second mode, Fig. 5 shows that there is proportional decrement in both $u(H)$ and $w(H)$ up to roughly 3.5 sec. After this, there is rapid proportional increment up to 5.037 sec (unreliable because of breakdown in the approximations). For periods exceeding 5.037 sec ($\xi = 1.172$) there is decrement in energy as in the first mode, and the broken line exactly as before shows that (4.6) remains quite small. A similar case will take place with higher modes also, except that the upper limits of period will be 3 sec, 2 sec, 1.6 sec etc. corresponding to $\xi = 1.172$, if the other ε terms are also retained. The phase velocities slightly exceed β_0 . These waves therefore appear as the vertical component of Lg-Waves.

5. On G-Waves

These are transverse waves in the upper part of the earth's surface in which the S-velocity increases slowly to a depth of 120 km and then decreases rapidly and again increases rapidly to a depth of 220 km, where its value becomes nearly equal to that at the top of the low velocity layer. Since these waves penetrate up to the low velocity layer only, we can consider the medium below it as rigid. Thus the medium can be divided into an upper layer $0 < z < H$ in which

$$\mu = \mu_0(1 + \varepsilon \cos mz), \quad mH = \pi \quad (5.1)$$

approximately, and the low velocity layer — $H' < z < 0$ in which

$$\mu' = \mu'_0(1 + \varepsilon' \cos m'z), \quad m'H' = 2\pi \quad (5.2)$$

The transverse displacements v, v' in the two layers are governed by equations of the type (2) of [BOSE 1965] subject to boundary conditions of the type (5) with continuity in rigidity at $z = 0$. Following the same procedure, it is found that on the surface of the earth

$$V(H, k) = -\frac{G}{\Delta} \left(1 + 2\varepsilon \frac{s_0^2 + k^2}{m^2 - 4s_0^2} \right) \left[\cos s_0 H \sin s'_0 H' \left(1 + \varepsilon \frac{s_0^2 + k^2}{m^2 - 4s_0^2} - \varepsilon' \frac{s_0'^2 + k^2}{m'^2 - 4s_0'^2} \right) + \frac{s'_0}{s_0} \sin s_0 H \cos s'_0 H' \left(1 - \varepsilon \frac{s_0^2 + k^2}{m^2 - 4s_0^2} + \varepsilon' \frac{s_0'^2 + k^2}{m'^2 - 4s_0'^2} \right) \right] \quad (5.3)$$

where

$$\Delta = s'_0 \cos s_0 H \cos s'_0 H' - s_0 \sin s_0 H \sin s'_0 H' - \left(\varepsilon \frac{s_0^2 + k^2}{m^2 - 4s_0^2} - \varepsilon' \frac{s_0'^2 + k^2}{m'^2 - 4s_0'^2} \right) (s'_0 \cos s_0 H \cos s'_0 H' + s_0 \sin s_0 H \sin s'_0 H') \quad (5.4)$$

The modified dispersion equation becomes

$$\tan(kH' \sqrt{\xi-1}) \tan\left(kH \sqrt{\frac{\beta_0'^2}{\beta_0^2} \xi - 1}\right) = \frac{\sqrt{\xi-1}}{\sqrt{\frac{\beta_0'^2}{\beta_0^2} - 1}} \quad (5.5)$$

where $\xi = c^2/\beta_0'^2$ and the proportional increment in $v(H)$ due to the low velocity layer (i. e. due to ϵ' terms) is

$$\epsilon' \frac{s_0'^2 + k^2}{m'^2 - 4s_0'^2} \frac{-\cos 2s_0'H'}{1 + \epsilon \frac{s_0^2 + k^2}{m^2 - 4s_0^2} \cos 2s_0H} \quad (5.6)$$

Now the S-velocity in the upper crust is 3.36 km/sec (cf. section 3) and in the upper mantle it increases from 4.60 to 4.65 km/sec at a depth of 120 km and at the lower boundary of the low velocity layer it is 4.70 km/sec which will be taken as 4.65 km/sec to fit with the theoretical model. We thus take $H = 120$ km, $H' = 100$ km, $\beta_0 = 4$ km/sec, $\beta_0' = 4.30$ km/sec, $\epsilon = 0.3514$, $\epsilon' = 0.1694$. The results for the first mode $0 < s_0H < \pi/2$, $0 < s_0'H' < \pi/2$ are presented in the following table:

ξ	kH'	period in sec	c in km/sec	(5.6)	ϵ, ϵ' terms of (5.4)
1.0	1.939	75.34	4.300	0.3887×10^{-2}	0
1.1	1.431	97.34	4.511	0.2045×10^{-2}	-0.9601×10^{-3}
1.3	1.036	124.00	4.903	0.1161×10^{-2}	-0.6972×10^{-3}

The period range agrees with that of G-Waves, but the increase with phase velocity is slightly quicker than the observed rate [cf. BATH and ARROYO 1962]. The calculated dispersion is still quite small and explains the pulse like character of G-Waves. The discrepancy is expected due to considerable simplification of the model. The last but one column shows that the increment in amplitude due to the low velocity layer is negligible. Thus the large amplitudes of G-Waves are due to their long wave lengths only and the low velocity layer guides their periods and phase velocities only.

References

- BÅTH, M., and A. L. ARROYO: Attenuation and dispersion of G-Waves, *J. Geophys. Res.* 67, 1933—1942, 1962
- BOSE, S. K.: A wave theory for the generation of Love, G and Sa waves, *Z. Geophys.* 29, 215—226, 1963
- : A Wave theory for the generation of T-Waves, *Z. Geophys.* 30, 235—244, 1964a
- : SOFAR and microseisms from storms over deep sea, *Gerl. Beitr. Geophys.* 73, 334—341, 1964b
- : The vertical component of Lg and Rg, *Z. Geophys.* 30, 294—300, 1964c
- : Lg-Waves and S-velocity distribution in the crust, *Z. Geophys.* 31, 1—6, 1965
- DORMAN, J., M. EWING and J. OLIVER: Study of shear velocity distribution in the upper mantle by mantle Rayleigh waves, *Bull-Seism. Soc. Amer.* 50, 87—115, 1960
- EWING, M., W. S. JARDETZKY and F. PRESS: *Elastic Waves in Layered Media*, McGraw-Hill, 1957
- PRESS, F., and M. EWING: A theory of microseisms with geologic applications, *Trans. Amer. Geophys. Union.* 29, 163—174, 1948