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A Method for Synthesis of the Seismic Coda of Local Earthquakes

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Abstract. The coda of the seismic signal due to local earthquakes is composed of scattered body and surface waves. An exact description of the coda in the time domain is very difficult. However, some general properties of the coda can be obtained from a study of real seismograms. The application of narrow band pass filtering to some central United States earthquakes showed that the coda envelope $a(t)$ of the filtered signal can be adequately described by

$$a(t)=\begin{cases}(t/t_j)^{-q_j} & t \geqq t_j\\ 0 & t<t_j,\end{cases}$$

where q_j and t_j are constants depending upon the filter center frequency and where t is the time after the P arrival. For the central United States $q \approx 1.5$ is a reasonable value in the 1–10 Hz frequency range.

This empirical description of the coda can be used for a realistic synthesis of the coda in the time domain. The steps used are the following: for each frequency range, a suitably random sequence is modulated by the envelope function for that frequency; the resultant time series is narrow band pass filtered; after the filtered time series are formed, they are added. By construction the coda so modeled has a frequency dependent envelope. A comparison between an observed seismogram and one predicted using this method shows good agreement.

Key words: Synthetic seismograms – Seismic coda – Local earthquake.

Introduction

The problem of generating realistic ground motion time histories at distances greater than a few tens of kilometers from a local earthquake source is hampered by the complexity of the earth as a transmission medium at frequencies greater than a few tenths of a Hertz. The seismic signal of a local earthquake is characterized by delineable P and S wave arrivals followed by a tail, or coda, of randomlike arrivals of decreasing amplitude with time. Because of the relatively large contribution of the coda to the total ground motion time history, an adequate description of this sequence of arrivals is required for a proper estimate of earthquake ground motion for the aseismic design of structures.

Aki (1969) and Aki and Chouet (1975) studied the coda of local earthquake signals from several source regions. These authors modeled the coda in terms of backscattered body and surface waves. The applicability of their models was demonstrated by their success in extracting source parameter information from observed coda. However, these models have not been extended to the problem of realistic time domain synthesis of the coda, given source parameters.

As another approach, Herrmann (1975) presented a time domain coda model based on empirical observations of the shape of the coda envelope with time. His technique consisted of specifying the shape of the coda envelope as a function of time, filling the envelope with a sequence of random amplitudes as a function of time, and then passing this time series through a series of filters to account for instrument response and the spectral characteristics of the seismic source. This model gave good agreement between observed and predicted coda duration as a function of seismic moment.

The model of Herrmann (1975) did not take into account the variation of the coda envelope with frequency and epicentral distance. Aki and Chouet (1975) showed that the shape of the coda is a function of the frequency band through which it is observed. This is expected since the earth should appear more coherent at low frequencies than at high frequencies for a propagating seismic signal if the transmission medium contains scatterers. The object of the present paper is to present a simple technique for incorporating this frequency dependence into the coda model.

Observations

During the period from 1970 to 1972, Saint Louis University operated a seismograph system in a lead mine at Flat River, Missouri (FRM). One of the systems consisted of an overdamped long period Sprengnether-Columbia vertical seismometer ($T_0 = 30$ s, $\zeta = 4$), whose output through a velocity transducer was recorded on analog magnetic tape. The relative response of this system to ground displacement is plotted as a function of frequency by the dashed curve in Figure 1. A search of earthquakes occurring during the 3 year period of operation was made to find suitable events for a study of coda properties. The event selected for presentation here is one which occurred in northeastern Arkansas on November 17, 1970. This event was 220 km distant from FRM and had a seismic moment of 1.1×10^{22} dyne-cm, an $m_b = 4.4$, and a corner frequendy of 1.4 Hz between the f^0 and f^{-2} asymptotes of the far-field L_g spectra (Street et al., 1975).

To study the frequency dependence of the coda shape, the recorded event was played back through a Krohn-Hite analog filter. The responses of some of the filters used, together with their center frequencies, are given by the solid curves of Figure 1. The filters were chosen such that their relative responses were the same when plotted as a function of the logarithm of frequency. Figure 2 shows some of the playbacks. Each playback is identified by the filter center frequency used. For four of the playbacks, the response of the analog filter to a step in voltage input is given by the inset to the upper right of each playback. Note the time scale in minutes. A cursory glance shows that there are distinct differences in the coda shape as a function of frequency. At first it seems that a distinct coda is assocítated with the frequencies of 0.05 and 0.08 Hz. However, note how similar these playbacks are to the filter step responses. This implies that the signal duration is in fact less than the step response of the filter and that little can be said about the coda shape at these low frequencies other than that the coda is of short duration. A conclusion from this set of filtered signals is that at high frequencies the transition from the *S* arrival to the coda is quite gradual, while at low frequencies the coda, if it exists, is very low in amplitude compared to a distinct *S* arrival.

To obtain an idea of the empirical shape of the coda envelope at various filter frequencies, the coda amplitude was plotted as a function of time after the *P* wave arrival. The results of these measurements are shown in Figure 3, where filter center frequencies of 0.5, 0.8, 1.25, 2, 3, 5, 8 and 12.5 Hz were used. When the data are plotted on log-log paper, it is found that the envelope can be described by a simple power law relationship, such that the amplitude is proportional to t^{-q}, where q is a constant and t is the time after the *P* wave arrival. These exponents are also presented in Figure 3. On the average $q=1.5$ for the frequency range of 0.5–12.5 Hz. These results differ from those of Aki and Chouet (1975) who found a strong dependence of coda shape on frequency in this frequency range. This difference may be due to the fact that they measured coda amplitude with respect to origin time rather than the *P* arrival time or possibly that the nature of scattering in the central United States differs from that in the more tectonic regions of central California and western Japan, the sources of their data. It must be stated that the empirical fit to a t^{-q} relationship is based on data at a distance of 220 km from the source. However, data from four other central United States earthquakes studied, observed at distances of 150–450 km, show similar coda characteristics.

Model

The process of incorporating frequency dependence of the coda envelope into a realistic ground motion time history is just the reverse of the process used in obtaining the filtered seismograms of Figure 2. A slight modification of the procedure introduced by Herrmann (1975) is all that is required to accomplish this task.

The first step is to form a time series $a_j(t)$ having the desired envelope shape in the frequency band $f_j \leqq f < f_{j+1}$ as follows:

$$a_j(t) = \begin{cases} 0 & t < t_{1j} \\ r(t) & t_{1j} \leqq t < t_{2j} \\ r(t)\,(t/t_{2j})^{-q_j} & t > t_{2j}, \end{cases} \qquad (1)$$

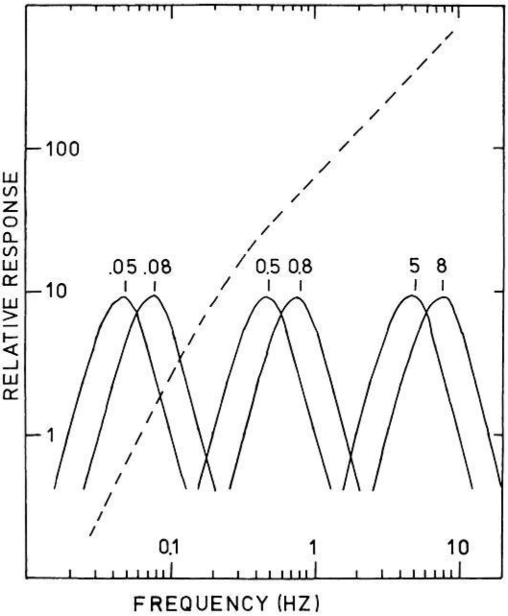

Fig. 1. Relative instrument magnification of the FRM analog recording system (dashed line) and relative response of the analog filters used (solid lines)

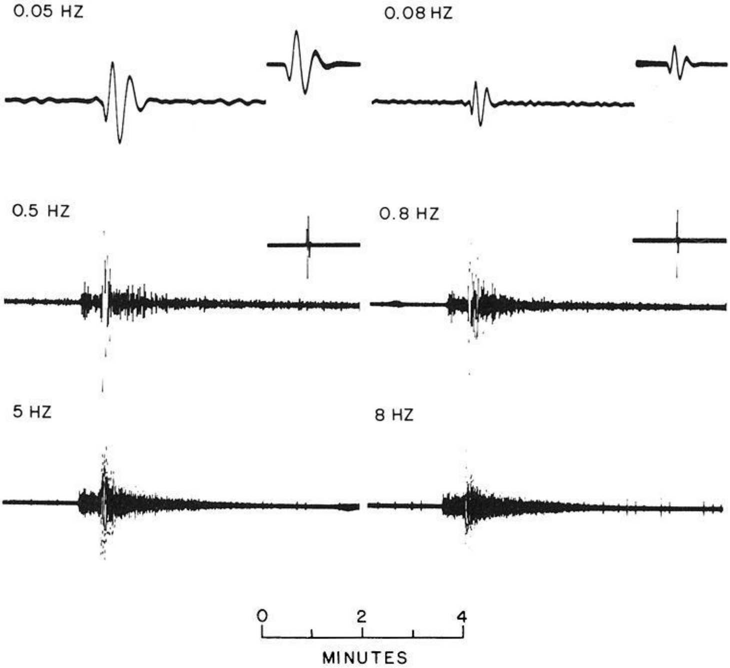

Fig. 2. Filtered playbacks of the November 17, 1970 event in northeastern Arkansas. The numbers refer to the center frequency of the filter used. The inserts show the step response of the filters for four of the center frequencies

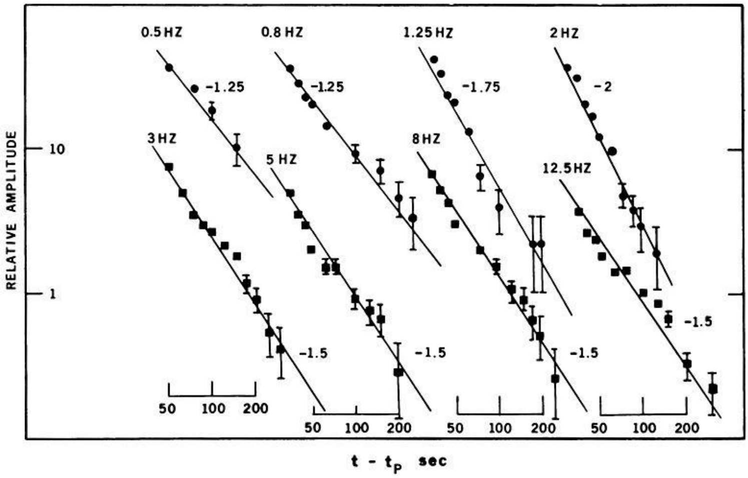

Fig. 3. Amplitude of filtered coda as a function of time after the P wave arrival for various filter center frequencies. The negative numbers are the slopes of the best visual fit to the data when plotted on a log-log scale. The error bars indicate the effect of a 1 mm error in reading the coda amplitudes

where $r(t)$ is a random number sequence uniformly distributed between -1 and $+1$, t is time after the P wave arrival, and t_{1j}, t_{2j} and q_j are constants. This form of the coda was chosen to account for the shape of the L_g higher mode surface arrivals between t_{1j} and t_{2j} and for the decay of the coda for $t > t_{2j}$.

The $a_j(t)$ time series is next filtered by convolving it with a filter function $g_j(t)$ whose Fourier Transform $G_j(f)$ is given by

$$G_j(f) = \begin{cases} 1 & f_j \leqq f < f_{j+1} \\ 0 & \text{elsewhere.} \end{cases} \tag{2}$$

The desired time series for the coda which has the proper frequency characteristics is given by a linear combination of the filtered functions $a_j(t) * g_j(t)$ as follows:

$$x(t) = \sum_j c_j a_j(t) * g_j(t), \tag{3}$$

where * denotes convolution, the summation runs over frequency bands, and c_j are constants chosen such that the amplitude spectrum of $x(t)$, $|X(f)|$, is suitably flat. Note that this procedure is in fact the inverse of the process used in obtaining Figure 2.

Because of the flatness of the amplitude spectrum of $x(t)$, the time series can thus be said to be representative of the response of the medium in which scattering takes place to a step dislocation. By convolving $x(t)$ with $s(t)$, the inverse Fourier Transform of the normalized source spectrum model

$$S(f) = \begin{cases} 1 & f \leqq f_c \\ (f_c/f)^2 & f \geqq f_c, \end{cases} \tag{4}$$

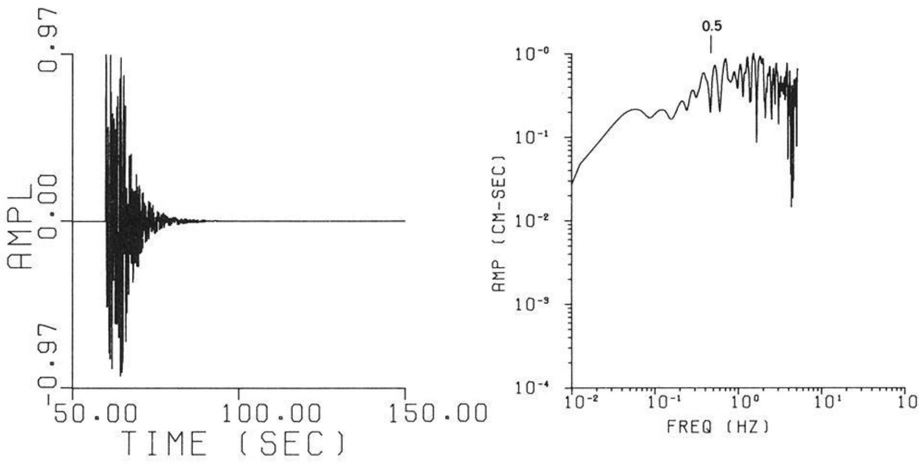

Fig. 4. Synthesized time series and corresponding amplitude spectrum used to represent low frequency character of the coda

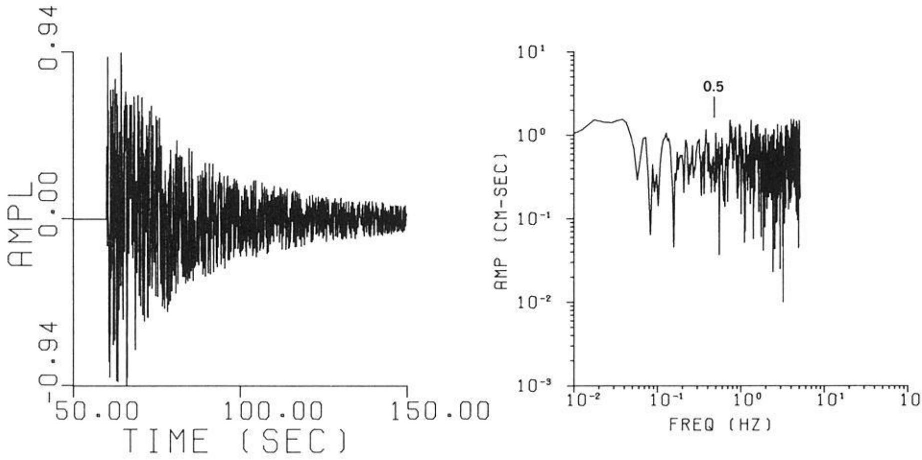

Fig. 5. Synthesized time series and corresponding amplitude spectrum used to represent the high frequency content of the coda

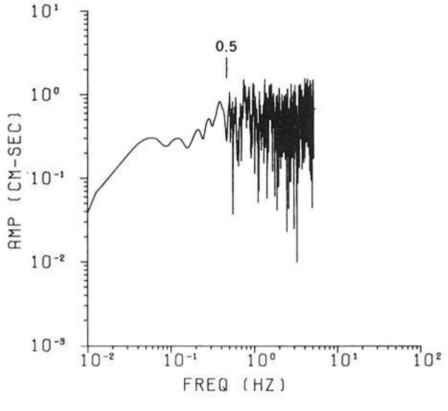

Fig. 6. Amplitude spectrum of composite time series $x(t)$ which has desired spectral content at low and high frequencies

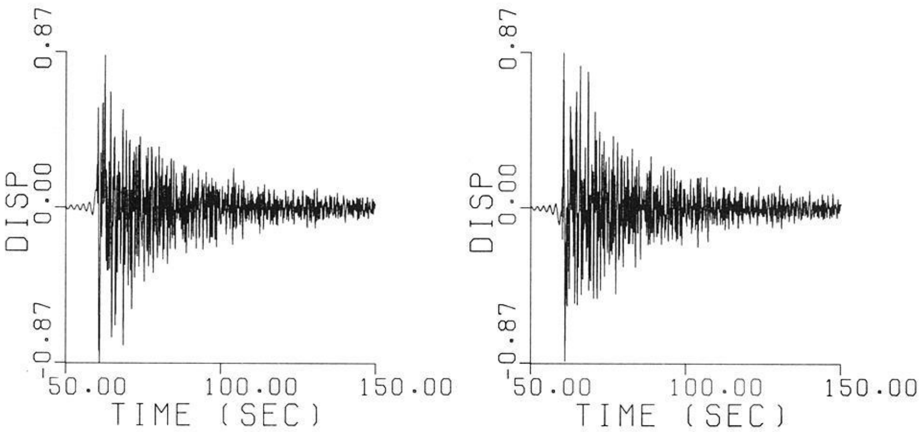

Fig. 7. Synthesized ground displacement (cm) time history having an amplitude spectrum with corner frequency of 2 Hz. Both the in-phase and quadrature components of ground displacement are shown

the variation of the coda envelope with source corner frequency f_c can be studied.

In the examples to follow, a time series of 4096 points was formed with a sample interval of DT = 400/4096 s. All filtering was done in the frequency domain using a Fast Fourier Transform to go from the time domain to the frequency domain and back. For simplicity of presentation, the coda is assumed to have only two shapes, at frequencies above and below 0.5 Hz. The examples are constructed to be representative of a seismic signal at an epicentral distance of about 200 km, with a *P* wave travel time of about 30 s and an *S* wave travel time of about 60 s.

For frequencies less than 0.5 Hz, it is assumed that there is little scattering. The input time series and its corresponding amplitude spectrum are shown in Figure 4. In this figure $a_1(t)$ is plotted as a function of time after the origin time. The coda envelope is assumed flat for times between 60 and 65 s, and falls off as ${t_{C-P}}^{-10}$, for times greater than 65 s, where t_{C-P} is the time of the coda arrival after the *P* wave arrival. The exponent was chosen to provide a short coda at low frequencies.

For the example here, it is assumed that scattering is important at frequencies greater than 0.5 Hz. To model this, $a_2(t)$ was formed such that the coda envelope is flat at times between 60 and 65 s after the origin time and that the coda envelope falls off as ${t_{C-P}}^{-2}$ for times greater than 65 s. This time series and its corresponding amplitude spectrum are shown in Figure 5.

The two spectra of Figures 4 and 5 were suitably band pass filtered by application of the filter of Equation (2) and weighted by the c_j coefficients of Equation (3). Using a simple averaging algorithm built into the computer program, the values used were $c_1 = 1.47$ and $c_2 = 1.00$. The composite amplitude spectrum $X(f)$ of the desired time series $x(t)$ is given in Figure 6. The vertical bar indicates the frequency of 0.5 Hz.

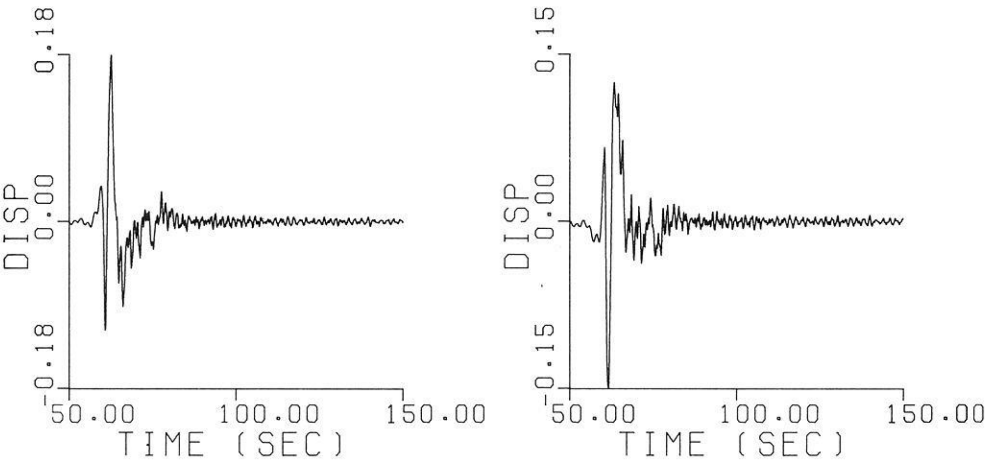

Fig. 8. Synthesized ground displacement (cm) time history having an amplitude spectrum with a corner frequency of 0.2 Hz. Both in-phase and quadrature components of ground displacement are shown

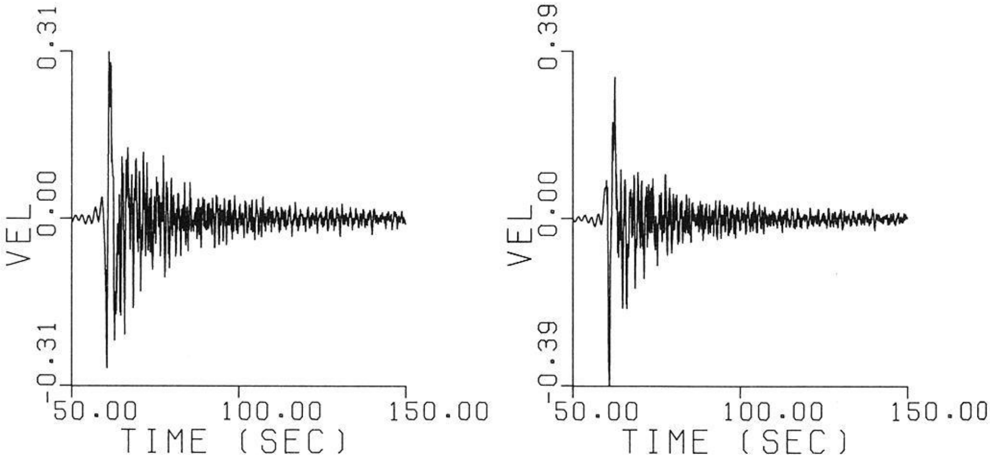

Fig. 9. Ground motion velocity (cm/s) corresponding to displacements of Figure 8

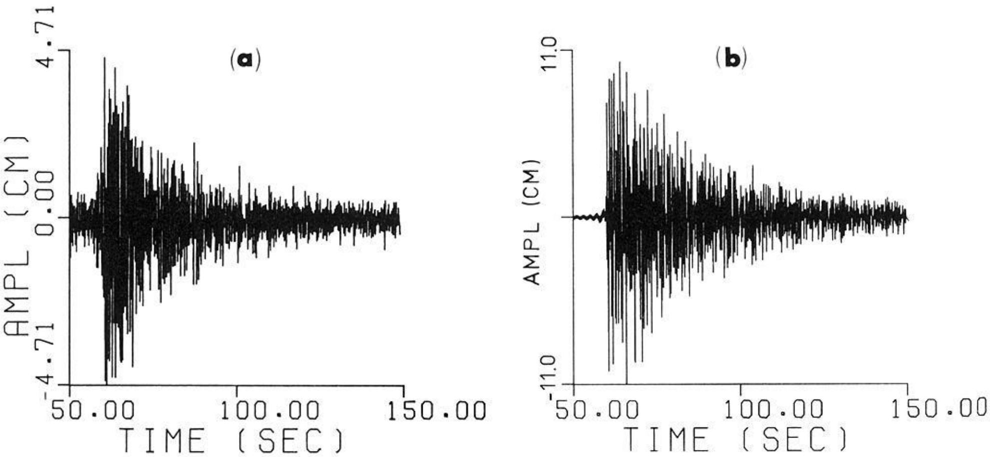

Fig. 10. a FRM broadband playback of the November 17, 1970 recording of the vertical component of ground motion. **b** Synthesized seismogram at FRM for the same event. Both amplitude scales are in units of cm

To see if the time series $x(t)$ has the desired coda properties, $x(t)$ was convolved with $s(t)$ for different choices of the corner frequency f_c. Figure 7 shows the synthesized ground displacement for a source spectrum with a corner frequency of 2 Hz. Two displacement time histories are given. The second is the quadrature component (phase spectrum differs by 90°) of the first record. Since the technique proposed here is based on initial random number sequences, a uniform phase shift of 90° still leaves a random number sequence. This example shows that the quadrature component also has the desired envelope properties. Thus two realistic ground motion time histories can be obtained for the price of a single Fast Fourier Transform. The noise arriving before the S wave arrival at 60 s is due to the fact that a non-causal source spectrum model, Equation (4), was used.

Figure 8 shows the effect of a source spectrum with a corner frequency of 0.2 Hz. Since this source is not as rich in high frequencies as the previous one and because of the 0.5 Hz frequency limit used in synthesizing $x(t)$, it is seen that the ground motion signal is one of relatively short duration. This figure demonstrates that the technique presented above succeeds in building frequency dependence into the coda. Figure 9 shows the ground velocity associated with the respective traces of Figure 8. Note how different these figures appear. This is due to the fact that time domain differentiation is equivalent to multiplication by $(2\pi jf)$ in the frequency domain, where j is the imaginary number. Thus the ground velocity emphasizes the higher frequency signal content and has a different coda shape.

Finally Figure 10 (a) shows the observed playback at FRM of the November 17, 1970 and the synthesized recording through the same instrument system in Figure 10 (b). To do this, the formula relating the L_g amplitude spectrum to seismic moment and distance for the central United States was used (Street et al., 1975). The seismogram of Figure 10 (b) is one to be expected for an event with a seismic moment of 1.0 E+22 dyne-cm, a corner frequency of 1.4 Hz at a station 220 km from the source having a flat velocity response of 4 K in the frequency range of 0.5–8 Hz. It is seen that the observed and predicted seismograms are within a factor of 2.5, which is quite acceptable considering the fact that the empirical formula of Street et al. (1975) predicts average spectral levels since it does not take into account radiation pattern, station corrections or other features which can affect the observed spectral amplitude at a recording site.

Discussion

In hindsight the method proposed here is both quite simple and obvious. The important result is that a technique is available for building the frequency dependence of the coda envelope into a realistic model of the ground motion. Even though this technical step has been made, theoretical and observational studies must yet be made in order to find a coda model which takes into account proper scaling of the coda with distance from the source.

The desire to build a frequency dependence into the coda model arose from practical difficulties in applying normal mode surface wave theory for earthquake sources in the crust to frequencies greater than 0.5 Hz (Herrmann and Nuttli, 1975). The technique proposed here will permit the development of a hybrid technique for predicting accurate ground motion time histories valid over a wide frequency range – the combination of an empirical coda model for frequencies greater than 0.5 Hz with normal mode surface wave theory for frequencies less than 0.5 Hz to construct a composite ground motion time history which exhibits signal coherence at low frequencies and scattering effects at high frequencies. This intriguing extension must await the resolution of the distance scaling problem of the coda.

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