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Investigation of Isostasy by Computing the Correlation Coefficients between Elevations and Bouguer Anomalies

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Abstract. A method for the investigation of isostasy by computing the correlation coefficients between elevations and Bouguer anomalies is presented. A tendency to an isostatic behaviour of the crust can be concluded from negative coefficients. Positive coefficients combined with a topographic high indicate mass surpluses; positive coefficients combined with a topographic low indicate mass deficiencies.

Applications of the computation of isostatic gravity anomalies and the correlation method to the Alps, Northern Italy, and Southern Scandinavia give in general similar results. Since the correlation method cannot resolve slight deviations from isostasy, it should be prefered in regions with little information concerning crustal density distribution and crustal thickness. In these regions it is difficult to derive a reasonable model for the computation of isostatic gravity anomalies. Therefore, the correlation method may be useful to investigate the isostatic state of the crusts of the terrestrial planets if Doppler gravity and elevation data are available.

Key words: Isostasy – Bouguer anomalies – Isostatic gravity anomalies – Correlation coefficients.

1. Introduction

Many methods have been developed to investigate the isostatic state of the Earth's crust. Considering the hypotheses of Pratt, Airy, and Vening Meinesz (Heiskanen and Vening Meinesz, 1958), the mean free-air anomalies ($\Delta g'$) should approach zero in areas of low reliefs and the Bouguer anomalies ($\Delta g''$) should decrease with increasing elevations in the isostatic case. Therefore, it is possible to get a general view of the isostatic behaviour of an area by free-air or Bouguer anomalies maps. To make the relations more evident, diagrams between elevations and gravity anomalies have been plotted (Woollard, 1959, 1969; Coron, 1969; Fig. 1).

The free-air anomalies used in Figure 1a are from Hilger (1968); the Bouguer anomalies used in Figure 1b and in the further investigations of the Alps and Northern Italy are from Makris (1971) and Bouvet (1971).

The free-air anomalies are influenced by two effects: (1) a scatter of ± 50 mgal caused mostly by variations in surface and near-surface geology, (Woollard, 1959) and (2) a dependence on relief for which Woollard (1969) derived emperical world wide relations for different elevation ranges. These relations are rather general; there may be deviations for different areas depending on the kind of isostatic compensation.

The most successful method for the investigation of isostasy is the computation of isostatic gravity anomalies. The computations are based on the model of compensation according to the hypotheses of Pratt, Airy, and Vening Meinesz. Regional computations of isostatic gravity anomalies using Vening Meinesz' hypothesis have seldom been carried out, because of the high uncertainty of the crustal elastic properties which are needed for the calculations.

Meissner and Vetter (1977) derived density-depth profiles from seismic velocity-depth profiles. Comparing density-depth curves of different areas permits conclusions in regard of the isostatic balance.

If the density distribution of the crust and the crust-mantle boundary are well known (e.g. on seismic profiles), the method of mass summations yields mass surpluses and deficiencies of an area (Janle, 1973; Goldflam et al., 1977).

In areas with little information on the crustal density distribution and crustal thickness it may be difficult to evaluate a reasonable density model for isostatic calculations. In addition to this the definition of a depth of compensation or the normal thickness of the crust according to the hypotheses of Pratt and Airy is often speculative.

All three hypotheses of isostasy mentioned above demand high Bouguer gravity above low relief (e.g. oceans) and low Bouguer gravity above high relief (e.g. mountainous regions). Proceeding from this anticorrelation between elevations and Bouguer anomalies in the isostatic case, a method for investigating isostasy will be presented in this paper.

2. Description of the Method

Diagrams between elevations and Bouguer anomalies constitute a first step for investigating the correlations between these two parameters. The correlation coefficients in Figure 1b are -0.8 for the Alps indicating a strong tendency for isostatic compensation, -0.6 for the Po Basin, and $+0.6$ for the Appennines and their northern foreland, indicating an anisostatic behaviour.

This method has two disadvantages: (1) one cannot see the geographical localization in the diagram; (2) it is difficult to choose from the Bouguer map samples of elevations and gravity which belong to the same relation. The ratio of mean elevations to mean Bouguer anomalies introduced by Makris (1971) may be of help to separate areas of equal relations. The correlation method, which will be described now, can be regarded as an advancement of the diagram method. This new method overcomes the disadvantages mentioned above.

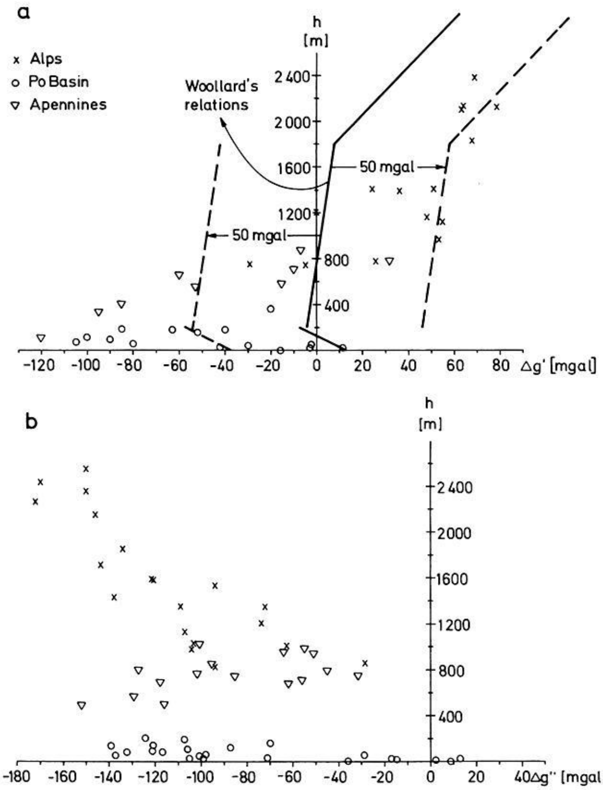

Fig. 1. a Diagram of mean free-air anomaly values to mean elevations (grid width: $\Delta\varphi=20'$, $\Delta\lambda=30'$); **b** Diagram of mean Bouguer anomaly values to mean elevations (grid width: $\Delta\varphi=12'$, $\Delta\lambda=20'$)

For preparation a grid system has to be chosen for the area to be investigated. Then the mean elevations and mean Bouguer anomalies of the areas of the grid elements must be estimated. After this the width of the sample area for the correlation must be determined, which depends on the assumption of local or regional compensation. The elevation and gravity values of the grid elements in the sample areas are used for the calculation of the correlation coefficient according to the following formulas:

$$S_x=\sqrt{\frac{\sum x_i^2-(\sum x_i)^2/n}{n-1}}$$

$$S_y=\sqrt{\frac{\sum y_i^2-(\sum y_i)^2/n}{n-1}} \qquad \text{standard deviations}$$

$$S_{xy}=\frac{1}{n-1}\left(\sum x_i y_i-\frac{1}{n}\sum x_i \sum y_i\right) \qquad \text{covariance}$$

$$c=\frac{S_{xy}}{S_x S_y} \quad -1\leqq c\leqq 1 \qquad \text{correlation coefficient}$$

(x_i = mean elevations, y_i = mean Bouger anomalies, n = number of the grid elements for the correlation).

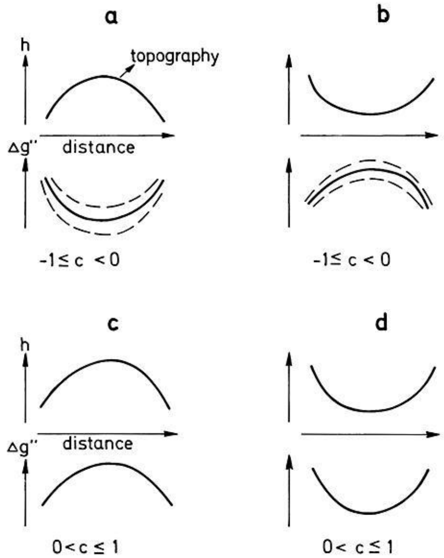

Fig. 2a–d. Illustration for the interpretation of the correlation coefficients (idealized cases): **a**, **c** topographic high (e.g. mountain range); **b**, **d** topographic low (e.g. basin); **a**, **b** significant anticorrelation between topography and Bouguer anomaly ($c<0$) implies a tendency to isostasy; the dashed $\Delta g''$ curves represent over- und undercompensation which cannot be concluded in these cases from the correlation coefficients; **c** positive coefficients combined with a topographic high indicate mass surpluses; **d** positive coefficients combined with a topographic low indicate mass deficiencies

Figure 2 should be of assistance in interpreting the correlation coefficients. In the case of a significant anticorrelation ($c<0$) a strong tendency to isostasy can be concluded. Unfortunately, slight over- or undercompensation gives negative coefficients also (Fig. 2a, b). Positive coefficients combined with a topographic high indicate mass surpluses, and positive coefficients combined with a topographic low indicate mass deficiencies (Fig. 2c, d).

3. Applications of the Correlation Method

The diagram method, the computation of isostatic gravity anomalies, and the correlation method have been applied to data measured in the Alps and Northern Italy. A grid system with a width of $\Delta\varphi = 12'$ and $\Delta\lambda = 20'$ was chosen for all three methods (except for the diagram of free-air anomaly values to mean elevations in Fig. 1a).

The isostatic gravity anomalies were computed assuming Airy compensation. The mean crustal density distribution used for the calculations was derived from crustal models of Makris (1971) and Snoek (1973; for the sea areas). The density model is included in Figure 3. The topographic – isostatic corrections for the Hayford zones 18-1 are from maps of Kärki et al. (1961).

For the calculation of the correlation coefficients a sample area was chosen with an extent of $\Delta\varphi = 60'$ and $\Delta\lambda = 80'$ which is about 110 km × 110 km. This area contains 20 grid elements, i.e. 20 pairs of elevations and Bouguer anomalies are available for the correlation. The lower significant lines of /0.2/ and /0.4/ have been left out in the figures of the correlation coefficients in order to simplify the maps. A 99.6 percent confidence results from 20 data pairs for a coefficient of /0.6/.

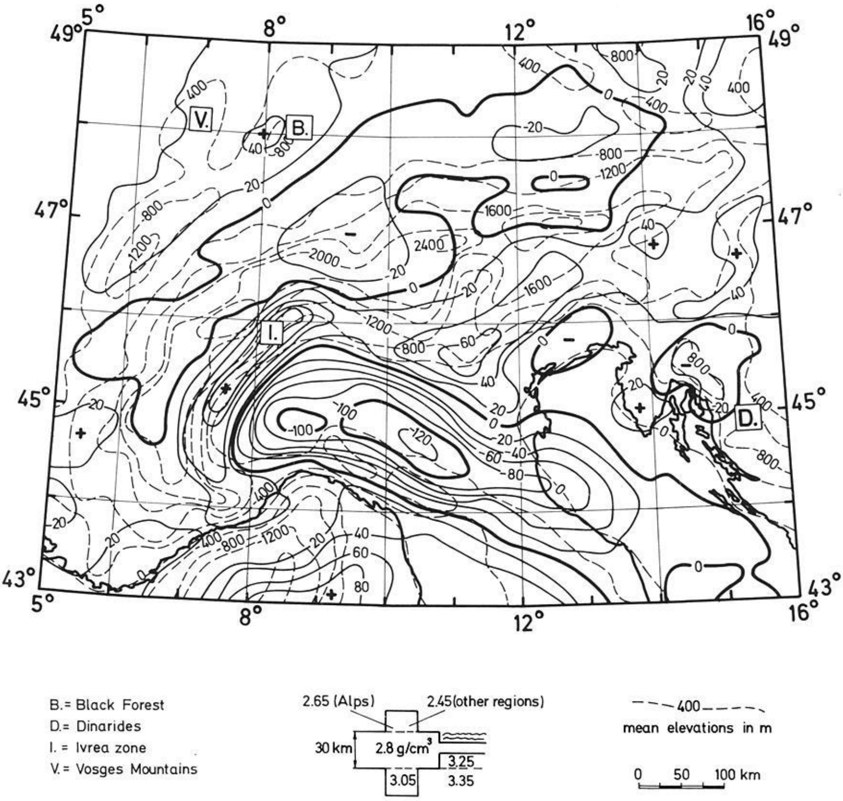

Fig. 3. Isostatic gravity anomalies of the Alps and Northern Italy. Reference plane: sea level; intervals of isolines: 20 mgal

Figure 1a shows the diagram of the elevation and the free-air anomaly values. The relations of Woollard for relief are also included. The dashed lines mark the ± 50 mgal scatter. Most of the points of the Alps and the points of the Appenines with high elevations lie within the ± 50 mgal scatter indicating an isostatic behaviour. Most of the points of the Po Basin and the points of the northern foreland of the Appenines are shifted strongly to negative values, indicating mass deficiencies for these regions. Some points of the Po Basin lie still within the ± 50 mgal scatter; i.e. some parts of the Basin show a tendency to an isostatic behaviour.

The diagram between elevations and Bouguer anomalies (Fig. 1b) has been discussed above. Note that the points of the Appenines with high elevations could belong to the group of the Alpine points. This indicates again a tendency to isostasy for the central Appennines. From very low gravity for many points of the Po Basin and for the foreland of the Apennines mass deficiencies can be concluded for these regions from this diagram also.

The isostatic gravity anomalies and the correlation coefficients will now be described together (Figs. 3, 4). The Alps and the Bavarian molasse show in general isostatic gravity anomalies below /40/ mgal, and the correlation coefficients are less than -0.6. Thus, both methodes show a tendency to an isostatic

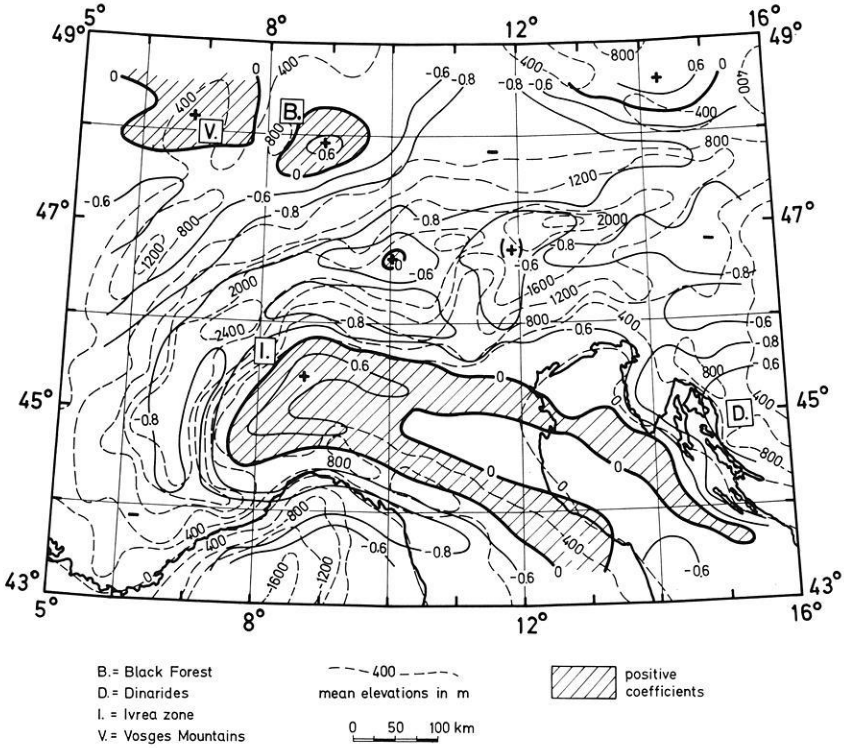

Fig. 4. Correlation coefficients between elevations and Bouguer anomalies of the Alps and Northern Italy. Intervals of isolines: 0.2; the lower significant lines of /0.2/ and /0.4/ have been left out in order to simplify the map

behaviour of these regions. In a more detailed view, the southern Alps show in contrast to the northern Alps positive isostatic gravity anomalies up to 40 mgal and relative low negative coefficients ($c > -0.6$). This may indicate slight mass surpluses. This result can be compared with modern concepts of orogenies. For instance the mass surpluses can be interpreted by the concept of Wunderlich (1966) which suggests intrusional formations and crustal thinning in the region of the backside of an orogene.

The southern Rhine Valley and its bordering mountains have positive isostatic gravity anomalies reaching more than 40 mgal. The correlation coefficients show positive values for the positive reliefs of the Black Forest and the Vosges Mountains. Thus, both methods indicate mass surpluses. This is in agreement with an updoming of the crust-mantle boundary in the southern Rhinegraben region (Edel et al., 1975).

The high isostatic gravity anomaly of the Ivrea zone (more than 80 mgal) finds no expression in the correlation method. The effects of this anomaly may be averaged out by the calculation of the correlation coefficients, because the Ivrea zone has a relative local extent. In this region high density, possibly mantle material intruded the crust causing a mass surplus (Berckhemer, 1968; Kaminski and Menzel, 1968; Makris, 1971).

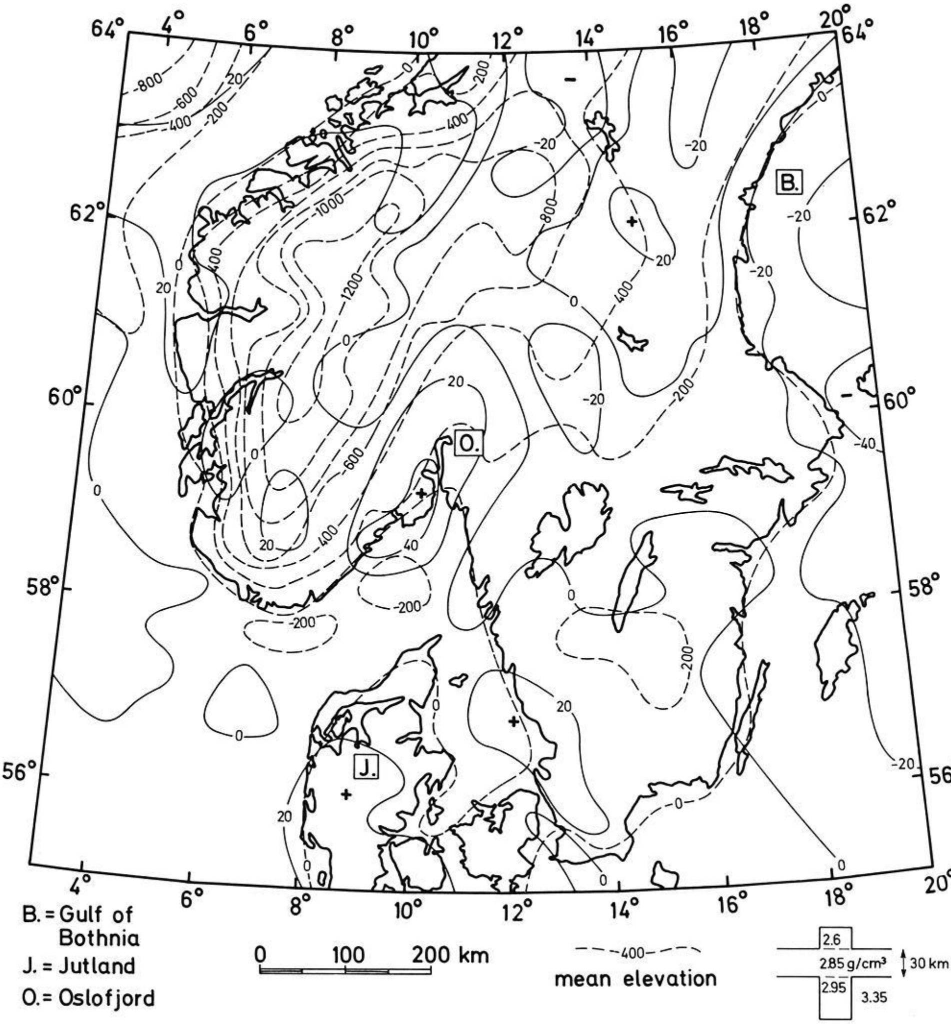

Fig. 5. Isostatic gravity anomalies of Southern Scandinavia. Reference plane: sea level; intervals of isolines: 20 mgal (from Janle, 1973)

The Po Basin and the foreland of the Apennines are characterized by strong negative isostatic gravity anomalies and positive or slightly negative correlation coefficients. Therefore, both methods show, in general, mass deficiencies of this region. The center of the isostatic gravity anomalies (−120 mgal) lies in the foreland of the Apennines. Mass deficiencies in the front of an orogene are in agreement with the conception of a still active orogeny of Wunderlich (1966).

The Dinarids, in the southeast of the area investigated, have low isostatic gravity anomalies and high negative correlation coefficients. Both methods show an isostatic state of this region.

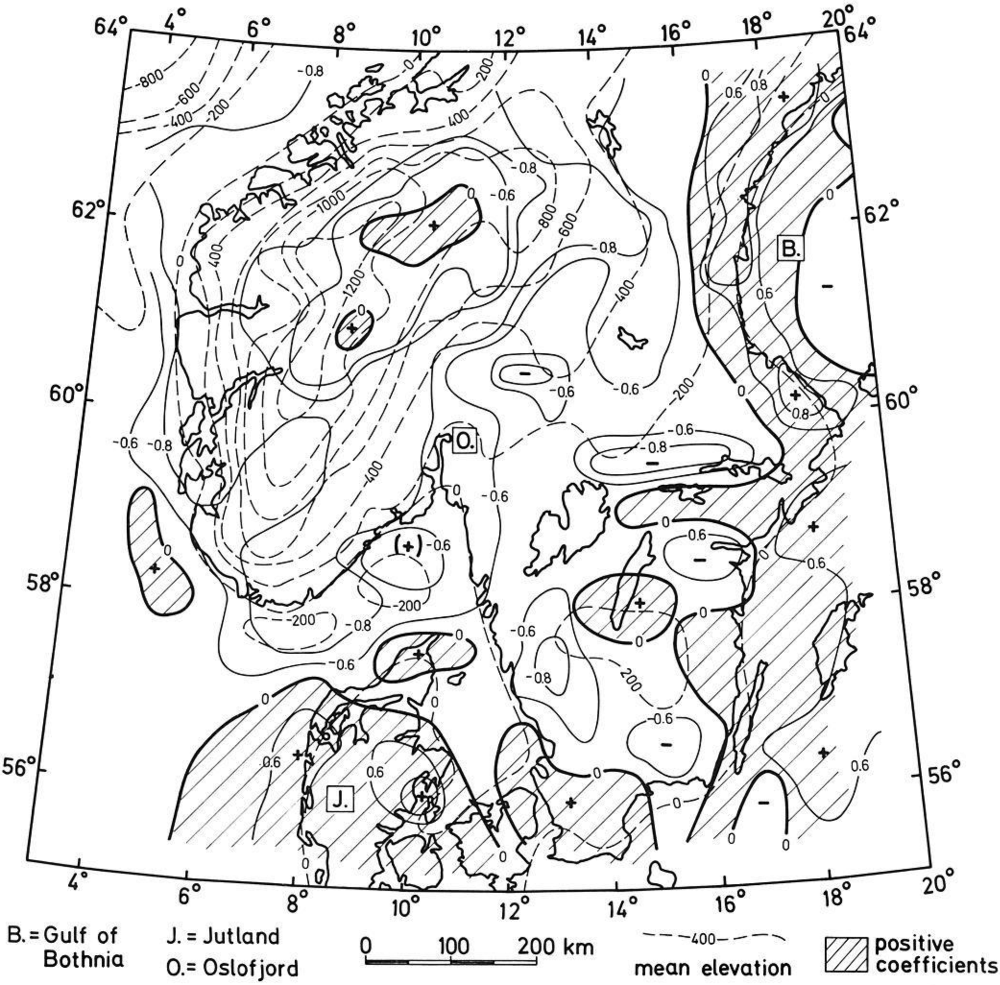

Fig. 6. Correlation coefficients between elevations and Bouguer anomalies of Southern Scandinavia. Intervals of isolines: 0.2; the lower significant lines of /0.2/ and /0.4/ have been left out in order to simplify the map

Comparing the results of the isostatic gravity anomalies and the correlation coefficients in the area of the Alps and Northern Italy, one can conclude a general agreement.

The correlation method has been also tested for Southern Scandinavia. The isostatic gravity anomalies and the gravity data are from Janle (1973). The computations of the isostatic gravity anomalies and of the correlation coefficients base both on a grid width of $30' \times 30'$.

For the calculation of the correlation coefficients a sample area was chosen with an extent of $\Delta\varphi = 1°$ and $\Delta\lambda = 2°$ which is about $110 \text{ km} \times 110 \text{ km}$. This area contains 8 grid elements. In this case the small number of 8 data pairs for the correlations results in an 88 percent confidence for a coefficient of /0.6/.

The Caledonian mountain range and Southern Sweden have low isostatic gravity anomalies and high negative coefficients indicating an isostatic behaviour of the crust (Figs. 5, 6).

The Gulf of Bothnia, parts of the Baltic Sea, and the eastern coast of Sweden show negative isostatic anomalies and high positive coefficients. Thus, both methods lead to mass deficiencies. This result is in agreement with the observation of the recent uplift of this area (Model, 1950).

The positive coefficients of Jutland combined with a slight positive relief indicate a mass surplus. Crustal models of Janle (1973) and Hirschleber (1975) confirm this result by upwellings of relative high density material in the region of Silkeborg and Ringkøbing-Fyn in central and southern Jutland.. The isostatic gravity anomalies are only slightly positive. The Oslofjord region is marked by high isostatic gravity anomalies. This can be explained by an upwelling of the mantle and intrusions of high density material into the crust (Ramberg and Smithson, 1971). The correlation coefficients show only a weak relative plus.

Summarizing the results for Southern Scandinavia, there is again a general agreement of both methods.

3. Conclusions

The isostatic gravity anomalies and the correlation coefficients show, in general, similar results. In the case of local anomalies, e.g. the Ivrea zone in the Alps and the Oslofjord region in Norway, the isostatic gravity anomalies have a greater resolving power. A further limitation of the correlation method is that a tendency to an isostatic behaviour can be concluded from negative coefficients, but slight deviations from isostasy cannot be resolved; on the other hand, high mass surpluses and deficiencies can clearly be interpreted from positive coefficients.

From these results it can be concluded that in areas with sufficient knowledge about the crustal density distribution and crustal thickness the computation of isostatic gravity anomalies gives more information than the correlation method. However, in regions with only sparse crustal data it is problematic to derive a reasonable model for the computations of isostatic gravity anomalies. In these cases the correlation coefficients method may be prefered.

It is technically relatively easy to get Doppler gravity data of the terrestrial planets aquired by means of orbiter missions. On the other hand, it is very difficult to gain sufficient seismic data for detailed crustal density models of planetary bodies. Therefore, the correlation method may be useful to give a general view of the isostatic state of the crusts of the terrestrial planets, if Doppler gravity and elevation data are available.

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