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Probability Distribution of Earthquake Accelerations with Applications to Sites in the Northern Rhine Area, Central Europe

L. Ahorner

Erdbebenstation der Universität Köln, D-5060 Bensberg, Vinzenz-Pallotti-Straße 26, Federal Republic of Germany

W. Rosenhauer

INTERATOM, Internationale Atomreaktorbau GmbH, D-5060 Bensberg, Friedrich-Ebert-Straße, Federal Republic of Germany

Abstract. Based on the observed earthquake activity of the period 1750–1970 and the distribution of neotectonic structural activity a regional seismicity model has been established to calculate the earthquake risk for sites in the Northern Rhine area by a computer program. The calculation procedure makes use of Gumbel's extreme value theory and gives the recurrence intervals of various earthquake magnitudes for finite volume elements of seismoactive regions and, in interdependence herewith, the probability distribution of earthquake accelerations at the earth surface as a function of distance from the entirety of finite focal volumes. Numerical calculations were made for 220 sites within an area of 160000 km^2. From the results, earthquake risk maps have been drawn giving isolines of equal probabilities for earthquake accelerations higher than 100 and 300 cm/s^2. The highest earthquake risk has been found in the western part of the Lower Rhine graben between Köln and Aachen, where two seismoactive zones, the Rhenish and Belgian earthquake zones, are intersecting. Methods and results presented are a first step towards an analysis of earthquake risk by probability considerations which is not state of the art in current licensing procedures *e.g.* for nuclear plants.

Key words: Earthquake Risk – Northern Rhine Area – Central Europe.

1. Introduction

As a result of the increasing industrial, commercial and residential developments, which cause the population centers to spread in ever-widening circles, reliable estimations of the local earthquake risk are of interest not only in the main earthquake countries but also in regions with minor seismicity such as Central Europe. This is especially true for sites of nuclear power plants and other industrial objects offering a potential source of environmental danger in regions with dense population.

An appropriate statistical tool to estimate the earthquake risk, i.e. the probability of the occurrence of a strong shock in a given time period, is the Gumbel extreme value theory (1935, 1967), which was applied to earthquake statistics first by Nordquist (1945) and later by Epstein and Lomnitz (1966), Kárník (1971) and others.

In the present paper the application of Gumbel's extreme value theory to earthquake data of Western Germany will be described. Based on the observed earthquake activity in the period 1750–1970 and the distribution of neotectonic structural activity a regional seismicity model has been established to calculate the probability distribution of earthquake accelerations at given sites by a computer program.

2. Seismicity of the Northern Rhine Area

The object of our study is the Northern Rhine area (4 °E to 10 °E, 49 °N to 52 °N) which covers the western part of Germany (Nordrhein-Westfalen, Hessen, Rheinland-Pfalz) and the adjoining regions of Luxembourg, Belgium and the Netherlands.

The seismicity and seismotectonics of this region have been studied in detail by Ahorner (1968, 1970). Seismicity data are based on historical earthquake catalogues of Sieberg (1940), Sponheuer (1952) and others, and – beginning with shocks later than 1900 – on a re-interpretation of all available macroseismic and microseismic original information. A chronological list has been prepared of earthquakes since 1500 giving for each shock the date of occurrence, the latitude and longitude of the epicenter, the macroseismic intensity I_0 (MSK-scale) near the epicenter, the magnitude M_{loc} (local magnitude according to the original Richter-scale) and an estimate of the focal depth h. No allowance was made of rockbursts and other seismic events of true artificial origin.

Fig. 1 shows the locations of epicenters classified according to magnitude, focal depth and accuracy of epicenter determination. For the period 1500–1749 only shocks with magnitudes $M \geqq 4.5$ were plotted, for 1750–1899 those with $M \geqq 3.5$ and for 1900–1969 those with $M \geqq 2.5$. It is obvious from the distribution of epicenters that nearly all foci being active since 1500 are situated within three zones (bounded by broken lines in the map):

– The Rhenish earthquake zone, which goes from Karlsruhe in the South to Nijmegen in the North, following the neotectonic fracture zones of the Upper Rhine graben, Middle Rhine valley and Lower Rhine graben.

– The Belgian earthquake zone, which trends in a westeastern direction across Belgium and meets the Rhenish zone in the region between Aachen and Köln.

– The Hunsrück earthquake zone, which is of minor importance and covers the south-western border region of the Hercynic Rhenish massif (Rheinisches Schiefergebirge) between Mainz and Luxembourg.

In the western part of the Lower Rhine graben near the intersection point between the Rhenish and Belgian earthquake zone the highest level of seismicity is observed. In this region, the strongest earthquake of the Northern Rhine area took place near Düren 1756 ($M \approx 6$, $I_0 = \mathrm{VIII}$). Another major shock with $M = 5.8$ occurred 1938 in the Belgian zone south-west of Bruxelles. Eight shocks in the last 200 years were of magnitude $M = 5.0 - 5.4$. From magnitude-frequency

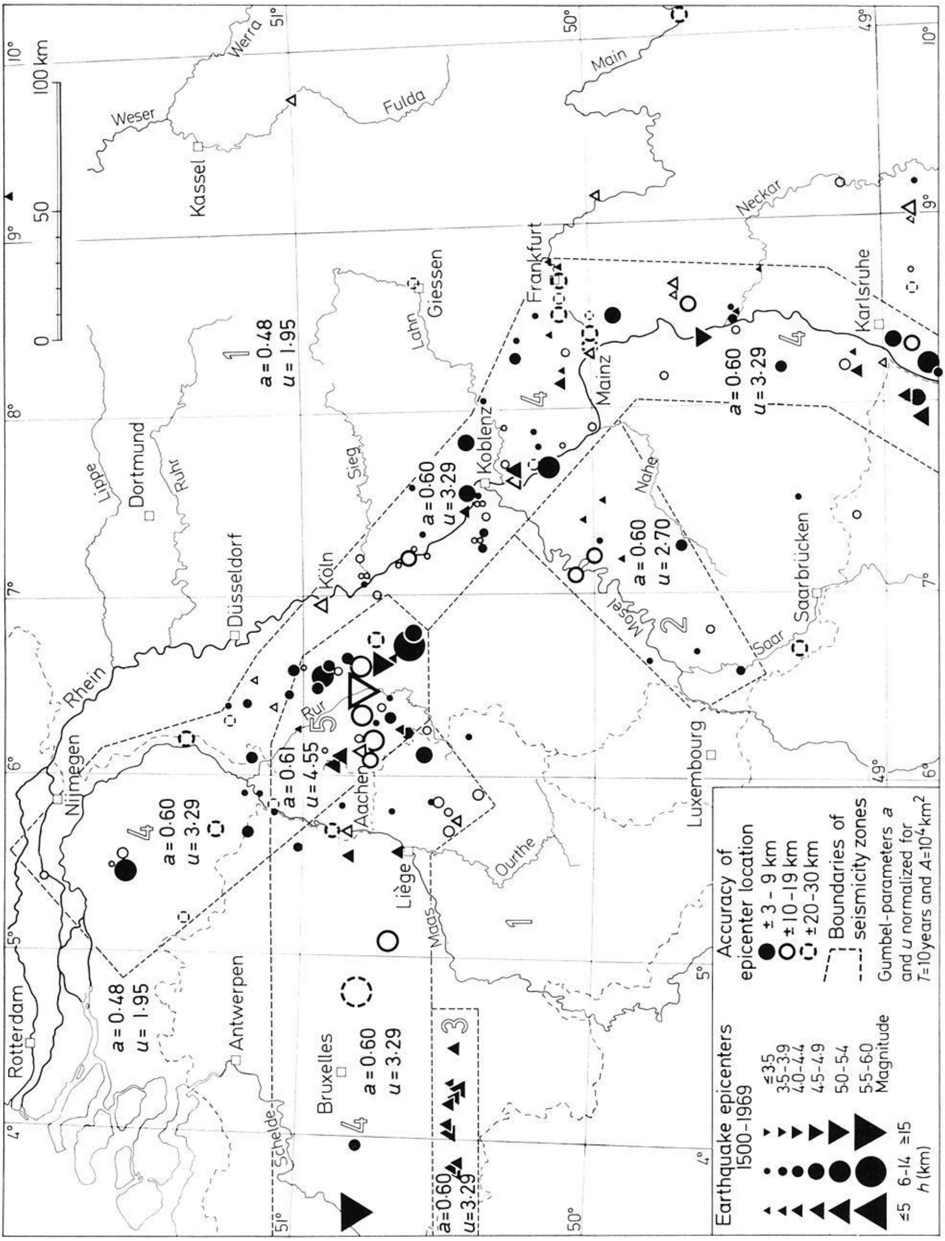

Fig. 1. Seismicity zones and earthquake epicenters of the Northern Rhine area. Normalized Gumbel parameters a and u are given for each region

analysis it becomes clear that the specific seismicity (magnitude frequency per unit area and time) of the Rhenish and Belgian earthquake zone is about one tenth of that of Southern California or the East African rift zone (Ahorner, 1975).

Focal depths of the Northern Rhine area, determined mostly macroseismically, vary between several and 25 km as a maximum. The great majority of shocks is released in the upper part of the crust within the depth range of $h = 6 - 12$ km

(Ahorner, Murawski, Schneider, 1972). Very superficial shocks with $h = 1 - 2$ km are predominant in the Hainaut region south of Bruxelles (Ahorner, 1972). Damaging earthquakes with intensities up to VIII are confined nearly exclusively to the Rhenish and Belgian earthquake zone (Ahorner, Murawski, Schneider, 1970). Outside of these zones only a few earthquakes with local damage have been observed.

In the light of regional seismotectonics the Rhenish and Belgian earthquake zones form together an intra-plate zone of crustal weakness, where – within the Eurasian lithospheric plate – recent block movements are going on with a mean slip rate of a few tenth of a mm per year (Ahorner, 1975). The simple large-scale deformation pattern and the close relationship between seismicity and neotectonic structural activity facilitates the seismic regionalisation of the Northern Rhine area as shown in Fig. 1.

3. Probability Description of Earthquakes

3.1. Conventional Statistics and Extreme Value Theory

Analysing a sample of N earthquakes in a specified region with respect to their Richter magnitudes M one might aim at the cumulative distribution $F(M)$, which is defined by the probability that an earthquake has a magnitude not greater than M:

$$\begin{aligned} &F(M) = P(\text{magnitude} \leqq M) \approx n(M)/N, \\ &n(M) = \text{number of sample members with magnitudes} \leqq M. \end{aligned} \tag{1}$$

$F(M)$ or closely related quantities like the return period $1/[1 - F(M)]$ or the probability density $dF(M)/dM$ are not easily applicable to risk estimations because

– reliable probabilities are required for higher values of M at which the small number of occurrences introduces large uncertainties of $F(M)$,

– an extrapolation of $F(M)$ relies upon the completeness of observations at low values of M,

– N ought to be the total number of earthquakes during the observation time; unfortunately, this number is not well defined as it grows with the improvement of the lower detection limit for smaller earthquakes.

The theory of extremes provides an approach avoiding these shortcomings by using a probability picture less detailed than Eq. (1), but still covering enough information for risk applications. Instead of $F(M)$ the cumulative distribution of the largest magnitude within a specified region A and time T is considered:

$$G(M) = P(\text{largest magnitude} \leqq M \text{ in } A, T). \tag{2}$$

$G(M)$ could in principle be determined analogously to Eq. (1). Looking only for the earthquakes with the largest magnitudes within certain intervals (e.g. the extremes for $T = 5$ years) of the whole observation time one gets:

$$\begin{aligned} &G(M) = ne(M)/Ne, \\ &ne(M) = \text{number of extremes with magnitudes} \leqq M, \\ &\quad Ne = \text{total number of extremes.} \end{aligned} \tag{3}$$

It is obvious that information concerning only the extremes, i.e. the largest earthquakes, is much better for the historical time than the observation material for the whole magnitude spectrum. A complete catalogue of earthquakes down to magnitude $M=4$ has been achieved in Western Germany only since about 1800 and down to $M=3$ only since 1950.

One of the main advantages of Eq. (2) is, however, that it is not necessary to determine the functional shape of $G(M)$, which would require much more data than available. The connection between $G(M)$ and $F(M)$ is easily derived from the fact that the extreme is not higher than M if every single event has magnitude $\leqq M$ and vice versa. The assumption of independence leads to

$$\begin{aligned} &G(M)=[F(M)]^n, \\ &\quad n=\text{total number of earthquakes in } A, T. \end{aligned} \tag{4}$$

Investigating the limit $n \to \infty$ it can be proved that for an extensive class of initial distributions $F(M)$ the extreme value distribution $G(M)$ approaches the Gumbel distribution as an asymptote:

$$\begin{aligned} &G(M) \approx \exp\left[-\exp\left(-\frac{M-u}{a}\right)\right] \\ &a=a(F), \qquad u=u(n,F). \end{aligned} \tag{5}$$

The interpretation of this result is that

– different distributions $F(M)$ lead to the same function $G(M)$ if n is not too small,

– n and the actual shape of $F(M)$ are contained in the numerical values of a and u,

– the knowledge of n and the functionals $a(F)$ and $u(n, F)$ can be dispensed with if the numerical values of a and u are deduced from extreme value observations directly.

As a proof of Eq. (5) the validity of the Gumbel distribution has to be tested against observed extremes.

3.2. Order Statistics and Gumbel Parameters

Using order statistics evaluation methods have been developed particularly suitable to the Gumbel distribution. Numerical results, best fits, tests, uncertainties etc. have been obtained from the INTERATOM program GUMBEL on a CDC 6400.

Let the Ne observed extremes be arranged in increasing order:

$$M_1 \leqq M_2 \leqq \cdots \leqq M_{Ne}. \tag{6}$$

Looking for the m-th observation counted from the bottom ($m=1, 2, \ldots, Ne$) it is evident that the value expected for it depends on the Gumbel parameters a and u. This dependence is found to be linear, containing a coefficient y_m which obviously has to be interpreted as the expectation of the m-th observation for a Gumbel distribution with $a=1$ and $u=0$:

$$\overline{M}_m=\overline{M}_m(a,u)=a\cdot y_m+u, \qquad y_m=\overline{M}_m(1,0). \tag{7}$$

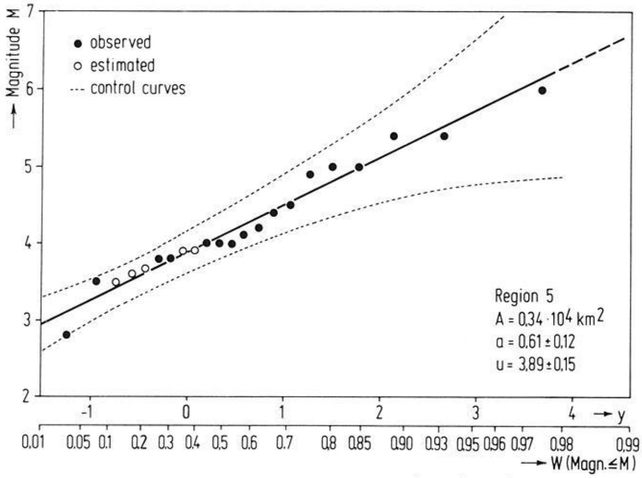

Fig. 2. Gumbel plot of 10-year extremes of earthquake magnitude for the western part of Lower Rhine graben (region 5, observation period 1750 – 1969). Control curves and not normalized parameters a and u calculated by the program GUMBEL are given

The numerical values for y_m are calculated once and for all from explicit formulae or by Monte Carlo simulation in program GUMBEL. Regarding the m-th observation M_m as an estimate of its expectation $\overline{M}_m$ one gets

$$\overline{M}_m \approx M_m, \qquad M_m \approx a \cdot y_m + u. \tag{8}$$

The last equation says that the points (y_m, M_m) scatter about a straight line the parameters of which are the Gumbel parameters wanted.

As an example, Fig. 2 shows the plotted 10-year extremes for the western part of the Lower Rhine graben (see Fig. 1).

The dashed control curves give the range of still acceptable deviations allowing the agreement between Eq. (5) and the observations to be judged by inspection, too. The GUMBEL estimates for a and u are without bias. A simulation subroutine provides the uncertainties of these estimates needed in Section 5.

A Gumbel parameter evaluation (Table 1) has been performed for the seismicity zones described in Section 2. The values a and u (in Table 1 and Fig. 1)

Table 1. Seismological subdivision of the Northern Rhine area

Region		Area $A/10^4$ km²	Gumbel parameters (normalized)	
			a	u
5	Western part of the Lower Rhine graben (between Aachen and Köln)	0.34	0.61 ± 0.12	4.55 ± 0.20
4	Rhenish and Belgian earthquake zone (except region 5)	3.20	0.60 ± 0.12	3.29 ± 0.21
3	Hainaut zone (shallow shocks)	0.15	0.60 ± 0.12	3.29 ± 0.21
2	Hunsrück zone	0.46	0.60 ± 0.12	2.70 ± 0.18
1	Regions with very low seismicity (outside regions 2–5)	12.8	0.48 ± 0.14	1.95 ± 0.68

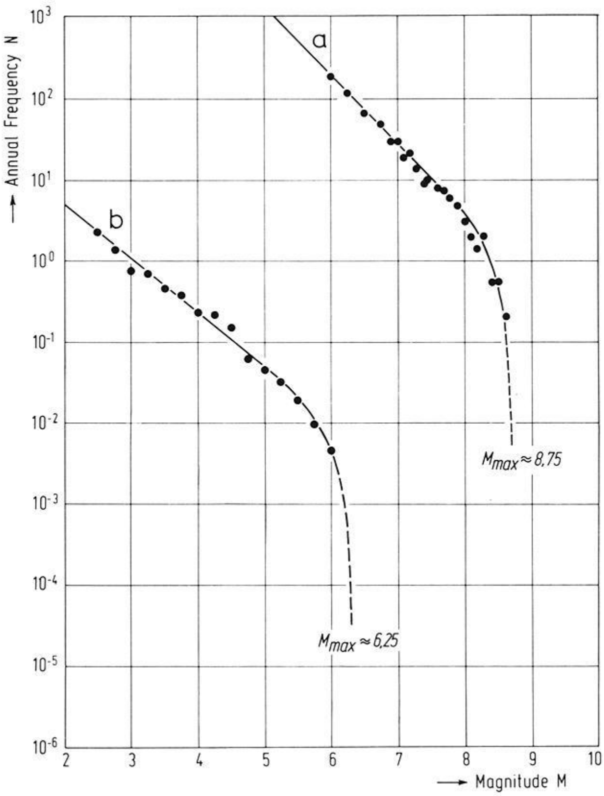

Fig. 3. Mean annual frequency distribution of earthquake magnitudes for the world (curve *a*, from Housner, 1970) and the Northern Rhine area (curve *b*, from Ahorner, 1972). Observed data allow an estimation of the maximum possible magnitude $M_{\max}$

are normalized to $T=10$ years and an area of $A=10^4$ km² respectively. The transformation between areas A_0 and A of different size is

$$a_0 = a, \qquad u_0 = u + a \cdot \ln(A_0/A). \tag{9}$$

3.3. Upper Bound for Magnitudes

Observed extremes give an excellent support for the Gumbel distribution up to $M \approx 6$ (see Fig. 2). Without further knowledge one had to accept its validity up to infinite values of M. This, however, is not reasonable because of lithophysical and seismotectonic boundary conditions. Absolute upper bounds seem to exist for the whole world at $M_{\max} \approx 8.75$ and for Central Europe at $M_{\max} = 6.5 \pm 0.25$ (see Fig. 3 and Housner (1970), Ahorner (1972)).

$M_{\max}$ is taken into account by using a conditional probability distribution $G_c(M)$ constructed from Eq. (5) by introducing the condition: M not greater than $M_{\max}$. This means a slight deformation of $G(M)$ in the range of observations and excludes values of M higher than $M_{\max}$:

$$G_c(M) = G(M/M \leqq M_{\max}) = \begin{cases} \dfrac{G(M)}{G(M_{\max})} & (M \leqq M_{\max}) \\ 1 & (M > M_{\max}). \end{cases} \tag{10}$$

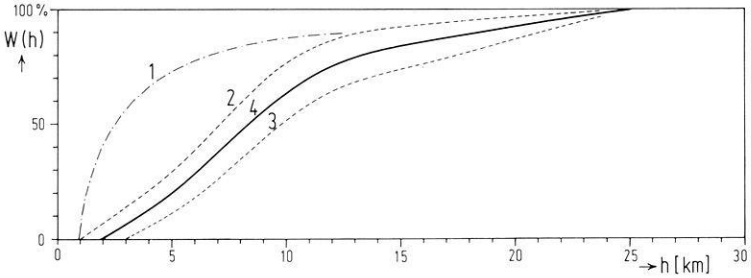

Fig. 4. Focal depth distribution $W(h)$ for different regions and magnitude ranges. Hainaut region: curve *1*. Other regions: curve *2* for $M \leqq 5$, curve *3* for $M > 5$, curve *4* model independent of M

3.4. Depth Distributions

In order to complete the statistical description of earthquakes the focal depth distribution $W(h)$ has to be considered. Several distributions have been found by evaluating observed data (see Fig. 4 and Ahorner, Murawski, Schneider (1972)). For the probability assessment it is sufficient as a first step to use curve *4*, which is the arithmetic mean of curve *2* (earthquakes with $M \leqq 5$) and curve *3* (earthquakes with $M > 5$). Curve *1* is used for the Belgium earthquake region of Hainaut (*S* of Bruxelles) where the focal depths are especially shallow (Ahorner, 1972).

It is planned to improve the methods by introducing a dependence of the focal depth distribution on magnitude, $W(h, M)$, in order to cover the increasing focal volume for higher magnitudes.

4. Calculation of Site Dependent Distributions for Accelerations

4.1. Transfer Function

The statistical description of earthquakes given in Section 3 is only a first step towards a risk assessment since it describes the causal events in the environs rather than the consequences at a site. A transfer function is needed as a link between causes and consequences, e.g. maximum ground acceleration B at the site.

B depends upon a great number of parameters. For a probability calculation it is justified to neglect a lot of them, e.g. the direction of the focal plane with respect to the site and the differences in the geological transfer medium between focus and site. Important parameters are the magnitude M and the hypocentral distance R between site and focus.

A transfer function of the following shape has been proposed by Esteva (1970) for North America:

$$B = B(M, R) = B_0 \cdot \exp(cM) \cdot \left(\frac{R + R_0}{\text{km}}\right)^{-d} \tag{11}$$

with

$$B_0 = 1230\ \text{cm/s}^2, \quad c = 0.8, \quad R_0 = 25\ \text{km}, \quad d = 2.$$

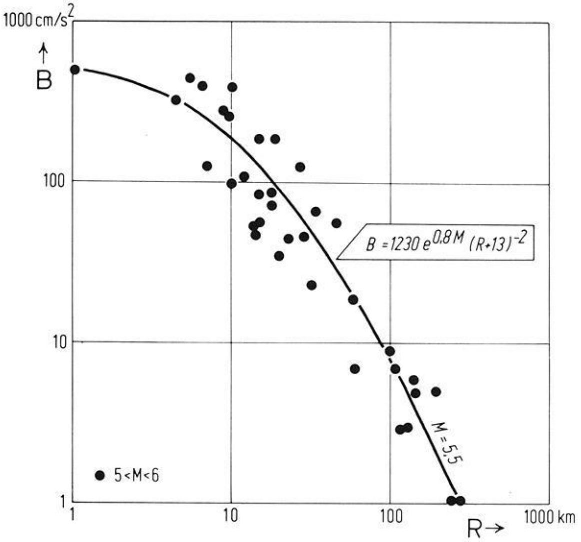

Fig. 5. Maximum ground acceleration B versus hypocentral distance R for earthquakes of magnitude $M=5.5$ calculated from Eqs. (11) and (12). Points give observed accelerations from events with $M=5-6$ after Page *et al.* (1972)

From a comparison with observed accelerations (Fig. 5) in the magnitude range $5<M<6$ given by Page *et al.* (1972) we get the following values and uncertainties which seem more proper for Central Europe:

$$\begin{aligned} &\log\left(\frac{B_0}{\mathrm{cm/s^2}}\right)=3.1\pm0.1, \qquad &&c=0.80\pm0.05, \\ &R_0=(13\pm2)\,\mathrm{km}, &&d=2.00\pm0.08. \end{aligned} \tag{12}$$

Eq. (11) allows one to find the magnitude $M=M(B,R)$ that must not be exceeded in a focal distance R in order to get a maximum acceleration less than B:

$$M=M(B,R)=\frac{1}{c}\left[\ln(B/B_0)+d\cdot\ln\left(\frac{R+R_0}{\mathrm{km}}\right)\right]. \tag{13}$$

4.2. *Program WASEW*

The computer program WASEW transforms probabilities of earthquakes to probabilities of accelerations at the site.

WASEW subdivides the environs of a site S into elements of volume ΔV small enough to represent their distances to S by a point for each of them (see Fig. 6):

$$R=R(\Delta V,S)=\sqrt{h^2+\Delta^2}\,. \tag{14}$$

Given a certain value for the maximum acceleration B the quantity wanted is the probability that B is not exceeded at S. This means that M determined by Eqs. (13) and (14) is not exceeded in any ΔV. The corresponding probability is the product of probabilities extended over the environs of S:

$$\begin{aligned} &W(\text{max. acceleration}\leqq B)=\prod W(\text{magnitude}\leqq M\text{ in }\Delta V) \\ &=\exp\left[\sum\ln W(\text{magnitude}\leqq M\text{ in }\Delta V)\right]. \end{aligned} \tag{15}$$

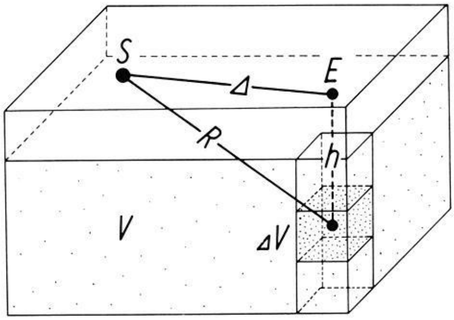

Fig. 6. Scheme to explain WASEW transformation of probabilities of earthquake magnitudes within a given crustal volume V to probabilities of accelerations at the site S. ΔV volume element, h depth of volume element, R distance between volume element and site. E epicenter, Δ epicentral distance

From Eq. (10) one finds that the contribution to the sum is zero for values of $M > M_{\max}$. For $M \leqq M_{\max}$ the probability is required that magnitudes are less than M in ΔV. Analogously to Eq. (10) one gets

$$W(\text{magnitude} \leqq M \text{ in } \Delta V) = \frac{G(M, \Delta V)}{G(M_{\max}, \Delta V)}$$

$$G(M, \Delta V) = \exp\left[-\exp\left(-\frac{M - u(\Delta V)}{a}\right)\right]$$

$$u(\Delta V) = u + a \cdot \ln\left(\frac{\Delta V \cdot W'(h)}{A}\right). \tag{16}$$

The last equation is analogous to Eq. (9) taking into account the depth distribution $W(h)$ (see Section 3.4). Using Eq. (16) one gets

$$\ln W(\text{max. acceleration} \leqq B)$$
$$= \frac{1}{A} \cdot \sum \Delta V \cdot W'(h) \cdot \exp\left(\frac{u}{a}\right) \cdot \left[\exp\left(-\frac{M_{\max}}{a}\right) - \exp\left(\frac{-M}{a}\right)\right]. \tag{17}$$

The sum has to be extended over the elements ΔV in the environs of S with $M \leqq M_{\max}$ determined by Eqs. (13) and (14) and has to take into account the different zones with different areas A and different values for a and u given in Fig. 1.

5. Results

The Northern Rhine area investigated has been covered by a regular net of 220 sites with 25 km grid width. For each site the probability distribution of the maximum ground acceleration has been calculated. The results for two typical sites are given in Fig. 7.

Much care has been devoted to the derivation of uncertainties for the input parameters and to the study of their influence upon the probability results. Monte-Carlo simulation using random numbers was applied to get the variation

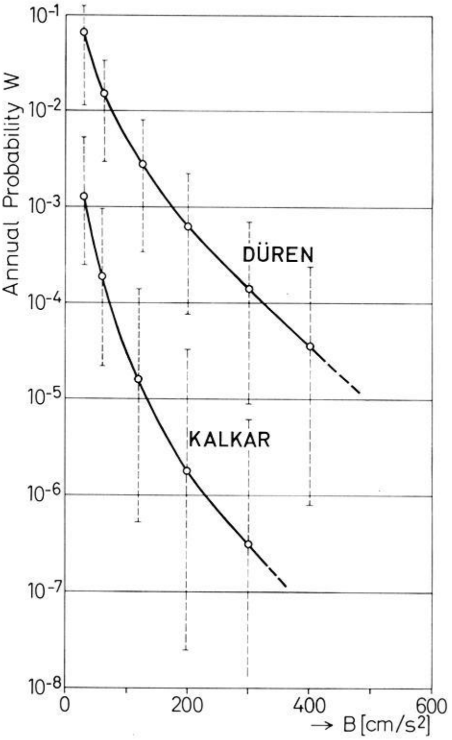

Fig. 7. Annual probability W of exceeding a maximum ground acceleration B at typical localities. Düren is situated in the western part of the Lower Rhine graben (region 5 with high seismicity). Kalkar is situated 40 km SE of Nijmegen (region 1 with very low seismicity, but still influenced by the nearby region 4). The given uncertainties are upper and lower standard deviations

of the resulting probability distribution caused by the uncertainties of the input parameters given in the text. The upper uncertainty of the final result – the probability of exceeding a given maximum ground acceleration at a site – was found to be less than the lower uncertainty (see Fig. 7), e.g. for Düren:

$$W(B > 120\ \mathrm{cm/s^2},\ 1\ \mathrm{year}) = 2.8 \cdot 10^{-3} \begin{cases} \leqq 4.6 \cdot 10^{-3} \\ \geqq 4 \cdot 10^{-4}. \end{cases} \tag{18}$$

The whole set of probability distributions for the 220 sites mentioned above is presented in two maps. Fig. 8 gives numerical results and isolines for the probability that $B = 100\ \mathrm{cm/s^2}$ is exceeded during one year. Fig. 9 shows the same for $B = 300\ \mathrm{cm/s^2}$.

6. Conclusions

A probability picture has been given describing quantitatively the earthquake risk relevant at a site. The numerical results are in good agreement with the more qualitative seismological concepts about earthquake risk which have been used

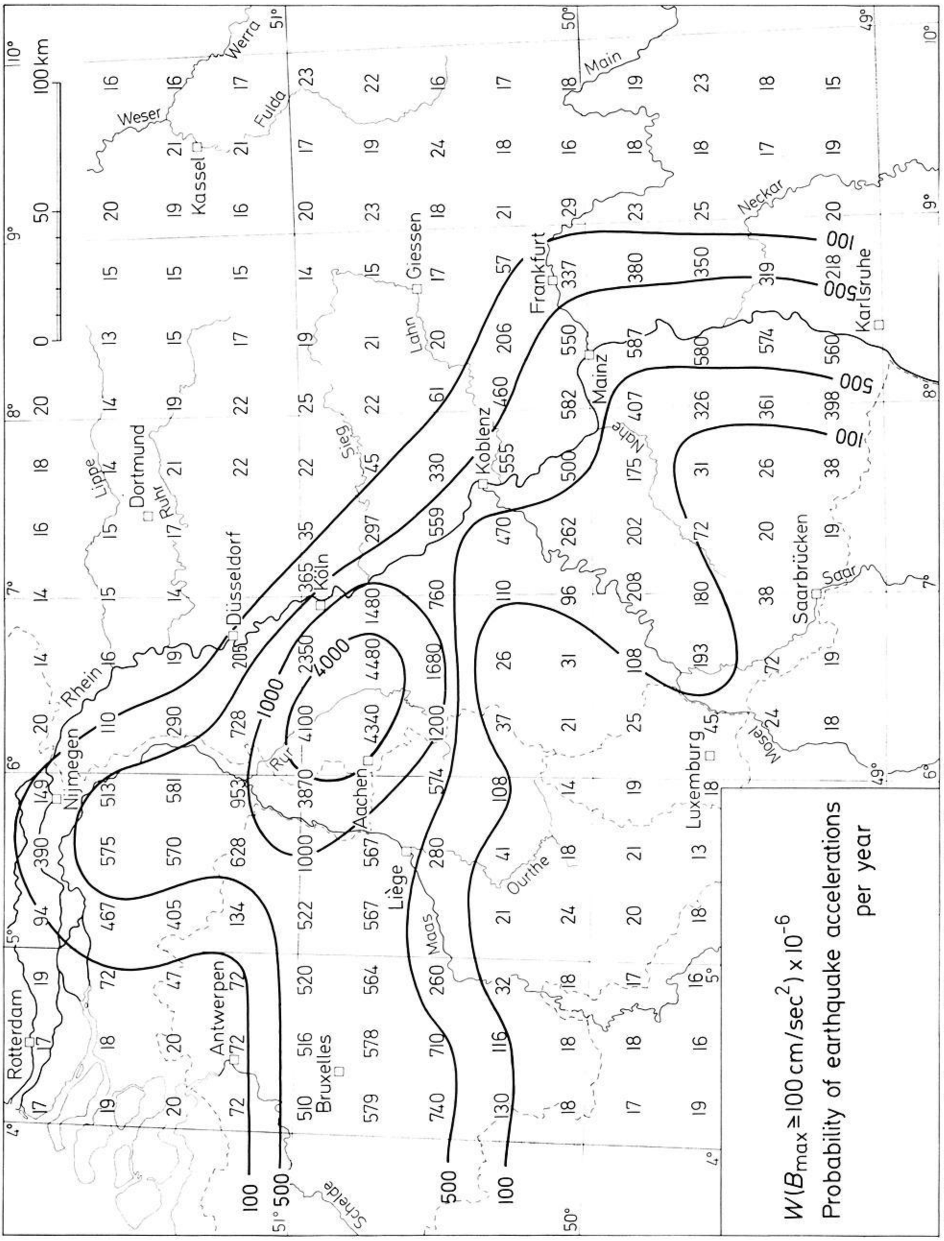

Fig. 8. Numerical probability values and isolines for exceeding $B = 100\,cm/s^2$ during one year

up to now. In spite of considerable uncertainties which have been calculated numerically the presented methods and results will help to improve the earthquake risk analysis on a consistent basis and will make possible comparisons with other risks.

To improve the probability results, further work is required with respect to the following topics:

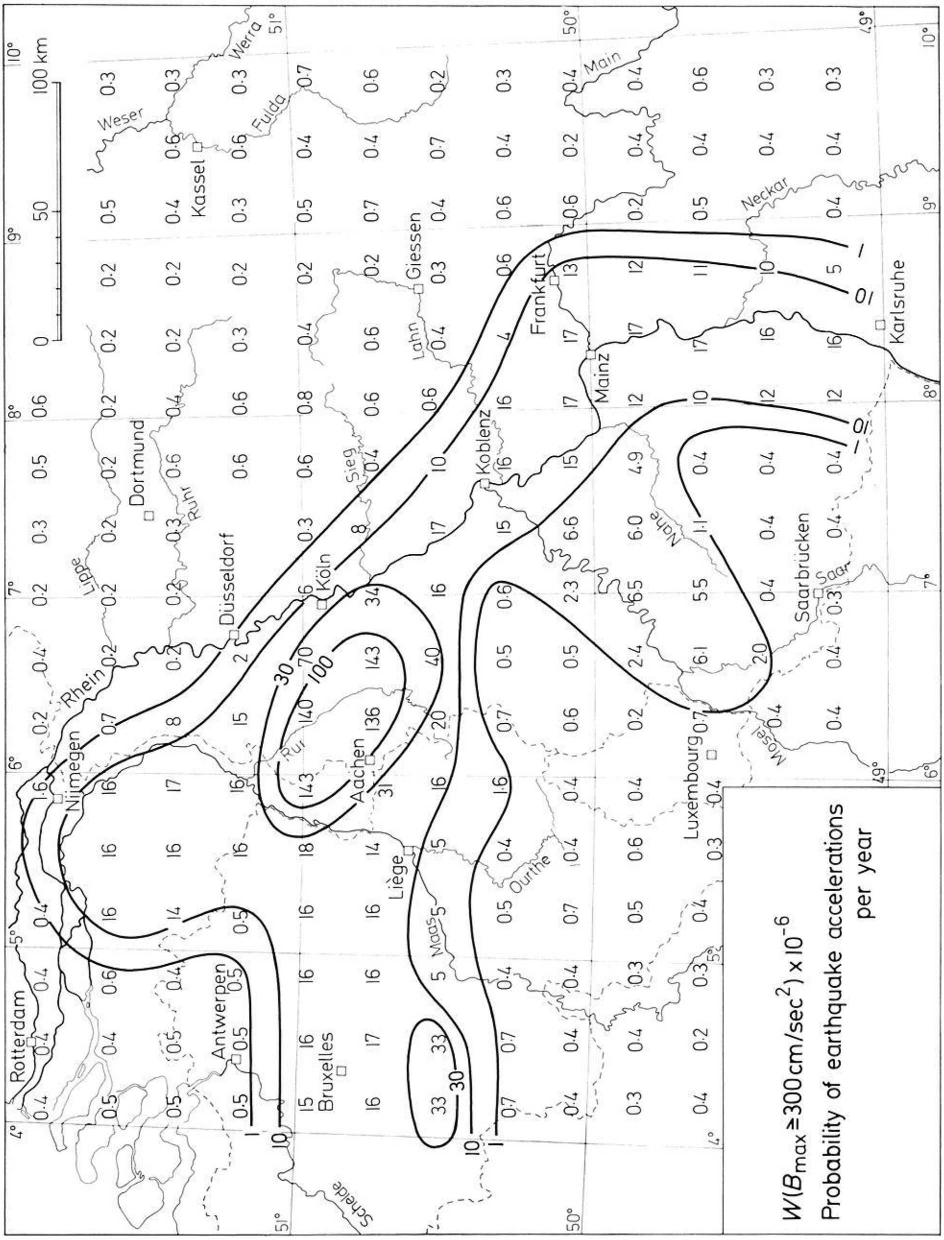

Fig. 9. Numerical probability values and isolines for exceeding $B = 300$ cm/s^2 during one year

– a more detailed subdivision of seismicity zones based on future seismological and seismotectonic investigation (e.g. micro-earthquake observations),

– inclusion of the focal volume increasing with magnitude (which has conservatively been neglected in the present model),

– derivation of a more exact transfer function for the maximum ground acceleration at sites in Central Europe,

– research concerning statistical models establishing a maximum magnitude in a region,

– expansion of the study to the whole area of the Federal Republic of Germany in order to get a unified treatment of earthquake risk.

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