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On the Propagation of Seismic Pulses in a Porous Elastic Solid *

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Abstract. On the basis of Biot's theory the one dimensional problem of the propagation of seismic pulses in a porous elastic half space is studied. Using the Laplace transform, the solution is obtained in the wave front and long time approximations. The S pulses are shown to be attenuated as in a particular visco-elastic solid, while the P pulses, in a quasi-compatibility condition, exhibit a purely elastic propagation.

Key words: Dynamic poroelasticity – Transient waves – Wavefront and long time approximations.

1. Introduction

The theory of wave propagation in fluid-saturated porous solids has been developed by Biot (1956a, b, 1962a, b) in a series of papers. Some geodynamic applications of Biot's equations have been considered by several authors, e.g. Geertsma and Smit (1961), Jones (1961), Deresiewicz (1960, 1961, 1962), Deresiewicz and Rice (1962, 1964), and recently by Paul (1976a, b) and Burridge and Vargas (1976).

In most of the works the geometrical effects, taken into account, have obscured the actual role of poroelasticity on the propagation of seismic pulses. In order to single out the effect of poroelasticity, we consider the simple boundary value problem of plane waves propagating in a half space with input conditions on the free surface.

We work in the most general framework of Biot's equations (1962b) which are discussed in Appendix A. The systems for the rotational and dilatational waves will be written in non-dimensional variables and treated by the Laplace transform. The analytic solutions cannot be obtained, and interest is focused on the wave speeds and on short and long time approximations.

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We show that the S pulses propagate as in a particular visco-elastic solid and the P pulses as in a perfectly elastic solid with suitable density and modulus.

As a consequence we suggest that the introduction of geometric effects can be better achieved starting from these simpler equivalent models.

2. Statement of the Problem

The basic equations for S and P waves derived in Appendix A can be put in a more convenient form by using nondimensional variables. It is natural to introduce a characteristic time $t_0 = a^2/v$ where a is the diameter of the pores and v the kinematic viscosity of the fluid, and two characteristic velocities for the S and P waves, $v_{os} = (\mu/\rho)^{1/2}$ and $v_{op} = (H/\rho)^{1/2}$, where ρ is the mass density of the aggregate and μ, H are elastic moduli. Finally, the new spacetime variables read

$$t' = t/t_0 \qquad x'_j = \begin{cases} x_j/(v_{os}\, t_0) & \text{for } S \text{ waves} \\ x_j/(v_{op}\, t_0) & \text{for } P \text{ waves} \end{cases} \qquad (j = 1, 2, 3) \tag{2-1}$$

ρ becomes unit density, μ and H become unit moduli for S and P waves, respectively.

With this new set of variables and dropping the superscripts the basic systems read

$$\begin{Bmatrix} \nabla^2 \omega \\ 0 \end{Bmatrix} = \begin{Bmatrix} 1 & -\rho_f \\ \rho_f & -\dfrac{\rho_f}{f} - K(s) \end{Bmatrix} \begin{Bmatrix} \dfrac{\partial^2 \omega}{\partial t^2} \\ \dfrac{\partial^2 \chi}{\partial t^2} \end{Bmatrix} \tag{2-2}$$

$$\begin{Bmatrix} 1 & -\alpha M \\ \alpha M & -M \end{Bmatrix} \begin{Bmatrix} \nabla^2 e \\ \nabla^2 \zeta \end{Bmatrix} = \begin{Bmatrix} 1 & -\rho_f \\ \rho_f & -\dfrac{\rho_f}{f} - K(s) \end{Bmatrix} \begin{Bmatrix} \dfrac{\partial^2 e}{\partial t^2} \\ \dfrac{\partial^2 \zeta}{\partial t^2} \end{Bmatrix} \tag{2-3}$$

where

$$K(s) = \frac{Y(s)}{s} - \frac{\rho_f}{f} = \frac{\rho_f}{f} \left\{ -1 + \left[1 - \frac{2}{\sqrt{s}} \frac{I_1(\sqrt{s})}{I_0(\sqrt{s})} \right]^{-1} \right\} \tag{2-4}$$

and s is the nondimensional Laplace parameter.

We consider a half space initially at rest, subjected at the free surface $x = 0$ to a known displacement condition for $t > 0$, and we look for the transient plane waves. The field variables ω, χ, e, ζ are functions of x and t and the field equations are supplemented by the following initial and boundary conditions:

$$\omega(x, 0) = \left. \frac{\partial \omega(x, t)}{\partial t} \right|_{t=0} = \chi(x, 0) = \left. \frac{\partial \chi(x, t)}{\partial t} \right|_{t=0} = 0 \tag{2-5}$$

$$\omega(0, t) = \omega_0(t)$$

$$\begin{aligned} e(x,0) &= \left.\frac{\partial e(x,t)}{\partial t}\right|_{t=0} = \zeta(x,0) = \left.\frac{\partial \zeta(x,t)}{\partial t}\right|_{t=0} = 0. \\ e(0,t) &= e_0(t) \qquad \zeta(0,t) = \zeta_0(t) \end{aligned} \tag{2-6}$$

The solutions will also be required to be bounded at infinity.

3. Shear Waves

The Laplace transform equation for the shear waves obtained from (2-2) with the initial conditions from (2-5), reads in obvious notation

$$\begin{Bmatrix} \hat{\omega}_{xx} \\ 0 \end{Bmatrix} = s^2 \begin{Bmatrix} 1 & -\rho_f \\ \rho_f & -\frac{\rho_f}{f} - K(s) \end{Bmatrix} \begin{Bmatrix} \hat{\omega} \\ \hat{\chi} \end{Bmatrix} \tag{3-1}$$

where $\hat{\omega} = \hat{\omega}(x,s)$, $\hat{\chi} = \hat{\chi}(x,s)$.

Imposing the boundary condition from (2-5), the transformed solution bounded at infinity reads

$$\begin{Bmatrix} \hat{\omega} \\ \hat{\chi} \end{Bmatrix} = \begin{Bmatrix} 1 \\ f[1-R(s)] \end{Bmatrix} e^{-x\mu(s)}\, \hat{\omega}_0(s), \tag{3-2}$$

where accounting for (3-1) and (2-4) we have

$$R(s) = \left\{ 1 - \left[1 + \frac{f}{\rho_f} K(s) \right]^{-1} \right\} = \frac{2}{\sqrt{s}} \frac{I_1(\sqrt{s})}{I_0(\sqrt{s})} \tag{3-3}$$

and

$$\mu(s) = \frac{s}{\beta} \{1 + (\beta^2 - 1) R(s)\}^{1/2}, \qquad \beta = (1 - f\rho_f)^{-1/2}. \tag{3-4}$$

Since $R(s) \to 0$ for $s \to \infty$, β represents the wave front velocity of the pulse. We shall consider short and long time responses to a step input, $\hat{\omega}_0(s) = 1/s$, in order to avoid inessential complications.

In order to compute the short-time solution we expand $R(s)$ and $\mu(s)$ in a Puiseux series at $s \to \infty$:

$$R(s) = \sum_{k=1}^{\infty} \rho_k s^{-k/2}, \qquad \mu(s) = \frac{1}{\beta}\{s + \mu_{1/2} s^{1/2} + \mu_0 + 0(s^{-1/2})\} \tag{3-5}$$

where from (3-3) and (3-4) we have

$$\rho_1 = 2, \quad \rho_2 = -1, \quad \rho_3 = -\tfrac{1}{4} \dots \quad \mu_{1/2} = \beta^2 - 1, \quad \mu_0 = -\frac{\beta^2}{2}(\beta^2 - 1). \tag{3-6}$$

According to the method proposed by Buchen and Mainardi (1975) for transient visco-elastic waves, we get

$$\hat{\omega}(x,s) = e^{-\frac{x}{\beta}(s + \mu_{1/2} s^{1/2} + \mu_0)} \sum_{k=0}^{\infty} s^{-(k/2+1)} \sum_{h=0}^{k} A_{k,h} \frac{x^h}{h!} \tag{3-7}$$

where the $A_{k,h}$ are given by the following recursive relation

$$A_{k,h}=\delta_{k0}$$

$$A_{k,h}=-\mu_{1/2}A_{k-1,h}+\frac{\mu_{1/2}}{\beta}(\mu_0+\tfrac{1}{8})A_{k-1,h-1}+\frac{\beta}{2}A_{k-2,h+1} \tag{3-8}$$

$$-\mu_0 A_{k-2,h}+\frac{1}{2\beta}(\mu_0^2+\tfrac{1}{4}\mu_{1/2})A_{k-2,h-1}-\frac{\mu_{1/2}}{2\beta}\sum_{J=3}^{k}\rho_{J+2}A_{k-J,h-1}.$$

The solution for $\hat{\chi}(x,s)$ is given by (3-8) provided that $A_{k0}=-f\rho_k$ with $\rho_0=-1$.

The inversion of (3-7) reads

$$\omega(x,t)=e^{-\frac{x\mu_0}{\beta}}\left\{F_0(z)+\sum_{k=1}^{\infty}F_k(z)\,2^k\left(t-\frac{x}{\beta}\right)^{k/2}\sum_{h=0}^{k}A_{kh}\frac{x^h}{h!}\right\}H(t-x/\beta) \tag{3-9}$$

where H denotes the Heaviside step-function, and

$$z=\frac{\mu_{1/2}x}{2\beta}\left(t-\frac{x}{\beta}\right)^{-1/2}, \qquad F_k(z)=I^k\,\mathrm{Erfc}(z) \tag{3-10}$$

where I^k is the k-th repeated integral and can also be obtained by a recursive method (see e.g. Abramowitz and Stegun, 1968, p. 239).

The initial character of the pulse is described by the leading term of (3-9) which, accounting for the asymptotic form of $\mathrm{Erfc}(z)$ for large z, can be written

$$\begin{Bmatrix}\omega(x,t)\\ \chi(x,t)\end{Bmatrix}\simeq\begin{Bmatrix}1\\ f\end{Bmatrix}\frac{1}{\sqrt{\pi}}$$

$$\cdot\frac{2\beta}{\beta^2-1}\,\frac{(t-x/\beta)^{1/2}}{x}\,\mathrm{Exp}\left(\frac{\beta(\beta^2-1)}{2}x-\frac{(\beta^2-1)^2}{4\beta^2}\,\frac{x^2}{(t-x/\beta)}\right). \tag{3-11}$$

The pulse exhibits no discontinuity on the wave front and evolves as in a diffusion process. This behaviour is similar to the one found in a visco-elastic solid with a creep function $J(t)=a+b\,t^{1/2}$ by Buchen and Mainardi (1975).

The long time behaviour is determined by the limit of (3-1) as $s\to 0$

$$\begin{Bmatrix}\hat{\omega}(x,s)\\ \hat{\chi}(x,s)\end{Bmatrix}\simeq\begin{Bmatrix}1\\ fs/8\end{Bmatrix}\frac{1}{s}\,\mathrm{Exp}\left(-x\,s+\frac{\beta^2-1}{16\beta^2}\,x\,s^2\right). \tag{3-12}$$

The inversion is carried out by the saddle-point method which yields:

$$\omega(x,t)\simeq\tfrac{1}{2}\left\{1+\mathrm{Erf}\left(\frac{2\beta}{\sqrt{\beta^2-1}}\,\frac{t-x}{\sqrt{x}}\right)\right\}$$

$$\chi(x,t)\simeq\frac{f}{4\sqrt{\pi}}\,\frac{\beta}{\sqrt{\beta^2-1}}\,\frac{1}{\sqrt{x}}\,\mathrm{Exp}\left(-\frac{4\beta^2}{\beta^2-1}\,\frac{(t-x)^2}{x}\right). \tag{3-13}$$

The pulses are centered at $t=x$ with a spread

$$\Delta t=\frac{2\sqrt{\beta^2-1}}{\beta}\sqrt{x}. \tag{3-14}$$

Since from the short-time analysis the pulses start at $t=x/\beta$ we expect the long-time approximation to hold when

$$\left(x-\frac{x}{\beta}\right)\gg\frac{\Delta t}{2}, \quad \text{i.e.,}\ x\gg\frac{\beta+1}{\beta-1}. \tag{3-15}$$

The long-time behaviour of the rotational wave $\omega(x,t)$ is the same as for a Maxwell visco-elastic solid with the creep function $J(t)=c+dt$ (see Chu, 1962; Mainardi, 1971).

4. Dilatational Waves

The transformed equation for the dilatational waves, obtained from (2-3) with the initial conditions from (2-6), reads in obvious notation

$$\begin{Bmatrix} 1 & -\alpha M \\ \alpha M & -M \end{Bmatrix} \begin{Bmatrix} \hat{e}_{xx} \\ \hat{\zeta}_{xx} \end{Bmatrix} = s^2 \begin{Bmatrix} 1 & -\rho_f \\ \rho_f & -\frac{\rho_f}{f}-K(s) \end{Bmatrix} \begin{Bmatrix} \hat{e} \\ \hat{\zeta} \end{Bmatrix} \tag{4-1}$$

where $\hat{e}=\hat{e}(x,s)$, $\hat{\zeta}=\hat{\zeta}(x,s)$. Imposing the boundary condition from (2-6) the transformed solutions bounded at infinity read

$$\begin{Bmatrix} \hat{e} \\ \hat{\zeta} \end{Bmatrix} = \begin{Bmatrix} \hat{e}_+ + \hat{e}_- \\ \hat{\zeta}_+ + \hat{\zeta}_- \end{Bmatrix}, \quad \begin{Bmatrix} \hat{e}_\pm \\ \hat{\zeta}_\pm \end{Bmatrix} = e^{-\mu_\pm(s)x}\, T_\pm(s) \begin{Bmatrix} \hat{e}_0 \\ \hat{\zeta}_0 \end{Bmatrix} \tag{4-2}$$

where $\hat{e}_0=\hat{e}_0(s)$ $\hat{\zeta}_0=\hat{\zeta}_0(s)$, and $\mu_\pm(s)$ are the solutions (positive for s positive) of the biquadratic equation

$$A\mu^4 - s^2[B+K(s)]\mu^2 + s^4[C+K(s)]=0 \tag{4-3}$$

with

$$A=M-(\alpha M)^2, \quad B=M-2\rho_f\alpha M+\frac{\rho_f}{f}, \quad C=\frac{\rho_f}{f}-\rho_f^2 \tag{4-4}$$

and $T_\pm(s)$ are 2×2 matrices determined by the boundary conditions. Explicitly we have

$$\mu_\pm(s)=\frac{s}{\alpha_\pm}[1+\psi_\pm(s)]^{1/2} \tag{4-5}$$

with

$$\alpha_\pm=\left(\frac{2A}{B\pm\Gamma}\right)^{1/2}, \quad \Gamma=(B^2-4AC)^{1/2} \tag{4-6}$$

$$\psi_\pm(s)=\frac{K(s)\pm\Gamma\{[1+W(s)]^{1/2}-1\}}{B\pm\Gamma}, \quad W(s)=\frac{K^2+2K(B-2A)}{\Gamma^2} \tag{4-7}$$

and

$$T_{\pm}(s)=\begin{Bmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{Bmatrix}\pm\frac{1}{\Gamma[1+W(s)]^{1/2}}\begin{Bmatrix}-\dfrac{F+K(s)}{2} & E+\alpha MK(s)\\ -D & \dfrac{F+K(s)}{2}\end{Bmatrix} \tag{4-8}$$

with

$$D=\rho_f-\alpha M \quad E=\frac{\rho_f}{f}\alpha M-\rho_f M \quad F=\frac{\rho_f}{f}-M. \tag{4-9}$$

The fundamental inequalities $\alpha^2 M<1$, $\rho_f f<1$, $f<\alpha$ insure the positivity of A, C, E. Moreover we notice that

$$B=A+C+D^2>0 \quad \Gamma^2=(A-C)^2+2D^2+D^2(A+C)+D^4>0. \tag{4-10}$$

We remark that the $(+)$ and $(-)$ solutions correspond to the two independent dilatational waves with velocities α_+ and α_- (since $\psi_{\pm}(s)\rightarrow 0$ for $s\rightarrow\infty$) and that they interchange for Γ replaced by $-\Gamma$. The $(-)$ solutions correspond to the so called waves of first kind (fast waves), the $(+)$ solutions to the waves of second kind (slow waves) according to Biot's notations. Furthermore we remark that the condition $D=0$ allows purely elastic wave propagation with unit velocity $e(x,t)=e_0(t-x)$ without relative motion between fluid and solid: $\zeta(x,t)=0$ when $\zeta_0(t)=0$. Therefore the condition $D=0$ corresponds to the dynamic compatibility relation mentioned by Biot (1956a) for harmonic waves.

In order to simplify the treatment without loosing the effect of coupling between the solid and fluid we shall work in the condition of quasi-dynamic compatibility, i.e., we neglect all powers of D higher than one. Such an approximation is justified in cases of physical interest.

Assuming $C>A$ we obtain from (4-5) to (4-10)

$$\Gamma\simeq B-2A\simeq C-A \tag{4-11}$$

$$\alpha_-\simeq 1 \quad \psi_-(s)\simeq 0 \tag{4-12}$$

$$\alpha_+\simeq\sqrt{A/C} \quad \psi_+(s)\simeq K(s)/C \tag{4-13}$$

and

$$T_{\pm}(s)\simeq\begin{Bmatrix}\frac{1}{2}\mp\frac{1}{2} & \pm\alpha M\\ 0 & \frac{1}{2}\pm\frac{1}{2}\end{Bmatrix}\mp\frac{D}{\Gamma+K(s)}\begin{Bmatrix}\alpha M & 0\\ 1 & -\alpha M\end{Bmatrix}. \tag{4-14}$$

As for the shear waves we restrict our analysis to the short and long time approximations. For this purpose we make explicit use of the Tauberian theorems as $s\rightarrow\infty$ and $s\rightarrow 0$, for the following input conditions: $\hat{e}_0(s)=1/s$, $\hat{\zeta}_0(s)=0$. From (2-4), (4-5), (4-12), (4-13) we get for $s\rightarrow\infty$

$$K(s)\simeq\frac{\rho_f}{f}[2s^{-1/2}+3s^{-1}+15/4\,s^{-3/2}+\cdots]$$

$$\mu_-(s)\simeq s, \qquad \mu_+(s)\simeq\frac{1}{\alpha_+}[s+\mu_{1/2}s^{1/2}+\mu_0+\mu_{-1/2}s^{-1/2}+\cdots] \tag{4-15}$$

where

$$\mu_{1/2}=\beta^2, \quad \mu_0=\tfrac{3}{2}\beta^2-\tfrac{1}{2}\beta^4, \quad \mu_{-1/2}=\tfrac{15}{8}\beta^2-\tfrac{3}{2}\beta^4+\tfrac{1}{2}\beta^6, \ldots \tag{4-16}$$

with β defined as for the S waves by $\beta=(1-f\rho_f)^{-1/2}$. Inverting (4-2) with $T_\pm(s)$ given by (4-14) and $[\Gamma+K(s)]^{-1}\simeq\frac{1}{\Gamma}\left(1-\frac{2}{f\Gamma}\rho_f s^{-1/2}\right)$ we obtain the short-time solution:

$$\begin{Bmatrix} e_-(x,t) \\ \zeta_-(x,t) \end{Bmatrix} \simeq H(t-x)\left[\begin{Bmatrix} A_{11}^- \\ A_{21}^- \end{Bmatrix} + \frac{2}{\sqrt{\pi}}\begin{Bmatrix} B_{11}^- \\ B_{21}^- \end{Bmatrix}(t-x)^{1/2}\right] \tag{4-17}$$

$$\begin{Bmatrix} e_+(x,t) \\ \zeta_+(x,t) \end{Bmatrix} \simeq \mathrm{Exp}[-x\mu_0/\alpha_+]\,H(t-x/\alpha_+)\left[\begin{Bmatrix} A_{11}^+ \\ A_{21}^+ \end{Bmatrix} U(\xi,\tau) + \begin{Bmatrix} B_{11}^+ \\ B_{21}^+ \end{Bmatrix} V(\xi,\tau)\right] \tag{4-18}$$

where

$$A_{11}^-=1+D\frac{\alpha M}{\Gamma} \qquad B_{11}^-=-D\frac{2\alpha M}{\Gamma^2}\frac{\rho_f}{f} \tag{4-19}$$

$$A_{21}^-=\frac{D}{\Gamma} \qquad B_{21}^-=-D\frac{2}{\Gamma^2}\frac{\rho_f}{f}$$

$$A_{11}^+=-D\frac{\alpha M}{\Gamma} \qquad B_{11}^+=\frac{D}{\Gamma^2}\left(2\alpha M\frac{\rho_f}{f}+\frac{\alpha M}{\Gamma}\frac{\mu_{-1/2}}{\alpha_+}x\right) \tag{4-20}$$

$$A_{21}^+=-\frac{D}{\Gamma} \qquad B_{21}^+=\frac{D}{\Gamma^2}\left(\frac{2\rho_f}{f}+\frac{1}{\Gamma}\frac{\mu_{-1/2}}{\alpha_+}x\right)$$

$$\xi=x\mu_{1/2}/\alpha_+, \qquad \tau=t-x/\alpha_+, \tag{4-21}$$

and $U(\xi,\tau)$, $V(\xi,\tau)$ are given by

$$\begin{aligned} U(\xi,\tau)&=\mathrm{Erfc}[\xi/(2\sqrt{\tau})] \\ V(\xi,\tau)&=2\sqrt{\tau/\pi}\,\exp[-\xi^2/(4\tau)]-\xi\,U(\xi,\tau) \end{aligned} \tag{4-22}$$

The fast waves $\{e_-, \zeta_-\}$ exhibit a discontinuity at the wave front while the slow waves $\{e_+, \zeta_+\}$ show a diffusive-like behaviour as the shear waves.

The long-time solution is easily computed from $K(s)\simeq 8\frac{\rho_f}{f}s^{-1}$ $(s\to 0)$ and reads

$$\begin{Bmatrix} e_-(x,t) \\ \zeta_-(x,t) \end{Bmatrix} \simeq \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} H(t-x) \tag{4-23}$$

$$\begin{Bmatrix} e_+(x,t) \\ \zeta_+(x,t) \end{Bmatrix} \simeq -\begin{Bmatrix} \alpha M \\ 1 \end{Bmatrix} D\frac{\beta f}{8\rho_f\alpha_+}\sqrt{\frac{2}{\pi}}\,x\,t^{-3/2}\exp[-2(\beta/\alpha_+)^2x^2/t]. \tag{4-24}$$

The fast wave solution corresponds to a perfectly elastic propagation while the slow wave is affected by the porous-elastic coupling. The pulse of the latter is centered at $t_M=4/3(\beta/\alpha_+)^2x^2$ and the rise time is $\Delta_R t\simeq 3t_M/4$ while the decay time is $\Delta_D t\simeq 10t_M$. Since from the short-time analysis the pulses start at $t=x/\alpha_+$

we expect the long-time approximation to hold when $x/\alpha_+ \ll t_M/4$, i.e., $x \gg 3\alpha_+/\beta^2$. We notice also that the amplitude at the maximum decays as x^{-2} and consequently is strongly damped[1].

5. Discussion and Geophysical Implications

From the previous analysis we see that the effect of poroelasticity on pulse propagation can be summarized as follows. The relative motion of a viscous fluid in the pores generally implies an attenuation in the motion of the solid. The rotational waves exhibit the same dispersion as if the attenuating mechanism were a particular type of linear viscoelasticity. The dilatational waves were investigated in the quasi-dynamical compatibility condition since we expect that the deviation from the elastic behaviour can be treated as a perturbation for most physical purposes. The deviation from pure elasticity is measured by the parameter D which can be considered as a coupling constant, in analogy with thermoelastic theory. The distortion of the fast wave is relevant only for very short times and the slow wave decays very rapidly so that even when the dynamic compatibility condition is not exactly satisfied the medium allows a perfectly elastic propagation without relative motion between fluid and solid.

We confine our numerical analysis to a particular liquid filled porous solid of geophysical interest (a kerosene-satured sandstone) whose parameters can be deduced from experimental data (Fatt, 1959) and are quoted in Appendix B.

For the S wave the nondimensional wave front velocity is $\beta = 1.05$, and the evolution of the pulse at fixed distances is shown in Figure 1 where the short and long-time solutions are compared. Since unit distance and time are very small (see Appendix B) the long-time approximation well describes the propagation of seismic pulses. As a consequence the (continuous) precursors do not start effectively at $t = x/\beta$ but rather at $t = x - \Delta t/2$, with Δt given by (3-14) so that the actual velocity appears to be $v \simeq 1 + [(\beta - 1)/(2x)]^{1/2}$.

In order to have an estimate of the porosity influence on the seismic pulse we point out that after one second the pulse has travelled $\simeq 1$ km and its spread is $\simeq 5$ m.

For the P waves the dynamic compatibility condition is fulfilled within the uncertainty of the experimental values of elastic coefficients and consequently we have a perfectly elastic propagation of the seismic pulses.

Appendix A. Discussion of Biot's Equations

The dynamical equations of poroelasticity obtained by Biot (1962b) in the case of isotropy and uniform rigidity are:

$$\mu \nabla^2 \boldsymbol{u} + \nabla [(\lambda_c + \mu) e - \alpha M \zeta] = \frac{\partial^2}{\partial t^2 t^2} (\rho \boldsymbol{u} + \rho_f \boldsymbol{w})$$

[1] It can be observed that for the most general input $e_0(t)$, $\zeta_0(t)$ the conclusions are essentially the same. The solution for $x \gg 1$ reads $\{e_-(x,t) \simeq [e_0(t-x) - \rho_f \zeta_0(t-x)], \zeta_-(x,t) \simeq 0\}$, i.e., we have elastic propagation in the solid corresponding to an effective input pulse $(e_0 - \rho_f \zeta_0)$. Any other disturbance has an amplitude decaying like x^{-2}

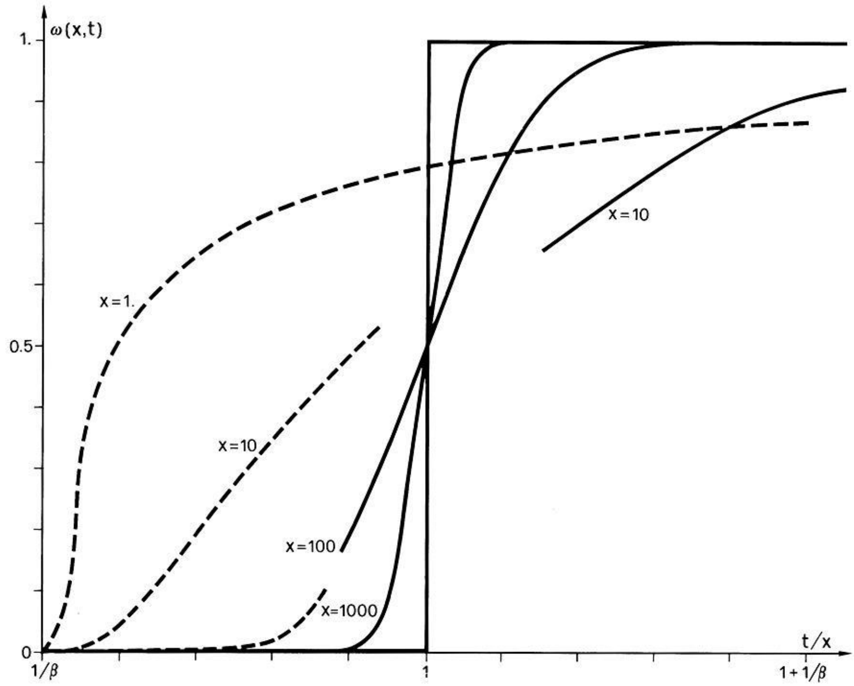

Fig. 1. The shear response $\omega(x,t)$ to a step input $\omega_0(t)=H(t)$ is shown at various distances. The wave front (dotted line) and long time response (continuous line) are plotted versus t/x and compared with the elastic response $H(t-x)$

$$\nabla(\alpha M e - M\zeta) = \frac{\partial^2}{\partial t^2}(\rho_f \boldsymbol{u}) + \hat{Y}\frac{\partial}{\partial t}\boldsymbol{w}. \tag{A.1}$$

We recall that: $e=\nabla\cdot\boldsymbol{u}$, $\zeta=-\nabla\cdot\boldsymbol{w}$, $\boldsymbol{w}=f(\boldsymbol{U}-\boldsymbol{u})$, where $\boldsymbol{u}$, $\boldsymbol{U}$ are the displacement vectors of the solid and fluid, respectively; μ, λ_c, α, M are elastic coefficients whose physical meaning is clarified by Biot and Willis (1957); ρ, ρ_f are the mass densities of the aggregate and fluid, respectively; and $\hat{Y}$ is the viscodynamic operator, a function of $s=d/dt$, which embodies the dynamics of the fluid in relative motion[2]. The specific form of $\hat{Y}(s)$ can be determined, assuming a simple geometry for the pores, by considering the flow of a viscous fluid under an oscillatory pressure gradient $e^{i\omega t}$ below the turbulent regime. From Biot (1956b) we deduce the following result for cylindrical pores of radius a:

$$\hat{Y}(s)=\frac{\rho_f}{f}s\left\{1-\frac{2}{a}\left(\frac{v}{s}\right)^{1/2}T\left[a\left(\frac{s}{v}\right)^{1/2}\right]\right\}^{-1}, \qquad T(z)=\frac{I_1(z)}{I_0(z)}, \tag{A.2}$$

where f is the porosity, v is the kinematic viscosity of the fluid and I_0, I_1 denote modified Bessel functions. The above-mentioned limit of validity can be ex-

[2] The viscodynamic operator $\hat{Y}(s)$ accounts for the deviation of the microvelocity field from the Poiseuille flow ($s=0$) as the frequency increases

pressed as an upper bound $\omega_{max}=\pi v_0/a$ on the frequency $\omega=s/i$, where v_0 is a characteristic phase velocity. The characteristic frequency of the system is given by $\omega_0=2\pi\nu/a^2$ and we notice that for our purposes $\omega_{max}/\omega_0=a v_0/(2\nu)$ is very large since $v_0\simeq 10^5$ cm/s, $a\simeq 10^{-3}$ cm, $\nu\simeq 10^{-2}$ cm²/s. The limits of the operator $\hat{Y}$ at low and high frequencies can be easily deduced from (A.2); we get

$$(s\to 0)\quad \hat{Y}(s)\simeq d_0+ms,\qquad d_0=\frac{8\nu}{a^2}\frac{\rho_f}{f},\qquad m=\frac{4}{3}\frac{\rho_f}{f} \tag{A.3}$$

$$(s\to\infty)\quad \hat{Y}(s)\simeq\frac{\rho_f}{f}s+d_\infty s^{\frac{1}{2}},\qquad d_\infty=\frac{2}{a}\nu^{\frac{1}{2}}\frac{\rho_f}{f}. \tag{A.4}$$

At zero frequency ($s=0$) the operator yields Darcy's law, i.e. $\hat{Y}=d_0=\mu/\kappa$, where $\mu=\rho_f\nu$ is the dynamic viscosity and $\kappa=a^2f/8$ is the permeability.

For the case of uniform porosity, using the variable $\boldsymbol{U}$ instead of $\boldsymbol{w}$ and introducing $\varepsilon=-\nabla\cdot\boldsymbol{U}$, the dynamical Equations (A.1) can be written in the alternative form

$$\begin{aligned} &N\nabla^2\boldsymbol{u}+\nabla[(A+N)e+Q\varepsilon]=\frac{\partial^2}{\partial t^2}(\hat{\rho}_{11}\boldsymbol{u}+\hat{\rho}_{12}\boldsymbol{U})\\ &\nabla(Qe+R\varepsilon)=\frac{\partial^2}{\partial t^2}(\hat{\rho}_{12}\boldsymbol{u}+\hat{\rho}_{22}\boldsymbol{U})\end{aligned} \tag{A.5}$$

where

$$N=\mu,\qquad A=\lambda_c-2f\alpha M+f^2M,\qquad Q=fM(\alpha-f),\qquad R=f^2M \tag{A.6}$$

$$\hat{\rho}_{11}=\rho-2f\rho_f+f^2\hat{Y}(s)/s,\qquad \hat{\rho}_{12}=f\rho_f-f^2\hat{Y}(s)/s,\qquad \hat{\rho}_{22}=f^2\hat{Y}(s)/s. \tag{A.7}$$

The Equations (A.5) are to be considered the correction of equations derived formerly by Biot (1956b) and usually adopted in geophysical applications (e.g. Deresiewicz and Rice, 1962)

$$\begin{aligned} &N\nabla^2\boldsymbol{u}+\nabla(A+N)e+Q\varepsilon=\frac{\partial^2}{\partial t^2}(\rho_{11}\boldsymbol{u}+\rho_{12}\boldsymbol{U})+b\hat{F}(s)\frac{\partial}{\partial t}(\boldsymbol{u}-\boldsymbol{U})\\ &\nabla(Qe+R\varepsilon)=\frac{\partial^2}{\partial t^2}(\rho_{12}\boldsymbol{u}+\rho_{22}\boldsymbol{U})-b\hat{F}(s)\frac{\partial}{\partial t}(\boldsymbol{u}-\boldsymbol{U}),\end{aligned} \tag{A.8}$$

where

$$\rho_{11}=\rho-2f\rho_f+f^2m,\qquad \rho_{12}=f\rho_f-f^2m,\qquad \rho_{22}=f^2m \tag{A.9}$$

and

$$b=\frac{8}{a^2}f\rho_f\nu=f^2d_0,\qquad \hat{F}(s)=\frac{1}{4}\,\frac{a(s/\nu)^{\frac{1}{2}}\,T[a(s/\nu)^{\frac{1}{2}}]}{1-\frac{2}{a}\left(\frac{\nu}{s}\right)^{\frac{1}{2}}T[a(s/\nu)^{\frac{1}{2}}]}. \tag{A.10}$$

Strictly speaking, the Equations (A.8) are correct only when $s\to 0\,(\hat{F}(s)\to 1)$ as considered by Biot (1956a) in the low frequency range. For higher frequencies a correction factor must also apply to the density parameters ρ_{11}, ρ_{12}, ρ_{22}, namely to the parameter m in (A.9). For $s\to\infty$ we get $m=\rho_f/f$ so that

$$\rho_{11}\simeq\rho-f\rho_f,\qquad \rho_{12}\simeq 0,\qquad \rho_{22}\simeq f\rho_f \tag{A.11}$$

and

$$b\hat{F}(s)\simeq\frac{2}{a}f\rho_f(\nu s)^{\frac{1}{2}}=f^2 d_\infty s^{\frac{1}{2}}. \tag{A.12}$$

We point out that Equations (A.1) are valid for any frequency range and that all the parameters are uniquely determined[3].

The equations for shear and dilational waves are obtained as usual by taking the curl and divergence of Equations (A.1), respectively, and read

$$\begin{aligned}\mu\nabla^2\omega&=\frac{\partial^2}{\partial t^2}(\rho\,\omega-\rho_f\chi)\\ 0&=\frac{\partial^2}{\partial t^2}(\rho_f\,\omega)-\hat{Y}(s)\frac{\partial\chi}{\partial t},\end{aligned} \tag{A.13}$$

where $\boldsymbol{\omega}=\nabla\wedge\boldsymbol{u}$, $\boldsymbol{\chi}=-\nabla\wedge\boldsymbol{w}$, and

$$\begin{aligned}\nabla^2(He-\alpha M\zeta)&=\frac{\partial^2}{\partial t^2}(\rho e-\rho_f\zeta)\\ \nabla^2(\alpha Me-M\zeta)&=\frac{\partial^2}{\partial t^2}(\rho_f e)-\hat{Y}(s)\frac{\partial\zeta}{\partial t},\end{aligned} \tag{A.14}$$

where $H=\lambda_c+2\mu=A+2N+2Q+R$.

Appendix B. Numerical Values

We quote the numerical values of the physical parameters of a kerosene-satured sandstone (Fatt, 1959). In *cgs* units we have:

$$a=10^{-3},\quad \nu=2.44\cdot10^{-2},\quad \rho_f=0.82,\quad \rho=2.137,\quad f=0.26$$

$$N=0.276\cdot10^{11},\quad H=1.178\cdot10^{11},\quad M=0.482\cdot10^{11},\quad \alpha=0.853.$$

The characteristic time is $t_0=a^2/\nu=4.1\cdot10^{-5}$. For S waves the elastic velocity is $v_{os}=(N/\rho)^{\frac{1}{2}}=1.14\cdot10^5$ and the characteristic distance $x_0=v_{os}t_0=4.6$. For the P waves the analogous quantities are $v_{op}=(H/\rho)^{\frac{1}{2}}=2.35\cdot10^5$, $x_0=v_{op}t_0=9.6$.

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[3] From the above analysis we see that the assumptions made by Deresiewicz and Rice (1962), $\rho_{12}=-10^{-3}\rho$ in the whole frequency range, is not strictly correct since ρ_{12} varies from $-1/3(\rho_f/f)$ at $\omega=0$ to 0 at $\omega=\infty$

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Note Added in Proof. Using the data available in a recent work (Yew, C.H., Jogi, P.N.: Study of wave motions in fluid-saturated porous rocks. J. Acoust. Soc. Amer. **60**, 2–8, 1976), we find that the quasi-compatibility condition holds for several water-saturated rocks ($0.05 \leqq D^2 \leqq 0.1$).