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Interpretation of Magnetic Anomalies With Fourier Transforms, Employing End Corrections

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Abstract. The method of Fourier transforms has been extended to interpret the magnetic anomalies of arbitrarily magnetised horizontal circular cylinders, dykes and faults. The Fourier transforms of the magnetic anomalies are derived without any apriori assumption of the position of the source and its magnetisation. They are usually evaluated by numerical integration of the available anomalies along the finite length of the profile. 'End corrections', which account for the contributions of the unknown anomalies outside the length of the profile, have been suggested to improve the reliability of these functions. It is also shown that without these corrections, the derived functions tend to be highly oscillatory and no useful interpretation can be arrived at.

Formulae useful in actual interpretation are derived for the three models mentioned above, making use of the amplitude spectrum and some auxilary functions derived from them. These new functions are found to be dependent on the shape of the body.

Key words: Fourier transforms – Magnetic interpretation – End corrections.

Introduction

Many articles have appeared in the recent literature on the use of Fourier transforms for interpreting gravity and magnetic anomalies. In this method, the anomalies in the space domain are transformed into the frequency domain and the various body parameters of the model are derived from the characteristic properties of the amplitude spectrum. Odegard and Berg (1965) derived expressions for the amplitude spectrum of the gravity anomalies of spheres, horizontal circular cylinders and vertical steps. Bhattacharya (1966), and Spector and Grant (1970) studied the continuous spectrum of the total field anomalies of prismatic bodies. Sharma and Geldart (1968) applied the method of Fourier transforms to interpret the gravity anomalies of two-dimensional faults. Rao and

Avasthi (1973) could determine all the body parameters of a symmetrical anticline from the amplitude spectrum of its gravity anomalies for a given density contrast.

Many authors (eg. Collins et al., 1974; Regan and Hinze, 1976) have recognised that the calculated spectra from the gravity and magnetic anomalies differ substantially from the true spectra so as to arrive at useful interpretation. The basic reason is that the anomalies are available only on a finite length of the profile, whereas the numerical integrations involved in the transformation require the data from $-\infty$ to $+\infty$ on the profile. During the course of the calculations, it is assumed or taken for granted that the data beyond the available length of the profile take zero values. In the present paper, corrections known as 'end corrections' are suggested for anomalies of two-dimensional bodies to improve the reliability of the spectrum calculated from them, when available over a finite length of the profile. The discussion is with special reference to the vertical magnetic anomalies of arbitrarily magnetised and arbitrarily striking cylinders, dykes and thick faults, but the same rules will apply to their total field anomalies also.

In the majority of papers published to date in this field it is assumed that the magnetisation is caused by induction. It is also assumed that the origin, with respect to which the anomaly expression is written, is known. In practice, the origin is not known and the magnetisation is not always caused by induction. Thus, in this paper, the anomaly expressions are written with reference to an arbitrary reference and methods are suggested to determine not only the body parameters of the model under question, but also the direction of magnetisation and the exact position of the body. Both the sine and cosine transforms are derived and used in interpretation. It is also found out that the shape of the body can be decided directly from the spectrum.

Derivation of Analytical Expressions

The Fourier transform $f^*(\omega)$ of a function $f(x)$ is defined by the relation

$$f^*(\omega)=\int_{-\infty}^{\infty} f(x)\exp(-i\omega x)\,dx$$

which is a complex quantity. $f(x)$ is the known geophysical data, and can be numerically integrated to give $f^*(\omega)$ for any given value of the spatial wave number ω. But the Fourier cosine and Fourier sine transforms are real quantities defined by the relations,

$$\mathrm{FCOS}(\omega)=\int_{-\infty}^{\infty} f(x)\cos\omega x\,dx \quad \text{and} \quad \mathrm{FSIN}(\omega)=\int_{-\infty}^{\infty} f(x)\sin\omega x\,dx.$$

The amplitude spectrum of $f(x)$ is defined as

$$\mathrm{FT}(\omega)=\sqrt{\mathrm{FCOS}^2(\omega)+\mathrm{FSIN}^2(\omega)}.$$

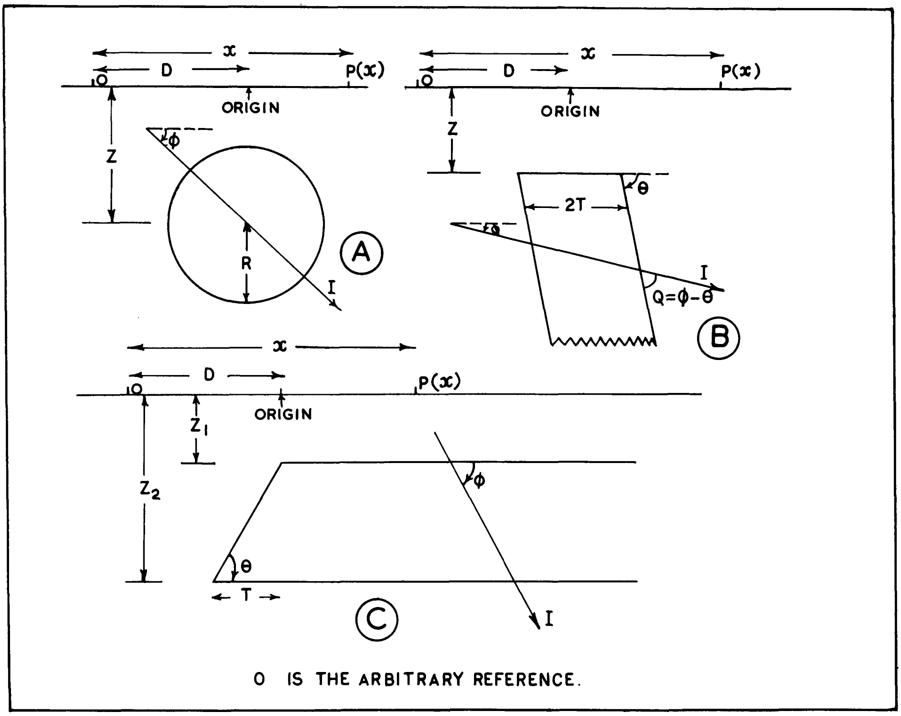

Fig. 1. Illustration of the various parameters. (*A*) Horizontal circular cylinder (*B*) Inclined dyke, and (*C*) Fault

i) Horizontal Circular Cylinder

The vertical magnetic anomaly over a horizontal circular cylinder is given by

$$\Delta V(x) = C_1(I_1 \sin \Phi - I_2 \cos \Phi) \tag{1}$$

where $I_1 = [Z^2 - (x-D)^2]/[(x-D)^2 + Z^2]^2$

$$I_2 = 2Z(x-D)/[(x-D)^2 + Z^2]^2$$

$$C_1 = 2\pi R^2 I$$

and the other parameters are as shown in Figure 1(A). Since the exact position of the cylinder is not known, the magnetic anomalies may be plotted from an arbitrary reference 0 on the profile. Thus C_1, D, Φ and Z are the parameters to be obtained from the Fourier amplitude spectrum. To evaluate the transform of Equarion (1), we may first evaluate the transforms of I_1 and I_2 separately, letting $D=0$ and finally employing the translation theorem. Accordingly I_1 and I_2 exhibit even and odd symmetry respectively, so that the sine transform of I_1 and cosine transform of I_2 are zero. Following from the Tables of integrals by Erdelyi et al. (1954), we have

$$\int_{-\infty}^{\infty} [1/(x^2 + Z^2)] \cos \omega x \, dx = (\pi/Z) \exp(-Z\omega)$$

and

$$\int_{-\infty}^{\infty} [x/(x^2+Z^2)] \sin \omega x \, dx = \pi \exp(-Z\omega).$$

Differentiating the above with respect to Z we can deduce that

$$\int_{-\infty}^{\infty} [(Z^2-x^2)/(x^2+Z^2)^2] \cos \omega x \, dx = \pi \omega \exp(-Z\omega)$$

and

$$\int_{-\infty}^{\infty} [2Zx/(x^2+Z^2)^2] \sin \omega x \, dx = \pi \omega \exp(-Z\omega).$$

Using the translation theorem i.e., the Fourier transform of $f(x-D)$ is $f^*(\omega) \exp(-iD\omega)$, we can finally obtain that

$$\begin{aligned} &\mathrm{FCOS}(\omega) = \pi C_1 \omega \exp(-Z\omega) \sin(\Phi + D\omega) \\ &\text{and} \\ &\mathrm{FSIN}(\omega) = -\pi C_1 \omega \exp(-Z\omega) \cos(\Phi + D\omega) \end{aligned} \tag{2}$$

ii) Dipping Dyke

Measuring the distance x from an arbitrary reference point 0 on the profile, the expression for the vertical magnetic anomaly of a dipping dyke can be written as

$$\Delta V(x) = C_2(I_3 \cos Q + I_4 \sin Q) \tag{3}$$

where $C_2 = 2I \sin Q \quad Q = \Phi - \theta$

$$I_3 = \arctan(\overline{x-D+T}/Z) - \arctan(\overline{x-D-T}/Z)$$

$$I_4 = \tfrac{1}{2}\{\ln[(x-D+T)^2+Z^2] - \ln[(x-D-T)^2+Z^2]\}$$

and the various other parameters are defined as in Figure 1(B). When $D=0$, I_3 and I_4 show even and odd symmetry respectively. To obtain the transforms of these functions, we may start from the following equations (Erdelyi et al., 1954)

$$\int_{-\infty}^{\infty} \left[\frac{T+x}{Z^2+(T+x)^2} + \frac{T-x}{Z^2+(T-x)^2}\right] \cos \omega x \, dx = -2\pi \sin T\omega \exp(-Z\omega)$$

and

$$\int_{-\infty}^{\infty} \left[\frac{Z}{Z^2+(T+x)^2} - \frac{Z}{Z^2+(T-x)^2}\right] \sin \omega x \, dx = -2\pi \sin T\omega \exp(-Z\omega).$$

Integrating the above with respect to Z, we have

$$\int_{-\infty}^{\infty} [\arctan(\overline{x+T}/Z) - \arctan(\overline{x-T}/Z)] \cos \omega x \, dx$$

$$= 2\pi(\sin T\omega/\omega) \exp(-Z\omega) + \lambda_1$$

and

$$\tfrac{1}{2}\int_{-\infty}^{\infty}[\ln((x+T)^2+Z^2)-\ln((x-T)^2+Z^2)]\sin\omega x\,dx$$
$$=2\pi(\sin T\omega/\omega)\exp(-Z\omega)+\lambda_2$$

where λ_1 and λ_2 are integration constants. The quantity

$$\arctan(\overline{x+T}/Z)-\arctan(\overline{x-T}/Z)$$

is actually the angle subtended by the top of the dyke at the point of observation. If $T=0$, the above quantity is zero for all values of x, so that the transform is also zero. Thus λ_1 is zero. Similarly it can be shown that λ_2 is also zero. Using translation theorem we can finally show that

$$\mathrm{FCOS}(\omega)=(2\pi\, C_2/\omega)\exp(-Z\omega)\sin T\omega\cos(Q+D\omega)$$

and (4)

$$\mathrm{FSIN}(\omega)=(2\pi\, C_2/\omega)\exp(-Z\omega)\sin T\omega\sin(Q+D\omega)$$

for an infinite dyke.

iii) Fault

The vertical magnetic anomaly due to an arbitrarily magnetised fault model is given by

$$\Delta V(x)=C_3\left[\tfrac{1}{2}\sin Q\ln\frac{Z_2^2+(x-D+T)^2}{Z_1^2+(x-D)^2}\right.$$
$$\left.+\cos Q\left(\arctan\frac{x-D+T}{Z_2}-\arctan\frac{x-D}{Z_1}\right)\right] \tag{5}$$

where $C_3=2I\sin\theta$, $Q=\Phi+\theta$, and the other parameters carry the meanings shown in Figure 1(C). Following the analysis similar to that of the dyke, we can finally deduce that

$$\mathrm{FCOS}(\omega)=(\pi\, C_3/\omega)[\exp(-Z_1\omega)\sin(Q+D\omega)$$
$$-\exp(-Z_2\omega)\sin(Q+\overline{D-T}\omega)]$$

and (6)

$$\mathrm{FSIN}(\omega)=-(\pi\, C_3/\omega)[\exp(-Z_1\omega)\cos(Q+D\omega)$$
$$-\exp(-Z_2\omega)\cos(Q+\overline{D-T}\omega)]$$

when the distances are measured from an arbitrary reference point.

End Corrections

Evaluation of the sine and cosine transformation is essentially a technique of numerical integration, and many methods exist in this direction. But the method used in evaluation of the integrals in the various examples cited in this paper is based on the method of Filon (1928–29). In any method, the integration is performed on the anomalies available on the finite length of the profile by writing, for example,

$$\mathrm{FCOS}(\omega) = \int_{x_1}^{x_{2N+1}} f(x) \cos \omega x \, dx,$$

where x_1 and x_{2N+1} are the initial and final values of x in the profile containing $2N+1$ observations. The arbitrary reference with which the values of x are measured may be anywhere, but can be selected as close to the origin as possible.

We will observe at a later stage that these 'finite transforms' tend to be highly oscillatory and that correct interpretation can not be obtained from them. These transforms can however be improved by applying corrections by calculating the contributions of the missing anomalies outside the length of the profile. The basis of this correction is as follows: A simple analytical expression can be fitted to the last few anomaly points, showing the trend or variation of the anomaly over this distance. This trend is assumed to prevail from the last anomaly point to infinity also. Knowing these expressions, one for each end of the profile, their Fourier transforms can be arrived at in a closed form. These may be called the correction terms and will represent the effects of the missing anomalies from $-\infty$ to x_1 and from x_{2N+1} to ∞. They may be added to finite $\mathrm{FCOS}(\omega)$ and finite $\mathrm{FSIN}(\omega)$ worked out above.

i) Horizontal Circular Cylinder

At sufficiently large values of x, we can neglect D and Z^2 in comparison with x and x^2 respectively in Equation (1). Thus,

$$\Delta V(x) = C_1(-\sin \Phi/x^2) - (2 C_1 Z \cos \Phi/x^3)$$

and the magnetic anomaly may be assumed to obey the equation $\Delta V(x) = A_1/x^2 + A_2/x^3$ from $x = -\infty$ to $x = x_2$, and $\Delta V(x) = A_3/x^2 + A_4/x^3$ from $x = x_{2N}$ to $x = \infty$. A_1 and A_2 can be solved from the magnetic anomalies $\Delta V(1)$ and $\Delta V(2)$, and A_3 and A_4 from $\Delta V(2N)$ and $\Delta V(2N+1)$. Knowing A_1, A_2, A_3 and A_4, the correction can be worked out as follows:

C_{FCOS} = correction for Fourier cosine transform

$$= \int_{-\infty}^{x_1} (A_1/x^2 + A_2/x^3) \cos \omega x \, dx + \int_{x_{2N+1}}^{\infty} (A_3/x^2 + A_4/x^3) \cos \omega x \, dx$$

$$= A_1[C(1) + \mathrm{SI}(1)] + A_3[C(N) + \mathrm{SI}(N)] + A_2[-C(1)/2\,|x_1| + \omega S(1)/2 - (\omega/2)\,\mathrm{CI}(1)] + A_4[C(N)/2 x_{2N+1} - \omega S(N)/2 + (\omega/2)\,\mathrm{CI}(N)]$$

C_{FSIN} = correction for Fourier sine transform

$$= \int_{-\infty}^{x_1} (A_1/x^2 + A_2/x^3) \sin \omega x \, dx + \int_{x_{2N+1}}^{\infty} (A_3/x^2 + A_4/x^3) \sin \omega x \, dx$$

$$= A_1[-S(1) + \mathrm{CI}(1)] + A_3[S(N) - \mathrm{CI}(N)] + A_2[S(1)/2\,|x_1| + \omega\, C(1)/2 + (\omega/2)\,\mathrm{SI}(1)] + A_4[S(N)/2x_{2N+1} + \omega\, C(N)/2 + (\omega/2)\,\mathrm{SI}(N)]$$

where

$$S(1) = \sin(\omega\,|x_1|)/|x_1| \qquad C(1) = \cos(\omega\,|x_1|)/|x_1|$$
$$S(N) = \sin(\omega\, x_{2N+1})/x_{2N+1} \qquad C(N) = \cos(\omega\, x_{2N+1})/x_{2N+1}$$
$$\mathrm{SI}(1) = \omega\, \mathrm{Si}(\omega\,|x_1|) \qquad \mathrm{CI}(1) = \omega\, \mathrm{Ci}(\omega\,|x_1|)$$
$$\mathrm{SI}(N) = \omega\, \mathrm{Si}(\omega\, x_{2N+1}) \qquad \mathrm{CI}(N) = \omega\, \mathrm{Ci}(\omega\, x_{2N+1}).$$

ii) Dipping Dyke

At large values of x, we can write that,

$$\arctan[(x - D + T)/Z] - \arctan[(x - D - T)/Z]$$
$$= \arctan[2TZ/(Z^2 + (x - D)^2 + T^2)]$$
$$= \arctan(2TZ/x^2) = 2TZ/x^2$$

and

$$\ln \frac{(x - D + T)^2 + Z^2}{(x - D - T)^2 + Z^2} = 2 \ln \frac{(x + T)}{(x - T)} = 2 \ln \frac{(x/T + 1)}{(x/T - 1)} = \frac{4T}{x}.$$

Substituting these in Equation (3), we may show that the magnetic anomaly due to a dyke varies according to the equation

$$\Delta V(x) = A_1/x + A_2/x^2 \quad \text{from} \quad x = -\infty \text{ to } x = x_2,$$

and

$$\Delta V(x) = A_3/x + A_4/x^2 \quad \text{from} \quad x = x_{2N} \text{ to } x = \infty.$$

Based on the first two and the last two anomalies on the profile, A_1 to A_4 can be solved. Then,

$$C_{FCOS} = A_1\, \mathrm{CI}(1)/\omega - A_3\, \mathrm{CI}(N)/\omega + A_2[C(1) + \mathrm{SI}(1)] + A_4[C(N) + \mathrm{SI}(N)]$$

$$C_{FSIN} = -A_1\, \mathrm{SI}(1)/\omega - A_3\, \mathrm{SI}(N)/\omega + A_2[-S(1) + \mathrm{CI}(1)] + A_4[S(N) - \mathrm{CI}(N)]$$

iii) Fault

For the fault model, it can be shown that the anomaly varies according to the equation

$$\Delta V(x) = A_1/x + A_2/x^3 \quad \text{from} \quad x = -\infty \text{ to } x = x_2, \text{ and}$$
$$\Delta V(x) = A_3/x + A_4/x^3 \quad \text{from} \quad x = x_{2N} \text{ to } x = \infty.$$

The corrections can be finally worked out as:

$$C_{\text{FCOS}} = A_1\,\text{CI}(1)/\omega - A_3\,\text{CI}(N)/\omega + A_2[-C(1)/2\,|x_1| + \omega\,S(1)/2 - (\omega/2)\,\text{CI}(1)] + A_4[C(N)/2x_{2N+1} - \omega\,S(N)/2 + (\omega/2)\,\text{CI}(N)]$$

$$C_{\text{FSIN}} = -A_1\,\text{SI}(1)/\omega - A_3\,\text{SI}(N)/\omega + A_2[-S(1)/2\,|x_1| + \omega\,C(1)/2 + (\omega/2)\,\text{SI}(1)] + A_4[S(N)/2x_{2N+1} + \omega\,C(N)/2 + (\omega/2)\,\text{SI}(N)]$$

Method of Analysis and Examples

For any given magnetic profile, the Fourier transforms FCOS(ω) and FSIN(ω) can be calculated by numerical integration for different values of ω. From these data we can determine the various parameters of the model.

i) Horizontal Circular Cylinder

The amplitude spectrum for this model can be obtained from Equation (2) as

$$FT(\omega) = \sqrt{\text{FCOS}^2(\omega) + \text{FSIN}^2(\omega)} = \pi\,C_1\,\omega\,\exp(-Z\,\omega). \tag{7}$$

The amplitude spectrum $FT(\omega)$ shows a maximum at $\omega = \omega_{\max}$ given by $Z = 1/\omega_{\max}$ from which the depth can be found out. If ω is expressed in radians per station spacing, Z is obtained in units of station spacing. The value of C_1 is then solved from any one value of $FT(\omega)$ by substituting the value of Z thus obtained in Equation (7). Alternatively, a new function $F(\omega)$ can be worked out from the amplitude spectrum as defined below:

$$F(\omega) = FT(\omega)/\omega = \pi\,C_1\,\exp(-Z\omega).$$

This, when drawn on a semi-logarithmic paper, appears as a straight line having a slope of $-Z$ and cutting the ordinate at $\pi\,C_1$. Φ may be determined from the relation, $\Phi = \arctan[-\text{FCOS}(0)/\text{FSIN}(0)]$. An alternative procedure may also be followed to calculate D and Φ. The ratio $-\text{FCOS}(\omega)/\text{FSIN}(\omega)$ is calculated for different values of ω. Then the quantity $\arctan[-\text{FCOS}(\omega)/\text{FSIN}(\omega)]$ when plotted against ω gives a straight line defined by $\Phi + D\omega$. This straight line has a slope of D and cuts the ordinate at Φ. Φ in the correct quadrant may be finally worked out from the signs of FCOS(ω) and FSIN(ω).

Figure 2 shows as an example the interpretation of magnetic anomalies over a cylinder. The magnetic profile, the transforms FCOS(ω) and FSIN(ω) along with the Fourier spectrum $FT(\omega)$ and the function $F(\omega)$ are also indicated in the figure. The theoretical values of these functions over the range of ω considered, coincide with the corrected transforms and consequently are not shown. The various parameters calculated from these functions are as follows: Depth = 5

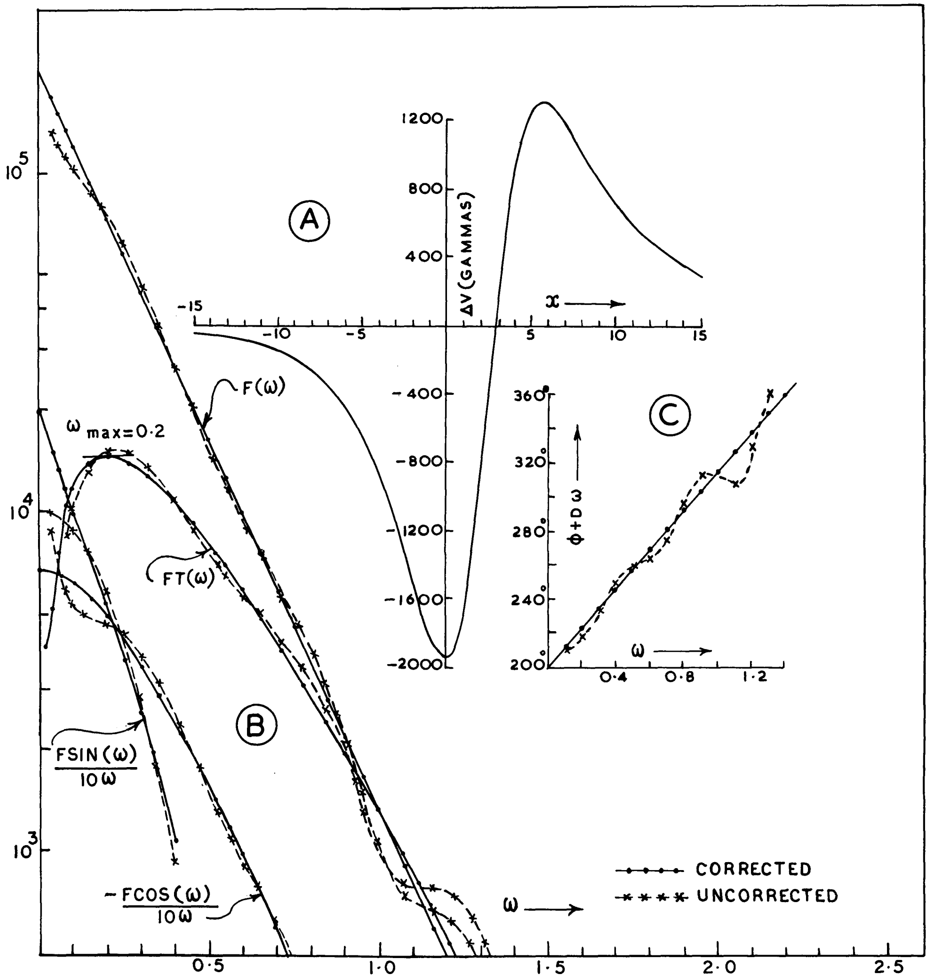

Fig. 2. Interpretation of a theoretical vertical magnetic anomaly profile over an arbitrarily magnetised horizontal circular cylinder. (*A*) Magnetic profile (*B*) Variation of FSIN(ω), FCOS(ω) and $F(\omega)$ against ω, and (*C*) Diagram for calculation of Φ and D

units (assumed value = 5 units), $\Phi = 199$ degrees (assumed value = 200 degrees) origin = 1.98 units (assumed value = 2.00 units).

ii) Dipping Dyke

For the dyke model, the amplitude spectrum is given by

$$FT(\omega) = (2\pi\, C_2/\omega)\exp(-Z\,\omega)\,|\sin(T\omega)|. \tag{8}$$

A new function $F(\omega)$ can be calculated as follows:

$$F(\omega) = FT(\omega)\times\omega = 2\pi\, C_2 \exp(-Z\,\omega)\,|\sin(T\omega)|. \tag{9}$$

The function $F(\omega)$ is zero at points defined by $T\omega_0 = n\pi\,(n=0, 1, 2, \ldots)$. In practice the zero points will not be brought out on the $F(\omega)$ curve because this function can never be negative and thus has no cross-overs. However, the zero-points will appear as 'sharp minima' on the $F(\omega)$ curve and can therefore be easily identified. Figure 3 shows the functions $F(\omega)$, FCOS(ω) and FSIN(ω) calculated for a theoretical anomaly profile over a dyke having $2T=4$ times the station spacing and $Z=2$ times the station spacing. The anomaly profile used for numerical integration is shown in Figure 3(A). In Figure 3(B), the functions $F(\omega)$, FCOS(ω) and FSIN(ω) for this profile are shown. The function $F(\omega)$ is calculated both from the corrected and uncorrected values of FCOS(ω) and FSIN(ω) and is plotted separately. The function $F(\omega)$ is also calculated theoretically by substituting $Z=2$ and $T=2$ in Equation (9), and shown as open circles in the same figure. It can be observed from this figure that the $F(\omega)-\omega$ curve as derived from the uncorrected values of FCOS(ω) and FSIN(ω) is highly oscillatory and does not give any useful information. Also the uncorrected values of $F(\omega)$ deviate too much from the theoretical values for all values of ω greater than one radian per station spacing. In contrast to this, the corrected values of $F(\omega)$ show a consistent trend. They also agree closely in magnitude to the theoretical values upto $\omega=3.2$ radians per grid spacing. Figure 3 thus brings out the importance and need of applying the correction to the numerically evaluated transforms, and shows the extent to which the evaluated transforms are improved.

To determine T, we can use any one of the values of ω_0, at which the function $F(\omega)$ vanishes. If ω_0 is the position of the first zero-point, then,

$$T = \pi/\omega_0. \tag{10}$$

If ω_0 is expressed in radians per station spacing, T will be given in station spacings. Alternatively, two consecutive values of $\omega_0 (=\omega_{01}$ and ω_{02} say) can be located to find out T by the formula, $T=\pi/(\omega_{02}-\omega_{01})$. The use of this formula may however be avoided because $F(\omega)$ cannot be calculated very accurately at higher values of ω. Either of the two formulae determines T independent of Z, D and C_2. Z can be determined from the position of the maximum value of $F(\omega)$. This is maximum or minimum if

$$Z = T\cot(T\omega_{max}). \tag{11}$$

Actually $F(\omega)$ shows many turning points and any value of ω_{max} corresponding to any one of these turning points may be used in the above equation to find out Z. Because the function $F(\omega)$ is obtained more accurately at lower values of ω, it is preferable to use the location of the first turning point only for more accurate values of Z. Z is thus determined independent of C_2, D and Q.

C_2 may be determined by substituting the values of T and Z thus obtained in Equation (9), knowing $F(\omega)$ for any given value of ω. Q and C_2 may also be determined from the values of FCOS(0) and FSIN(0) because, from Equation (4) it can be shown that, FCOS$(0)=2\pi\,C_2\,T\cos Q$ and FSIN$(0)=2\pi\,C_2\,T\sin Q$. Alternatively, the relation $\arctan[\mathrm{FSIN}(\omega)/\mathrm{FCOS}(\omega)]=Q+D\omega$ is a straight line having a slope D and intersecting the ordinate at Q. Although this prop-

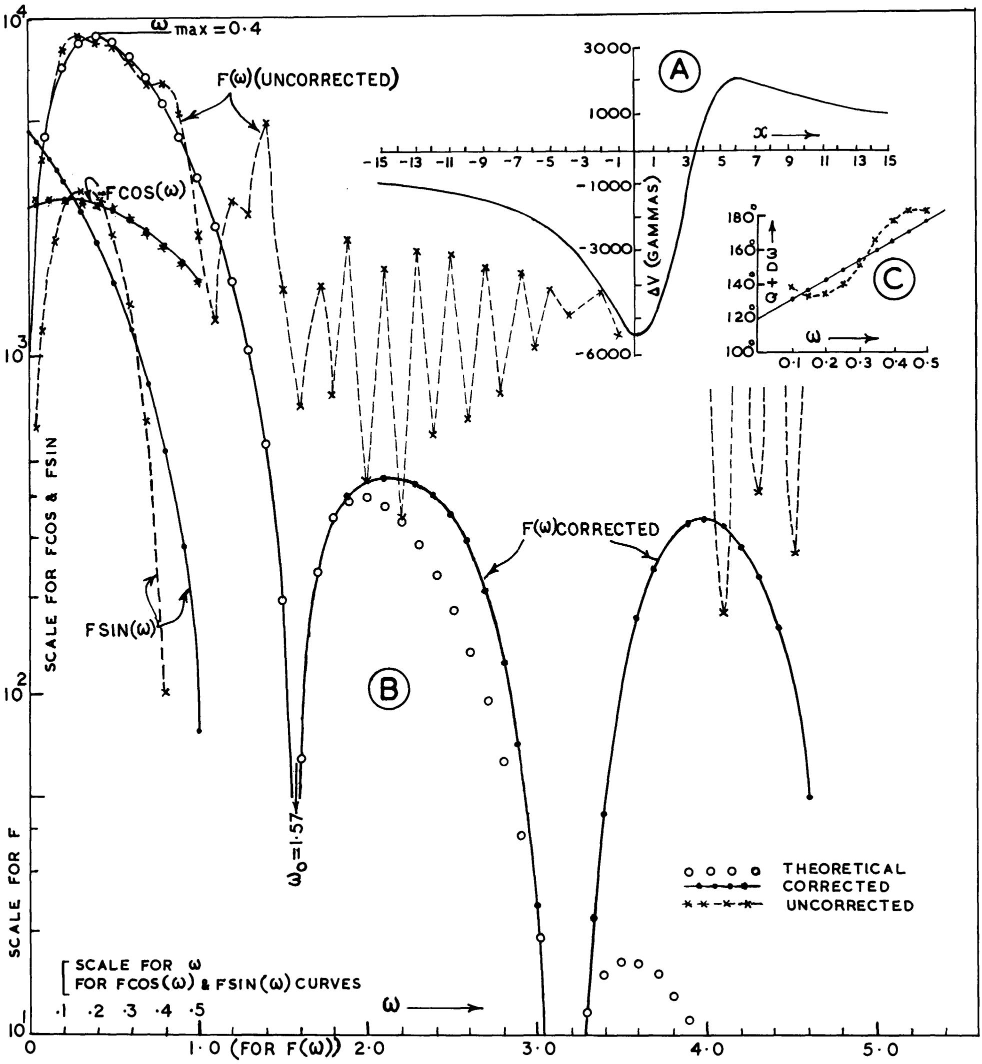

Fig. 3. Interpretation of a theoretical vertical magnetic anomaly profile over an arbitrarily magnetised dyke. (*A*) Magnetic profile (*B*) Variation of FSIN(ω), FCOS(ω) and $F(\omega)$ against ω, and (*C*) Diagram for calculation of Q and D

erty is valid for all values of ω, calculation of the quantity arctan[FSIN(ω)/FCOS(ω)] shall be limited to small values of ω, particularly in the interpretation of field profiles. This is because the functions FSIN(ω) and FCOS(ω) are not very accurate at higher values of ω, as we have already noted above. Also geological bodies do not fit perfectly to a dyke model and do not have a uniform magnetisation throughout their volume.

The values of Z, T, Q and D as calculated from the above rules of interpretation for the profile shown in Figure 3(A) are as follows: Thickness

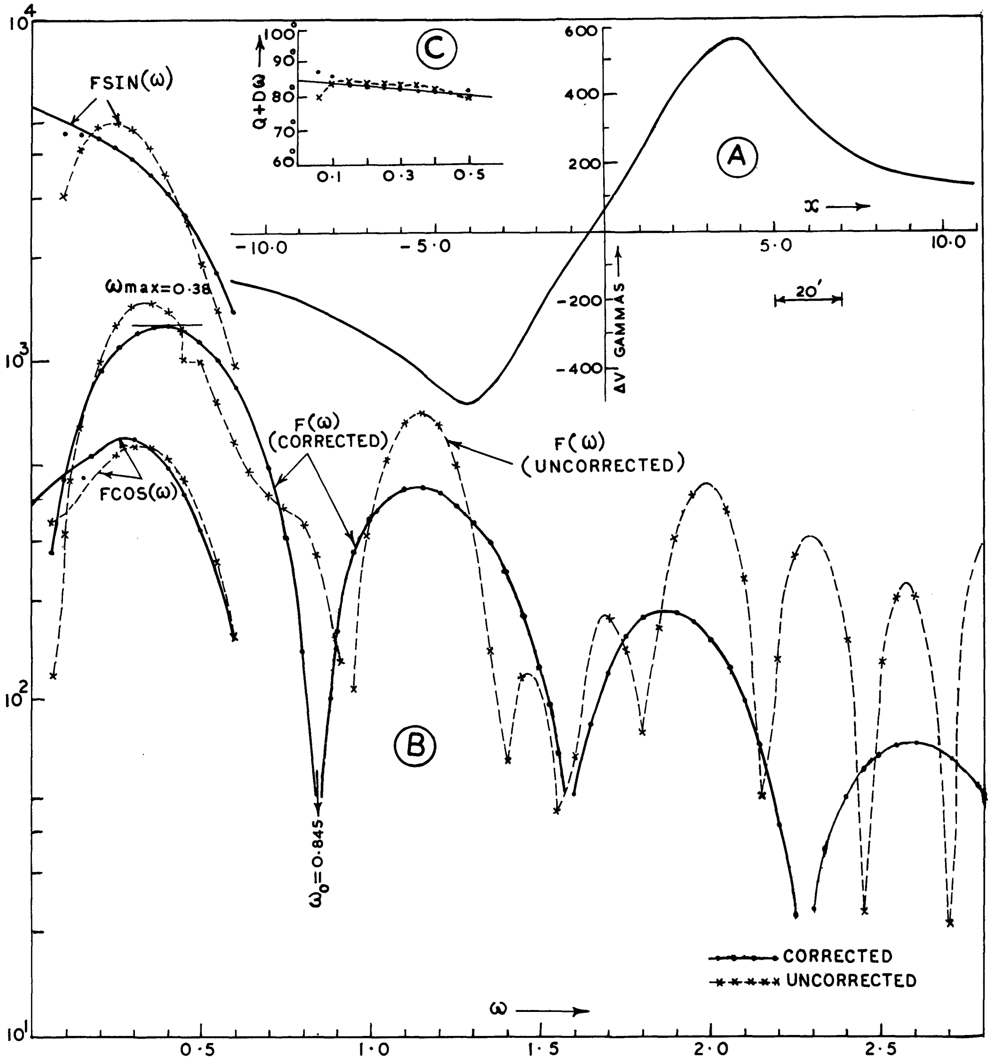

Fig. 4. Interpretation of a vertical magnetic anomaly profile over an outcropping quartz-magnetite dyke-like body, Karimnagar area, Andhra Pradesh. (*A*) Magnetic Profile (*B*) Variation of FSIN(ω), FCOS(ω) and $F(\omega)$ against ω, and (*C*) Diagram for calculation of Q and D

=4.0 units (assumed value=4.0 units), Depth=1.94 units (assumed value=2.00 units), $Q=120$ degrees (assumed value=120 degrees), and $D=2.01$ units (assumed value=2.00 units).

Figure 4 shows the interpretation of a field example of vertical magnetic anomalies observed over a dyke-like body by the method of Fourier transforms. Figure 4(A) is a magnetic anomaly profile taken over an outcropping quartz-magnetite dyke-like body of thickness 80 ft in Karimnagar area, Andhra Pradesh (Subba Rao, 1974). The transformed anomalies of this profile, both corrected and uncorrected, and the function $F(\omega)$ are also shown in the figure.

The corrected transforms are interpreted by the application of the rules cited above. The profile is also interpreted on the computer by the method of iteration of Rao and Radhakrishna Murthy (1973). The values obtained for various parameters are as follows: Thickness = 74 ft (by iteration 75 ft), Depth = 6 ft (by iteration 9 ft), $Q = 86$ degrees (by iteration 85 degrees), and $D = -1.8$ ft (by iteration -2.3 ft). It may be observed that the results obtained by Fourier transformation and also by the method of iteration mentioned above are close to each other.

iii) Fault

The expressions for the Fourier cosine and sine transforms of the magnetic anomalies over faults are given in Equation (6). To calculate the various parameters, we evolve a new function $F(\omega)$ defined as

$$\begin{aligned} F(\omega) &= \omega\, FT(\omega) = \omega\,[\mathrm{FCOS}^2(\omega) + \mathrm{FSIN}^2(\omega)]^{1/2} \\ &= \pi\, C_3\,[\exp(-2Z_2\,\omega) + \exp(-2Z_1\,\omega) \\ &\quad -2\exp(-\overline{Z_1 + Z_2}\,\omega)\cos(T\omega)]^{1/2}. \end{aligned}$$

At large values of ω, this becomes, $F(\omega) = \pi\, C_3 \exp(-Z_1\,\omega)$, showing that the $F(\omega)$ versus ω curve, when drawn on a semi-logarithmic paper, degenerates into a straight line. The slope of this straight line gives $-Z_1$. The intercept of this straight line on the ordinate is equal to $\pi\, C_3$ and hence C_3 can be calculated. The relation

$$\arctan[-\mathrm{FCOS}(\omega)/\mathrm{FSIN}(\omega)] = Q + D\,\omega$$

is a straight line at large values of ω, corresponding to the straight line portion of the function $F(\omega)$. From this straight line, Q and D can be calculated. To find out the other parameters, we may determine the values of FCOS(0) and FSIN(0), which are given by

$$\mathrm{FCOS}(0) = \pi\, C_3\,[T\cos Q + (Z_2 - Z_1)\sin Q] = \pi\, C_3 (Z_2 - Z_1)\operatorname{cosec}\theta\cos\Phi$$

$$\mathrm{FSIN}(0) = \pi\, C_3\,[T\sin Q - (Z_2 - Z_1)\cos Q] = \pi\, C_3 (Z_2 - Z_1)\operatorname{cosec}\theta\sin\Phi.$$

From these, the parameters ϕ, θ and Z_2 can be worked out by the relations

$$\Phi = \arctan[\mathrm{FSIN}(0)/\mathrm{FCOS}(0)], \qquad \theta = Q - \Phi$$

and

$$Z_2 = (\sin\theta/\pi\, C_3)\sqrt{\mathrm{FCOS}^2(0) + \mathrm{FSIN}^2(0)} + Z_1.$$

Figure 5 shows an actual example of magnetic fault interpretation worked out by the method of Fourier transforms. The function $F(\omega)$ as obtained from the uncorrected values of $\mathrm{FCOS}(\omega)$ and $\mathrm{FSIN}(\omega)$ is also plotted to bring out again the need of applying the 'end corrections'. The values of the various parameters interpreted from the corrected transforms along with the assumed

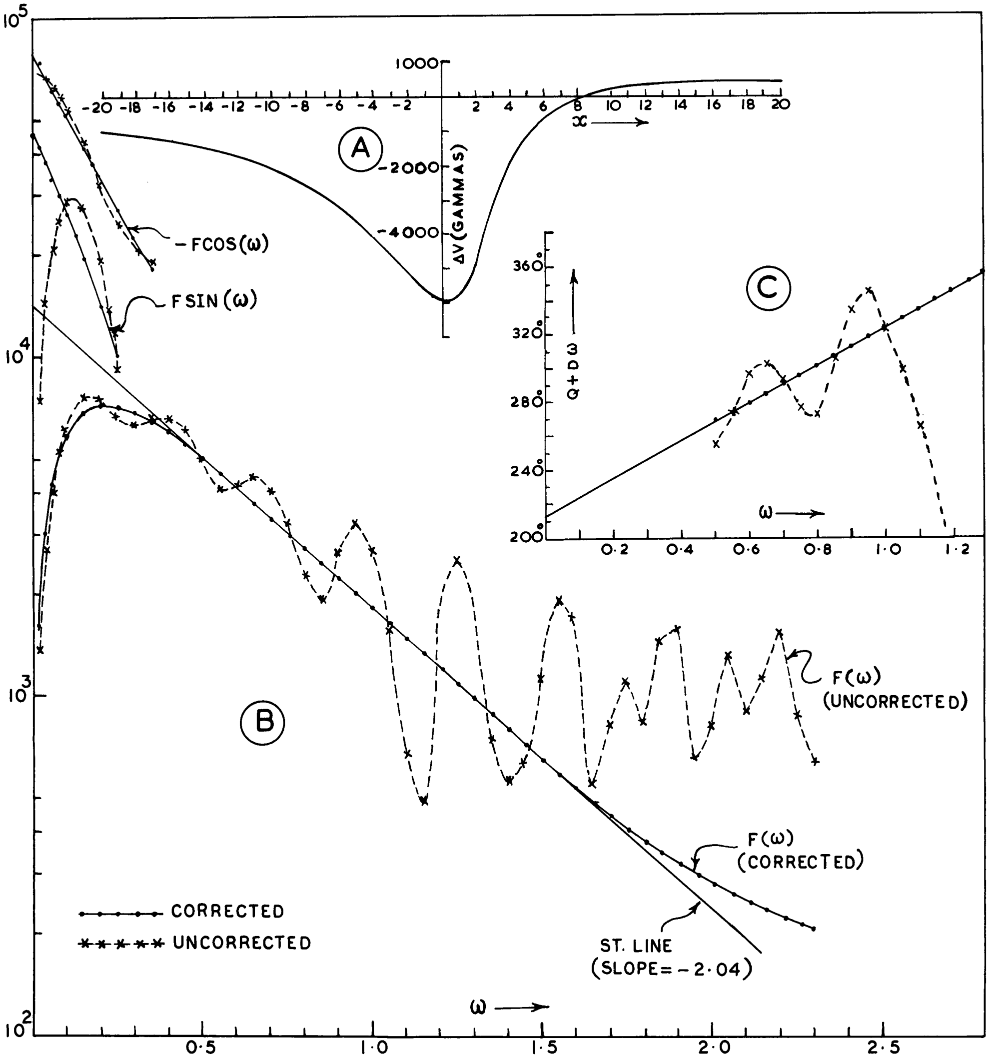

Fig. 5. Interpretation of a theoretical vertical magnetic anomaly profile over an arbitrarily magnetised fault. (*A*) Magnetic profile (*B*) Variation of FSIN(ω), FCOS(ω) and $F(\omega)$ against ω, and (*C*) Diagram for calculation of Q and D

values are as follows: $Z_1 = 2.04$ units (assumed value = 2.00 units), $Z_2 = 7.90$ units (assumed value = 8.00 units), $D = 1.92$ units (assumed value = 2.00 units), $\Phi = 149.8$ degrees (assumed value = 150 degrees), and $\theta = 62.7$ degrees (assumed value = 60 degrees).

Discussion

This paper does not advocate any superiority of the method of Fourier transforms over the other methods of interpretation of magnetic anomalies of simple geometric models. It only studies the possibility of interpreting these anomalies in a generalised case where the magnetisation is not by induction and when the position of the body is not known. By deriving both the sine and cosine transforms, it is shown that both the direction of magnetisation and the position of the body can be easily determined. 'End corrections' are suggested, probably for the first time, to improve the sine and cosine transforms obtained by numerical integration of the anomalies over a limited length of the profile. It is observed that without application of these corrections, the derived transforms tend to be highly oscillatory and any information obtained, there from may not be reliable. It is also observed in this analysis that even the corrected transforms tend to deviate from true transforms for $\omega > 2.0$. However, this will not affect the interpretation because all body parameters are deduced at lower frequencies. The validity of the expressions, representing the anomalies outside the length of the profile is verified by calculating a few anomalies from these expressions and comparing them with those by the exact formulae. Further, the constants A_1 to A_4 were determined from the edge anomalies, with and without application of the procedure of least-squares, and it was found that the application of the least-square procedure does not improve the values of these constants.

The following advantages are usually mentioned for the method of Fourier transforms: (a) All anomalous field values are taken into consideration during analysis, (b) calculations of derivatives and continuation to different levels are easily carried out in the frequency domain than in the space domain. In addition to the above, we observe in this paper two interesting applications of the method of Fourier transforms. The first is that the shape of the function $F(\omega)$ depends on the shape of the body, and consequently it can be determined. $F(\omega)$, when plotted, against ω on semi-logarithmic paper is a straight line for the cylinder, a curve showing a series of maxima for the dyke, and a curve showing a maximum and then degenerating into a straight line for a fault. The second relates to the interpretation of magnetic anomalies of faults. We are not aware of any simple and standardised method of interpreting the magnetic anomalies of arbitrarily magnetised faults. The difficulty is due to the large number of parameters to be obtained from the anomalies. In contrast to this, all the parameters of the model can be solved very easily, at least theoretically, by the method of Fourier transforms.

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