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Estimates of *in situ* Thermal Diffusivity of the Ore-Bearing Rocks in Some Drillholes in the Skellefte Field (N. Sweden) Using the Annual Temperature Wave

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Abstract. The damping of the annual temperature wave down to depths of 20—50 m in some drillholes has been used to estimate the *in situ* thermal diffusivity of the rocks, and the wavelength of the annual temperature wave.

Ten estimates yield a value of $(4.90 \pm 1.80).10^{-6}$ m^2/s for the *in situ* thermal diffusivity of the mineralized crystalline rocks (average 10 percent S) and one estimate gives a value of $0.63 \cdot 10^{-6}$ m^2/s for the glacial overburden. Estimates of wavelength in individual holes range between 30 and 55 m. Assuming a specific heat of 800 J/kg. °C and a density of 2700 kg/m^3 for the mineralized rocks in question, their *in situ* thermal conductivity is estimated as 10.6 W/m. °C. This relatively high value may be due to sulphide impregnation. A mean value for the phase velocity of the annual thermal wave in the rocks under consideration is 0.11 m/day.

Key words: Thermal Diffusivity — Ore-Bearing Rocks — Temperature

1. Introduction

Investigations into the possibilities of developing the geothermal method for ore prospecting have been underway at the Boliden Company, Sweden for a long time. A large number of temperature measurements were made in the Skellefte ore field in relatively shallow holes (reaching depths of about 100 m) as early as 1949—1950 and this work has now been extended to drillholes reaching vertical depths of 700 m and more below ground surface. While a discussion of these measurements from the heat-flow point of view is planned to appear in a separate paper, the object of the present paper is to report some estimates of the *in situ* thermal diffusivity of rocks made during the course of the above work by using the annual temperature variation. Other related information obtained in the study reported here includes the wavelength and the phase velocity of the annual temperature wave.

Very few *in situ* geothermal parameter determinations have been hitherto reported in the literature.

2. *Theory*

As the basic theory behind the procedure used here is simple and well known it is only briefly recapitulated below.

If the flat surface of a semi-infinite, homogeneous earth, throughout which the undisturbed temperature is T_s, be subject to a sinusoidal temperature variation $V_1 \cos(2\pi t/P)$ of period P and amplitude V_1, the variation penetrates the medium as a damped thermal wave. The temperature can be shown to be a periodic function of t and the depth z, given by (see, for example, Ingersoll and Zobell, 1948, p. 45):

$$T = T_s + V_1 \exp\left(-\frac{2\pi}{\lambda} z\right) \cos 2\pi \left(\frac{t}{P} - \frac{z}{\lambda}\right) \quad (1)$$

where λ, the wavelength, is related to the thermal diffusivity (α) of the medium by the equation

$$\lambda = 2(\pi \alpha P)^{\frac{1}{2}} \quad (2)$$

Further, the diffusivity, thermal conductivity (k), the specific heat (c) and the density (ϱ) are related by the equation

$$k = \alpha c \varrho \quad (3)$$

T_s in Eq. (1) is also the mean temperature of the surface over one period

SI units will be used throughout in this paper. Thus α is expressed in m^2/s, c in J/kg.°C, ϱ in kg/m^3 and k in W/m.°C. The conversion to c.g.s units, adopting 4.18 J as the mechanical equivalent of one calorie, is as follows:

α m^2/s $= 10^4 \alpha$ cm^2/s
c J/kg.°C $= 0.2392.10^{-3}\, c$ cal/g.°C
ϱ kg/m^3 $= 10^{-3} \varrho$ g/cm^3
k W/m.°C $= 0.2392.10^{-2} k$ cal/cm.s.°C

It should be noted here that, in general, there should be an additional term $C(z).\,z$ on the right-hand side of Eq. (1), representing the "geothermal" variation of temperature with depth z. If $C(z)$ is constant the observed temperatures may be replaced by T_{obs}—$C.z$ for calculating purposes.

In general, however C may vary with depth, depending upon the thermal history of the area, and calculations such as the ones in this paper cannot be validly made unless the function $C(z)$ can be determined. In the Skellefte district of N. Sweden, the annual temperature variation is superimposed on a general temperature-depth trend as shown in Fig. 1, from which we see that the steady thermal gradient is not constant at shallow depths. This is the result of past climatic changes, a topic which will be dealt with in the separate paper referred to above, but for our present

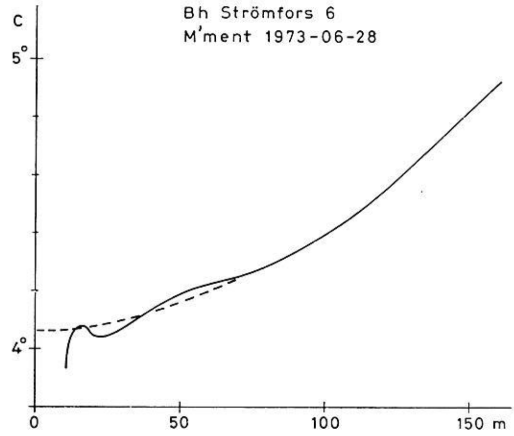

Fig. 1. Typical temperature-depth curve in the Skellefte field, N. Sweden

purpose it is sufficient to note that the main trend suggests that the steady thermal gradient is almost zero for depths upto about 50 m. That this is the case has been checked for all holes included in the study reported below. Therefore Eq. (1) may be used in the present case as it stands.

The penetration depth, that is, the depth at which the amplitude of the sinusoidal variation of period P falls to 1/e of its surface value, is

$$d = \lambda/2\pi = (\alpha P/\pi)^{\frac{1}{2}} \tag{4}$$

Putting $\alpha = 2.10^{-6}$ m²/s, a reasonable order of magnitude for the thermal diffusivity of rocks, and $P = 86400$s (1 day) we find $d = 0.24$ m. At a depth of $10d$ the amplitude due to diurnal variation is V_1 exp(—10) or approximately $4.5.10^{-5} V_1$. It follows that the temperature at depths greater than, say, 2.5 m, will not be affected to a measurable degree by the diurnal variation, even if the surface amplitude V_1 is as large as 50 °C. For the annual temperature variation ($P = 31.536 \times 10^6$s) we get $d = 4.59$ m. In this case even for V_1 as small as 3 °C, the amplitude of the temperature variation at depths as much as 23 m ($5d$) will still be about 0.02 °C, that is, well above the accuracy with which temperature differences can be measured in drillholes.

By taking $P = 1$ year, and choosing suitable values of T_s, V_1, λ and t, we can fit the measured temperatures between the depths, say, 5 and 50 m in a drillhole (the zone where the annual temperature variation is measurable and the diurnal negligible) to an expression of the type (1). If the fit is sufficiently good we can obtain an estimate of the *in situ* thermal diffusivity from Eq. (2). It will be seen below that very accurate fits between the observed temperatures and those calculated from Eq. (1) are possible in practice.

3. The Drillholes

The 8 holes used for diffusivity estimates are all in the Skellefte ore field in N. Sweden (approx. 65°N 20°EGr). They penetrate through a moderate thickness of glacial moraine (minimum 2 m and maximum 25 m) into the precambrian rock, either ore-bearing sericite-quartzite, as in 6 of the holes, or black shales and graywackes (2 holes). The holes are in some cases drilled to considerable depths (one of them to a depth of 700 m below ground surface) but only the measurements in the upper parts (to depths of 20—50 m depending upon the hole) are of relevance in this paper.

All the holes are slant holes, five of them drilled at an angle of 50°, one at 55° and two at 80° with the horizontal. The holes are cased in moraine but not in the bedrock. The strength of the precambrian rock in Sweden is such that drillholes in it generally remain open and unimpaired for a long time.

The diameter of the holes is 52 mm in the moraine and 33 mm or less in the bedrock. The holes are entirely water filled from the groundwater level, which is only a few metres below surface, down to the bottom of each hole. The water in the holes is still and undisturbed and its temperature can safely be taken to be the local temperature of the surrounding rock. In some holes (but not in any of the measurements treated in this paper) there can sometimes be a movement of water through bedrock fissures cutting the hole. Such cases are generally easy to recognize owing to the locally erratic temperature readings to which the water movement gives rise.

The ground in the area is frozen during each winter to a depth of about 1—2 m, forming a layer locally known as "tjäle", which thaws each spring. The effect of the frozen layer is to blanket the ground below, screening it from the severe extremes of air temperature during the winter. The upper decimetres of the moraine form a loose, porous layer that also acts more or less like a screen during the spring, summer and autumn months.

The result is that whereas the mean annual temperature of the air immediately above the ground surface varies by as much as 40—50 °C, the temperature variation a couple of metres below the surface is only a few degrees. This will also be evident from the calculations presented later. The mean annual temperature of the ground in the area is slightly less than 4 °C.

4. The Measurements

Some of the measurements treated here were made in 1950 under the supervision of D. Malmqvist but have not been analyzed from the point of view of this paper previously. Other measurements were made more recently (summer 1973) under the author's supervision.

The 1950 measurements were made using resistance thermometers and a Wheatstone bridge. The absolute accuracy of the temperature measurements in this case was not better than 0.1 °C but the relative accuracy, which is of primary interest here, was about 0.01 °C although perhaps not as high as 0.005 °C.

The 1973 measurements were made using a thermistor (FS 23 B) mounted and sealed by means of O-rings in a rod of brass, about 25 cm long and about 10 mm in overall diameter, at the end of a 4-conductor cable. The thermistor was the unknown resistance in one arm of a Wheatstone bridge. The rod-thermistor probe was placed in a water bath in close contact with a Hewlett-Packard oceanographic quartz temperature probe (model 2800 A), and the bridge readings were calibrated against the readings of the Hewlett-Packard thermometer. The HP thermometer utilizes the effect that the piezoelectric frequency of a suitably cut quartz crystal is temperature-dependent. The thermometer is a direct-reading one and has been calibrated against a platinum thermometer by the manufacturers.

A least-squares polynomial of the type $T = a + bR + cR^2 + dR^3$, where T is the temperature and R the resistance reading, was found to fit the calibration measurements within 0.003 °C.

Although the accuracy of a single temperature measurement with the thermistor arrangement is better than 0,005 °C, the overall relative accuracy of the 1973 measurements, as shown by some repeat measurements, has been about 0.01 °C, that is, approximately the same as in the 1950 measurements. However, the absolute accuracy is now considerably better, being also about 0.01 °C.

The heat capacity of the brass rod-thermistor combination is sufficiently small for the final reading to be obtained within less than half a minute after the probe is in place in a water-filled hole. In the upper few metres of a hole, where there is only air, the readings are unstable owing to the poor thermal contact between the probe and the surrounding. The water level is indicated very clearly as one lowers the probe in the hole, since the readings become immediately stable when the probe enters water.

The measured temperatures in the various holes are shown in Figs. 2—8 by open circles. The x-coordinate in these figures is the distance along the hole but in applying Eq. (1), of course, the vertical depth has been used. Measurements in nearby holes suggest that horizontal variations in the temperature, at the short distances of relevance here, are not so large as to alter the estimates reported below.

5. Thermal Estimates

The best estimates of the parameters T_s, V_1, δ $(=2\pi/\lambda)$ and ε $(=2\pi t/P)$ for each hole are to be made by minimizing the sum

$$\sum (T_{\text{obs}} - T_s - V_1 \exp(-\delta z) \cos(\varepsilon - \delta z))^2$$

where T_{obs} is the observed temperature. The procedure for least-squares adjustment when the parameters to be adjusted are not linearly related is well known (see, for example, Newcomb, 1960). Its outline for the present case is as follows.

Let $\varepsilon = \varepsilon_0 + \Delta\varepsilon$, $\delta = \delta_0 + \Delta\delta$, $V_1 = V_0 + \Delta V$, $T_s = T_0 + \Delta\theta$ where ε_0, δ_0, V_0, T_0 are suitably estimated initial approximations and $\Delta\varepsilon$, $\Delta\delta$, ΔV and $\Delta\theta$ are the corrections sought.

By Taylor's theorem,

$$T - T(\varepsilon_0, \delta_0, V_0, T_0) = \Delta\varepsilon \frac{\partial T}{\partial \varepsilon} + \Delta\delta \frac{\partial T}{\partial \delta} + \Delta V \frac{\partial T}{\partial V_1} + \Delta\theta \frac{\partial T}{\partial T_s}$$

where the derivatives are evaluated at ε_0, δ_0, V_0, T_0. The difference $\Delta T = T - T(\varepsilon_0, \delta_0, V_0, T_0)$ is thus a linear function of $\Delta\varepsilon$, $\Delta\delta$, ΔV and $\Delta\theta$ and minimizing

$$\sum \Delta T^2$$

we obtain the normal equations to be solved for $\Delta\varepsilon$, $\Delta\delta$, ΔV, $\Delta\theta$. These are

$$\begin{array}{lllll}
(\Sigma a^2)\Delta\varepsilon + & (\Sigma ab)\Delta\delta + & (\Sigma ac)\Delta V + & (\Sigma ad)\Delta\theta & = (\Sigma a)\Delta T \\
\Sigma ba & \Sigma b^2 & \Sigma bc & \Sigma bd & = \Sigma b \\
\Sigma ca & \Sigma cb & \Sigma c^2 & \Sigma cd & = \Sigma c \\
\Sigma da & \Sigma db & \Sigma dc & \Sigma d^2 & = \Sigma d
\end{array}$$

where

$$\begin{aligned}
a &= -V_0 \exp(-\delta_0 z) \sin(\varepsilon_0 - \delta_0 z) \\
b &= V_0 \sqrt{2}\, z \exp(-\delta_0 z) \sin(\varepsilon_0 - \delta_0 z - (\pi/4)) \\
c &= \exp(-\delta_0 z) \cos(\varepsilon_0 - \delta_0 z) \\
d &= 1
\end{aligned}$$

It remains therefore to choose suitable initial approximations ε_0, δ_0, V_0, T_0.

T_0 can be chosen by inspection as the constant temperature towards which the values tend at "large" depths. Similarly, V_0 can be chosen by an inspection of the principal oscillation in the observed curve while in choosing δ_0 and ε_0 we note the following.

Differentiating Eq. (1) with respect to z it is easily shown that if z_1 is the depth of the first extremum in the T—z curve and z_2 the depth of the first inflection point, then

$$\frac{t}{P} - \frac{z_1}{\lambda} = \frac{1}{8}$$

$$\frac{t}{P} - \frac{z_2}{\lambda} = 0$$

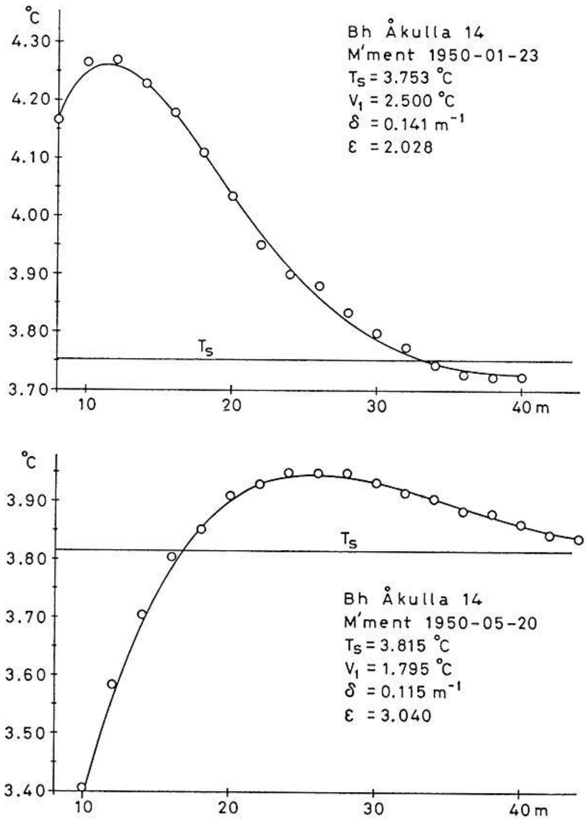

Fig. 2. Measured temperatures (open circles) and least-squares calculated curves (continuous lines) in borehole Åkulla 14

Hence, $\lambda = 8(z_2 - z_1)$ and $t/P = z_2/8(z_2 - z_1)$ giving us $\pi/4(z_2 - z_1)$ and $\pi z_2/4(z_2 - z_1)$ as the initial approximations δ_0 and ε_0 respectively.

Figs. 2—8 show the least-squares adjusted values of T_s, V_1, δ and ε, and the temperature curves calculated according to Eq. (1) (continuous line).

It will be seen that except for borehole Åkulla 42, measurement of 1950—05—20 (Fig. 6) the agreement between the observed and the calculated temperatures is excellent. The measurements shown in all the holes, except Bh Strömfors 6 (Fig. 8), are made in the bedrock. Those in Bh Strömfors 6 are in moraine, the thickness of which is unusually great here, namely, 25 m.

Using Eq. (2) and noting (4) we can estimate the *in situ* thermal diffusivity α by putting $P = 31.536 \times 10^6$s ($= 1$ year) from the data in the figures. Table 1 summarizes the estimates obtained.

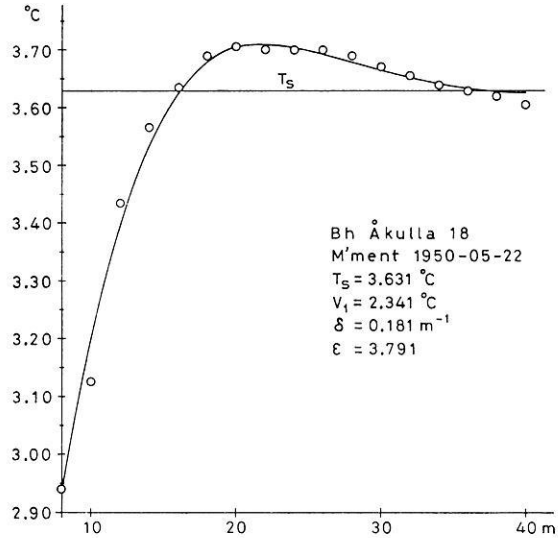

Fig. 3. Measured temperatures (open circles) and least-squares calculated curve (continuous line) in borehole Åkulla 18

Table 1. Borehole data and geothermal estimates

Hole and Date of measurement	Angle	Over-burden thickness	T_s °C	V_1 °C	δ m^{-1}	λ m	$10^6\alpha$ m^2/s
Åkulla 14	50°	9.35 m					
1950–01–23			3.753	2.500	0.141	44.6	5.02
1950–05–20			3.815	1.795	0.115	54.7	7.55
Åkulla 18	50°	12.20 m					
1950–05–22			3.631	2.341	0.181	34.7	3.04
Åkulla 32	50°	11.14					
1950–01–26			3.509	1.871	0.120	52.4	6.91
1950–05–23			3.614	1.610	0.120	52.4	6.91
Åkulla 36	50°	13.38					
1950–05–22			3.485	1.343	0.131	48.0	5.81
Åkulla 42	50°	9.38					
1950–01–23			3.687	3.219	0.172	36.6	3.37
1950–05–20			3.780	1.55	0.205	30.7	2.38
Åkulla 84	55°	5.96					
1950–01–18			3.454	2.771	0.158	39.8	4.00
1950–01–28			3.564	2.723	0.158	39.8	4.00
Strömfors 6	80°	25.00					
1973–06–28			4.055	10.25	0.398	15.8	0.63
Nyholm 6	80°	2.50	3.426	2.121	0.203	31.0	2.42

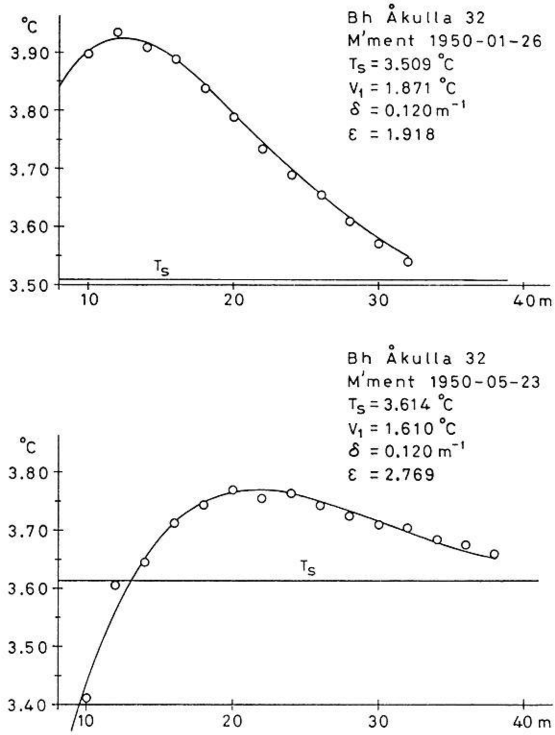

Fig. 4. Measured temperatures (open circles) and least-squares calculated curves (continuous lines) in borehole Åkulla 32

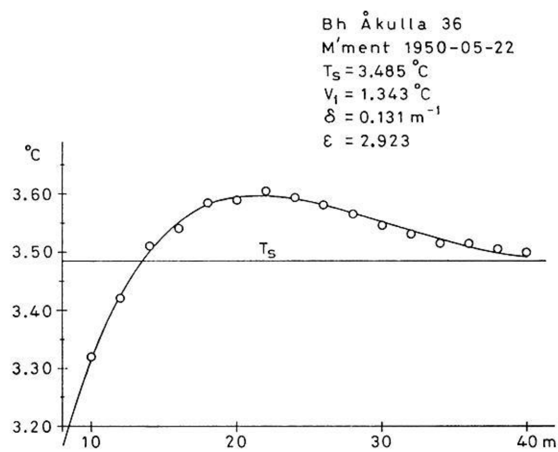

Fig. 5. Measured temperatures (open circles) and least-squares calculated curve (continuous line) in borehole Åkulla 36

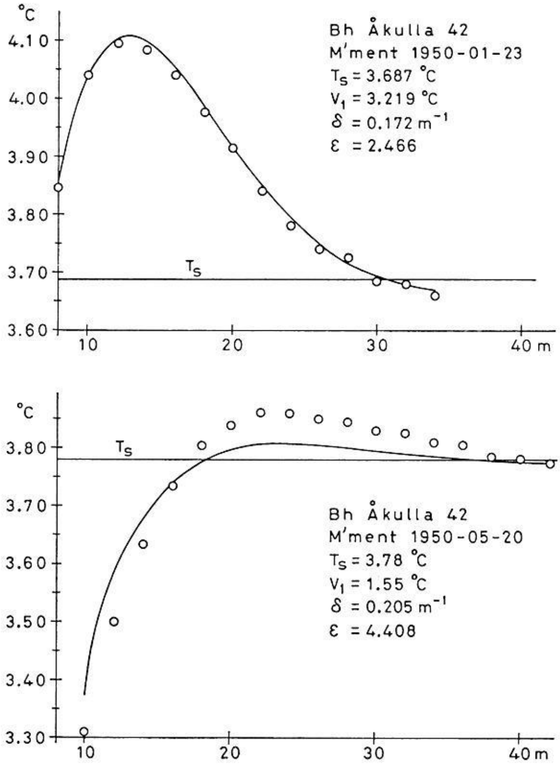

Fig. 6. Measured temperatures (open circles) and least-squares calculated curves (continuous lines) in borehole Åkulla 42

The Åkulla boreholes fall in one geological (geographical) group. The precambrian bedrock in all of them is sericite-quartzite with sulphide mineralization while the bedrock in Bh Nyholm 6 is shales and graywackes. The mean *in situ* diffusivity of the bedrock in the Åkulla holes is found to be $(4.90 \pm 1.80).10^{-6}$ m^2/s.

Estimates of the phase velocity v $(=\lambda/P)$ of the thermal wave can be made from the λ values in Table 1 but in four of the Åkulla boreholes in which two temperature measurements separated by 117—120 days have been made (Figs. 2, 4, 6, 7) it is also possible to make direct estimates by noting the vertical depth of the temperature maximum in a hole at the time of each measurements. In table 2 are given the direct as well as the indirect estimates made from Table 1.

Although the estimates agree in order of magnitude, the direct estimates systematically give a lower phase velocity. It must be pointed out, however, that the time of maximum temperature due to the annual wave is not

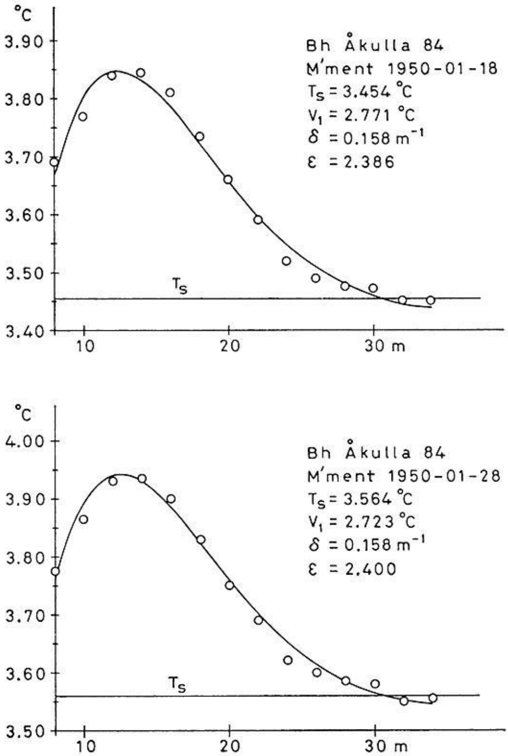

Fig. 7. Measured temperatures on two different days in winter (open circles) and least-squares calculated curves (continuous lines) in borehole Åkulla 84

Table 2. Comparison of direct and indirect estimates of phase velocity v

Hole	Direct estimate m/day	Indirect estimate m/day
Åkulla 18	0.077	0.095
Åkulla 32	0.059	0.144
Åkulla 42	0.071	0.092
Åkulla 14	0.096	0.136

accurately identifiable at any depth because pseudo-periodic cold or warm spells can shift the actual maximum in time. From this point of view the direct estimates are not particularly reliable.

The *in situ* thermal conductivity of rocks can be estimated from Eq. (3) provided c and ϱ are known. The specific heat of crystalline rocks seems to vary within the very narrow range of 750/800 J/kg. °C or 0.18—0.20 c.g.s. units (Clark, 1966).

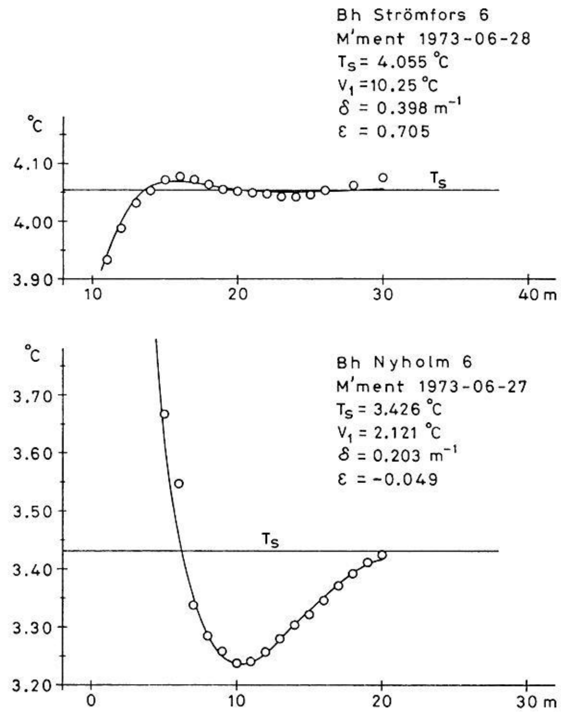

Fig. 8. Measured temperatures (open circles) and least-squares calculated curves (continuous lines) in boreholes Strömfors 6 and Nyholm 6

Taking a value of 800 J/kg. °C and using the estimate of 4.90×10^{-6} m^2/s for the *in situ* diffusivity, we obtain an estimate of 10.6 W/m. °C for the *in situ* thermal conductivity of the rocks in the Åkulla holes ($\varrho = 2700$ kg/m^3). This value is almost twice the generally accepted value for quartzitic shield rocks and may be the result of the sulphide impregnation in the holes.

6. *Conclusions*

From a detailed analysis of the annual temperature wave observed in shallow drillholes it is possible to make estimates of the *in situ* thermal diffusivity of the rocks and the phase velocity of the wave. One finding of the above investigation that may have an important bearing on the question of the utility of geothermal measurements for prospecting, at least in the precambrian shield areas of Sweden, is that the wavelength of the annual wave in the crystalline rocks is found to be as much as 30—50 m.

This means that temperature measurements in holes whose depth is of this order of magnitude cannot be used for geothermal prospecting purposes unless adequate and accurate corrections for the effect of the annual temperature wave are made.

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