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P Wave Amplitude Variability at NORSAR

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Abstract. A detailed study of *P*-wave amplitude variability within the Norwegian Seismic Array (NORSAR) has revealed that the amplitude pattern is repeatable for events from the same area, but can change rapidly as a function of source region. The difference between the largest and smallest amplitude recorded is on the average 14 dB, with 9.5 and 22 dB as minimum and maximum, respectively. The standard deviation of the amplitudes recorded inside this 110 km aperture array is of the same order as that of a world-wide network. There is no significant variation in noise level inside the array. Because of the large variations in signal amplitudes the performance for a particular region is crucially dependent upon the array configuration. Thus when selecting sites for new installations (single instrument or arrays) care should be taken in order to ensure that the site(s) selected optimizes the event detection performance with respect to the seismic regions of most interest. The rapid amplitude fluctuations observed demonstrate the necessity for adequate spatial sampling prior to inversion of amplitude observations in terms of Earth structure; for NORSAR and LASA (Montana, USA) instrument-to-instrument distances down to 5 to 10 kilometers are required. Currently only models of the random medium type or a block type structure seem able to explain the data satisfactorily.

Key words: Amplitude variations – Amplitude loss – Beamforming – Array configuration – Network density – Spatial sampling.

Introduction

The large scatter in short period *P*-wave amplitude observations has always been problematic for seismologists. In order to reduce this scatter in magnitude determinations using seismograph networks, refined and regionalized versions of the Gutenberg (1945) magnitude formula have been widely used. The statistical aspects of magnitude determinations, such as the problem of relevant sampling, has been studied by Husebye *et al.* (1974), who showed that the magnitude correction for a station may be a function of magnitude. In addition, *P*-wave amplitudes have also been widely exploited in studies of the distribution of anelastic parameters within the earth (*e.g.*, Anderson and Kovach, 1964; Anderson *et al.*, 1965; Kanamori, 1967), as well as studies of local upper mantle and crustal structures (Mereu, 1969; Larner, 1970; Landers, 1971). These studies generally assume that the amplitudes vary slowly and regularly in space. The large seismic arrays provide a valuable opportunity to examine the validity of this assumption.

P-wave amplitude variations are especially important insofar as they bear on the event detection capability of single stations or arrays. A large number of papers have been concerned with the problem of noise suppression, while only few works have drawn attention to the effect of amplitude variations. That this effect can be of considerable importance has recently been shown by Christoffersson and Husebye (1974).

This paper is directed to an analysis of NORSAR *P*-wave amplitude anomalies as a function of event location or seismic regions. In addition, an analysis of the noise level variations within the array aperture is presented. The impact of the results on the array's event detectability is examined. Special attention is given to the problem of deciding which array configuration maximizes the event detectability for different regions. The analysis presented demonstrates that when selecting sites for arrays or single stations, due consideration should be given to relative signal amplitude levels as well as the ambient noise field.

As mentioned previously, the fact that the large seismic arrays provide a dense spatial sampling of earthquake signal amplitudes gives an excellent opportunity to check different geophysical models proposed in order to explain the variations of these amplitudes. For instance, in this paper it will be demonstrated that "aliasing effects" may seriously affect the inversion of amplitude data in terms of earth structures unless the spatial sampling is adequate.

Array Beamforming Theory

The operational principles of seismic arrays are usually based on the assumption that the *P* signals are identical across the array while the noise is Gaussian and approximately uncorrelated from one sensor to another. For a signal/noise model of the above kind, simple delay-and-sum processing (beamforming) provides the optimum signal processing method. The expected gain in SNR is proportional to $\sqrt{N}$ where N is number of sensors in the array. In practice, the above assumptions on noise and signal properties are only approximately valid, resulting in an SNR beamforming gain a few dB less than that corresponding to $\sqrt{N}$. (As a general introduction to array data processing theory, we refer to Birtill and Whiteway (1965).)

An important task of the NORSAR array (for description, see Bungum *et al.*, 1971; Bungum and Husebye, 1974) is seismic surveillance, that is, real-time signal processing for the detection of seismic events. In this application, the simple beamforming technique is preferred on account of its computational simplicity, which is currently a requirement for real-time analysis of array data. In this section we will give a brief presentation of beamforming theory, emphasizing the gain in SNR when the basic assumption of identical signals is not strictly valid.

Assume we have a spatial arrangement of N sensors, and the output from these is sampled at certain fixed intervals. If the j-th sample from the i-th sensor is denoted a_{ij}, then the power of this sensor may be written

$$A_i^2 = \frac{1}{\mathrm{STA}} \sum_{j=1}^{\mathrm{STA}} a_{ij}^2$$

where STA ("short term average") is number of samples. The power of the sum of the N traces, S_{ab}^2, is the given by

$$S_{ab}^2 = \frac{1}{\mathrm{STA}} \sum_{i=1}^{\mathrm{STA}} (a_{1i} + a_{2i} + \cdots + a_{Ni})^2. \tag{1}$$

It is assumed that proper time delays have been introduced so the signals are in phase or correctly lined up. Performing the squaring operation in Eq. (1) one gets

$$\begin{aligned} S_{ab}^2 = \frac{1}{\mathrm{STA}} \sum_{i=1}^{\mathrm{STA}} (a_{1i}^2 &+ a_{1i} \cdot a_{2i} + \cdots + a_{1i} a_{Ni} + \\ a_{2i} \cdot a_{1i} &+ a_{2i}^2 + \cdots + a_{2i} a_{Ni} + \\ &\vdots \\ a_{Ni} a_{1i} &+ a_{Ni} a_{2i} + \cdots + a_{Ni}^2). \end{aligned} \tag{2}$$

Performing the summation one gets

$$\begin{aligned} S_{ab}^2 = (&A_1^2 + A_1 A_2 \rho_{12} + \cdots + A_1 A_N \rho_{1N} + \\ &A_2 A_1 \rho_{21} + A_2^2 + \cdots + A_2 A_N \rho_{2N} + \\ &\vdots \\ &A_N A_1 \rho_{N1} + A_N A_2 \rho_{N2} + \cdots + A_N^2) \end{aligned} \tag{3}$$

where A_i^2 denotes the power of trace i, and ρ_{ij} is the normalized correlation coefficient between sensor i and j. Eq. (3) may be written in matrix form

$$\begin{aligned} S_{ab}^2 &= \{A_1, A_2, \ldots, A_N\} \begin{Bmatrix} 1, \rho_{12}, \ldots, \rho_{1N} \\ \rho_{21}, 1, \ldots, \rho_{2N} \\ \vdots \\ \rho_{N1}, \rho_{N2}, \ldots, 1 \end{Bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ \vdots \\ A_N \end{Bmatrix} \\ &= A R_S A' \end{aligned} \tag{4}$$

where A is a row vector consisting of the elements A_i, $i=1$, N and R_S is the signal correlation matrix. A' is the transpose of A. If noise is present, the same formula still applies. For example, letting B be a row vector consisting of N elements where B_i^2 represents noise power for instrument i, and letting R_N be the noise correlation matrix, we find that the signal-to-noise ratio, SNR, on the summed trace may be calculated from the ratio

$$\mathrm{SNR}^2=\frac{A\cdot R_S\cdot A'}{B\cdot R_N\cdot B'}. \tag{5}$$

If all elements in A are equal to $\sqrt{\bar{a}^2}$ and $\rho_{ij}=\rho_s$ for $i\neq j$ in the matrix R_S, Eq. (3) gives

$$S_{ab}^2=\bar{a}^2(N+(N-1)\cdot N\cdot\bar{\rho}_S)=\bar{a}^2\cdot N(1+(N-1)\bar{\rho}_S). \tag{6}$$

If the same assumptions are valid also for noise, we get the familiar expression (Denham, 1963)

$$\mathrm{SNR}_{ab}^2=\frac{\bar{a}^2\cdot N\cdot(1+(N-1)\rho_S)}{\bar{n}^2\cdot N\cdot(1+(N-1)\rho_N)}. \tag{7}$$

For $\rho_s=1$ and $\rho_N=0$, this again reduces to $\mathrm{SNR}_{ab}^2=\bar{a}^2/\bar{n}^2\cdot N$; that is, the gain in SNR increases with $\sqrt{N}$.

Example 1. Let us assume we are summing two traces whose signal amplitude is given by A_1 and A_2 respectively. ρ_{12} is assumed equal to one for the signals and zero for the noise, and the noise level is supposed equal to $\bar{n}^2$ on both traces. The signal-to-noise ratio of the summed trace is then found from Eq. (5)

$$\mathrm{SNR}_{ab}^2=\frac{(A_1+A_2)^2}{2\bar{n}^2} \tag{8}$$

while the corresponding value for the first trace is $A_1^2/\bar{n}^2$. In order to have a gain in SNR by summing the two traces, one must have

$$\frac{(A_1+A_2)^2}{2\bar{n}^2}>\frac{A_1^2}{\bar{n}^2} \tag{9}$$

i.e., $A_2>(\sqrt{2}-1)\cdot A_1\approx 0.41\cdot A_1$. That is, if the amplitude of the second trace is less than 0.41 (7.7 dB) times the amplitude of trace one, adding the two traces does not give a gain in signal-to-noise ratio, even though they have exactly the same signal shape. If $\rho_{12}=0.7$, which is a reasonable value for P-signals recorded at NORSAR, the criterion is $A_2>0.51\cdot A_1$ in order to have a gain in signal-to-noise ratio by summing the two traces.

Example 2. Let us assume we have already summed 21 traces, all with the same amplitude, $\bar{A}$, and shape. The noise level is as before assumed equal on all traces and completely uncorrelated from one trace to another. The signal-to-noise ratio would then be $\mathrm{SNR}_{21}^2=\dfrac{\bar{A}^2\cdot 21\cdot 21}{n^2\cdot 21}$.

Adding to this a trace no. 22 of the same shape but with amplitude A_{22} gives:

$$\mathrm{SNR}_{22}^2=\frac{(\bar{A}\cdot 21+A_{22})^2}{n^2\cdot 22}. \tag{10}$$

Requiring SNR_{22} greater than SNR_{21} gives:

$$A_{22} > (\sqrt{22 \cdot 21} - 21) \cdot \bar{A} \approx 0.49 \cdot \bar{A}. \tag{11}$$

That is, in order to have a gain in signal-to-noise ratio, A_{22} must have an amplitude which is not less than 0.49 (6.12 dB) times the amplitude of the other traces.

Example 3. Let us assume that in the above case A_{22} is equal to zero. This gives $SNR_{22} = SNR_{21}\sqrt{21/22}$, that is, the result would be a loss of 0.2 dB in signal-to-noise ratio.

Example 4. Let A_{22} in Ex. 2 be equal to $\bar{A}$, but exactly 180° out of phase; this gives $SNR_{22}^2 = \frac{(\bar{A} \cdot 21 - \bar{A})^2}{n^2 \cdot 22}$. That is, the result would be a loss of 0.6 dB in signal-to-noise ratio.

Data

NORSAR is located in southeastern Norway (centered at 60.8° N, 10.83° E) and comprises 22 subarrays each containing three long period (V, NS, EW) and six vertical short period seismometers. The array configuration is shown in (Berteussen, 1975, p. 72). The short period data is searched in real time for earthquake and explosion signals. This is done by first using a recursive bandpass filter and then forming beams steered towards the most active seismic regions and the known underground nuclear test sites. Two types of beamforming (envelope and conventional) are used in order to ensure overall surveillance together with maximum performance in the most interesting regions. Conventional ("coherent") beamforming is the most important of these and involves simple delay-and-sum procedures using all instruments. Totally there are 318 such beams, their deployment in slowness space being shown in Fig. 1.

All *P*-waves recorded at NORSAR during 1972 and 1973 which did not saturate the recording system and had a signal-to-noise ratio (SNR) above 17 dB have been used in the present study. Before calculating the amplitudes the traces were filtered with a 1.2–3.2 Hz 3rd order Butterworth bandpass filter. In total, the data base consists of some 964 events. The computerized method of calculating amplitudes is described in Husebye *et al.* (1974). The data have been grouped according to which of the (coherent) Detection Processor (DP) beam locations (Fig. 1) the events were closest to. No events located more than 0.5 sec/deg (measured in slowness space) from any of the DP beams were used. As was observed by Husebye *et al.* (1974) the amplitude pattern is approximately stationary from one event to the other as long as the events are close enough in slowness and azimuth. This is demonstrated in Table 1, which lists the amplitude loss in dB relative to the best subarray for a set of events detected on DP beam location 188, *i.e.*, events close to the location 37° N, 71° E (Afghanistan-USSR border).

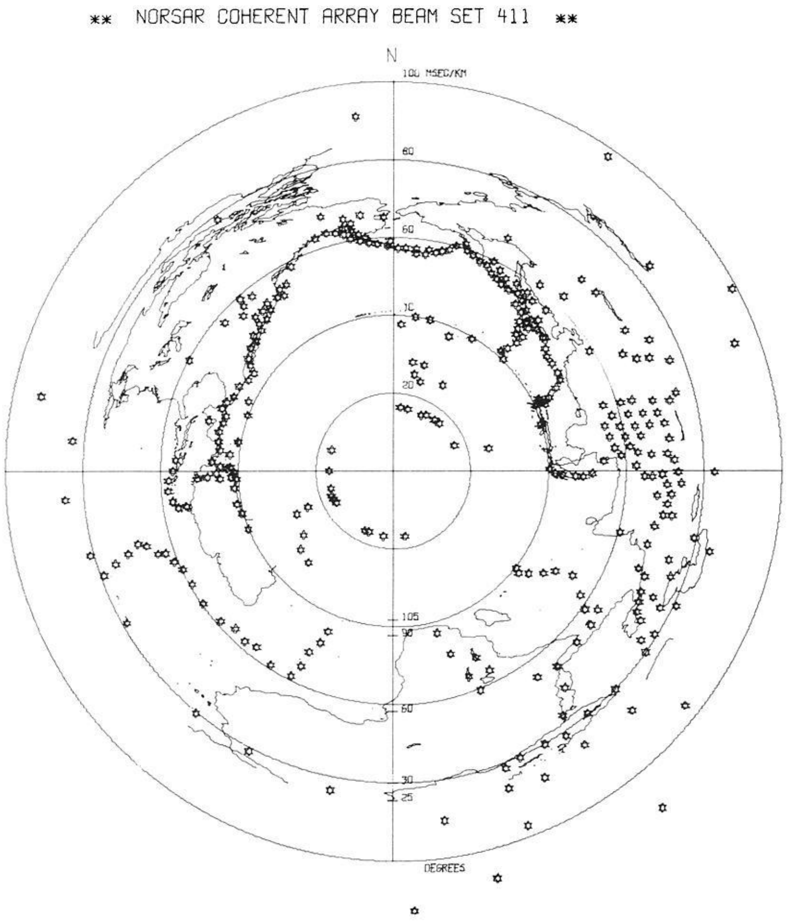

Fig. 1. NORSAR coherent Detector Processor beam deployment. The contours drawn represent the world map as seen in slowness space at NORSAR

The stability of the observed amplitude patterns has been tested using the Kendall coefficient of concordance (Siegel, 1956). Subarray 22 was excluded from analysis, and also those events where any of the other subarrays were out of operation. This Kendall coefficient is equal to 1.0 if the amplitude ranking pattern is reproduced from one event to another, but would be zero if the pattern is completely random. The value obtained was 0.83, while the chance of randomly getting such a large value is less than one in a thousand (0.001). The coefficient has also been calculated for the five DP beam locations where most data was available, namely, 52° N, 160° E; 43° N, 147° E; 11° N, 125° E; 28° S, 177° W; and 32° S, 179° W, respectively. The average value is 0.8 ± 0.1, and for any of the five beams the chance of randomly getting a value as extreme as that obtained is less than 0.001.

Although the amplitude pattern is stable for events coming from approximately the same location, the pattern may change drastically as one moves from

Table 1. Subarray and array beam (AB) amplitude losses in dB for events detected on NORSAR Detection Processor (DP) beam no. 188. This beam is pointed towards 37N, 71E (Afghanistan-USSR border). As seen from the table subarry 11 is best for all events analyzed. The loss values are relative to the best subarray

Event I.D.	Subarray Number																						
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	AB
40,800	18	6	8	–	16	16	18	11	5	4	0	10	7	18	9	19	21	16	12	5	10	15	11.2
41,530	15	5	8	9	14	14	–	12	4	4	0	10	9	18	9	14	18	16	12	4	10	14	10.4
78,890	16	6	8	10	13	12	22	14	5	1	0	11	7	16	10	15	19	17	12	8	9	13	11.3
17,310	14	6	8	6	12	19	18	11	7	6	0	14	6	17	11	13	14	16	–	6	13	11	11.2
65,000	14	7	7	10	15	14	14	10	4	7	0	14	9	14	9	15	15	13	11	5	11	–	11.8
74,660	13	8	7	4	12	11	18	13	4	5	0	13	7	16	11	9	12	14	13	9	7	15	13.6
49,570	17	6	8	8	16	15	17	11	5	3	0	10	7	16	9	20	18	–	12	4	12	18	10.8
53,910	14	5	8	7	16	18	15	10	4	8	0	8	6	14	9	15	21	15	9	4	7	–	10.0
79,370	14	6	5	9	13	14	13	11	5	7	0	11	9	15	6	13	15	18	12	6	11	13	11.8
2,180	12	6	7	8	13	14	14	10	5	6	0	14	8	18	8	–	18	18	11	6	12	14	12.0
32,550	15	5	6	5	14	11	13	8	2	5	0	11	7	16	2	15	14	15	7	1	12	18	9.3
38,270	18	6	8	9	15	14	17	11	4	5	0	12	7	17	9	17	19	17	13	6	11	16	11.7
79,230	12	6	9	8	–	14	17	10	3	5	0	10	6	13	10	14	17	14	12	8	9	9	11.2
85,910	15	5	7	8	18	14	17	9	4	5	0	11	7	18	7	15	15	15	12	5	11	15	10.7
52,240	11	7	7	7	16	15	17	10	8	8	0	7	7	17	11	13	–	13	9	6	12	11	11.1
17,630	14	9	9	9	18	16	15	12	5	6	0	6	–	17	12	12	18	15	13	5	12	12	12.4
69,380	13	–	9	10	16	15	12	14	6	5	0	9	7	14	11	13	13	17	13	4	12	12	12.7
53,970	14	7	9	10	16	13	15	11	6	3	0	8	6	14	11	13	14	16	11	5	9	–	11.1
58,810	12	4	7	8	16	12	17	11	7	1	0	10	6	14	10	14	20	15	12	5	10	–	11.7

one beam location to the next. This is demonstrated in Fig. 2, where the amplitude pattern for two DP beams steered towards Iran are shown. Both are steered towards a point with distance 35.1 degrees from NORSAR, but with azimuth slightly different, 117.0° and 123.7°, respectively, and as can be seen from the figure, this difference in azimuth has produced two entirely different amplitude populations. Also note that for these beams the difference in amplitude between the best and the poorest subarray is 15 and 19 dB, respectively, *i.e.*, a factor of 6 and 9 in amplitude ratio inside an array of radius 50 km. These are representative values for events coming from east and north, while events from the west usually have somewhat less amplitude differences. This can also be seen from Table 2 where we have listed the average of the observed amplitude values measured in dB down from the best subarray for some selected DP beams. A maximum difference of 22 dB, corresponding to a factor of 13 in ratio between the highest and lowest amplitude, was observed for a beam steered towards the Kirgiz-Sinkiang border region, while the minimum difference of 9 dB, corresponding to a factor of 3, was observed for a PKP beam steered towards South Sanwich Is. region. As an average for all regions there is 14 dB (a factor of 5) in amplitude differences within the array aperture. It should also be mentioned that we here are measuring subarray beam amplitudes. If we measured single sensor amplitudes, the results would be even more extreme. The standard deviation of the amplitude recordings for the different events are (4.4 ± 2.1) dB, corre-

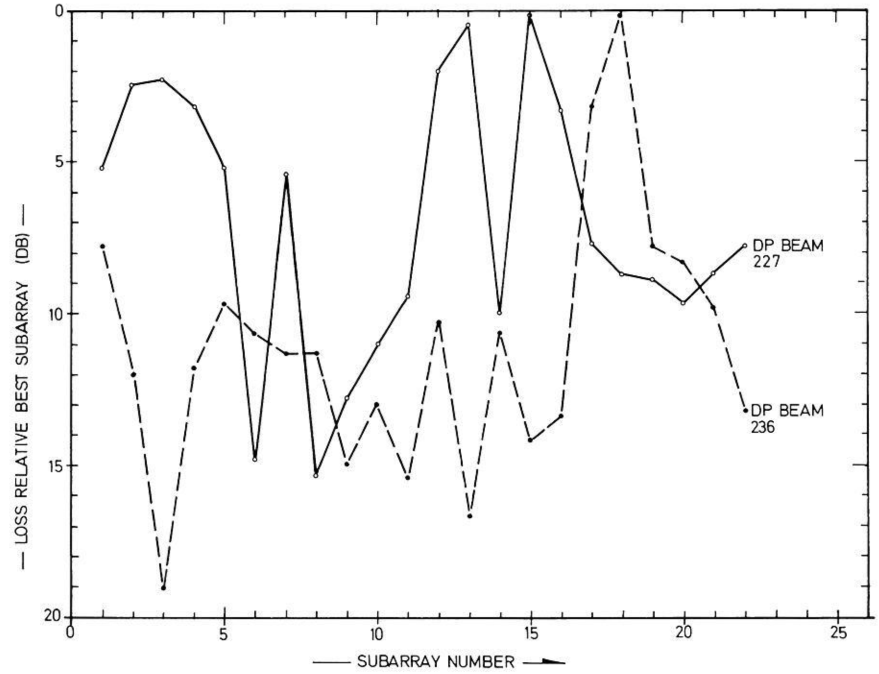

Fig. 2. Average signal amplitude measured in dB down from best subarray for signals coming from two different areas in Iran, DP beams 227 and 236 respectively. DP beam 227, distance = 35.1, Azimuth = 117.0, --------- DP beam 236, distance = 35.1, Azimuth = 123.7

sponding to (0.22 ± 0.11) MB units. Using all instruments Dahle (1975) reports an average standard deviation of 0.4 MB units. These values should be compared with for example the standard deviation of 0.356 MB units Veith and Clawson (1972) found using the amplitudes from different explosions as recorded by a world-wide network of stations. The last row in Table 2 gives the amplitude values of the full array beam relative to the best subarray. As an average the normalized amplitude of the full array beam is 8.8 dB below the best subarray. But because the array beam has a beamforming gain of 13.4 dB (due to noise suppression) it has in average 4.6 dB better SNR than the best subarray beam.

As a further example of the variability of the amplitude pattern, it is found that all the subarrays have at least one DP beam location where it is the best subarray, and also at least one location where it is the poorest subarray. This can be seen from Fig. 3, which is a histogram showing for each subarray how many DP beams it has the maximum and how many it has the minimum amplitude, respectively. Thus, although some of the subarrays are good for most of the seismic regions, there always exists a region where they are the poorest subarrays. In Fig. 4 the amplitude pattern when averaged over all DP beams where any data was available is shown. Because of the variability of the amplitude pattern, the difference between the best-in-average subarray (05C or no. 13) and the poorest (14C or no. 22) is only 5 dB, while the difference between the best and the poorest subarray for single beams was 14 dB on the average.

Table 2. Average subarray and array beam (AB) amplitude losses in dB for some selected DP beams. It should be noted that the beams selected for each region are only one out of several

Beam no.	91	175	195	23	254	13	115	227	236
Region (Flinn and Engdahl)	S. of Honshu	Northern Colombia	Afghanistan	Fox Is. Aleutians	C. Mid-Atlantic	S.E. Alaska	Samar. Philip.	Iran	Western Iran
Distance	81.1	80.8	46.4	66.5	64.6	58.2	91.8	35.1	35.1
Azimuth	44.2	268.0	99.6	358.6	212.3	346.5	63.7	117.0	123.7
01A 1	7	1	17	14	7	3	4	5	8
01B 2	6	1	4	13	4	9	11	3	12
02B 3	5	10	12	7	7	6	9	2	19
03B 4	13	7	12	13	5	13	9	3	12
04B 5	7	4	7	13	13	8	7	5	10
05B 6	13	5	11	15	11	9	9	15	11
06B 7	10	3	15	4	3	9	6	5	11
07B 8	6	6	12	9	7	6	5	15	11
01C 9	2	9	5	7	12	7	3	13	15
02C 10	3	5	7	8	5	8	3	11	13
03C 11	7	10	0	0	2	7	3	9	15
04C 12	2	3	6	3	3	8	3	2	10
05C 13	0	9	3	0	12	5	6	1	17
06C 14	13	7	8	10	10	12	10	10	11
07C 15	11	8	9	8	11	12	5	0	14
08C 16	14	2	10	5	8	7	3	3	13
09C 17	12	11	14	7	7	7	11	8	3
10C 18	11	10	18	8	6	9	3	9	0
11C 19	10	11	11	3	1	8	2	9	8
12C 20	12	8	9	5	9	5	8	10	8
13C 21	14	6	6	9	9	10	3	9	6
14C 22	18	10	16	5	10	0	10	8	14
AB	9	8	10	8	10	9	8	9	13

If one wishes to use Eq. (5) to estimate how the SNR will vary as a function of array configuration, one also needs information about the noise fluctuations across the array, in addition to signal amplitude and correlation information. For approximately half of the events used in this study noise estimates were therefore also obtained. This was done by calculating for each subarray the power inside a $^1/_2$-minute long window, ending 5 seconds before the signal onset. The loss value for each subarray was calculated by subtracting its power (in dB) from the power of the subarray with highest noise level. These values were then averaged over the number of events available and found to range from 0.49 ± 0.30 dB to 0.54 ± 0.31 dB, the maximum difference in noise level across the array is in average only 0.05 dB.

Cultural noise is known to be significant at NORSAR (Bungum and Ringdal, 1974); one could therefore expect that certain subarrays in the neighborhood of towns and major roads could have higher noise levels than the rest at certain

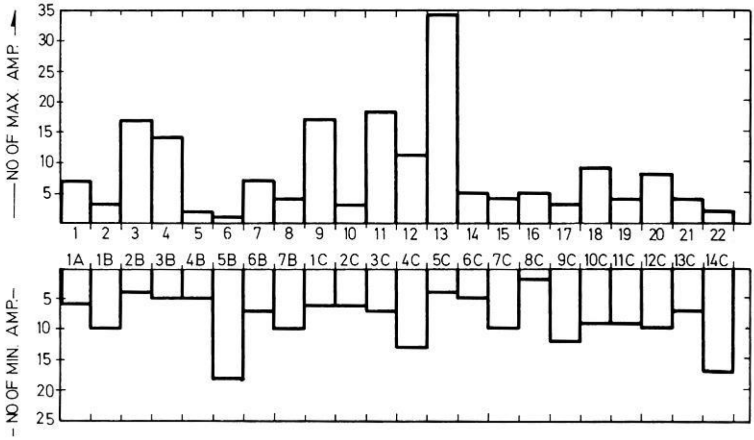

Fig. 3. Histogram showing for how many regions (DP beam locations) a subarray has the highest and for how many it has the lowest amplitude respectively

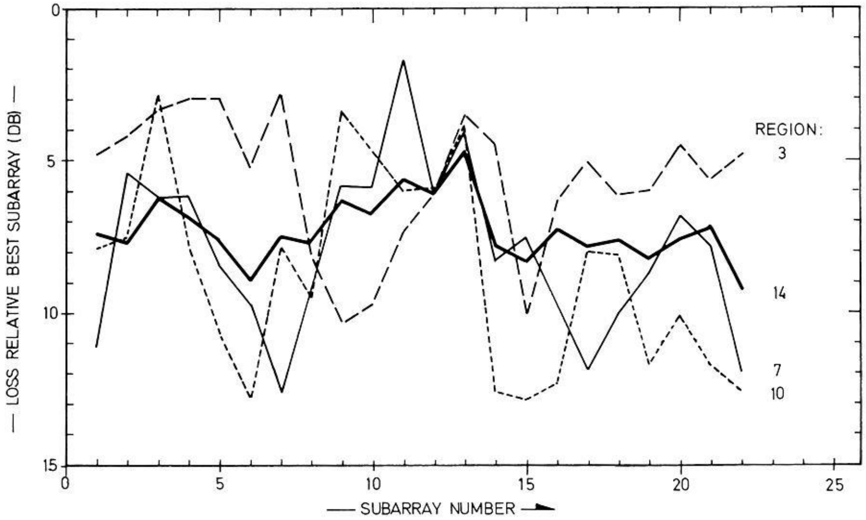

Fig. 4. Average amplitude loss values for some of the regions used in Table 3. The thick line is for Region 14, which covers distance range 30° 180° from NORSAR

times during the day. However, by grouping the noise measurements according to time of day, and then averaging within four hour intervals, no such effect was found. It should here be mentioned that this is bandpass filtered data (1.2–3.2 Hz); if no filter had been applied, the above statement may not have been true. Also subarray 22 (14C) is known to have high noise level due to a neighboring power plant (Hjortenberg and Risbo, 1975). This effect could not be observed; the reason for this is believed to be partly the filter applied and partly that this subarray has been masked in the noisiest periods.

The conclusion is thus that compared with the signal amplitude differences, the effect of the noise level variations across the array safely can be neglected. Thus the vector *B* in Eq. (5) can be considered to consist of only equal elements. It should be kept in mind that we here have been considering only noise level variations inside the array using filtered data. There are undoubtedly significant variations both in absolute noise level and in the noise spectra as a function of time. These effects are discussed in detail by Bungum and Ringdal (1974) and also Steinert *et al.* (1975).

Event Detectability

As mentioned in the previous section, the array beam amplitude has been calculated and compared with the best (*i.e.*, largest amplitude) subarray. Using Eq. (4) it is possible to calculate the average signal correlation ($\bar{\rho}_s$) between the different subarrays, assuming that the array beam loss is caused solely by imperfect correlation. With the $\bar{\rho}_s$ value known and under the additional assumptions that the noise is uncorrelated from one subarray to another ($\bar{\rho}_n = 0$) and that the noise level is equal at all subarrays, it is possible, by using Eq. (5), to calculate expected SNR loss on the array beam for the case that one or more subarrays are excluded from the beamforming. The procedure has been to calculate the expected SNR for each DP beam when successively more subarrays are excluded, starting with the exclusion of only the smallest amplitude subarray.

In Fig. 5 the expected performance for beam no. 91 (South of Honshu) is shown as a function of number of subarrays, where the subarrays have been ranked according to their amplitudes. For this beam, using only the best subarray (05C), one would have a loss of 4.6 dB relative to the current array beam. Using the 15 best subarrays would give a gain of 0.15 dB. For this beam it is seen from Table 2 that the difference in amplitude between the best subarray (05C) and the poorest one (14C) is as much as 18 dB. In contrast, for beam 175 the difference in amplitude between the best (01A) and the poorest (09C) is only 10 dB. Fig. 6 shows that there is no gain by excluding any of the subarrays in this case.

It has been found that for more than 90% of the DP beams one or more subarrays could be excluded without decreasing the SNR of the array beam signal. For 70% of the beam locations three or more subarrays could be excluded. Only 9% of the beam signals would suffer a loss in SNR by excluding the poorest subarray. As could be expected from the previous discussions (see Examples 2, 3 and 4), there is seldom any significant gain in SNR by deleting subarrays. Only for 2% of the DP beams is it possible to obtain an SNR gain of 0.4 dB or more, while for 18% of the cases a gain of 0.2 dB or more is possible.

Fig. 7 shows the variation in SNR of the array beam signal if a certain number of subarrays are permanently excluded. Because of the small subarray amplitude variations when averaging over all beam locations, a net loss would be observed even by excluding only the very worst subarray.

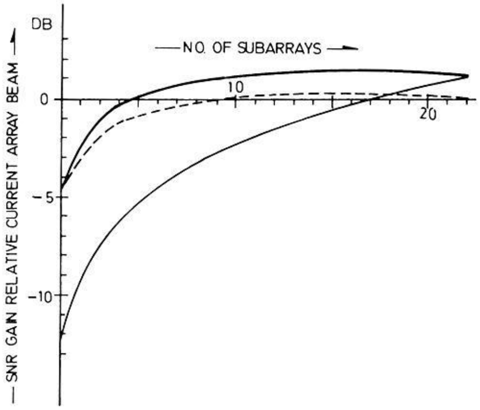

Fig. 5. Expected signal-to-noise ratio as a function of number of subarrays for DP beam 91, pointing towards 29N, 139E (South of Honshu, Japan). The subarrays have been ranked according to their amplitude. The first point on the horizontal axis corresponds to subarray no. 13, which has the highest amplitude in this case. The bottom line shows the theoretical $\sqrt{N}$ performance for the case of identical signals. The relative gain here using all 22 subarrays is 1.2 dB, which implies that the observed array beam signal suffers an average loss of 1.2 dB. The upper line gives the SNR improvement for the case of identical subarray signal shapes, but with the amplitude distribution listed for beam no. 91 in Table 2. The broken line gives the gain in SNR when the signal correlation is in average 0.75

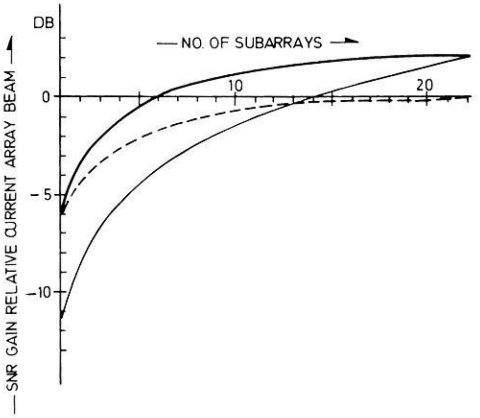

Fig. 6. Same as Fig. 5 for DP beam no. 175 pointing towards 7N, 73W (Northern Colombia). Subarray no. 1, which is best, is 5.9 dB below the array beam. The maximum difference in amplitude between two subarrays is 10 dB. The broken line shows that nothing would be gained by excluding any of the poorest subarrays. The amplitude loss of the currently used array beam is 2.1 dB

Excluding only subarray 14C will, in this case, give a loss of 0.1 dB. If both subarrays 14C and 05B (which has on the average the second smallest amplitude) are excluded, the average loss would be 0.2 dB. Excluding half of the subarrays gives a loss of 2.1 dB on average. For some particular regions, the exclusion of some subarrays would of course be more detrimental.

Fig. 7. Same as Fig. 5, but the data has been average over all beam locations. The array beam loss is seen to be 1.6 dB. Subarray no. 13 is best and is 9.3 dB below the full array beam signal. The broken line shows that if only the 11 best subarrays were used, the loss would be 2.1 dB

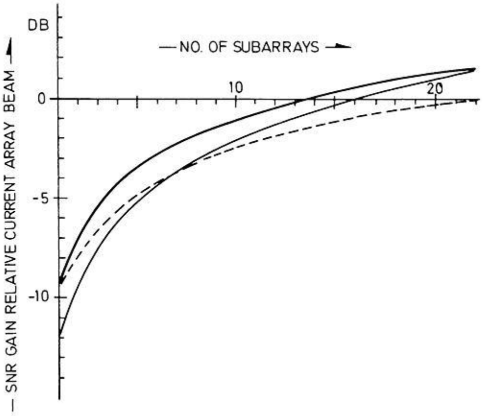

Table 3. Expected SNR loss for a set of large regions (defined by Bungum and Husebye, 1974) as a fuction of no. of subarrays removed, starting with the on average smallest amplitude subarray. The bottom row gives the corresponding theoretical values

Region	Area of coverage	No. of subarrays removed				
		1	2	5	10	21
1	Aleutians-Alaska	0.22	0.30	0.64	1.12	7.22
2	Western North America	0.15	0.25	0.78	2.44	13.71
3	Central America	0.22	0.44	0.94	2.46	10.12
4	Mid-Atlantic Ridge	0.15	0.23	0.89	2.57	13.62
5	Mediterranean-Middle East	0.22	0.47	0.97	2.03	8.62
6	Iran-Western Russia	0.10	0.24	0.74	2.17	8.72
7	Central Asia	0.04	0.14	0.67	1.53	8.02
8	Southern-Eastern Asia	0.05	0.10	0.78	1.62	7.52
9	Ryukuo-Philippines	0.06	0.09	0.63	1.23	6.72
10	Japan-Kamchatka	0.04	0.06	0.15	1.05	7.82
11	New Guinea-Hebrides	0.09	0.48	1.27	2.90	16.42
12	Fiji-Kermadec	0.12	0.57	1.35	2.61	17.02
13	South America	0.06	0.47	0.95	2.66	11.62
14	Distance range 30°-180°	0.11	0.24	0.75	1.90	9.32
	Theoretical ($\sqrt{N}$) loss	0.20	0.41	1.12	2.63	13.42

In Table 3 some of these loss values are listed for a set of larger regions (defined by Bungum and Husebye, 1974). Excluding only subarray 14C would in average give a loss considerable lower than the 0.2 dB expected from the $\sqrt{N}$ performance. However, for the Aleutian-Alaska region (1) and the Central American region (3) the loss values are slightly above the theoretical values, because subarray 14C tends to have relatively high amplitudes for these regions. Actually subarray 14C which on average has the smallest amplitudes, has the very highest amplitudes for DP beam nos. 11 and 13 which are both steered towards southeastern Alaska. Generally it can be seen that excluding from one to twenty-one subarrays starting with the poorest gives loss values below the theoretical values; however, because of the rapid variation in the amplitude pattern, there are always some regions where this is not true. For example, using only the very best subarray, 05C (*i.e.,* masking the other 21), one would for the Western North America, the Mid-Atlantic Ridge, the New Guinea-Hebrides and the Fiji-Kermadec regions (2, 4, 11 and 12) have a performance which is poorer than that expected from the theoretical loss values. Thus, if one were to use only one subarray, and if one were particularly interested in, for example, region 2 (Western North America), this subarray (05C) would not be a good site to choose, even though it in average is the best one.

Discussion

By measuring the subarray and array beam amplitudes of a large number of events, a set of relative amplitude values has been established for most of the NORSAR Detection Processor beams. In addition, these amplitudes have been used to establish those subarrays which could be beneficially masked and the gain in SNR achieved by such masking. Similar results have also been obtained on a regional basis. These results derived depend on the validity of certain simplifying assumptions; these assumptions are examined in turn in the following paragraphs.

The assumptions that the relative subarray amplitudes for a particular beam location were independent of event magnitude is not necessarily true due to the combined effect of the spectrum scaling law (Aki, 1967) and a frequency dependent crust and upper mantle transfer function. For example, Husebye *et al.* (1974) found that the body wave magnitude correction for conventional seismograph stations could vary with event magnitude. To test whether such effects are significant in the amplitude variation across NORSAR, events corresponding to the five most active beam locations were separated in two equally large populations according to their magnitude. The hypothesis that the relative amplitudes for these groups were identical was tested using both the sign test and the Wilcoxon matched-pairs sign-rank test (Siegel, 1956). These tests revealed that this hypothesis had to be rejected at the 0.05 confidence level for all the five samples considered, thus supporting the hypothesis that the relative amplitudes may change as a function of magnitude. The data indicated that the large events had more extreme amplitude differences than the small events. However, by excluding the 3–5 weakest subarrays, we could no longer reject the hypothesis. The latter result seemingly contradicts the first one, but can be explained as follows. For the groups of small magnitude events, the amplitudes of the weakest subarray signals are less than or of the same order as those of interfering noise wavelets, so the observational data for these particular subarray may be erroneous. Thus, when removing the most unreliable observations, the hypothesis of identical relative amplitudes for the two groups cannot be rejected. From

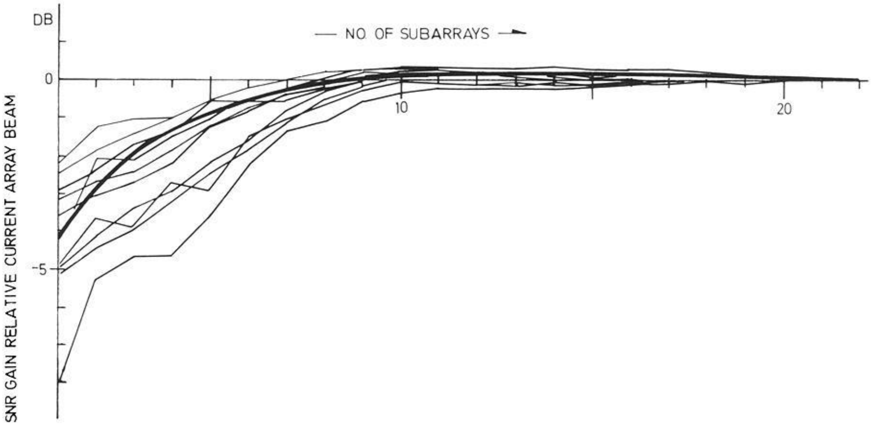

Fig. 8. Expected and obtained performance as a function of number of subarrays for DP beam no. 36. This beam is pointed towards 52N, 160E (off east coast of Kamchatka). The subarrays used have been ranked according to their amplitude. The first one is the one with the highest amplitude, in this case no. 3. The thick line shows the expected performance from the observed amplitude pattern, and corresponds to the dotted line on Figs. 5, 6 and 7. The thin lines give the observed values for 10 different events

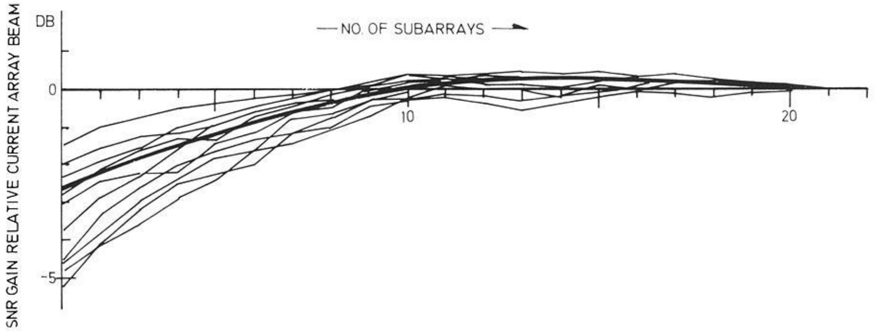

Fig. 9. Same as Fig. 8 for DP beam 63. This beam is pointed towards 43N, 147E (Kurile Islands)

the above results, we concluded that the relative subarray amplitudes do not vary significantly with event magnitude for the type of data considered.

It has also been assumed that all the off-diagonal elements in the matrix R_S in Eq. (4) were equal to the average signal correlation value. This is undoubtedly an oversimplification, as demonstrated in Figs. 8 and 9. The procedure here was first to measure SNR for the best subarray signal, then to add the second best subarray and measure SNR again. The process was repeated until all 22 subarrays were included and thus corresponding to the full array beam. Figs. 8 and 9 show the results obtained for beam locations 36 and 63 using 10 different events in each case. All events are from 1974, *i.e.*, none of them have been used in the previous data collection. The SNR variation is large when less than 10 subarrays are used in the beamforming process, but levels off rapidly as more subarrays are included. For example, the observed SNR variation was always less than 0.5 dB when the array beam is based on minimum 15 subarrays. The corresponding value for the case of 18 subarrays was 0.3 dB. These results have been interpreted as follows. The signal correlation matrix (Eq. (4)) does not consist of only equal elements in the off-diagonal locations, thus the SNR of the summed traces is not a function of relative amplitude alone but also of the varying correlation between subarrays. When many instruments are used, say 15 or more, the effect of the scatter in the correlation values becomes less important, and for this case the assumption that all off-diagonal elements are equal is acceptable. When all 22 subarrays are used, the two different matrices have the same effect in this context.

Finally, it has been assumed that the noise level is constant across the array, and that the noise is uncorrelated from one subarray to the other. This latest assumption is reasonable in view of previous NORSAR array studies, see for example, Felix *et al.* (1972) and Harley (1972), as well as the noise level measurements made in this study.

So far we have discussed the gain in SNR on the array beam level with respect to a 1/0 (one-zero) subarray weighting scheme. The reason for this is that the beamforming algorithm in the NORSAR on-line Detection Processor is limited to 1/0 weights. However, more flexible models could improve the gain in SNR of the array beam even more. This problem has recently been discussed by Christoffersson and Husebye (1974) who also described different weighting procedures which all are optimal under certain conditions. See also Birtill and Whiteway (1965). For example, using a model based on identical signals except for an unknown amplitude scaling factor, Christoffersson and Husebye (1974) obtained a relative gain in SNR of approx. 2.5 dB for events located in Japan and Central Asia. In order to illustrate this weighting technique, we consider a case with two subarrays having signal amplitudes $A_1=1$ and $A_2=2$. Straight summation of the two traces gives an SNR value of:

$$\mathrm{SNR}_{ab}=\left[\frac{(1+2)^2}{1^2+1^2}\right]^{\frac{1}{2}}=2.12. \tag{12}$$

Assigning weights of 0.45 and 0.89 to the traces gives:

$$\mathrm{SNR}_{wab}=\left[\frac{(0.45\cdot 1+0.89\cdot 2)^2}{0.45^2+0.89^2}\right]^{\frac{1}{2}}=2.24. \tag{13}$$

That is, there is a relative gain in SNR of 0.46 dB by introducing individual subarray weights. These weights have the same general characteristics as the previously discussed 1/0 weights, which means that the gain is largest when the amplitude variations are most extreme. The procedure for calculating such signal amplitude weights is usually too complicated for on-line data processing, but instead we may introduce predetermined subarray amplitude weights. This alternative is not optimum but should still yield a relative gain in SNR; for the single beam locations an average gain of 0.72 dB is to be expected. For several of the areas with large amplitude values this gain can be as large as 1.3 to 1.8 dB. As mentioned above, calculating individual amplitude weights for each event may give significantly better results, and this more sophisticated version of array beam forming has been implemented in the NORSAR off-line Event Processor.

For several regions the amplitude variations within the NORSAR array are as much as 20 dB, but only exceptionally is a relative gain of more than 0.3 dB obtained in SNR by excluding one or more of the subarrays in beamforming. Moreover, as the subarray amplitude pattern may change drastically within a small seismic region, any type of weighted array beamforming should be a function of the individual beam locations, *i.e.*, different array configurations for different regions. Although some of the subarrays are bad for most of the seismic regions covered by NORSAR, they all have several regions where they contribute positively to the array beam. Therefore, in average there will always be some loss by excluding any of the subarrays consistently. It should be noted that we have been measuring subarray beam amplitudes. If we instead had measured the amplitudes on single sensors, even more drastic variations would have been found. Also, it should be added that there is no good reason to believe that the NORSAR site is unique in regard to the large amplitude variability; data from the LASA array exhibits variations of the same order of magnitude (Berteussen *et al.*, 1975).

When planning the installation of seismic arrays a large effort is invariably devoted to the problem of noise suppression; measurements are generally made of noise correlation as a function of frequency and spatial lag. The array configuration is then made such that maximum noise suppression is achieved when other factors like signal correlation and event location capabilities, etc., also have been considered. However, the fact that signal amplitude variations inside the area of interest may down or upgrade the capabilities of the array significantly are usually not given much attention. Most works show that the short period seismic noise behaves fairly well as expected, *i.e.*, for instruments located more than a specific distance (usually 2–4 km) from each other the $\sqrt{N}$ noise reduction is approached. Also, at least for NORSAR, there is not any significant noise level variation across the array. Thus it is reasonable to believe that the noise behavior does not depend much on the exact instrument location, as long as certain instrument-to-instrument distance rules are obeyed. As has been shown, however, this is not so for the teleseismic signals.

Conclusion

It has been found that there may be more than 20 dB amplitude variations inside the NORSAR array, and as much as 10 dB in amplitude difference between instruments located less than 15 km from each other. In order to say something about the signal amplitudes at a certain site, it is thus possible to extrapolate only from very close-by instruments. If one is constructing an array where one of the main concerns is good signal detectability, the problem of finding the particular instrument sites which give best amplitude performance should be investigated. As it turns out, the relative signal amplitude may depend highly on the seismic region. Thus the event detection capability of a station will vary with region. This pattern may, however, be determined before the array configuration is fixed if careful amplitude studies using a lot of instruments (or by moving them around) are performed. It has been demonstrated that the amplitude pattern is very repeatable from one event to the other for events from the same location. Therefore, each test site would have to be operated just long enough to get a few good signals from the most interesting regions.

As mentioned previously, a large number of papers have been concerned with the investigation of observed *P*-wave amplitudes. Except for a few studies of array data, most of these works have been on data recorded at conventional stations which have had separations much larger than those of the NORSAR subarrays. This is a very important point because this type of sampling may not reveal the true character of the space-variations of the amplitudes. From the data presented herein it is obvious that an amplitude measurement may often not be representative for distances as large as 10–15 km, also the standard deviation of the amplitudes recorded at NORSAR has been found to be of the same order as that of a world-wide seismic network. Thus observations which are inverted, for example to determine Q structures, may be crucially dependent on the density of the observation network. This conclusion is consistent with the results of Aki (1973) (LASA), Capon (1974) (LASA), Dahle (1975) (NORSAR), and Berteussen *et al.* (1975) (NORSAR and LASA). These authors have performed a random medium (Chernov, 1960) analysis for NORSAR and/or LASA and all have reported that the transverse autocorrelation both for amplitude and phase falls to $1/e$ (0.37) of its maximum for a spatial lag of 10 to 20 km. While the random medium interpretation of the results is not necessarily unique, the observational fact that the anomalies are correlated only for distance of 10 to 20 km can, however, not be disputed. If this is taken as the minimum anomaly wavelength one would like to cover, one thus needs a spatial sampling rate of 5 to 10 km to ensure that no aliasing occurs. Finally we remark that only models of the random medium type or a block type structure as proposed by Aki *et al.* (1975) currently seem able to account for the rapid fluctuations of the observed amplitudes.

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