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On Equivalent Models of Seismic Sources*

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Abstract. The Green matrices for a general excitation (dislocation or body force) of an elastic layered medium are expressed by means of 3 matrices corresponding respectively to *P*, *SV* and *SH* waves. Two methods are used to derive the solution corresponding to a point stress glut. It is shown that a continuous medium cannot sustain the action of a dipole source without faulting and the expression of the stress glut equivalent to this source is given. It is shown that a dislocation is equivalent to an infinity of systems of 3 dipoles.

Key words: Green matrices – Seismic dipole – Seismic dislocation – Stress glut.

1. Introduction

The difficult problem of the representation of seismic sources has been the subject of many research papers. The classical method of Volterra for the study of dislocations has been applied to prove the equivalence of the fields of displacements due to rupture in an elastic medium or to a system of body forces (Burridge and Knopoff, 1964; Kostrov, 1970). The use of the propagator matrix in the Thomson-Haskell method also allows to derive the displacement field produced by a point dislocation (Jobert, 1975 noted below Paper I). A new approach, which may provide a decisive progress in this domain, has been recently proposed by Backus and Mulcahy (1976).

In the present work we try to show how the idea of stress glut introduced in the last mentioned paper may be used in the Thomson-Haskell method to clarify the equivalence of dislocation and system of dipoles.

2. Notation

References to the formula (10) of Paper I or of the Appendix will be noted respectively (I, 10) or (A.10). The matrix transposed of a matrix A is noted A^T. I is the unit matrix.

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We introduce a fixed orthonormed basis (h_0, θ_0, k) and cylindrical coordinates (η, φ, ζ) with respect to the k-axis. We shall attach a mobile orthonormed basis (h, θ, k) to the (real) wave vector p such that

$$p = \eta h + \zeta k.$$

The Fourier-Laplace transform (F.L.T.) of a function f will be defined by the following expressions

complete F.L.T. $\bar{f}(p, v) = \int_0^\infty \iiint_{-\infty}^{+\infty} dt\, dx \exp[\varpi(p^T x + i v t)]\, f(x, t)$

partial F.L.T. $\tilde{f}(\varepsilon, \eta, \varphi, v) = \int_0^\infty \iint_{-\infty}^{\infty} dt\, d\tilde{x} \exp[\varpi(\eta h^T x + i v t)]\, f$

L.T. $\hat{f}(x, v) = \int_0^\infty dt \exp(\varpi i v t)\, f$

where x is the location vector, dx the volume element, $\mathfrak{z} = k^T x$ the vertical component, $\tilde{x}$ the horizontal projection of x, $d\tilde{x}$ the horizontal area element, $\varpi = 2i\pi$. $\delta(\mathfrak{z})$ is the Dirac distribution, H the Heaviside function, $\varepsilon = \operatorname{sign} \mathfrak{z} = 2H - 1$.
ρ is the density, λ, μ the Lamé's parameters.

$$\kappa = \lambda/(\lambda + 2\mu), \quad \kappa' = \mu/(\lambda + 2\mu), \quad a = \kappa + \kappa', \quad m^2 = \rho v^2/\mu\eta^2 > 0$$

$$s^2 = m^2 + 1, \quad r^2 = m^2 \kappa' + 1 \quad (r, s > 0), \quad b = s^2 + 1.$$

∂_u represents a derivation with respect to the variable u.

The Thomson-Haskell (T.H.) vector is defined by

$$V = \operatorname{col}(u, \tau).$$

where u is the displacement, τ the stress vector acting on a $\mathfrak{z} = C^t$ plane.

We shall use in fact the modified F.L.T.

$$\bar{X} = \operatorname{col}(\bar{u}, \bar{\tau}/\mu\varpi\eta)$$

3. Structure of the Green Matrices

We shall first recall some results concerning the solution of the differential system

$$\partial_{\mathfrak{z}} \tilde{X} = M\tilde{X} + \tilde{E} \tag{1}$$

where $\partial_{\mathfrak{z}} M = 0$. Let $P(\mathfrak{z})$ be the propagator of M, i.e. the matrix such that

$$\partial_{\mathfrak{z}} P = MP, \quad P(0) = \mathrm{I}.$$

For an excitation

$$\tilde{E} = \tilde{G}\,\delta(\mathfrak{z}) \tag{2}$$

the solution is

$$\tilde{X} = P(\tilde{G}H(\mathfrak{z}) + \tilde{G}_1), \tag{3}$$

with $\partial_{\mathfrak{z}} \tilde{G}_1 = 0$. This solution presents for $\mathfrak{z} = 0$ a discontinuity equal to $\tilde{G}$.

If

$$\tilde{E} = \tilde{G}\,\delta'(\mathfrak{z}) \tag{4}$$

the solution becomes

$$\begin{aligned}\tilde{X}_1 &= \partial_{\mathfrak{z}} X = P\tilde{G}\,\delta(\mathfrak{z}) + (\partial_{\mathfrak{z}} P)(\tilde{G}H + \tilde{G}_1)\\ &= \tilde{G}\,\delta(\mathfrak{z}) + MP(\tilde{G}H + \tilde{G}_1)\end{aligned} \tag{5}$$

as

$$P\,\delta(\mathfrak{z}) = P(0)\,\delta(\mathfrak{z}) = \mathrm{I}\,\delta(\mathfrak{z}).$$

As shown in Paper I the eigenvalues of the matrix M corresponding to the T.H. system may be noted $\pm\varpi\eta i r$, $\pm\varpi\eta i s$ so that its spectral representation is given by

$$M = \varpi\eta\,[ir(\Pi_P^+ - \Pi_P^-) + is(\Pi_S^+ - \Pi_S^-)], \tag{6}$$

where the Π are orthogonal eigenprojectors, the indices P and S corresponding respectively to P and S waves, the signs $\pm$ to down and up-going waves. The unity matrix I may be decomposed in their sum

$$\mathrm{I} = \Pi_p^+ + \Pi_p^- + \Pi_s^+ + \Pi_s^- \tag{7}$$

The propagator P is then given by

$$P = \Pi_p^+\, e_p^+ + \Pi_p^-\, e_p^- + \Pi_s^+\, e_s^+ + \Pi_s^-\, e_s^-$$

where $e_p^\pm = \exp(\pm\varpi\eta i r \mathfrak{z})$, $e_s^\pm = \exp(\pm\varpi\eta i s \mathfrak{z})$.

In terms of converging (C) or diverging (D) waves we may write

$$P = \Pi_p^C e_p^C + \Pi_s^C e_s^C + \Pi_p^D e_p^D + \Pi_s^D e_s^D$$

where

$$\begin{pmatrix}\Pi^D\\ \Pi^C\end{pmatrix} = J\begin{pmatrix}\Pi^+\\ \Pi^-\end{pmatrix} \quad \text{with} \quad J = J^{-1} = \begin{pmatrix} H & 1-H\\ 1-H & H\end{pmatrix} \tag{8}$$

are also orthogonal projectors. The wave functions are given by

$$e_P^C = \exp(-\varpi\eta i r \mathfrak{z}) = \exp(2\pi\eta r|\mathfrak{z}|)$$
$$e_P^D = \exp(+\varpi\eta i r \mathfrak{z}) = \exp(-2\pi\eta r|\mathfrak{z}|)$$

with similar expressions for S waves. These projectors $\Pi^{C,D}$ may be considered as composed eigenprojectors of M as they change discontinuously across the plane $z=0$. Using (8) we have for example

$$\begin{aligned}M\Pi_P^D &= M(H\Pi_P^+ + (1-H)\Pi_P^-) = \varpi\eta i r(H\Pi_P^+ - (1-H)\Pi_P^-)\\ &= \varepsilon\varpi\eta i r\,\Pi_P^D.\end{aligned} \tag{9}$$

One obtains the result for the converging wave by changing ε into $-\varepsilon$, for the S waves by changing r into s.

In order to destroy the converging waves in the solution (3) we have to take

$$\tilde{G}_1 = -(\Pi_p^- + \Pi_s^-)\tilde{G}.$$

Using (7) we have indeed

$$(\Pi^+ + \Pi^-)H - \Pi^- = \varepsilon(\Pi^+ H + (1-H)\Pi^-) = \varepsilon \Pi^D.$$

The solution of (1) (2) diverging from the source is then given by

$$\tilde{X}^D = \varepsilon(\Pi_P^D e_P^D + \Pi_S^D e_S^D)\tilde{G}. \tag{10}$$

Using (9) and similar formulae in (5) we find the diverging solution for the excitation (4)

$$\tilde{X}_1^D = \tilde{G}\,\delta(\mathfrak{z}) + \varpi\eta(ir\,\Pi_P^D e_P^D + is\,\Pi_S^D e_S^D)\tilde{G}, \tag{11}$$

but this solution is δ-singular at the origin.

The discontinuity for $\mathfrak{z}=0$ of the diverging wave in (11) is according (8) and (6)

$$\varpi\eta[ir(\Pi_P^+ - \Pi_P^-) + is(\Pi_S^+ - \Pi_S^-)]\tilde{G} = M\tilde{G}. \tag{12}$$

We shall need later the expressions of the products $\Pi^D \begin{pmatrix} h \\ 0 \end{pmatrix}$, $\Pi^D \begin{pmatrix} 0 \\ h \end{pmatrix}$ ... A very simple result is obtained: As shown in the Appendix (A.4, 5) these 6 quantities may be expressed by means of 3 matrices R, S, T only corresponding respectively to P, SV and SH waves.

$$R = \begin{pmatrix} \varepsilon h - irk \\ \varepsilon bk + 2irh \end{pmatrix}, \quad S = \begin{pmatrix} \varepsilon k + ish \\ -\varepsilon bh + 2isk \end{pmatrix}, \quad T = m^2 \begin{pmatrix} \varepsilon \\ is \end{pmatrix} \theta. \tag{13}$$

4. Thomson-Haskell Equations and Stress Glut

Let us now examine how the concept of stress glut introduced by Backus and Mulcahy (1976) may be applied in the T.H. treatment of point sources. Let σ_0 be the tensor of the stresses really present in the medium and σ a model of σ_0, for example the tensor derived from the displacement using everywhere Hooke's law.

In the region of the source – at the origin for a concentrated source – these two tensors differ. Their difference has been called stress glut by Backus and Mulcahy.

$$\Gamma = \sigma_0 - \sigma. \tag{14}$$

The equation of motion is always given by

$$\operatorname{div} \sigma_0 + F = \rho\,\partial_{tt} u \tag{15}$$

where F is the extraneous body force. The T.H. equations may be derived by 2 methods.

(a) If the source conditions contain some information about the real stress field, for example if they imply the continuity of the stress vector $\tau_0 = \sigma_0 k$ acting on the fault plane $\mathfrak{z}=0$, we may use $\tilde{X} = \operatorname{col}(\tilde{u}, \tilde{\tau}_0/\mu\varpi\eta)$ as the modified T.H. vector. $\tilde{\tau}_0$ has to be evaluated from the stress model vector $\tilde{\tau}$, the expression of which may be deduced from (I.9)

$$\tilde{\tau} = A\,\partial_{\mathfrak{z}}\tilde{u} - \varpi\eta B\tilde{u} \tag{16}$$

where

$$A=(\lambda+2\mu)kk^T+\mu(hh^T+\theta\theta^T), \qquad B=\lambda kh^T+\mu hk^T$$

we therefore have

$$\tilde{\tau}_0=\tilde{\sigma}_0 k=(\tilde{\sigma}+\tilde{\Gamma})k=\tilde{\tau}+\tilde{\Gamma}k.$$

The first T.H. equation becomes

$$\partial_{\mathfrak{z}}\tilde{u}=A^{-1}\tilde{\tau}_0+\varpi\eta A^{-1}B\tilde{u}-A^{-1}\tilde{\Gamma}k.$$

The second equation is derived from (15) as had been (I.8) for τ. The excitation due to the stress glut is thus

$$\tilde{E}=-\begin{pmatrix}1\\0\end{pmatrix}A^{-1}\tilde{\Gamma}k.$$

As the stress glut must be concentrated at the origin, it may be expressed as a sum of Dirac and Dirac derivatives distributions. We shall limit ourselves to the case of a Dirac

$$\Gamma=\Gamma_0\delta(\mathfrak{z}).$$

The corresponding diverging solution is then given by (3)

$$\tilde{X}_1^D=-\varepsilon(\Pi_p^D e_p^D+\Pi_s^D e_s^D)\begin{pmatrix}1\\0\end{pmatrix}A^{-1}\tilde{\Gamma}k.$$

It corresponds to a classical dislocation depending only on the stress glut vector $\tilde{\Gamma}_0 k=\gamma=\gamma_1 h+\gamma_2\theta+\gamma_3 k$. The discontinuity of $\tilde{u}$ for $\mathfrak{z}=0$ is

$$d=d_1h+d_2\theta+d_3k=-A^{-1}\tilde{\Gamma}_0k=-(\gamma_1h+\gamma_2\theta+\kappa'\gamma_3k)/\mu$$
$$\text{as } A^{-1}=(hh^T+\theta\theta^T+\kappa'kk^T)/\mu. \tag{17}$$

Using (A.4) we obtain for the solution the following expressions

$$\tilde{X}_1^D=[(2\gamma_1+\varepsilon b\kappa'\gamma_3/ir)Re_p^D+(2\kappa'\gamma_3-\varepsilon b\gamma_1/is)Se_s^D-\gamma_2Te_s^D]/2\mu m^2 \tag{18}$$

in terms of the stress glut vector components, or

$$\tilde{X}_1^D=-[(2d_1+\varepsilon bd_3/ir)Re_p^D+(2d_3-\varepsilon bd_1/ir)Se_s^D-d_2Te_s^D]/2m^2 \tag{18 bis}$$

in terms of the dislocation d components.

b) We may also use the model stress σ as unknown. The first T.H. equation is then simply (16). The equation of motion (15) becomes

$$\operatorname{div}(\sigma+\Gamma)+F=\rho\partial_{tt}u,$$

so that an apparent body force $\operatorname{div}\Gamma$ is introduced. As

$$\overline{\operatorname{div}\Gamma}=-\varpi\bar{\Gamma}p=-\varpi\bar{\Gamma}(\eta h+\zeta k)$$

the stress glut excitation is given by

$$\tilde{E} = -\begin{pmatrix}0\\1\end{pmatrix} \widetilde{\operatorname{div}\Gamma}/\mu\varpi\eta = -\begin{pmatrix}0\\1\end{pmatrix}[\partial_3 \tilde{\Gamma} k - \varpi\eta\tilde{\Gamma} h]/\mu\varpi\eta$$

$$= -\begin{pmatrix}0\\1\end{pmatrix}[\tilde{\Gamma}_0 k\delta' - \varpi\eta\tilde{\Gamma}_0 h\delta]/\mu\varpi\eta.$$

In absence of extraneous body force the corresponding diverging solution is then deduced from (10) (11)

$$\mu\varpi\eta\tilde{X}_0^D = \varepsilon[\Pi_p^D e_p^D + \Pi_s^D e_s^D]\begin{pmatrix}0\\1\end{pmatrix}\varpi\eta\tilde{\Gamma}_0 h - \begin{pmatrix}0\\1\end{pmatrix}\tilde{\Gamma}_0 k\delta$$
$$-\varpi\eta[ir\Pi_p^D e_p^D + is\Pi_s^D e_s^D]\begin{pmatrix}0\\1\end{pmatrix}\tilde{\Gamma}_0 k. \tag{19}$$

The real stress T.H. vector contains only diverging waves as it is given by

$$\tilde{X}_2^D = \tilde{X}_0^D + \begin{pmatrix}0\\1\end{pmatrix}\tilde{\Gamma} k/\mu\varpi\eta = \varepsilon[\Pi_p^D e_p^D + \Pi_s^D e_s^D]\begin{pmatrix}0\\1\end{pmatrix}\tilde{\Gamma}_0 h/\mu$$
$$-[ir\Pi_p^D e_p^D + is\Pi_s^D e_s^D]\begin{pmatrix}0\\1\end{pmatrix}\tilde{\Gamma}_0 k/\mu. \tag{19 bis}$$

To compare to the solution $\tilde{X}_1^D$ obtained in (a) we must take into account the continuity of τ_0. From (12) and (A.1) we obtain the condition

$$\tilde{\Gamma}_0 h - (0\ 1)M\begin{pmatrix}0\\1\end{pmatrix}\tilde{\Gamma}_0 k/\varpi\eta = 0, \qquad \text{or}$$

$$\tilde{\Gamma}_0 h = (\kappa h k^T + k h^T)\tilde{\Gamma}_0 k. \tag{20}$$

The solution depends then only on $\gamma = \tilde{\Gamma}_0 k$.

It is easy to prove the identity of the expressions of X_1^D and X_2^D using (A.4).

Let us now compare these results with those obtained for classical body force sources. For a force $f(t)\,\delta(x)\,l$ we deduce from (10)

$$\tilde{X}_F^D = -\varepsilon(\Pi_p^D e_p^D + \Pi_s^D e_s^D)\begin{pmatrix}0\\1\end{pmatrix}\bar{f}l/\mu\varpi\eta. \tag{21}$$

The L.T. of X_F is

$$\hat{X}_F^D = \int_0^\infty \eta\, d\eta \int_0^{2\pi} d\varphi \exp(-\varpi\eta h^T x)\,\tilde{X}_F^D(x). \tag{22}$$

A momentless couple or dipole, of unit vector l, may be considered as the limit for $d \to 0$ of two forces $\pm fl/2d$ acting at $x_0 = ld$ and $-x_0$. To obtain the corresponding solution $\hat{X}_c^D$ we must replace in (22) x by $x \pm ld$. The limit is then

$$\hat{X}_C^D = -\partial_d \int_0^\infty \eta\, d\eta \int_0^{2\pi} d\varphi \exp(-\varpi\eta\, h^T(x - ld)\,\tilde{X}_F^D(x - ld)|_{d=0}.$$

We obtain finally

$$\mu\varpi\eta\tilde{X}_C^D = [k^T l\delta - \varepsilon\varpi\eta[(h^T - \varepsilon irk^T) l\Pi_p^D e_p^D + (h^T - \varepsilon isk^T) l\Pi_s^D e_s^D]]\begin{pmatrix}0\\1\end{pmatrix}\bar{f}l \tag{23}$$

This solution has a δ-singular stress vector. This is *impossible* in a continuous medium, as the displacement must be continuous and therefore the stress vector is at most discontinuous. A continuous medium cannot sustain a dipole action.

We note the similarity between (19) and (23). A dipole is equivalent to a stress glut defined by

$$\tilde{\Gamma}_0 k = -l l^T k \bar{f}, \quad \tilde{\Gamma}_0 h = -l l^T h \bar{f}$$

or

$$\tilde{\Gamma}_0 = -(l l^T + n\theta^T)\bar{f}.$$

As $\tilde{\Gamma}_0$ is symmetric and fixed in space we must take $n=0$ so that the stress glut equivalent to the dipole $f(t)\,\delta(x)\,l$ is

$$\Gamma = -f(t)\,\delta(x)\,l l^T. \tag{24}$$

This stress glut cannot satisfy the condition (20) and the real stress vector cannot be continuous. We have indeed if

$$l = \alpha h + \beta\theta + \gamma k$$

$$\tilde{\Gamma}_0 h = -\alpha l \bar{f} \qquad \tilde{\Gamma}_0 k = -\gamma l \bar{f}.$$

The condition (20) becomes

$$\alpha l = (\kappa h k^T + k h^T)\gamma l = \kappa\gamma^2 h + \alpha\gamma k$$

or

$$\begin{aligned} \alpha^2 - \kappa\gamma^2 &= 0 \\ \alpha\beta &= 0 \end{aligned} \tag{25}$$

but α and β are either both zero, or functions of φ whereas γ is constant.

5. Equivalence of a Dislocation and of a System of Dipoles

The dislocation $d = d_1 h_0 + d_2\theta_0 + d_3 k$ is from (17) produced by the stress glut vector

$$\tilde{\Gamma}_0 k = -\mu(d_1 h_0 + d_2\theta_0) - (\lambda + 2\mu) d_3 k.$$

Let us introduce three dipoles defined by their intensities and directions $l_j = \alpha_j h + \beta_j\theta + \gamma_j k = \alpha_{j0} h_0 + \beta_{j0}\theta_0 + \gamma_j k$ ($j=1,3$).

Using (24) we have the conditions of equivalence

$$\bar{f}_j l_j l_j^T k = \mu(d_1 h_0 + d_2\theta_0) - (\lambda + 2\mu) d_3 k,$$

or

$$\begin{aligned} \bar{f}_j \alpha_{j0}\gamma_j &= \mu d_1 \\ \bar{f}_j \beta_{j0}\gamma_j &= \mu d_2 \\ \bar{f}_j \gamma_j^2 \quad &= (\lambda + 2\mu) d_3. \end{aligned} \tag{26}$$

On the other hand the continuity of the stress vector implies from (25) that

$$\begin{aligned} \bar{f}_j \alpha_j^2 \quad &= \kappa \bar{f}_j \gamma_j^2 = \lambda d_3 \\ \bar{f}_j \alpha_j \beta_j &= 0. \end{aligned} \tag{27}$$

The relations (27) must be satisfied for any value of $\varphi = (h_0, h)$.

We thus obtain

$$\bar{f}_j \alpha_{j0}^2 = \bar{f}_j \beta_{j0}^2 = \lambda d_3$$
$$\bar{f}_j \alpha_{j0} \beta_{j0} = 0. \tag{27 bis}$$

(a) Unit Dislocation Normal to the Fault Plane ($d_1 = d_2 = 0$, $d_3 = 1$). We introduce the vectors $A = \mathrm{col}(\alpha_{10}\alpha_{20}\alpha_{30})$, $B = \mathrm{col}(\beta_{10}\beta_{20}\beta_{30})$, $C = \mathrm{col}(\gamma_1\gamma_2\gamma_3)$, the matrix $F = \mathrm{diag}(\bar{f}_1\ \bar{f}_2\ \bar{f}_3)$ ($\bar{f}_i$ are supposed here positive) and the vectors

$$X = F^{1/2} A \qquad Y = F^{1/2} B \qquad Z = F^{1/2} C.$$

The system (26) (27 bis) may then be written

$$X^T Y = Y^T Z = Z^T X = 0$$
$$X^T X = Y^T Y = \lambda, \qquad Z^T Z = (\lambda + 2\mu).$$

Its solution is therefore

$$X = \sqrt{\lambda}\, \mathbb{R} h_0 \qquad Y = \sqrt{\lambda}\, \mathbb{R} \theta_0 \qquad Z = \sqrt{(\lambda + 2\mu)}\, \mathbb{R} k,$$

where $\mathbb{R}$ is an arbitrary orthogonal matrix, and so

$$A = \sqrt{\lambda}\, F^{-1/2} \mathbb{R} h_0 \qquad B = \sqrt{\lambda}\, F^{-1/2} \mathbb{R} \theta_0 \qquad C = \sqrt{\lambda + 2\mu}\, F^{-1/2} \mathbb{R} k. \tag{28}$$

The values of the intensities may be deduced from the condition of unitarity of the vectors l_j. As

$$(ABC) = (l_1\, l_2\, l_3)^T$$

We must have

$$\mathrm{diag}[(ABC)(ABC)^T] = I \quad \text{or}$$
$$\mathrm{diag}(\lambda F^{-1} + 2\mu F^{-1/2} \mathbb{R} k k^T \mathbb{R}^T F^{-1/2}) = I$$
$$F = \mathrm{diag}(\lambda I + 2\mu \mathbb{R} k k^T \mathbb{R}^T). \tag{29}$$

In particular for $\mathbb{R} = I$ we find the classical representation deduced from Betti's formula

$$\bar{f}_1 = \bar{f}_2 = \lambda \qquad \bar{f}_3 = (\lambda + 2\mu)$$
$$l_2 = h_0 \qquad l_2 = \theta_0 \qquad l_3 = k.$$

(b) Unit Shear Dislocation along h_0 ($d_1 = 1$, $d_2 = 0$, $d_3 = 0$). The system (26) (27 bis) becomes

$$\bar{f}_j \alpha_{j0} \gamma_j = \mu$$
$$\bar{f}_j \beta_{j0} \gamma_j = \bar{f}_j \alpha_{j0} \beta_{j0} = 0$$
$$\bar{f}_j \gamma_j^2 = \bar{f}_j \alpha_{j0}^2 = \bar{f}_j \beta_{j0}^2 = 0.$$

As $|l_j| = 1$ the three last relations imply that $\Sigma \bar{f}_j = 0$. It is always possible to choose $\bar{f}_1 < 0, \bar{f}_2 > 0, \bar{f}_3 \geqq 0$. We take now $F = \mathrm{diag}(-\bar{f}_1, \bar{f}_2, \bar{f}_3)$. Using the same other notations as before we obtain now

$$Z^T Z = \bar{f}_j \gamma_j^2 - 2\bar{f}_1 \gamma_1^2 = -2\bar{f}_1 \gamma_1^2.$$

Similarly

$$X^T X = -2\bar{f}_1 \alpha_{10}^2 \qquad Y^T Y = -2\bar{f}_1 \beta_{10}^2$$
$$X^T Z = \mu - 2\bar{f}_1 \alpha_{10} \gamma_1 \qquad Y^T Z = -2\bar{f}_1 \beta_{10} \gamma_1 \qquad Y^T X = -2\bar{f}_1 \alpha_{10} \beta_1$$

We see that

$$(Y^T Z)^2 = (Y^T Y)(Z^T Z)$$
$$(Y^T X)^2 = (Y^T Y)(X^T X)$$

which proves that $Y = aX = cZ$. Either $Y=0$ or the three vectors are proportional. The last case corresponds in fact to a simple couple and gives no solution (from (25)). To find the solution corresponding to $Y=0$ we introduce the angles $\omega_j = (h_0, l_j)$

$$\alpha_{j0} = \cos\omega_j \qquad \gamma_j = \sin\omega_j.$$

The system reduces then to

$$\bar{f}_j \sin\omega_j \cos\omega_j = \mu$$
$$\bar{f}_j \sin^2\omega_j = \bar{f}_j \cos^2\omega_j = 0.$$

This system gives $\bar{f}_j$ for any orientation of the three coplanar dipoles. It is however possible to obtain a solution with only two couples by taking

$$\bar{f}_3 = 0 \qquad \omega_1 + \omega_2 = \pi \qquad \bar{f}_2 = -\bar{f}_1 = 1/\sin 2\omega_1.$$

In particular for $\omega_2 = \pi/4$ we find the classical representation by means of two orthogonal dipoles of intensity μ inclined at $\pi/4$ to the normal to the fault plane and to the displacement.

(c) Dislocation of Arbitrary Direction. A similar method could be developed for the case of an arbitrary direction. A given dislocation is therefore equivalent to a continuous infinity of systems of three dipoles.

6. Conclusion

The main results of this paper are the following:

The diverging waves produced by an arbitrary point source may be described by only three simples matrices corresponding to *P*, *SV*, *SH* waves.

The introduction of the stress glut tensor in the Thomson-Haskell method clarifies the notion of equivalence between dislocation and system of dipoles.

A continuous elastic medium cannot sustain without faulting the action of a dipole. This last appears as an apparent force system derived from a stress glut tensor.

A given dislocation is equivalent to a continuous infinity of systems of three dipoles. Orthogonal dipoles are therefore a particular case without special physical meaning.

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Appendix

We shall use notations analogous to, but slightly different from, those used in Paper I. The matrix M of (1) is simply proportional to the matrix M of (I.12)

$$M=\varpi\eta\begin{pmatrix}\kappa k h^T+h k^T & I-a k k^T\\ -m^2 I-4 a h h^T-\theta\theta^T & \kappa h k^T+k h^T\end{pmatrix}. \tag{A.1}$$

From this expression we deduce

$$M^2=\varpi^2\eta^2\begin{pmatrix}D & aQ\\ -2abQ & D\end{pmatrix},$$

with

$$D=-(s^2+2a)hh^T-(r^2-2a)kk^T-s^2\theta\theta^T$$

$$Q=hk^T+kh^T \quad (Q^2=I-\theta\theta^T).$$

It is easy to see that we may write

$$m^2 I=\begin{pmatrix}K & Q\\ -2bQ & K\end{pmatrix}+\begin{pmatrix}L & -Q\\ 2bQ & L\end{pmatrix}=m^2(E_p+E_s) \tag{A.2}$$

$$m^2 M^2=-\varpi^2\eta^2(r^2 E_p+s^2 E_s)$$

$$E_p E_s=E_s E_p=0, \quad E_p^2=E_p,\ E_s^2=E_s$$

if

$$K=-2hh^T+b\,kk^T$$

$$L=-2kk^T+b\,hh^T+m^2\theta\theta^T.$$

The orthogonal projectors E_p and E_s are therefore the eigenprojectors of M^2.

The eigenprojectors of M are deduced from them

$$\Pi_p^{\pm}=[\pm\varpi\eta i r E_p+ME_p]/(\pm 2\varpi\eta i r)$$

$$\Pi_s^{\pm}=[\pm\varpi\eta i s E_s+ME_s]/(\pm 2\varpi\eta i s) \tag{A.3}$$

After some algebra we obtain

$$m^2 ME_p=-\varpi\eta\begin{pmatrix}+bhk^T+2r^2 kh^T & hh^T-r^2 kk^T\\ -4r^2 hh^T+b^2 kk^T & bkh^T+2r^2 hk^T\end{pmatrix}$$

$$m^2 ME_s=-\varpi\eta\begin{pmatrix}bkh^T+2s^2 hk^T & s^2 hh^T-kk^T+m^2\theta\theta^T\\ 4s^2 kk^T-b^2 hh^T-s^2\theta\theta^T & bhk^T+2s^2 kh^T\end{pmatrix}$$

According to (8) the diverging projectors are given by

$$2\Pi^D=\Pi^+ +\Pi^- +\varepsilon(\Pi^+ -\Pi^-)=E+\varepsilon ME/\varpi\eta i|\lambda|$$

where $|\lambda|$ is r or s. We thus obtain

$$2m^2 ir\Pi_p^D = \begin{pmatrix} irK-\varepsilon(bhk^T+2r^2kh^T) & irQ+\varepsilon(r^2kk^T-hh^T) \\ -2irbQ+\varepsilon(4r^2hh^T-b^2kk^T) & irK-\varepsilon(bkh^T+2r^2hk^T) \end{pmatrix}$$

$$2m^2 is\Pi_s^D = \begin{pmatrix} isL+\varepsilon(bkh^T+2s^2hk^T) & -isQ+\varepsilon(s^2hh^T-kk^T+m^2\theta\theta^T) \\ 2isbQ+\varepsilon(4s^2kk^T-b^2hh^T-s^2\theta\theta^T) & isL+\varepsilon(bhk^T+2s^2kh^T) \end{pmatrix}$$

expressions which correspond to (I.29, 30).

We remark that the products $\Pi^D\begin{pmatrix}h\\0\end{pmatrix}$, $\Pi^D\begin{pmatrix}0\\h\end{pmatrix}$ ... may be easily deduced from $\Pi^D\begin{pmatrix}h&0\\0&h\end{pmatrix}=\Pi^D h\ldots$

We thus obtain

$$2m^2 ir\Pi_p^D\begin{pmatrix}h&0\\0&h\end{pmatrix} = -\begin{pmatrix}2irh+2\varepsilon r^2k & \varepsilon h-irk\\ 2irbk-4\varepsilon r^2h & 2irh+\varepsilon bk\end{pmatrix} = -(2ir\varepsilon|1)R$$

$$2m^2 ir\Pi_p^D\begin{pmatrix}k&0\\0&k\end{pmatrix} = \begin{pmatrix}irbk-\varepsilon bh & irh+\varepsilon r^2k\\ -2irbh-\varepsilon b^2k & irbk-2\varepsilon r^2h\end{pmatrix} = (-b|ir)R$$

$$2m^2 ir\Pi_p^D\begin{pmatrix}\theta&0\\0&\theta\end{pmatrix}=0$$

$$2m^2 is\Pi_s^D\begin{pmatrix}h&0\\0&h\end{pmatrix} = \begin{pmatrix}isbh+\varepsilon bk & -isk+\varepsilon s^2h\\ 2isbk-\varepsilon b^2h & isbh+2s^2\varepsilon k\end{pmatrix} = (b|-is\varepsilon)S$$

$$2m^2 is\Pi_s^D\begin{pmatrix}k&0\\0&k\end{pmatrix} = \begin{pmatrix}-2isk+2\varepsilon s^2h & -ish-\varepsilon k\\ 2bish+4s^2\varepsilon k & -2isk+\varepsilon bh\end{pmatrix} = -(2is\varepsilon|1)S$$

$$2m^2 is\Pi_s^D\begin{pmatrix}\theta&0\\0&\theta\end{pmatrix} = \begin{pmatrix}ism^2\theta & \varepsilon m^2\theta\\ -\varepsilon s^2\theta & ism^2\theta\end{pmatrix} = (is\varepsilon|1)T \tag{A.4}$$

where the matrices R, S, T correspond respectively to P, SV and SH waves:

$$R=\begin{pmatrix}\varepsilon h-irk\\ \varepsilon bk+2irh\end{pmatrix}\quad S=\begin{pmatrix}\varepsilon k+ish\\ -\varepsilon bh+2isk\end{pmatrix}\quad T=m^2\begin{pmatrix}\varepsilon\\ is\end{pmatrix}\theta. \tag{A.5}$$

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