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Hydromagnetic Waves in a Non-Uniform Plasma

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Abstract. It is shown, that a small δ-like density perturbation in a cold uniform plasma cannot explain the latitude dependence of the poloidal mode of geomagnetic micropulsations.

Key words: Micropulsations — Hydromagnetic Waves.

Observations of geomagnetic micropulsations of typ Pc3 show a pronounced latitude dependence of the frequency of the north-south component of the perturbed earth's magnetic field (Voelker, 1963; Zelwer and Morrison, 1972). Assuming that non-interacting density lamellae along the field lines exist, Siebert (1965) attributes these observations to the poloidal mode of hydromagnetic waves in the magnetosphere. Abandoning the assumption of absolute density decoupling the influence of a density perturbation on the poloidal mode is considered in the present paper. Below we discuss in more detail the connection of the present treatment to Siebert's model.

The propagation of hydromagnetic disturbances in a column of a cold perfectly conducting plasma possessing a nonuniform mass density distribution:

$$\varrho = \varrho_0 \cdot \left[1 + \lambda \cdot \delta\left(\frac{r}{r_a} - \frac{r_0}{r_a}\right)\right] \tag{1}$$

(ϱ_0: unperturbed mass density, λ: parameter of the δ-like mass density disturbance at radius r_0, r_a: radius of the cylindrical column) is investigated. The plasma column of length L is subject to a uniform axial magnetic field $\mathbf{H}_0$.

The fundamental equations used are Maxwell's equations omitting the displacement current, the equation of motion and Ohm's law. The equations are linearized by assuming the wave field to be small compared to the steady magnetic field.

Under the assumption of axial symmetry the wave equation decomposes into two independent equations for the field components E_ϕ and E_r, both of which are of the form $R(r) \cdot \exp[i(\omega t - kz)]$. Only the isotropic mode is considered below. The equation for the azimuthal electric field E_ϕ is:

$$\frac{\mu_0 \varrho}{B_0^2} \cdot \frac{\partial^2 E_\phi}{\partial t^2} = \frac{\partial^2 E_\phi}{\partial z^2} + \frac{1}{r} \cdot \frac{\partial}{\partial r}(r \cdot E_\phi) \,. \tag{2}$$

For a uniform plasma in a column with perfectly conducting walls the solution of the radial part of (2) can be reduced to an eigenvalue problem by expanding $R(r)$ in terms of first-order Bessel functions J_1:

$$R = \sum_m R_m^{(0)} = \sum_{m=1}^{\infty} A_m^{(0)} \cdot J_1\left(j_m \frac{r}{r_a}\right) \tag{3}$$

where j_m is the m-th zero of J_1.

The eigenvalues are given by

$$\omega_{mk}^{(0)2} = \frac{B_0^2}{\mu_0 \varrho_0} \cdot \left[k^2 + \frac{j_m^2}{r_a^2}\right] . \tag{4}$$

For a parabolic density distribution the solutions are confluent hypergeometric functions (Pneuman, 1965; Cross and Lehane, 1968).

In the model just described meridional planes correspond to planes $\phi = \text{const}$. Assuming axial symmetry the equation of motion for the poloidal modes is given by (2) and corresponds to Siebert's Eq. (5.17). Lamellae are defined by decomposing the meridian plane parallel to the z axis into a discontinuous sequence of strips of zero and nonzero density. Siebert's assumption (5.20a) that the field parameters do not significantly change over a given lamella-cross section would read $\partial E_\phi / \partial r = 0$ in our model. In order to investigate the effect of incomplete density decoupling on the poloidal modes we have introduced the density distribution (1).

In this case we have to solve the following equation

$$\left\{ r^2 \frac{d^2}{dr^2} + r \frac{d}{dr} - 1 - k^2 r^2 + r^2 \cdot \frac{\omega^2 \mu_0 \varrho_0}{B_0^2} \cdot \left[1 + \lambda \cdot \delta\left(\frac{r}{r_a} - \frac{r_0}{r_a}\right)\right]\right\} R = 0 \,. \tag{5}$$

If the density $\varrho_0 \lambda \delta(r/r_a - r_0/r_a)$ is sufficently small the solution of (5) can be found by first-order perturbation theory yielding

$$R = \sum_m R_m \tag{6}$$

$$R_m = A_m^{(0)} \cdot \left[J_1\left(j_m \frac{r}{r_a}\right) - \lambda \omega_{km}^{(0)2} \cdot \frac{\mu_0 \varrho_0}{B_0^2} \cdot 2 r_0 r_a \right.$$
$$\left. \times J_1\left(j_m \frac{r_0}{r_a}\right) \sum_{l \neq m} \frac{J_1\left(j_l \frac{r_0}{r_a}\right)}{J_1'^2(j_l)} \cdot \frac{J_1\left(j_l \frac{r_0}{r_a}\right)}{j_m^2 - j_l^2} \right] + 0(\lambda^2) \tag{7}$$

$$\omega_{mk}^2 = \omega_{mk}^{(0)2} \cdot \left[1 - 2\lambda \frac{r_0}{r_a} \frac{J_1^2\left(j_m \frac{r_0}{r_a}\right)}{J_1'^2(j_m)} \right] + 0(\lambda^2) \,. \tag{8}$$

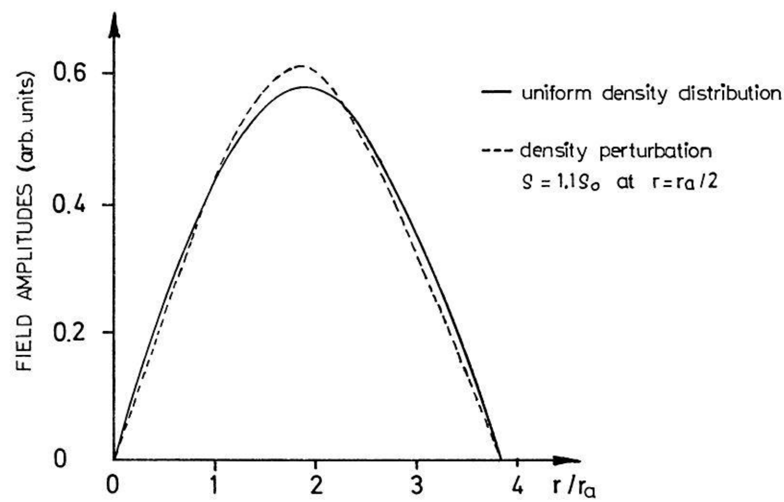

Fig. 1. Radial distribution of the azimuthal electric field E_ϕ in a plasma column of radius r_a and length L

Let us assume that fundamental mode excitation is dominant. Fig. 1 then shows the influence of the density perturbation on the radial part of the field for reasonable parameter values ($r_a = 3 \cdot 10^3$ km, $L = 6 \cdot 10^4$ km, $V_A = (B_0^2/\mu_0\varrho_0)^{1/2} = 10^3$ km s^{-1}, density enhancement 10% of ϱ_0 at $r_0 = r_a/2$). The frequency is shifted to $\omega_{11} = 0.89\ \omega_{11}^{(0)}$. Only a small increase of R_1 compared to $R_1^{(0)}$ is observed. We thus conclude that the observed latitude dependence of the poloidal mode of the micropulsations at the surface of the earth cannot be explained by a small density variation.

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