J. Geophys. 40, 269–282, 1974

Original Investigations

Transverse Plasma Waves in the Solar Wind Close to the Proton Gyrofrequency

H.W. Rehn

Institut für Geophysik und Meteorologie der Technischen Universität, Braunschweig

Received January 25, 1974

Abstract. Transverse, right-hand polarized waves propagating along a magnetic field in the hot plasma of the solar wind consisting of protons and electrons are discussed for frequencies in the vicinity of the proton gyrofrequency. The thermal anisotropies of the particles are able to produce a nonresonant and a resonant instability. The first one, called firehose instability, is driven by electrons and protons, while the extension of this instability is produced by the electrons only. The second one, called cyclotron instability, is maintained by the protons. The same is true for the extension of this instability. Two different approximations of the dispersion relation allow the analytic discussion of the firehose and the cyclotron instability. In case the resonant velocities are not much larger than the thermal velocities, the approximations are not valid, and it is necessary to discuss the dispersion relation numerically. It is shown that enhanced electron or proton anisotropies as well as greater $\beta_{\| p}$-ratios increase the growth rate and the unstable frequency range. For low anisotropies and $\beta_{\| p}$-ratios only the cyclotron instability does exist. For higher values of the parameter the firehose instability arises in addition. Further increase of the electron anisotropy or of the $\beta_{\| p}$-ratio leads to an extension of the firehose instability and to a switching of the polarization-sense, while the wave is propagating antiparallel to the magnetic field — even if the protons are isotropic. This change in the polarization-sense was not found in the analytic approximations, which are possible if the resonant velocities are much larger than the thermal velocities.

Key words: Plasma Waves — Solar Wind — Parallel Propagation of Waves in Bi-Maxwellian Magnetized Plasma.

1. Introduction

Parker (1963) discussed the expansion of the solar wind with the aid of the hydromagnetic theory. By reasons of the conservation of the energy of a particle and of the first adiabatic invariant Parker predicted a thermal anisotropy near the earth. Measurements, which confirmed an anisotropy of the protons (Hundhausen, 1968; Hundhausen *et al.*, 1970) and electrons (Hundhausen, 1968; Montgomery *et al.*, 1968), nevertheless gave smaller values than those to be expected by an exospheric model. Instabilities, caused by the anisotropies, may produce the observed values.

In this paper we want to investigate the transversal right-hand circularly polarized electromagnetic waves propagating along a uniform magnetic field in a hot plasma close to the proton gyrofrequency.

The thermal anisotropies in the solar wind $T_{\|} > T_{\perp}$ (T = temperature, the subscripts $\|$, $\perp$ refer to the directions parallel and perpendicular to the average magnetic field, respectively) lead to two instabilities. The Alfvén waves deform the magnetic field. If the anisotropies are strong enough, the particles are subject to centrifugal forces while running along the bended field, causing the "firehose" instability. Protons with a velocity so high that they "see" the electric field of the wave with their own gyrofrequency, may drive a resonant instability in the vicinity of the proton gyrofrequency: the cyclotron instability. Electron anisotropy and high β-ratios (β is the ratio of the thermal to the magnetic pressure) increase the growth rates.

If damping or an instability occurs the energy exchange alters the anisotropies of electrons and protons. Resonant instabilities have only a small influence in varying anisotropies, because there are only few particles which produce the instability. The resonant particles have a great influence, however, if they are near the maximum of the distribution function.

For the analytic treatment of the firehose instability and of the proton cyclotron instability one can use an asymptotic expansion of the dispersion relation. The asymptotic expansion is possible in case the resonant velocities are much larger than the thermal velocities. In case the resonant velocity of the protons is much smaller than their thermal velocity a different expansion is possible yielding another limiting case.

2. *The Dispersion Relation*

It is assumed that the plasma is collisionless and consists of protons and electrons moving in a static and uniform magnetic field. The particles obey an anisotropic Maxwellian distribution function. Therefore the dispersion equation can be derived from Maxwell's equations and the Vlasov equation. In the linear approximation the result for transversal waves propagating parallel to the static magnetic field is the following

$$-\frac{\hat{\omega}^2}{\beta_{\|p}}\frac{v_{\|p}^2}{c^2}+\frac{\hat{k}^2}{\beta_{\|p}}-\left[M\left(\frac{\hat{\omega}}{\hat{k}\sqrt{M\cdot TV}}Z(z_e)-K_e\big(1+z_eZ(z_e)\big)\right)+\right.$$
$$\left.+\left(\frac{\hat{\omega}}{\hat{k}}Z(z_p)-K_p\big(1+z_pZ(z_p)\big)\right)\right]=0 \qquad (1)$$

(Montgomery and Tidman, 1964; Rehn, 1972)
where $\hat{\omega}$ is the complex frequency normalized by the proton gyrofrequency. Growth or damping of the waves are indicated by a positive or negative

imaginary part of the frequency $\hat{\omega}$, respectively. $\beta_{\parallel p}$ is the ratio of the parallel thermal pressure component of protons to the magnetic pressure. The thermal proton velocity is defined by

$$v_{\parallel p}^2 = \frac{2\varkappa T_{\parallel p}}{m_p}$$

where m_p is the proton mass and $\varkappa$ is Boltzmann's constant. c is the speed of light, and $K_e = 1 - (T_\perp / T_\parallel)_e$, $K_p = 1 - (T_\perp / T_\parallel)_p$ (the indices e, p refer to the electrons or to the protons, respectively) while M is the mass ratio of protons to electrons, and TV is the ratio of the temperature parallel to the magnetic field of the electrons to that of the protons.

Finally Z is the plasma dispersion function given by

$$Z(z) = \exp(-z^2)(i\sqrt{\pi} - \int_0^z \exp(t^2)\,\mathrm{d}t)$$

where the arguments are

$$z_e = (\hat{\omega} \mp M)/(\hat{k}\sqrt{M \cdot TV})$$

and

$$z_p = (\hat{\omega} \pm 1)/\hat{k}$$

If the real part of $\hat{\omega}$ is positive, the upper signs refer to a right-hand and the lower signs to a left-hand polarized mode. In this paper we only consider the upper sign. Numerical values for the function $Z(z)$ are found in Fried and Conte (1961).

3. Approximations of the Dispersion Relation

In order to discuss some properties of the instabilities, it is convenient to obtain an analytical solution of the complicated dispersion relation (1). Therefore it is useful to consider approximations of the dispersion relation.

To obtain an approximation one can use the asymptotic expansion of the plasma dispersion function, which is valid in the case $|z| >> 1$, for both protons and electrons.

$$Z(z) = i\sigma\sqrt{\pi}\exp(-z^2) - \left[\frac{1}{2} + \frac{1}{2z^3} + \frac{1 \cdot 3}{2^2 z^5} + \dots\right] \tag{2}$$

where

$$\sigma = \begin{cases} 0 & > 0 \\ 1 \text{ for } \mathrm{Im}(z) & = 0 \\ 2 & < 0 \end{cases}$$

$|z| >> 1$ means that the velocity of the resonant particles is large compared to the velocity of the thermal particles, i.e. there are only few resonant particles.

Another critical case is given by $|z| << 1$, which means that the particles are near the maximum of the distribution function. This case allows us to expand the plasma dispersion function into a Taylor series:

$$Z(z) = i\sqrt{\pi}\exp(-z^2) - 2z(1 - 2z^2/3 \pm \ldots) \tag{3}$$

3.1. The Firehose Instability

Using the asymptotic expansion (2) in the dispersion relation (1) and neglecting the exponential term in (2) because of few resonance particles and terms higher than second order, we get the approximate dispersion relation

$$\hat{\omega}^2 v_{\parallel p}(1 + \omega_{pp}^2/\Omega_p^2 + \omega_{pe}^2/\Omega_e^2) - c^2 \hat{k}^2 (1 - \beta_\parallel/2 + \beta_\perp/2) = 0 \tag{4}$$

where $\beta_\parallel$ ($\beta_\perp$) is the ratio of plasma pressure parallel (perpendicular) to the magnetic field B_0 to the magnetic pressure. ω_{pp}, ω_{pe} are the plasma frequencies for protons, and for electrons respectively. Furthermore Ω_p is the gyrofrequency of protons, and Ω_e that of electrons. (4) is the dispersion relation for Alfvén waves if $\beta_\parallel = \beta_\perp = 0$. The condition for an instability, i.e. Im $(\hat{\omega}) > 0$, $k > 0$ is determined by

$$1 - \beta_\parallel/2 + \beta_\perp/2 < 0. \tag{5}$$

Transforming (5) leads to

$$\beta_{\parallel p}\left(1 - \frac{1}{T_{\parallel p}/T_{\perp p}}\right) + \beta_{\parallel e}\left(1 - \frac{1}{T_{\parallel e}/T_{\perp e}}\right) > 2 \tag{6}$$

with the definitions

$$\beta_{\parallel p} = 8\pi n_0 \varkappa T_{\parallel p}/B_0^2\,, \qquad \beta_{\parallel e} = 8\pi n_0 \varkappa T_{\parallel e}/B_0^2$$

n_0 is the average particle density of protons and electrons.

This well known instability is called "firehose" instability because the magnetic field lines become deformed like a firehose which is not fixed. Running along the curved field lines the particles experience a centrifugal force which tends to enlarge the curvature. For this reason the firehose instability arises for frequencies lower than the proton gyrofrequency. Close to the proton gyrofrequency the instability vanishes because of the inertia of protons which no longer follow the curvature of the magnetic field.

For high values of $\beta_{\parallel p}$, small proton anisotropies satify (6). Since in most cases the electron temperature is higher than the proton temperature in the solar wind, smaller electron anisotropies are sufficient to produce firehose instability.

The condition (5) or (6) is determined by the fluid-like behaviour of the plasma only, and does not contain specific properties of the distribution function like a resonance instability.

Shapiro and Shevchenko (1964) considered the quasi-linear stabilization of firehose unstable Alfvén waves in a plasma with anisotropic velocity distribution. They obtained equations for the changes in the longitudinal and transverse ion thermal energies.

Hollweg and Völk (1971) have shown that the firehose instability always increases the perpendicular temperature, but lowers the parallel temperature if only $T_{\parallel}/T_{\perp} > 1/2$. This process leads to the situation that (5) is not fulfilled, and therefore a stabilization takes place. They pointed out that in the case of a growing wave the thermal energy of particles decreases. Often in the solar wind these energy exchanges are valid for the electrons only. For this reason the observed electron temperatures may be lower than the temperatures predicted by Hartle andSturrock (1968).

For the resonant protons near the maximum of the distribution function, i.e. $|z_p| << 1$, while for the electrons $|z_e| >> 1$, one can use the Taylor expansion for $Z(z_p)$ and the asymptotic expansion for $Z(z_e)$ to find an approximate dispersion relation. Under these assumptions Hollweg and Völk (1970) obtained an extension to the firehose instability. This instability is driven by anisotropic electrons firehoselike and has significant growth rates, if $K_p > 1/2$ or $T_{\parallel p}/T_{\perp p} > 2$ for frequencies $\hat{\omega}_r - K_p > 0$. $\hat{\omega}_r$ is the real part of $\hat{\omega}$. Since this instability is produced by the electrons only, the electron temperature decreases, while the temperature of the damping protons increases.

3.2. The Cyclotron Instability

The fluid-like behaviour of the plasma determines the firehose instability. Treating this instability we could therefore neglect the exponential term in (2). On the other hand the cyclotron instability is a resonance instability determined by the distribution function. Here it is necessary to consider the exponential term.

Under the condition $|\mathrm{Im}(z)| << |\mathrm{Re}(z)|$ it is possible to expand $Z(z)$ in a Taylor series around a real point z_0. The first two coefficients are obtained utilizing the asymptotic expansion (2) for real arguments. Neglecting terms higher than third order in $\hat{\omega}_r$ we get from (1)

$$-\frac{\hat{\omega}_r}{\beta_{\parallel p}} \frac{v_{\parallel p}^2}{c^2} + \frac{\hat{k}^2}{\beta_{\parallel p}} + M \frac{\hat{\omega}_r}{\hat{\omega}_r - M} + \frac{\hat{\omega}_r}{\hat{\omega}_r + 1} = 0 \tag{7}$$

and

$$\hat{\omega}_i = -\frac{\sqrt{\pi}}{\hat{k}} \frac{\hat{\omega}_r[\hat{\omega}_r - K_p(\hat{\omega}_r + 1)]\exp[-(\hat{\omega}_r + 1)^2/\hat{k}^2]}{\hat{k}^2/\beta_{\parallel p} + M/(\hat{\omega}_r - M)^2 + 1/(\hat{\omega}_r + 1)^2} \tag{8}$$

where $\hat{\omega}_i = \mathrm{Im}\,(\hat{\omega})$.

Eq. (8) is the well known (e.g. Kennel and Petschek, 1966) growth rate of the cyclotron instability in the neighbourhood of the proton gyrofrequency. This instability arises, if the right hand side of (8) is positive, i.e., if

$$\hat{\omega}_r - K_p(\hat{\omega}_r + 1) < 0. \tag{9}$$

Transformations of (9) are

$$K_p > \hat{\omega}_r/(\hat{\omega}_r + 1) \tag{10}$$

and

$$\hat{\omega}_r < K_p/(1 - K_p). \tag{11}$$

The thermal anisotropies in the solar wind near the earth are such that $T_\parallel > T_\perp$, i.e. $K_p > 0$. Eq. (10) shows that the cyclotron instability is possible in the solar wind. Isotropic protons ($K_p = 0$) cannot drive an instability because of (11), while $K_p > 0$ always produces an instability in the frequence range $0 < \hat{\omega}_r < K_p/(1 - K_p)$.

The sign of (8) is determined by $-[\hat{\omega}_r - K_p(\hat{\omega}_r + 1)] = 1 - (\hat{\omega}_r + 1) \cdot \cdot T_{\perp p}/T_{\parallel p}$. Since $\hat{\omega}_i$ is proportional to $1 - (\hat{\omega}_r + 1)\,T_{\perp p}/T_{\parallel p}$, the higher the growth rate the lower the ratio $T_{\perp p}/T_{\parallel p}$ is.

Brice (1964) pointed out that the cyclotron instability diminishes the anisotropy — the ratio $T_{\parallel p}/T_{\perp p}$ —, which produces the instability, while $T_{\parallel p}$ decreases. On the other hand cyclotron damping creates or amplifies the anisotropy of protons, while $T_{\perp p}$ decreases. Therefore the marginal stability is given by a finite thermal anisotropy with $T_{\parallel p} > T_{\perp p}$. Since by assumption there are few resonance protons, these can influence the anisotropy, which is produced by all protons, slightly only.

In the framework of quasi-linear theory Kennel and Engelmann (1966) showed resonant and nonresonant instabilities force the particle distributions towards marginal stability. While stabilization takes place only resonant particles with velocities of the order of the phase velocity or less are scattered in energy at a rate comparable with their pitch angle scattering rate. The other resonant particles with velocities much larger than the phase velocity suffer pitch angle scattering only.

Looking at the resonant energies of the protons it can be shown that high $\beta_{\parallel p}$-ratios or not too small electron anisotropies are able to lower this resonance energy (Kennel and Scarf, 1968). Because of this lower limit many thermal protons now can maintain the instability.

Using the asymptotic expansion for $Z(z_e)$ and the Taylor expansion for $Z(z_p)$, which presuppose that the resonance protons are near to the maximum of the distribution function, Hollweg and Völk (1970) found an extension to the firehose instability. This instability appears if (11) is fulfilled and is sustained over a broader frequency range if $T_{\parallel p} > 4 T_{\perp p}$. Now the resonant protons are thermal particles, and they are able to reduce the proton anisotropy.

4. Numerical Solutions of the Dispersion Equation

The possible analytical approximations are applicable for special parameter sets only. In order to generally investigate the influence of various thermal anisotropies and $\beta_{\parallel p}$-ratios as a function of frequency, it is necessary to compute the dispersion relation without any approximation.

The plasma dispersion function $Z(z)$ can be calculated in the complex z-plane by various methods (Rehn, 1974).

The first method is a numerical integration for small arguments. Because of the round-off errors for great arguments the function $Z(z)$ is then computed by an asymptotic expansion and, if the imaginary part of z is large enough, by a continued fraction.

In the numerical work typical physical parameters of the solar wind near 1 AU have been chosen as well as a pair of low and high values for each quantity. The values for $T_{\parallel p}$ and $T_{\parallel e}/T_{\parallel p}$ are always the same.

The abbreviations in the figures mean: TPE/TSE is $T_{\parallel e}/T_{\perp e}$, TPP/TSP is the same for protons, TV is $T_{\parallel e}/T_{\parallel p}$, BETA is $\beta_{\parallel p}$, while FIREH is defined by the left hand side of (6). FIREH > 2 indicates that the firehose instability is possible. Nearly each figure contains several curves, which are marked by several symbols. The curves are computed by the parameter values characterized by the same symbol, while the other parameters are valid for all curves.

In Fig. 1. the imaginary part of the frequency, normalized by the gyrofrequency of the protons, $\hat{\omega}_i = \mathrm{WI/GYP}$ is plotted versus the real part of the frequency $\hat{\omega}_r = \mathrm{WR/GYP}$ which is also normalized by the gyrofrequency of the protons. Protons and electrons are assumed to be isotropic (TPE/TSE = TPP/TSP = 1). TV = 5 and TPP = 40000 have been chosen for all other figures. Because of the isotropy of protons and electrons the criteria of instability (5) and (6) are not met, and the waves are damped only. The damping rate and frequency range, in which damping exists, increase with increasing $\beta_{\parallel p}$.

If the protons are anisotropic (Fig. 2), the positive imaginary part of the frequency WI/GYP shows that instabilities exist. The $\beta_{\parallel p}$-ratio influences the growth rate in the same sense as the damping rate: the growth rate increases with increasing $\beta_{\parallel p}$. This becomes clear, if one considers the resonant energy of protons. The three curves belonging to low $\beta_{\parallel p}$-ratios show two properties of the cyclotron instability: both, the unstable frequency range and the growth rate, increase with increasing $\beta_{\parallel p}$. The value $\beta_{\parallel p}=5$ leads to FIREH >2 indicating the rising of the firehose instability. Now the cyclotron instability goes over into the firehose instability existing also at low frequencies. The unstable frequency range is very broad in case of a firehose instability, while a resonant instability alone like the cyclotron instability produces a small unstable range.

The growth rate and the unstable frequency range increases with increasing proton anisotropy (Fig. 3). Fig. 3 shows that the enhancement of the proton anisotropy enhances the growth rate and the unstable frequency range in the same sense as the enhancement of $\beta_{\parallel p}$ does.

Anisotropic electrons increase the growth rate of the cyclotron instability the greater the ratio TPE/TSE is (Fig. 4). Since they lower the phase velocity, the number of resonant protons becomes greater, which means that the growth rate increases.

The results shown in the first four figures can also be obtained analytically using the approximation of finite gyroradius (Kennel and Scarf, 1968).

In Fig. 5 the phase velocity VPH, normalized to the speed of light C, is plotted versus the normalized frequency WR/GYP. The protons are isotropic, and the electrons are assumed to be anisotropic. As we have seen above the phase velocity decreases, while $\beta_{\parallel p}$ increases. For $\beta_{\parallel p}=5$ negative frequencies appear, a result, which has not been found in the approximation of Kennel and Scarf, but which was confirmed by Watanabe (1970) and by Pillip and Völk (1971). Negative frequencies mean that the wave changes its sense of polarization from right to left while propagating antiparallel to the magnetic field. For small $\beta_{\parallel p}$-ratios a damping of the wave occurs (Fig. 6), but when $\beta_{\parallel p}=5$ there are positive growth rates. Since the protons are isotropic, only the electrons are able to drive this nonresonant instability. Though the isotropic protons tend to damp the wave, the energy delivered by the electrons is sufficient to make the wave unstable.

The switching of the sense of polarization already appears for smaller electron anisotropies, if the $\beta_{\parallel p}$-ratio increases (Figs. 7 and 8).

The protons and the electrons are assumed to be anisotropic in Fig. 9. Because of FIREH >2 the firehose instability arises for low frequencies. Near the proton gyrofrequency the cyclotron instability increases the growth rate. For increasing frequency the number of resonant protons decreases, and therefore the growth rate is diminished. Now the nonresonant electrons in a firehose-like mechanism cause a positive growth rate and the switching of the polarization. Finally the damping protons suppress the contribution of the electrons to the extension of the firehose instability and the wave becomes damped.

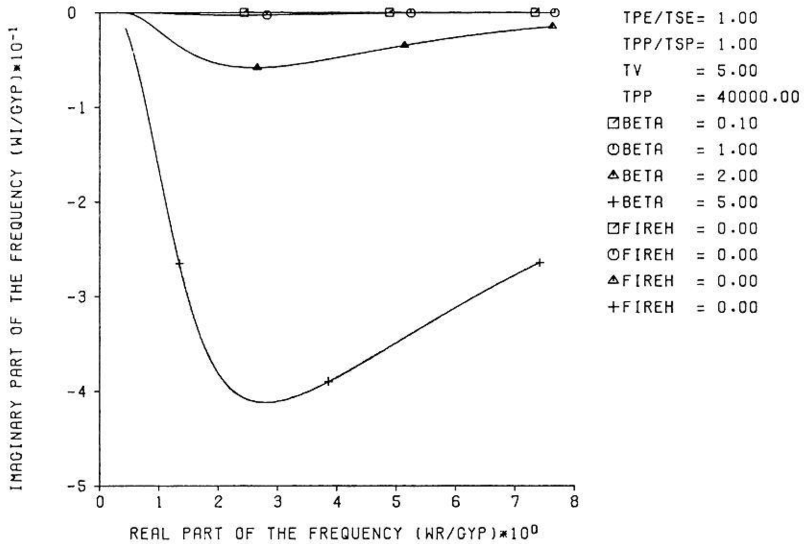

Fig. 1. Imaginary part WI/GYP ($\hat{\omega}_i$) versus the real part WR/GYP ($\hat{\omega}_r$) of the normalized frequency for isotropic protons and electrons and for various values of $\beta_{\parallel p}$. For further explanation see text

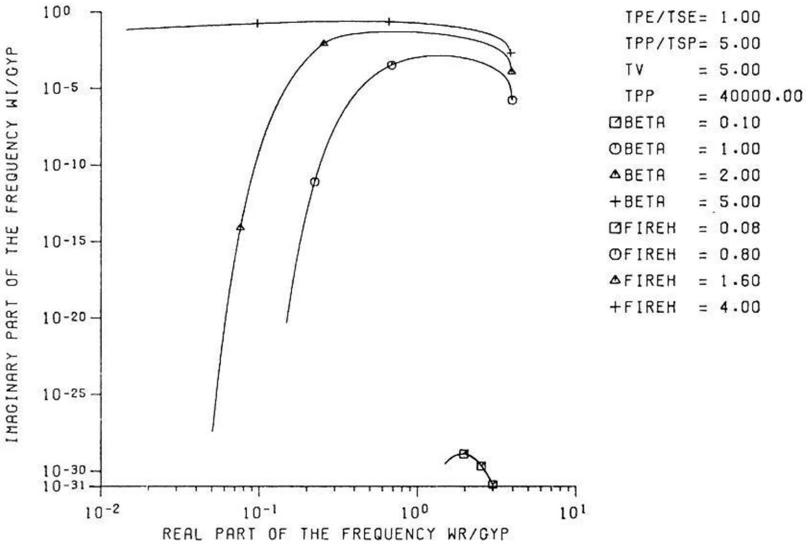

Fig. 2. Imaginary part WI/GYP ($\hat{\omega}_i$) versus the real part WR/GYP ($\hat{\omega}_r$) of the normalized frequency for anisotropic protons and isotropic electrons and for various values of $\beta_{\parallel p}$

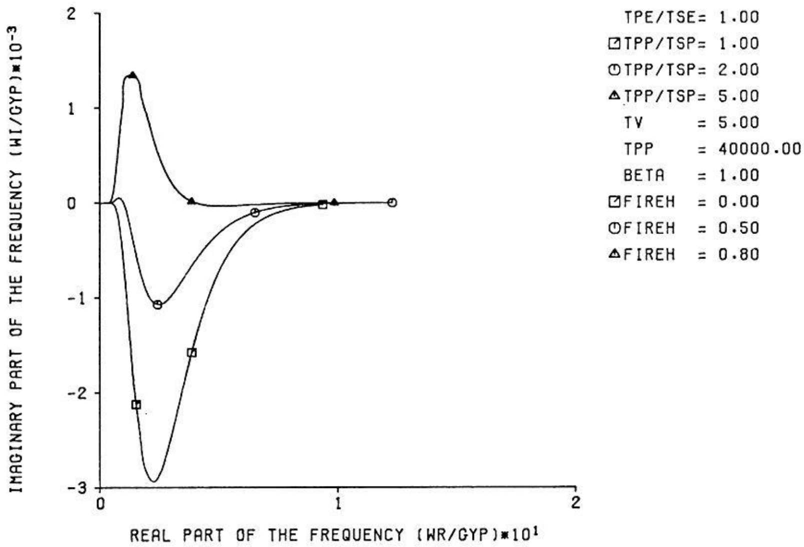

Fig. 3. Imaginary part WI/GYP ($\hat{\omega}_i$) versus the real part WR/GYP ($\hat{\omega}_r$) of the normalized frequency for various anisotropies of the protons and isotropic electrons and for moderate $\beta_{\parallel p}$

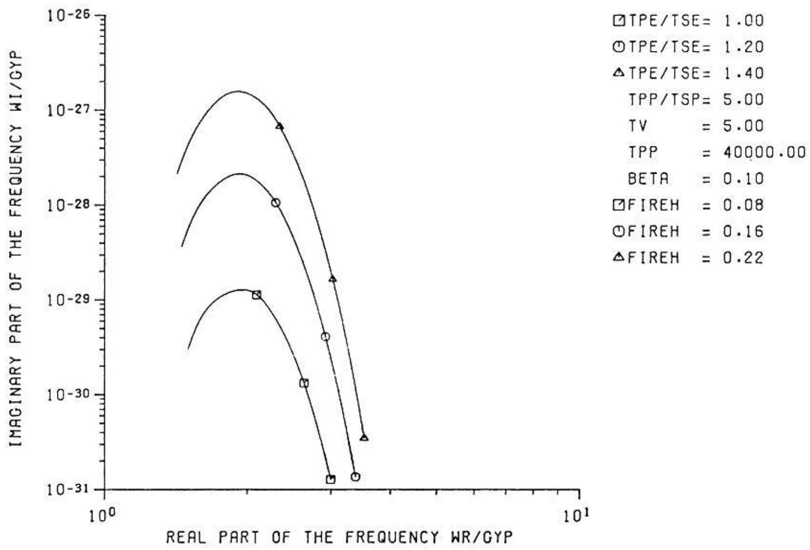

Fig. 4. Imaginary part WI/GYP ($\hat{\omega}_i$) versus the real part WR/GYP ($\hat{\omega}_r$) of the normalized frequency for various anisotropies of the electrons and anisotropic protons and for a low $\beta_{\parallel p}$-ratio

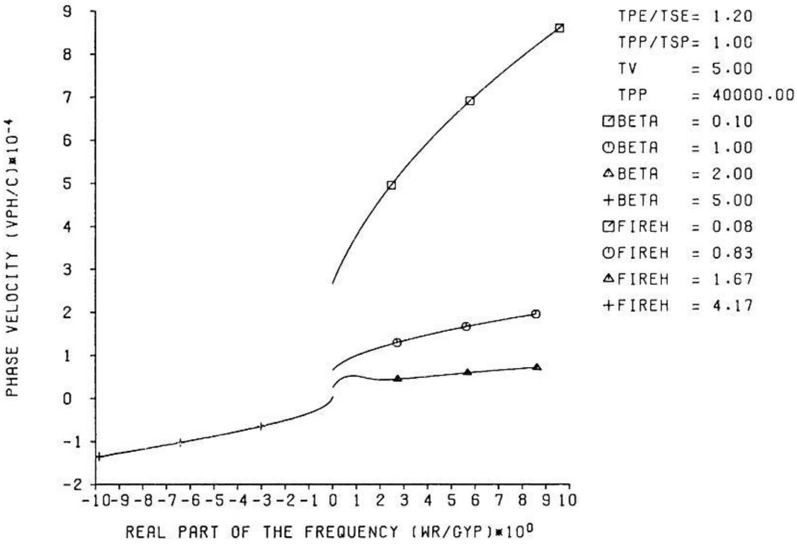

Fig. 5. Normalized phase velocity VPH/C versus the real part of the normalized frequency WR/GYP ($\hat{\omega}_r$) for anisotropic electrons and isotropic protons and for various values of $\beta_{\parallel p}$

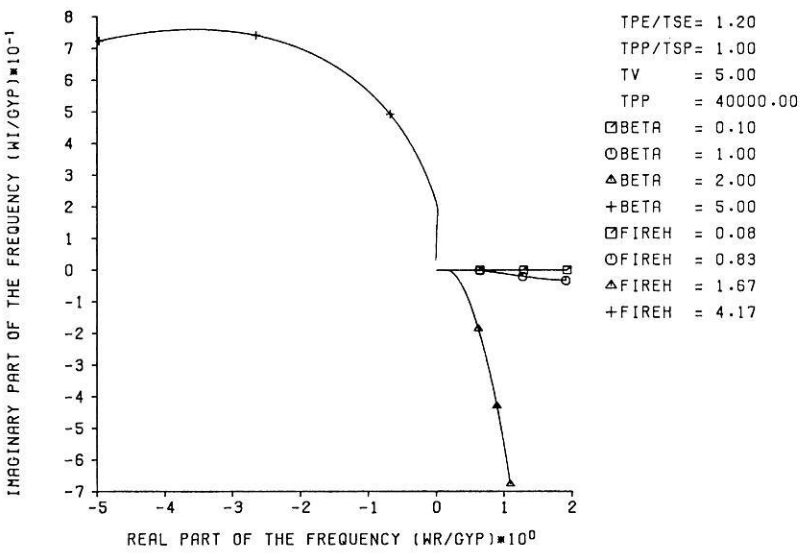

Fig. 6. Imaginary part WI/GYP ($\hat{\omega}_i$) versus the real part WR/GYP ($\hat{\omega}_r$) of the normalized frequency for the same parameters used in Fig. 5

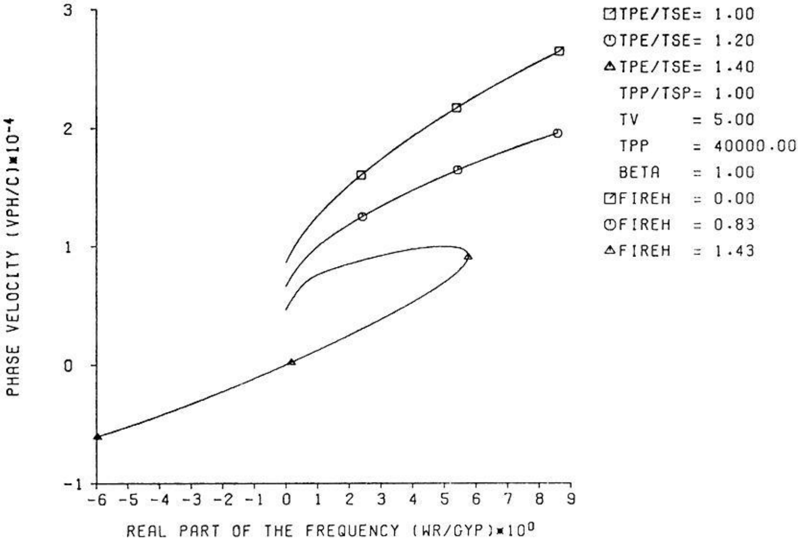

Fig. 7. Normalized phase velocity VPH/C versus the real part of the normalized frequency WR/GYP ($\hat{\omega}_r$) for isotropic protons and various anisotropies of the electrons and for moderate $\beta_{\parallel p}$

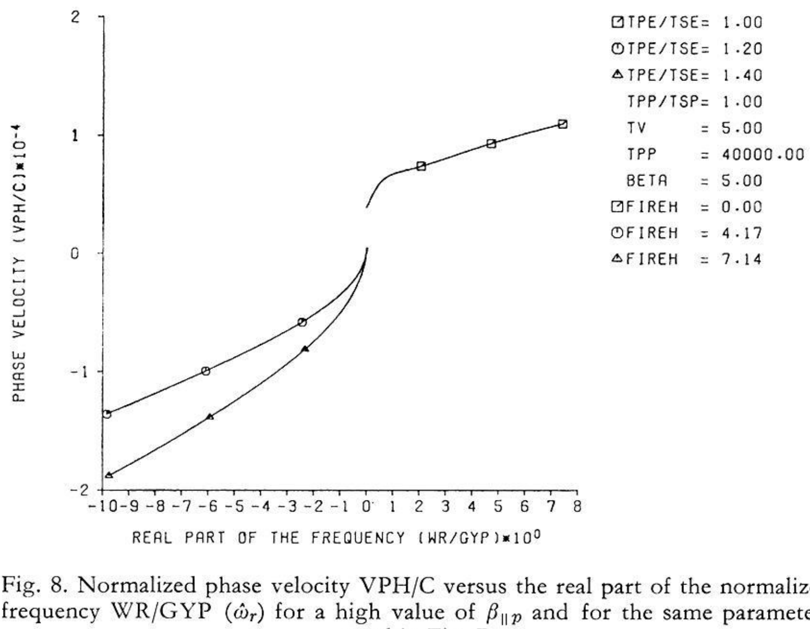

Fig. 8. Normalized phase velocity VPH/C versus the real part of the normalized frequency WR/GYP ($\hat{\omega}_r$) for a high value of $\beta_{\parallel p}$ and for the same parameters used in Fig. 7

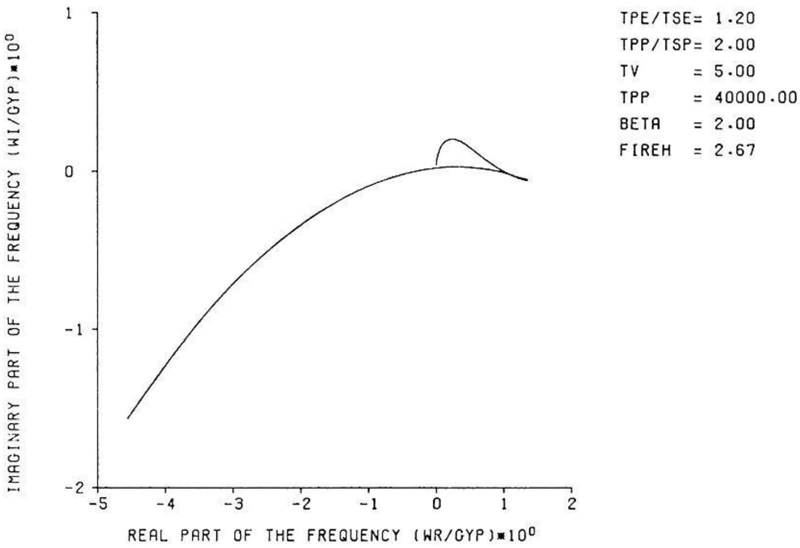

Fig. 9. Imaginary part WI/GYP ($\hat{\omega}_i$) versus real part of the normalized frequency for anisotropic electrons and protons and for moderate $\beta_{\parallel p}$

5. Summary

For right-hand polarized waves propagating parallel to the magnetic field two instabilities can exist in the vicinity of the proton gyrofrequency, if $T_{\parallel} > T_{\perp}$. The first one is the firehose instability and the second one the cyclotron instability, which in contrast to the first one is a resonant instability. If the phase velocity is low enough, then extensions of these instabilities exist.

Isotropic protons and electrons lead to damped waves only. Anisotropic protons produce the cyclotron instability the growth rate of which increases with increasing $\beta_{\parallel p}$ and increasing proton anisotropy. While increasing these two parameters one reaches a point where FIREH >2: the firehose instability can exist. Now it is impossible to distinguish between the two instabilities. The enhancement of the electron anisotropy increases the growth rates of both instabilities and finally leads to a switch in the sense of polarization. If the protons are isotropic, the electrons alone are able to produce a positive growth rate in the firehose manner.

This last instability causes a heating of protons and a cooling of electrons, while the cyclotron instability cools the protons. Furthermore, the instabilities decrease the parallel temperatures and increase the perpendicular temperatures. Cyclotron damping of the waves causes the inverse effects for protons. Therefore it is difficult to predict theoretically in what direction instabilities influence temperatures and thermal anisotropies.

References

Fried, B. D., Conte, S. D.: The plasma dispersion function. New York: Academic Press 1961

Hartle, R. E., Sturrock, P. A.: Two fluid model of the solar wind. Astrophys. J. *151*, 1155–1170, 1968

Hollweg, J. V., Völk, H. J.: New plasma instabilities in the solar wind. J. Geophys. Res. *75*, 5297–5309, 1970

Hollweg, J. V., Völk, H. J.: Energy and momentum exchange in transverse plasma waves. J. Geophys. Res. *76*, 7527–7541, 1971

Hundhausen, A. J.: Direct observations of solar-wind particles. Space Sci. Rev. *8*, 690–749, 1968

Hundhausen, A. J., Bame, S. J., Asbridge, J. R., Sydoriak, S. J.: Solar wind proton properties: Vela 3 observations from july 1965 to june 1967. J. Geophys. Res. *75*, 4643–4657, 1970

Kennel, C. F., Engelmann, F.: Velocity space diffusion from weak plasma turbulence in a magnetic field. Phys. Fluids *9*, 2377–2388, 1966

Kennel, C. F., Petschek, H. E.: Limit on stably trapped particle fluxes. J. Geophys. Res. *71*, 1–28, 1966

Kennel, C. F., Scarf, F. L.: Thermal anisotropies and electromagnetic instabilities in the solar wind. J. Geophys. Res. *73*, 6149–6165, 1968

Montgomery, M. D., Bame, S. J., Hundhausen, A. J.: Solar wind electrons: Vela 4 measurements. J. Geophys. Res. *73*, 4999–5003, 1968

Montgomery, D. C., Tidman, D. A.: Plasma kinetic theory. New York: McGraw-Hill Book Company 1964

Parker, E. N.: Interplanetary dynamical processes in: Monographs and texts in physics and astronomy. Vol. 8. New York: Interscience Publ. 1963

Pillip, W., Völk, H. J.: Analysis of electromagnetic instabilities parallel to the magnetic field. J. Plasma Phys. *6*, part 1, 1–17, 1971

Rehn, H.-W.: Wellen im heißen Plasma. Diplomarbeit, Inst. f. Geophys. u. Meteorol., TU Braunschweig, 1972

Rehn, H.-W.: Die Berechnung der Plasmadispersionsfunktion. In: Methoden und Verfahren der mathematischen Physik, B. Brosowski and E. Martensen, eds., Mannheim: Bibliographisches Institut, in press 1974

Shapiro, V. D., Shevchenko, V. J.: Quasilinear theory of instability of a plasma with an anisotropic ion velocity distribution. Soviet Phys. JETP *18*, 1109–1116, 1964

Watanabe, S.: Anomalous dispersion relation and instability in the solar wind plasma with thermal anisotropy. Rept. Ionosphere Space Res. Japan *24*, 298–312, 1970

H.-W. Rehn
Institut für Geophysik und Meteorologie
der Technischen Universität
D-3300 Braunschweig
Mendelssohnstraße 1
Federal Republic of Germany