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Signatur: 8 Z NAT 2148:41

Werk Id: PPN1015067948_0041

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J. Geophys. 41, 513–521, 1975

An Economically Working Method for Computing the Gravimetric Terrain Correction

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Abstract. The described method for the calculation of the terrain correction (deviation of the actual topography from a Bouguer plate) and the entire topographical reduction combines a station-independent orthogonal grid for the outer zones and a station-centered template for the inner zones as subdivision of the topography. This combination seems to have some practical advantages. Furthermore the question of the heights to be used with respect to the curvature of the geoid is discussed. The conventional split of the Bouguer-reduction into terrain reduction and plate reduction seems to be no longer advisable.

Key words: Terrain correction – Bouguer reduction – Simplified digital terrain model.

1. Introduction

To compare measured gravity values at different points of the physical surface of the earth, we first have to eliminate the known gravity-effecting influences, e.g. different heights, geographical latitude, topographical masses, changing densities etc.

The problem of removing the irregularly shaped topographical masses has already brought about a lot of solutions (Ehrismann, 1973). Some of them shall be repeated in a few words.

Most of the authors propose to replace the topography by a sum of mathematically simple bodies, either horizontal layers or vertical prisms, the basal surfaces of which are bounded by cylinder – (α, r) or by parallel (x, y) resp. geographical (φ, λ) coordinates. The upper bound of these prisms may be represented by a plane either horizontal or inclined or by curved surfaces.

It is also possible to approximate the terrain by steady functions for contour lines or vertical profiles or by a mathematically defined surface. Mathematical tools are e.g. polynomials or least squares prediction. Afterwards this artificial topography also may be cut into discrete bodies.

With respect to the practical performance we can distinguish mechanical, graphical and digital methods.

The method described in the following sections may not be the most elegant one from the mathematical point of view, but it has some practical advantages and the formulas and the computer program are quite simple.

Before going into details, we should clarify some basic terms: "Reduction" shall be used in the sense of a computational procedure to get any regularized gravity values from measured gravity data, whereas "correction" really means a correction of a reduction. "Topography" consists of all masses outside the geoid (except atmosphere), whereas "terrain" means the deviation of the actual topography from a plane surface passing through a measuring point P and parallel to the geoid. If any other reference level is used than the geoid, this should be stated by a remark. The terrain correction is therefore a correction to be added to the simple Bouguer reduction (Bouguer plate plus free-air reduction) in order to get the refined Bouguer reduction. The terrain reduction is used as a part of the refined free-air reduction. In this system the topographical reduction denotes the same as Bouguer plate reduction plus terrain correction. Of course we do not need these terms at all; we always can generally talk about shifting of masses or mass reduction and specify these masses.

Originally mostly used was the terrain correction. Therefore it serves as a title of this essay too.

2. The Conception

The topography is subdivided into prisms with horizontal upper surfaces. The subdivision is a combination of a station-independent orthogonal 1 km × 1 km-grid and a station-centered template for the surrounding 4 km × 4 km-area. The template consists of trapezoids, triangles and rectangles, which provide for a connection of orthogonal grid and template without any gaps.

All different types of fields are replaced by ring sectors of the same area, which permits the evaluation of the gravity effect by means of the following wellknown simple formula:

$$\delta g^1 = G\rho\Delta\alpha\left(r_a - r_i + \sqrt{r_i^2 + h^2} - \sqrt{r_a^2 + h^2}\right) \quad (2\text{-}1)$$

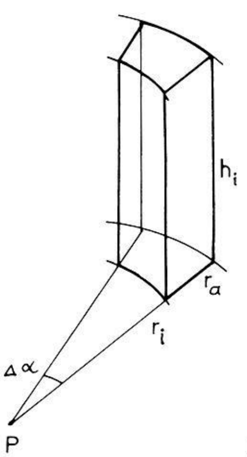

Fig. 1. Variables used in Eq. (2-1) (cf. text)

[1] We shall use the symbol δg for the effect of one compartment as well as for the whole reduction. This will not cause any difficulty.

with:

G gravitational constant 6.67×10^{-8} [cm^3 g^{-1} s^{-2}]
ρ density of the prism [g cm^{-3}].

The other variables are explained by Fig. 1.

The value to be inserted for h depends on the type of the reduction.

2.1. Topographical Reduction

As a part of the complete Bouguer reduction (to obtain Bouguer anomalies) the topographical reduction provides the (computational) removal of all masses outside the geoid or another reference level respectively. The height h always refers to the tangential plane T. Therefore because of the bent surface of the geoid two portions for each prism must be considered: Filling up or removing the terrain to the plane T and afterwards removing down to the geoid G (cf. Fig. 2):

$$\delta g_{\mathrm{Top}} = \delta g'_{\mathrm{Top}} + \delta g''_{\mathrm{Top}} = f(h') + f(h'') = f(|H - H_P - D|) - f(D + H_P). \qquad (2\text{-}2)$$

In this case we assume that H is not measured rectangular to the geoid or the ellipsoid, but parallel to $\overline{P_0 P}$. This neglection causes at e.g. $r = 100$ km an error of less than 0.02% of the height, i.e. a very small error with respect to the accuracy of height estimation.

D represents the horizon-depression:

$$D \approx \frac{r^2}{2R} \qquad (2\text{-}3)$$

with:

R mean radius of the earth.

It shall be pointed out, that the topographical reduction no longer is split into the levelling of the terrain at station elevation and the Bouguer-plate reduction. Varying density values easily can be regarded without introducing an artificial "effective Bouguer density".

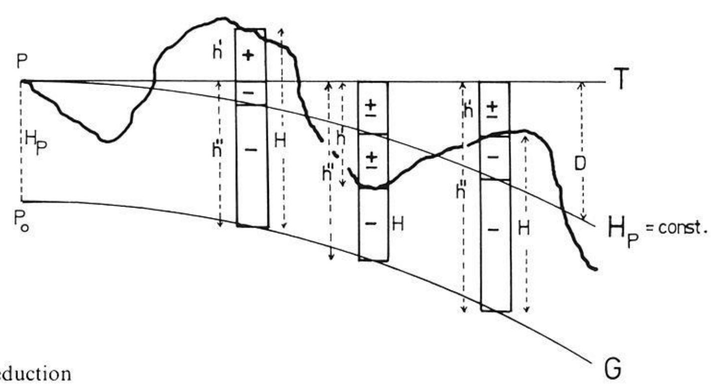

Fig. 2. Topographical reduction

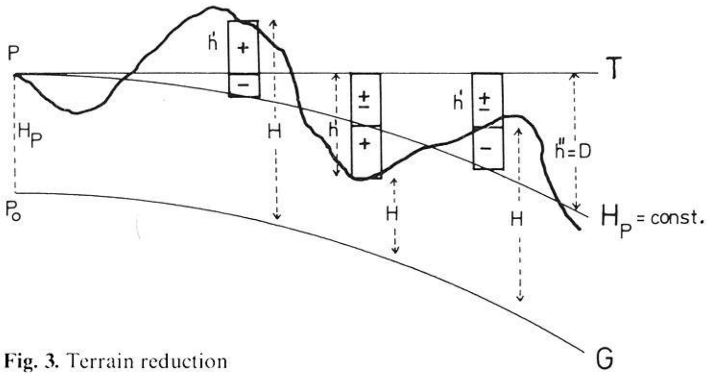

Fig. 3. Terrain reduction

2.2. *Terrain Reduction*

In order to compute free air anomalies we need as a part of the whole reduction the terrain reduction, which term means the (computational) levelling of the terrain. Corresponding to (2-2) we get:

$$\delta g_{\mathrm{Ter}} = \delta g'_{\mathrm{Ter}} + \delta g''_{\mathrm{Ter}} = f(h') + f(h'') = f(|H - H_P - D|) - f(D). \tag{2-4}$$

The difference between (2-2) and (2-4) consists only of H_P in the expression for h''.

If and only if the density is homogeneous throughout the reduction region we can convert Bouguer anomalies to free-air anomalies and vice versa by applying the Bouguer plate of thickness H_P.

3. Computational Formulas, Outlines of the Program

All computations were carried out by means of a FORTRAN-program with the CDC CYBER 73/76 of the "Regionales Rechenzentrum Niedersachsen" in Hannover. The computation-expenses for one point were less than 1 DM.

3.1. *Outer Region*

The orthogonal grid for the outer region is constituted by the full-km-values of the ordinary UTM- or Gauss-Krueger-coordinates X, Y. We establish a local coordinates system x, y, so that the center of the extreme south-west square obtains the coordinates ($x = 1$ km, $y = 1$ km), cf. Fig. 4. These local coordinates also represent the indices for the data storage of all heights of the squares $H(x, y)$ in the core memory of a digital computer. This has the great advantage, that for storing x, y, H only one comfortably organizable core-word is necessary. The density values $\rho(x, y)$ are stored in the same manner. The data for the outer region is fed into the computer only once for all stations in the region. Control statements take care of erroneous and missing data. Afterwards an optional num-

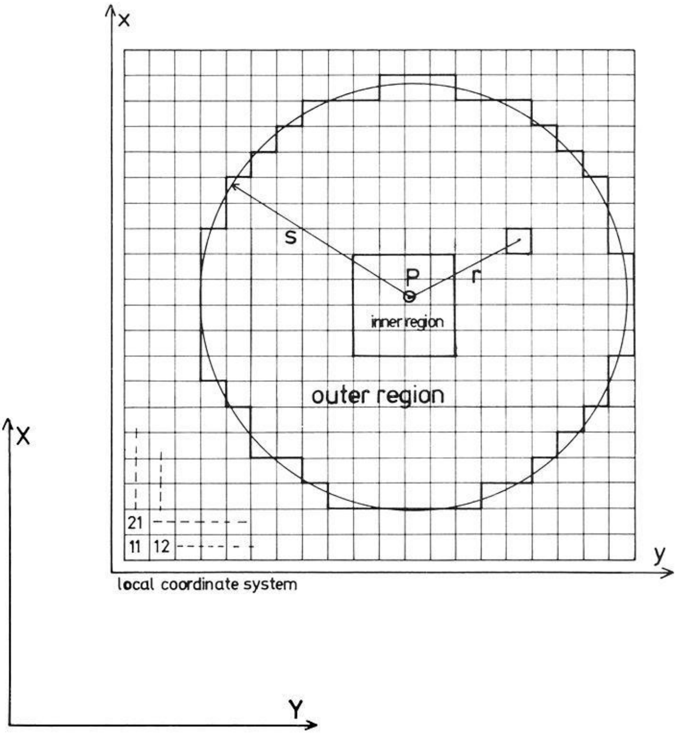

Fig. 4. Outer region

ber of data referring to the stations may be fed in. Then according to a codeword the topographical resp. the terrain reduction is computed.

The data referring to a station consists of point number, coordinates X_P, Y_P, H_P, radius s of reduction area and mean heights for the inner region. The coordinates X_P, Y_P are transformed to x_P, y_P.

For all squares the centers of which are inside the radius s, the gravity effect is evaluated in the following way:

The distance of a square is:

$$r=\sqrt{(x-x_P)^2+(y-y_P)^2}\,. \tag{3-1}$$

The area of a square is always $F_Q=1$ [km^2].

Under the condition, that the substitution area also should be of approximately quadratic shape, we obtain:

$$\Delta\alpha=\frac{1}{r}, \qquad r\,[\mathrm{km}] \tag{3-2}$$

and:

$$r_i=r-500\ \mathrm{m}$$

$$r_a=r+500\ \mathrm{m}\,. \tag{3-3}$$

These values are inserted in (2-1). All single gravity effects δg are summed.

3.2. Inner Region

The inner region of $4 \times 4\,\mathrm{km}^2$ is covered by a template with 44 compartments, centered to the station, and the innermost 100 m zone (cf. Fig. 5). The compartments 1 ... 24 are shaped under the conditions, that on one hand the gravity effect is constant if the terrain has a mean constant inclination and that on the other hand the height-estimation is facilitated. The triangles 25 ... 28 serve for the transfer to the orthogonal grid. The area of the compartments 29 ... 44 is variable according to the position of the station in the grid. When estimating mean heights of these compartments, only the inner parts (cf. Fig. 5) are regarded. All compartments are replaced by areas bounded by polar coordinates.

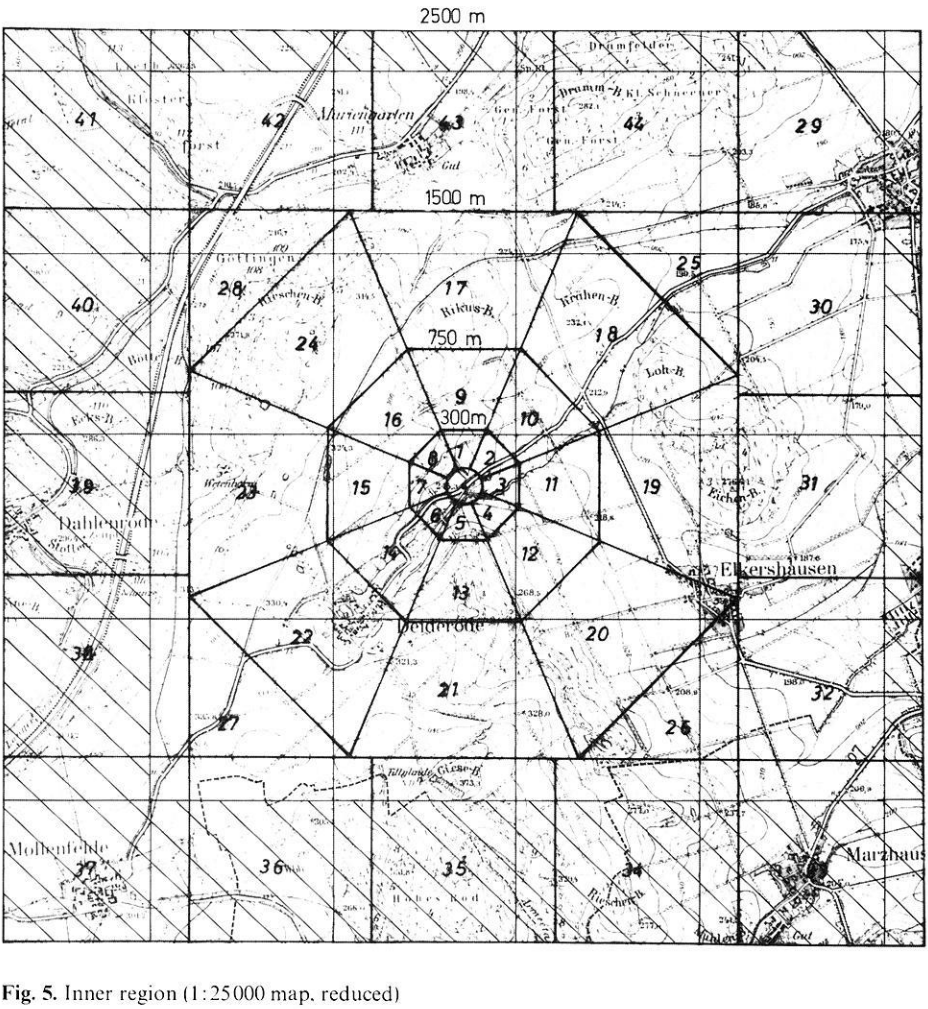

Fig. 5. Inner region (1:25000 map, reduced)

The values to be introduced into (2-1) are:

	$\Delta\alpha$ [R]	r_i [m]	r_a [m]
$i=1 \dots 8$	0.7854	100.00	308.10
$i=9 \dots 16$	0.7854	308.10	770.27
$i=17 \dots 24$	0.7854	770.27	1540.55
$i=25 \dots 28$	0.3537	1446.69	2068.00

$i=29 \dots 44$: Area F and distance r are computed from the station-coordinates within the grid. Setting

$$r_a - r_i = \sqrt{F},$$

we obtain

$$r_a = r + \tfrac{1}{2}\sqrt{F}$$
$$r_i = r - \tfrac{1}{2}\sqrt{F} \tag{3-4}$$

and

$$\Delta\alpha = \frac{\sqrt{F}}{r}. \tag{3-5}$$

For the computer program, only the 44 mean height- and density values are to be fed in.

3.2.1. 100 m-zone. If the inmost surrounding of the station is not approximately horizontal, we may use e.g. the method of a "two-dimensional" mass. The profile in the direction of steepest inclination is recorded on a diagram. The inclination and curvature rectangular to this direction should be negligible. One diagram of this type may be seen in (Jung, 1961, p. 152). We get the terrain reduction by means of this diagram. For the topographical reduction we additionally use (2-1), inserting $r_i=0$, $r_a=100$ m, $\rho=\rho_P$ and $h=h''=H_P$.

4. Error Analysis

We shall estimate the influence of the substitution of the original quadrangles and triangles through ring-sectors. It is not our aim, however, to compare the quality of terrain representation of some conventional templates and our template. The representation always depends on the density of estimated height values. Above all we shall compare our results of some practical computations with those obtained by means of a conventional template.

Inner Region

We divide the inner 1500 m × 1500 m-area into small sectors. If we assume constant height and constant density, we can restrict ourselves to the range $\alpha = 0° \dots 45°$. We use formula (2-1), inserting $r_i = 100$ m, $r_a = \dfrac{1500\text{ m}}{\cos\alpha_m}$, where α_m is the mean azimuth of the resp. sector.

For $\Delta\alpha$ we take values of $\Delta\alpha = 5°$ decreasing down to $\Delta\alpha = 1°$. This procedure gives us the limit for $\Delta\alpha \to 0$ with a sufficient accuracy. The results differ from our results not more than 1% up to $h = 1000$ m.

Outer Region

The nearest km-square has been subdivided into small sectors like in the inner region with $\Delta\alpha = 1{,}5° \ldots 0{,}5°$. The radii r_i, r_a have been computed according to the boundaries of the square and again we used formula (2-1). The difference to our computation does not exceed 3% ($100\,\text{m} \leqq h \leqq 5000\,\text{m}$, $h = \text{const.}$) and decreases quickly with increasing distance r.

Comparison with a Conventional Template

Computations of the terrain reduction with constant density were carried out for 38 stations in the Harz mountains ($50\,\text{m} < H < 1000\,\text{m}$) using a reduction radius $s = 30000$ m. Mean height estimations and computations were completely independent. Assuming the same accuracy for both methods we get a standard error of

$$m = \pm\sqrt{\frac{[dd]}{2n}} = \pm 0.12\ \text{mgal},$$

whereas the mean reduction amounts to 1.5 mgal. The standard error is less than 10% of the terrain reduction, which according to experience is the presumable error caused by estimation of mean heights.

5. Field of Application

We shall compare the number of squares to be estimated with the number of compartments when using a conventional template. We restrict ourselves to the outer region, because in the inner region our method works with a station-centered template too. Considering the accuracy in the outer region, our new method is better in any case, because the grid is denser than any conventional template.

We assume a quadratic shape of the measuring region. We shall compare the outer region with $2260\,\text{m} \leqq r \leqq 29000\,\text{m}$ (cf. Tab. 1). The lower bound corresponds to the region of $4 \times 4\ \text{km}^2$. The number of squares to be estimated is computed after $l = (K + 2 \times 29)^2$. The bounds of r mentioned above fairly correspond with the Hayford zones $G \ldots L$ (resp. $2290\,\text{m} \leqq r \leqq 28800\,\text{m}$) consisting of 108 compartments (Heiskanen/Moritz, 1967, p. 140).

We recognize the growing advantage with growing area and growing station-density. Moreover it is advantageous, that we only need one scale for the inner region, and that the estimation of the mean heights for the outer region is facilitated by the constant shape and size of the squares.

Table 1. Number of compartments to be estimated

Area km^2	Total number of compartments (Hayford)		Number of squares independent of station-distance
$K \times K$	1 station per 5 km^2	1 station per 10 km^2	l
10×10	2160	1080	4624
20×20	8640	4320	6084
30×30	19440	9720	7744
40×40	34560	17280	9604

But it is clear, that this method works favourable only for regional computations, not for global ones because of the distortion of the X, Y-coordinates and because of the great number of squares to be estimated for a great reduction radius.

6. Conclusions

The method reported represents a satisfying compromise of station-centered (inner region) and orthogonal station-independent subdivision of terrain for the gravimetric terrain correction, both terrain reduction for free-air anomalies and topographical reduction for Bouguer anomalies. It is no longer advisable, to split the Bouguerreduction into terrain reduction and Bouguer-plate reduction. It is easy to regard variable density-values.

Acknowledgements. I thank Dipl.-Ing. H. Drewes for leaving the computations of the conventional terrain reduction for the purpose of estimation of accuracy.

This work is part of the research project "Testnetz Westharz", which is sponsored by the "Deutsche Forschungsgemeinschaft" and performed at the "Institut für Theoretische Geodäsie" (Director: Prof. Torge).

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Received September 2, 1974; Revised Version April 24, 1975