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An Example of Nonlinear Dynamo Action

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Abstract. The earlier magnetohydrodynamic model of a convection driven geodynamo (Busse, 1975) is extended to include Lorentz force effects of higher order than those included before. It is shown that the action of the Lorentz force is such that the magnetic field enhances its own generation when the magnetic field strength is below a certain critical strength. Above that value the magnetic field tends to stabilize the dynamo process and the magnetic energy attains a stable equilibrium value.

Key words: Magnetohydrodynamics – Dynamo theory.

1. Introduction

During the past decades it has become generally accepted that the earth's magnetic field is generated by motions in the liquid outer core of the earth. The nature of these motions and their energy source, however, is still poorly understood. A large amount of potential information about the dynamic state of the earth's core is contained in the observed form of the magnetic field and its secular variations. The development of mathematical models which are sufficiently accurate to permit comparisons with observational data is therefore a foremost challenge of dynamo theory.

Most of the work in dynamo theory has been concerned with the kinematic dynamo problem which determines the critical amplitude of an arbitrary solenoidal velocity field at which generation of magnetic fields becomes possible. Since it has become evident in the past decade that nearly all velocity fields lead to generation of magnetic field if the amplitude is sufficiently high, the kinematic dynamo problem has lost some of its attraction for solving the problem of the geodynamo. It appears to be necessary to consider the magnetohydrodynamic dynamo problem described by the equations of motion together with the dynamo equation in order to restrict the manifold of solutions which reproduce the main features of the geomagnetic field.

The parameter of primary interest in any nonlinear magnetohydrodynamic dynamo model is the strength of the magnetic field. It is not known what determines the equilibration value of the magnetic energy in planetary and stellar dynamos although it is widely believed that the release of the dynamic constraint of the Coriolis force by the Lorentz force is a determining factor (Malkus, 1959; Chandrasekhar, 1961; Eltayeb, 1972; Roberts and Stewartson, 1975; Busse, 1976). Unfortunately it is not possible to test the various hypotheses on this subject by rigorous nonlinear theories. The perturbation approach used in most nonlinear dynamo models does not permit a quantitative application to situations of geophysical and astrophysical interest. However, a qualitative understanding of the various possibilities for the equilibration of magnetic energy may be gained from analytical models. It is the purpose of this paper to contribute towards this more modest goal.

The simplest case of equilibration of magnetic energy corresponds to Lenz' rule: The Lorentz force exerted by the growing magnetic field acts in the direction opposite to the velocity field and reduces the amplitude of the latter. The corresponding decrease of the magnetic Reynolds number leads to a decrease of the growth rate of the magnetic field. This process leads to an asymptotically vanishing growth rate and a corresponding equilibrium value for the magnetic energy. The generation of magnetic fields by convection considered in an earlier paper (Busse, 1973) describes a typical example of this process.

However, Lenz' rule is not a general law in magnetohydrodynamic dynamo theory. In a rotating system the Coriolis force may prevent the Lorentz force from reducing the amplitude of the velocity field. Soward's (1974) analysis of generation of magnetic fields by convection in a rotating system indicates that once the Lorentz force exceeds a threshold value the magnetic field tends to grow out of the range of validity of the expansion indicating a highly nonlinear process of equilibration which has not yet been investigated. In this paper we shall study a more simple case in which a particular component of the velocity field is increased such that the growth rate of the magnetic field initially increases with the Lorentz force and decreases only after a certain value of the magnetic energy has been exceeded.

The analysis of this paper is an extension of the theoretical model of the geodynamo considered in an earlier paper (Busse, 1975), which will be referred to as I. In that work the effect of the Lorentz force was taken into account only as far as it affected the amplitude of the convection motions. The resulting balance for the magnetic energy is not entirely satisfactory since in cases of geophysical interest the heat transport, and thus the amplitude of convection, rather than the temperature difference driving the motion must be regarded as the given parameter. It will be shown in this paper that a different balance, based on a higher-order effect of the Lorentz force, can be obtained without changing the model in any significant way.

2. Mathematical Formulation

As in I we consider a rotating cylindrical annulus, as shown in Figure 1. Because of the symmetry of the problem with respect to the equatorial plane $z=0$ we have omitted the lower part of the annulus shown in Figure 3 of I. Instead of

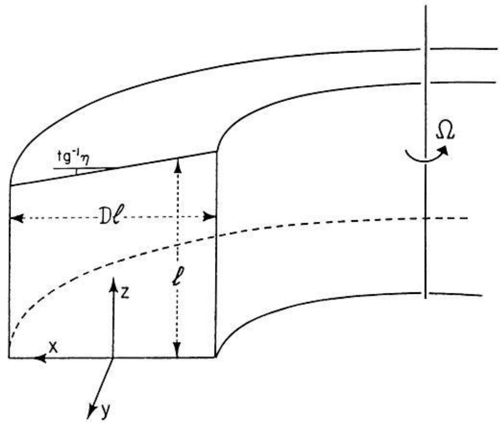

Fig. 1. Sketch of the annulus configuration used in the theoretical analysis

using $L=2l$ as the basic length scale, we use l in this paper. This has the result that the width-to-height ratio D of the annulus corresponds to $2D$ in I. In all other respects we shall use the same notation as in I. The assumption of the small gap approximation allows us to introduce a Cartesian system of coordinates (x, y, z) with the corresponding unit vectors **i**, **j**, **k** pointing in the radial, azimuthal, and axial directions, respectively. As in I it is assumed that the temperatures T_1 and $T_2\,(T_2>T_1)$ are prescribed on the outer and inner cylindrical walls, respectively, and that the gravity vector points in the direction of $-\mathbf{i}$.

We shall use the inverse of the angular velocity Ω as the time scale; $(T_2-T_1)/D$ as the temperature scale; and $(\rho_0\,\mu)^{\frac{1}{2}}\,L\Omega$ as the scale for the magnetic field, where ρ_0 is the mean density of the fluid contained by the annulus and μ is its permeability. After subtracting the equations for the static solution of the problem from the basic equations we obtain as equations for the velocity field $\mathbf{v}$ and the deviation θ of the temperature from the static temperature distribution

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\mathbf{v}+2\mathbf{k}\times\mathbf{v}=-\nabla\pi+B\,\theta\mathbf{i}+E\nabla^2\mathbf{v}+(\nabla\times\mathbf{H})\times\mathbf{H}, \tag{1a}$$

$$\nabla\cdot\mathbf{v}=0 \tag{1b}$$

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\theta=\mathbf{i}\cdot\mathbf{v}+P^{-1}E\nabla^2\theta \tag{1c}$$

where the three non-dimensional parameters B, E, and P are defined by

$$B=\frac{\beta(T_2-T_1)\,g}{\Omega^2 D\,l},\qquad E=\frac{\nu}{\Omega l^2},\qquad P=\frac{\nu}{\kappa}$$

β, ν, and κ represent the coefficient of thermal expansion, kinematic viscosity, and thermal diffusivity, respectively. g is the acceleration of gravity. The dynamo equation for the non-dimensional magnetic field **H** is given by

$$\left(\frac{\partial}{\partial t}-\tau\nabla^2\right)\mathbf{H}=\nabla\times(\mathbf{v}\times\mathbf{H}) \tag{2}$$

where τ represents the non-dimensional equivalent of the magnetic diffusivity λ,

$$\tau = \lambda/\Omega l^2.$$

In order to eliminate the need for equation (1b) we shall use the general representation

$$\mathbf{v} = \nabla \times \mathbf{k}\,\psi + \nabla \times (\nabla \times \mathbf{k}\,\phi)$$

for the solenoidal vector field $\mathbf{v}$. The solutions for ψ, ϕ, θ are obtained by perturbation methods based on the expansion

$$\begin{aligned}\psi &= \{\psi_0 + \eta\,\psi_1 + \eta^2\,\psi_2 + \cdots + M\,\psi^{(1)} + \cdots\} \exp\{i\omega t + i\alpha y\} \\ \phi &= \{\eta\,\phi_1 + \eta^2\,\phi_2 + \cdots + M\,\phi^{(1)} + \cdots - E^{\frac{1}{2}} z\,\psi_0/2 + \cdots\} \exp\{i\omega t + i\alpha y\} \\ \theta &= \{\theta_0 + \eta\theta_1 + \eta^2\,\theta_2 + \cdots + M\,\theta^{(1)} + \cdots\} \exp\{i\omega t + i\alpha y\}\end{aligned} \tag{3}$$

in terms of 3 small parameters of the problem. η is the mean angle of inclination of the top surface of the annulus, as shown in Figure 1. M represents the square of the amplitude H_A of the magnetic field $\mathbf{H}$. The third small parameter, $E^{\frac{1}{2}}$, represents the effect of the viscous Ekman layer at the rigid top boundary of the annulus, $z = \exp\{-\eta x\}$. In order to obtain a solution of the form (3) we must neglect the terms $\mathbf{v} \cdot \nabla \mathbf{v}$ and $\mathbf{v} \cdot \nabla \theta$ in equations (1). These terms determine the amplitude A of convection and will not be discussed in this paper. Expansions analogous to (3) must be assumed for B and ω,

$$\begin{aligned} B &= B_0 + \eta\,B_1 + \cdots + MB^{(1)} + \cdots \\ \omega &= \omega_0 + \eta\,\omega_1 + \cdots + M\,\omega^{(1)} + \cdots. \end{aligned} \tag{4}$$

The analysis in I considered the case

$$\text{“}M\text{”} \ll \text{“}\eta^2\text{”} \tag{5a}$$

while in this paper we shall consider the case

$$\text{“}\eta^2\text{”} \ll \text{“}E^{\frac{1}{2}}\text{”} \approx \text{“}M\text{”} \ll \text{“}\eta\text{”}. \tag{5b}$$

By "M" we mean "terms of the order M" in the above expansions, since the correct order of magnitude depends on other factors in the solutions. For example, it was found in I that $\eta^2\,\phi_2$ is actually of the order $\eta^{8/3} E^{1/3}$. This suggests a rescaling of the problem. However, in order to keep the changes of notation to a minimum and because the assumptions of the perturbation analysis can be easily checked by inspection of the results, we have omitted the complicated process of rescaling. We note that the basic assumption of the analysis in I, $\eta \gg E^{\frac{1}{4}}$, is compatible with relationship (5b).

It should be emphasized that the limit (5b) does not represent the case of direct geophysical interest since $E^{\frac{1}{2}}$ is rather small in the earth's core. Terms of the order "η^2" serve a similar function, however, as those of the order "$E^{\frac{1}{2}}$" as is shown in I. Because the analysis in the case "$E^{\frac{1}{2}}$"≪"η^2" is much more cumbersome we have chosen the limit (5b) in order to illuminate the basic process of nonlinear dynamo action.

3. Previous Results

In lowest order Equations (1) yield the solution (see I)

$$\psi_0 = A \sin \gamma (x + D/2), \qquad \theta_0 = \frac{i\alpha}{EP^{-1} a^2 + i\omega_0} \psi_0 \tag{6}$$

where a^2 is defined by

$$a^2 = \gamma^2 + \alpha^2$$

and γ is determined by

$$\gamma = \frac{n\pi}{D}, \qquad n = 1, 2, \ldots$$

in order that the boundary condition

$$\psi = \theta = 0 \qquad \text{at } x = \pm\tfrac{1}{2} D$$

is satisfied. As in I we shall use the property that for the physically relevant solution

$$1 \ll \gamma^2 \lesssim \alpha^2 \tag{7}$$

can be assumed. For the following discussion it is not necessary to consider the results of the equations of higher order explicitly. It is sufficient to note that the solution for ϕ satisfies the relationship

$$\Delta_2 \phi = -a^2 \phi \tag{8}$$

As was shown in I the task of solving the dynamo equation (2) can be simplified by assuming a 'flat' annulus,

$$D \gg 1. \tag{9}$$

In this case an average over the x- and y-dependences can be defined (indicated by a bar) and a representation of the form

$$\mathbf{H} = H_A \{ -\mathbf{j}\, G(z) + \mathbf{i}\, F(z) + \nabla \times (\nabla \times \mathbf{k}\, h) + \nabla \times \mathbf{k}\, g \} \tag{10}$$

can be assumed, where $H_A \equiv M^{\frac{1}{2}}$ and where h and g are functions of space and time with vanishing average,

$$\bar{h} = \bar{g} = 0.$$

The boundary conditions for G and F are given by

$$G(0)=F(0)=G(1)=0, \tag{11}$$

as discussed in I. Using property (7) and the fact that ϕ is small in comparison with ψ the solutions for g and h can be written in the form

$$g=\left(F\frac{\partial}{\partial x}\psi-G\frac{\partial}{\partial y}\psi\right)\quad(i\omega+\tau a^2)^{-1},$$

$$h=\left(F\frac{\partial}{\partial x}\phi-G\frac{\partial}{\partial y}\phi\right)\quad(i\omega+\tau a^2)^{-1}.$$

The equations for the mean magnetic field are given by

$$\tau\frac{d^2}{dz^2}F=\frac{d}{dx}\left\{\tfrac{1}{4}G\left[-\Delta_2\phi^+\frac{\alpha^2\psi}{i\omega+\tau a^2}+\frac{\partial}{\partial y}\psi^+\frac{a^2}{i\omega_0+\tau a^2}\frac{\partial}{\partial y}\phi+\text{c.c.}\right]\right\}, \tag{12a}$$

$$\tau\frac{d^2}{dz^2}G=\frac{d}{dz}\left\{\tfrac{1}{4}F\left[\Delta_2\phi^+\frac{\gamma^2\psi}{i\omega+\tau a^2}-\frac{\partial}{\partial x}\psi^+\frac{a^2}{i\omega+\tau a^2}\frac{\partial}{\partial x}\phi+\text{c.c.}\right]\right\} \tag{12b}$$

where ϕ^+ indicates the complex conjugate (c.c.) of ϕ and where Δ_2 is defined by $\Delta_2\equiv\frac{\partial}{\partial x^2}-\alpha^2$. Again, terms of higher order have been neglected in Equations (12).

It was shown in I that the lowest-order term proportional to $\eta\,\phi_1\,\psi_0$ vanishes on the right hand side of (12ab). Thus the dynamo action depends on terms of the order $\eta^2 A^2$, $E^{\frac{1}{2}}A^2$ or MA^2. While in I the first two possibilities were considered we shall concentrate in the following on the third possibility. Although it will turn out that the term of the order MA^2 cannot provide dynamo action by itself, in conjunction with other terms, in particular with the Ekman layer-induced term, it leads to an interesting nonlinear form of dynamo action.

4. Action of the Lorentz Force

It was shown in I that the Lorentz force arising from the vector product of the mean current and the mean magnetic field is balanced by the pressure. For this reason only the Lorentz force arising from the fluctuating current and the mean magnetic field is of interest in lowest order. By taking the z-components of the curl and the curl of Equation (2a) we obtain 2 equations for $\psi^{(1)}$ and $\phi^{(1)}$ which were previously derived in I,

$$(E\nabla^2-i\omega_0)\Delta_2\psi^{(1)}+B_0\frac{\partial}{\partial y}\theta^{(1)}+2\mathbf{k}\cdot\nabla\Delta_2\phi^{(1)}$$

$$=B^{(1)}\frac{\partial}{\partial y}\theta_0+i\omega^{(1)}\Delta_2\psi_0+L_1, \tag{13a}$$

$$2\mathbf{k}\cdot\nabla\Delta_2\psi^{(1)}=L_2 \tag{13b}$$

where L_1 and L_2 are given by

$$L_1=\left(G\frac{\partial}{\partial y}-F\frac{\partial}{\partial x}\right)\Delta_2 g, \tag{14a}$$

$$L_2=\left(F\frac{\partial}{\partial x}-G\frac{\partial}{\partial y}\right)\nabla^2\Delta_2 h. \tag{14b}$$

In I the solvability condition for the system (13) of equations was derived and expressions for $B^{(1)}$ and $\omega^{(1)}$ were determined. In this section we shall proceed beyond that stage and actually solve Equations (13). The solution becomes simple if we use the fact that L_2 is of the order η smaller than L_1 and thus can be neglected. Using in addition the solvability condition derived in I we find

$$\psi^{(1)}\equiv 0, \tag{15a}$$

$$\Delta_2\phi^{(1)}=\tfrac{1}{2}\left(\int_0^z L_1\,dz-z\int_0^1 L_1\,dz\right). \tag{15b}$$

The evaluation of expression (15b) yields

$$\Delta_2\phi^{(1)}=\frac{-a^2}{i\omega_0+\tau a^2}\left\{\int_0^z G^2\alpha^2+F^2\gamma^2\,dz-z\int_0^1 G^2\alpha^2+F^2\gamma^2\,dz\right\}\tfrac{1}{2}\psi_0$$
$$-\frac{i a^2\alpha}{i\omega_0+\tau a^2}\frac{\partial}{\partial x}\psi_0\int_0^z GF\,dz. \tag{16}$$

5. The Nonlinear Dynamo Problem

When expressions (6) and

$$\phi=(\eta\phi_1+M\phi^{(1)}-E^{\frac{1}{2}}z\psi)\exp\{i\alpha y+i\omega t\} \tag{17}$$

are inserted on the right hand side of equations (12) the contribution of $\eta\phi_1$ vanishes, as shown in I. For the same reason the second term on the right hand side of (16) does not yield a finite contribution. Thus we obtain after integrating equations (12) once

$$\frac{d}{dz}F=-\frac{\alpha}{\gamma}Kf(G,F,z)\,G, \tag{18a}$$

$$\frac{d}{dz}G=\frac{\gamma}{\alpha}Kf(G,F,z)\,F+c \tag{18b}$$

where $f(G,F,z)$ and K are defined by

$$f\equiv\int_0^z\left(\frac{\alpha}{\gamma}G^2+\frac{\gamma}{\alpha}F^2\right)dz-z\int_0^1\left(\frac{\alpha}{\gamma}G^2+\frac{\gamma}{\alpha}F^2\right)dz-\frac{\tau E^{\frac{1}{2}}}{M\alpha\gamma}z,$$

$$K\equiv\frac{\alpha^2\gamma^2a^2A^2M}{4\tau(\omega^2+\tau^2a^4)}.$$

The constant of integration c denotes the electrical field in the radial direction. Because the problem is periodic in the azimuthal direction a mean electrical field in the y-direction cannot exist, with the consequence that the constant of integration in Equation (18a) must vanish.

Equations (18) can be simplified by the transformation

$$\hat{F} \equiv \left(\frac{\gamma}{\alpha} K\right)^{\frac{1}{2}} F, \quad \hat{G} \equiv \left(\frac{\alpha}{\gamma} K\right)^{\frac{1}{2}} G, \tag{19a}$$

$$\delta = K \frac{\tau E^{\frac{1}{2}}}{M \alpha \gamma} \quad d = \left(K \frac{\alpha}{\gamma}\right)^{\frac{1}{2}} c \tag{19b}$$

with the result

$$\frac{d}{dz} \hat{F} = \hat{G} \left\{ \int_0^z (\hat{G}^2 + \hat{F}^2)\, dz - z \left(\delta + \int_0^1 (\hat{G}^2 + \hat{F}^2)\, dz \right) \right\}, \tag{20a}$$

$$\frac{d}{dz} \hat{G} = -\hat{F} \left\{ \int_0^z (\hat{G}^2 + \hat{F}^2)\, dz - z \left(\delta + \int_0^1 (\hat{G}^2 + \hat{F}^2)\, dz \right) \right\} + d. \tag{20b}$$

An explicit solution of these equations subject to the boundary condition (11) was derived in I in the limit where the amplitudes of G and F tend to zero. The lowest eigenvalue μ for which a solution exists is given by

$$\delta = \tfrac{1}{4} \mu_1 = 4.60. \tag{21}$$

At finite amplitudes of the magnetic field numerical methods are required for the solution of the nonlinear boundary value problem.

6. Numerical Results

Starting at $z=0$ Equations (20) can be integrated by the Runge-Kutta method for any prescribed value of d if the expression

$$\Gamma \equiv \delta + \int_0^1 (\hat{G}^2 + \hat{F}^2)\, \mathrm{d}z \tag{22}$$

is regarded as an adjustable parameter of the problem. A subsequent Newton-Raphson iteration on the boundary value $\hat{G}(1)$ as a function of Γ allows us to determine the value Γ for which $\hat{G}(1)$ vanishes. In this fashion the lowest values of $|\delta|$ for which solutions of the boundary value problem exist have been determined as a function of d. The results are shown in Figure 2.

The most interesting result is that solutions are possible for values of δ less than a quarter of the value of (21). Initially it was thought that solutions with $\delta = 0$ were possible. This would have been remarkable in that dynamo action would have occured at a finite strength of the magnetic field in a case when the purely hydrodynamic solution corresponds to a toroidal velocity field. However solutions with $\delta = 0$ do not appear to be possible.

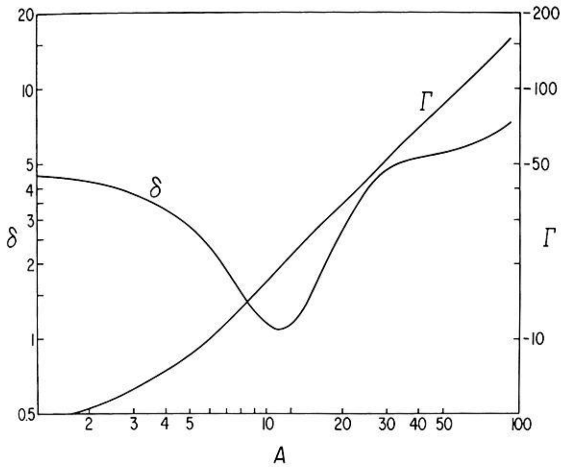

Fig. 2. Γ and δ as functions of the parameter d for the solution of Equations (20) corresponding to the lowest value of $|\delta|$

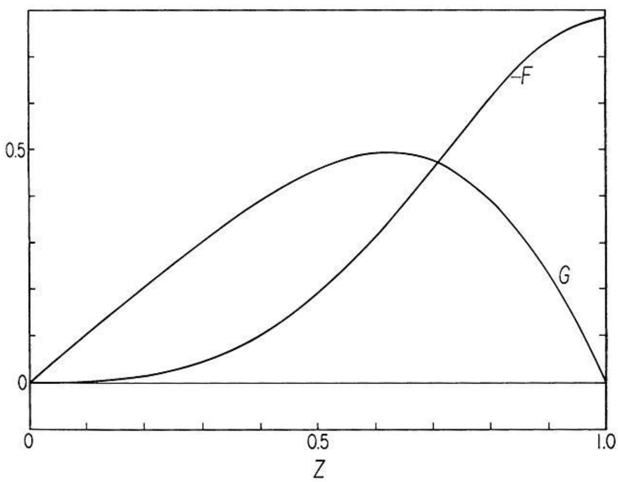

Fig. 3. $\hat{F}$ and $\hat{G}$ as functions of z for $\delta = 4.5$, $\Gamma = 4.775$, $d = 1.005$

The form of solutions $\hat{G}$ and $\hat{F}$ is plotted for some characteristic values of d in Figures 3–5. As d increases the number of zeros of $\hat{F}$ and $\hat{G}$ increases. For large values of d, $\hat{G}$ and $\hat{F}$ assume a boundary layer character near $z=0$ and for larger z, $\hat{F}$ becomes a constant $\hat{F}_c$ while $\hat{G}$ nearly vanishes. An approximate expression for $\hat{F}_c$ is

$$\hat{F}_c \approx d/\delta \tag{23}$$

which follows from the fact that $d\hat{G}/dz$ tends to vanish at $z=1$. It is evident from Figure 5 that relationship (23) is nearly satisfied. The asymptotic properties of

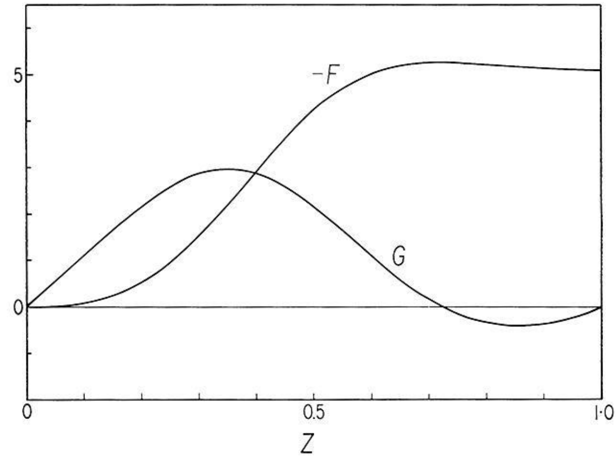

Fig. 4. $\hat{F}$ and $\hat{G}$ as functions of z for $\delta=1.08$, $\Gamma=18.74$, $d=11.09$

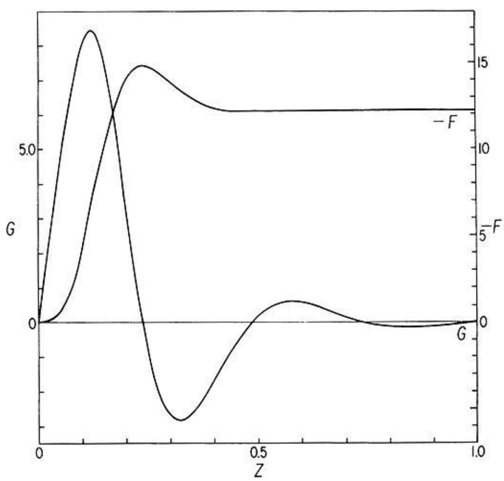

Fig. 5. $\hat{F}$ and $\hat{G}$ as functions of z for $\delta=7.3$, $\Gamma=157.4$, $d=92.1$

the functions $\hat{G}$ and $\hat{F}$ suggest the application of boundary layer methods. This has not yet been attempted.

Without an analytical theory for guidance in the interpretation of the numerical results it is not easy to understand the reason for the minimum of δ

$$\delta_{\min}=1.08 \quad \text{at} \quad d=11.13.$$

It appears that this minimum represents the optimal balance between the influence of the term $\Gamma-\delta$ which tends to decrease δ and the effect of the first term in the wavy brackets of (20), which tends to increase it.

There are other branches of solutions to equations (20), all of which appear to correspond to values of $|\delta|$ of the order 10 or larger. Since solutions corresponding to higher values of $|\delta|$ are of lesser physical interest we shall not discuss them at this point. We simply mention that the next higher branch corresponds to negative value of δ and is characterized by a zero of $\hat{F}$ in the interval $0<z<1$.

7. Discussion

It is apparent when the foregoing analysis is compared with the analysis in I that the first relationship in (5b) is not essential. The term proportional to δ in Equation (20ab) which originated from the Ekman layer suction could be replaced by the term arising from $\eta^2(\psi_0 \phi_2+\psi_1 \phi_1)$, the dynamo effect of which was considered in I. Since the latter term possesses a more complicated z-dependence we have chosen the case of (5b) in order to isolate the nonlinear dynamo effect in its simplest form.

One of the most interesting aspects of the nonlinear dynamo action is the tendency towards boundary layer formation. Boundary layer formation was observed in an earlier magnetohydrodynamic dynamo model (Busse, 1973) when it was caused by the expulsion of magnetic flux from the convection eddies. In the present case the boundary layer formation is an intrinsic part of the dynamo process, which reduces its characteristic length scale in order to keep the magnetic Reynolds number close to its optimal value. Whether the property that the azimuthal or toroidal component G becomes reduced relative to the radial or poloidal component F applies to the geodynamo is a matter of speculation. At least it demonstrates that a large toroidal field is not a necessary feature of the earth's dynamo.

As was emphasized in the introduction the objective of this paper is to illuminate a basic nonlinear aspect of dynamo action rather than to explain features of the geodynamo. In an earlier paper (Busse, 1976) it was pointed out that it is likely that the release of the Coriolis constraint by the magnetic field in the earth's core will lead to much larger scales of motion than those for which the analysis of I and this paper is directly applicable. However, since there does not seem to be any reason to change the basic geostrophic balance of the convective motion, it is possible that a process similar to the one analyzed in this paper is occurring in the earth's core. Hopefully, more detailed numerical investigations of the dynamo process in a sphere will provide realistic models for the geodynamo in the future. The simple analysis of the present paper may then be useful to interpret features of the numerical results.

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