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Relative Errors in Group Velocity Measurements*

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Abstract. The estimates of error in measurement of group velocities for a dispersed surface wave train are shown to be a strong function of the nature of the dispersion curve. At periods near a group velocity extremum, group velocity measurements give greater resolution of earth structure than do phase velocity measurements; for periods removed from such extrema, group velocity measurements give unacceptably large errors, at ordinary distances from an earthquake focus.

Key words: Error-estimation – Group velocities – Surface waves.

The postulate that a dispersed wave train has superimposed noise with random phase gives rise to the conclusion that the ratio of standard errors in the estimates of phase and group velocities is strongly dependent on the frequency interval used in the process of differentiation to derive group velocities from phase velocities (Knopoff and Chang, 1977). The expression obtained is

$$\sigma_U/\sigma_c = 2^{\frac{1}{2}}(\omega/\Delta\omega)(U^2/c^2) \tag{1}$$

where σ_U and σ_c are the standard errors in the group and phase velocities, ω is the frequency of the seismic signal and $\Delta\omega$ is the frequency difference between sample values. If the group velocities are derived from numerical differentiation of the phase velocity curve, the value of $\Delta\omega$ is obtained directly. The factor $2^{\frac{1}{2}}$ arises because of the assumption that the phases are assumed to be uncorrelated at the two frequencies. If the group velocities are obtained from a single seismogram by measuring the time of arrival of waves of a given period, the definition of the value of $\Delta\omega$ is not immediately obvious. In this case the factor $2^{\frac{1}{2}}$ in (1) may be replaced by 1, since the origin time of the earthquake is assumed to be known and the apparent group origin time can be determined directly for a postulated earth structure and focal mechanism (Knopoff and Schwab, 1968).

Let it be supposed that we wish to determine the value of the ratio σ_U/σ_c for the case of the direct reading of group velocities from a recording of a dispersed wave train. In the visual processing of a dispersed record, we note the time of arrival t of waves of a given period; the period is often determined by noting the time interval

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between successive crests of a more-or-less sinuous, frequency-modulated wave-train. A signal of instantaneous period T will appear on the seismogram at the time $t=x/U(T)$ where x is the epicentral distance. The difference in period ΔT between two neighbouring parts of a well-dispersed wave train is

$$\Delta T \approx \left|\frac{\mathrm{dT}}{\mathrm{dU}}\right| \Delta U.$$

The time interval between two successive maxima is

$$\Delta t = \frac{x}{U^2} \Delta U.$$

Hence

$$\Delta T = \left|\frac{\mathrm{dT}}{\mathrm{dU}}\right| \frac{U^2}{x} \Delta t.$$

Since the time interval between two maxima is about equal to the period of the oscillation, we have for the period difference corresponding to two successive maxima,

$$\Delta T = \left|\frac{\mathrm{dT}}{\mathrm{dU}}\right| \frac{U^2}{x} T. \tag{2}$$

The frequency difference is $\Delta\omega/\omega = -\Delta T/T$ if ΔT is small enough. Whence formula (1) becomes

$$\sigma_U/\sigma_c = \frac{x}{c^2}\left|\frac{\mathrm{dU}}{\mathrm{dT}}\right| \tag{3}$$

Thus σ_U/σ_c is small where dU/dT is small, i.e. near a group velocity extremum, and σ_U/σ_c increases with increasing distance, i.e. as the wave train becomes more and more dispersed. Evidently, since dU/dT is a strong function of period, σ_U is a strong function of T even if σ_c were independent of T.

For the case of Rayleigh waves on a continental structure (Fig. 2, Knopoff and Chang, 1977), the value of dU/dT is largest near a period of about 34 s; at this period $\sigma_U/\sigma_c = x\,(\mathrm{km})/370$. At the group velocity maximum near 75 s, $\sigma_U = 0$. Thus at the usual distances for determination of group velocities, phase velocity information provides significantly greater resolution than group velocity data. This is not true at periods near group velocity extrema; here the advantage lies with group velocity dispersion data.

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