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LOG Titel: The effect of earth structure on radial oscillations
LOG Typ: article

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J. Geophys. 45, 129–145, 1979

Journal of
Geophysics

The Effect of Earth Structure on Radial Oscillations

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Abstract. Quasi-periodic variations observed in the spacings between successive eigenfrequencies of radial oscillations and of spheroidal oscillations of PKIKP type are produced by the inner and outer core discontinuities and by the transition zone in the upper mantle. The individual effects of these regions have been separated by modelling experiments. The inner and outer core boundaries cause persistent oscillations in the eigenfrequency spacings, having periods (with respect to overtone number) of about 5.5 and 2.4 respectively. The oscillation produced by the upper mantle transition zone has a period of about 8. A discontinuous upper mantle causes a persistent oscillation of this kind, while that produced by a continuous upper mantle decays with increasing overtone number. In principle, differences between continuous and discontinuous upper mantle models are detectable in the currently observable eigenfrequency range. The analysis shows that the extent of presently available data is insufficient to determine structure at the core boundaries within adequate limits.

Key words: Radial oscillations – Earth structure – Eigenfrequency spacing.

1. Introduction

Anderssen, Cleary and Dziewonski (1975) have shown that the observed periods of the Earth's free oscillation modes ${}_nS_0$ and ${}_nS_1$ exhibit an oscillatory behaviour consistent with the existence of a solotone effect (cf. McNabb et al., 1976). They attributed this behaviour to the presence of the core discontinuities, but did not explore the extent to which the observed periods of these modes contain information about the detail of the discontinuities.

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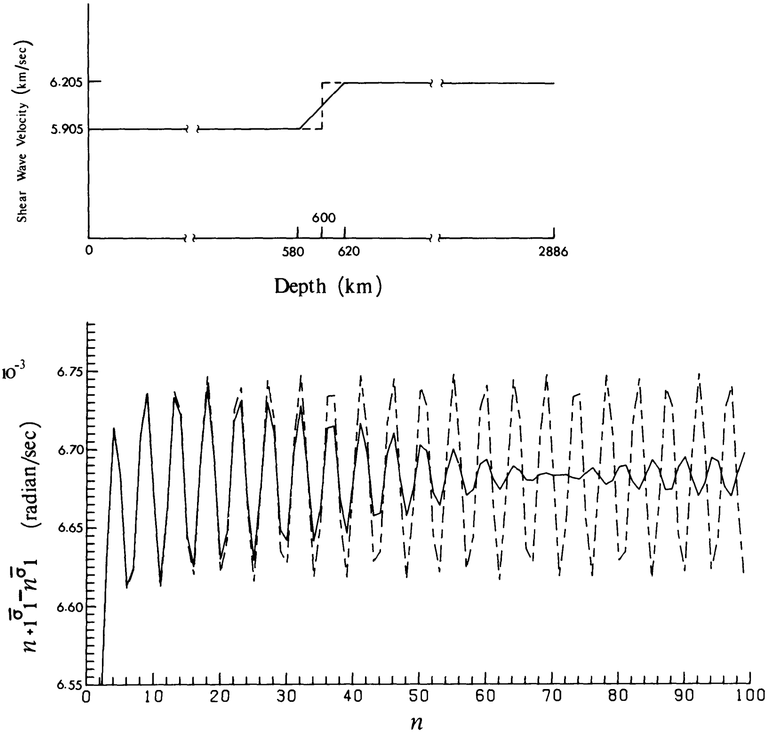

Fig. 1. Comparison between amplitudes of the solotone oscillations corresponding to a discontinuity *(dashed line)* and to a transition layer *(solid line)*, respectively, at 600 km depth (where ${}_n\bar{\sigma}_1$ is a torsional eigenfrequency)

It is clear from observations of short-period reflections from the core-mantle and inner-outer core boundaries that the changes in elastic properties at both boundaries are abrupt. On the other hand, little is known at present about lateral variations in the radii of the boundaries, although there has been speculation about variations of both short and long wave length in the radius of the core-mantle boundary (cf. Hide and Horai, 1968; Hales and Roberts, 1970; Haddon, 1972; Chang and Cleary, 1978). If such variations were present, it would be expected that the globally-averaging properties of spheroidal and radial oscillations would result in observed periods similar to those of a spherically symmetric Earth with transition regions at the core boundaries.

Transition regions in the Earth are important structural features, but their effect on the Earth's free oscillations has not yet been examined in sufficient detail. However, at least for torsional oscillations, it is clear that in this case the solotone effect is modified such that its amplitude decreases with increasing overtone number. This is a consequence of the fact that the amplitudes of waves reflected from transitions of finite width decrease as the frequency of the incident wave increases (cf. Wolf, 1937). This point is illustrated in Fig. 1.

Since (Anderssen et al., 1975) the differential equation for radial free oscillations can be written as a Sturm-Liouville system of the type examined by

McNabb et al. (1975), the above results should remain valid for radial oscillations. Anderssen et al. (1975) showed that the spacings between observed radial (and, for $l=1$, spheroidal) eigenfrequencies of successive overtone numbers contain an oscillatory component. In this paper the effects of the core-mantle (C/M) and inner-outer core (I/O) boundaries, and of the crust and upper mantle structure, on radial oscillations are studied, in order to confirm that results for torsional free oscillations are applicable to radial oscillations, and to provide a general basis for further Earth modelling.

Anderssen et al. (1975) also showed, in their Fig. 1, that the spacings between observed normal modes and between the corresponding eigenfrequencies of model 1066A or 1066B (Gilbert and Dziewonski, 1975) have some minor differences. We have examined the sensitivity of these differences to changes in the structure of the core boundaries, and the possibility of obtaining further information of the Earth's structure from the observed radial normal modes.

We restrict attention to radial oscillations, since they correspond to vertical PKIKP rays with no P/SV conversion at interfaces. The results show that each of the structural regions considered produces an oscillation of a particular period in the spacings between eigenfrequencies of successive overtone numbers. The period is determined by the location of the region, whereas the amplitude of the oscillation is closely related to the velocity and density contrast across the region and decreases at a rate related to its width.

2. Terminology

2.1. ${}_n\bar{\gamma}_l$ Curves

The effect of Earth structure on free oscillations is exhibited in the spacing of eigenfrequencies. Therefore, any parameter which depends on the overtone eigenfrequency spacing will reflect the effect of Earth structure. For fixed angular order number l, one such parameter is ${}_n\bar{\gamma}_l(\kappa)=\pi/[{}_{n+1}\sigma_l(\kappa)-{}_n\sigma_l(\kappa)]$, where ${}_n\sigma_l(\kappa)$ are the eigenfrequencies of any of the $(\mathrm{ScS})_\mathrm{H}$, $(\mathrm{ScS})_\mathrm{V}$, PKIKP, and J_V sequences (Anderssen et al., 1975; Gilbert, 1975). This parameter was first used by Anderssen, Cleary and Dziewonski (1975) and has the advantage that, when l is small, the baseline for the ${}_n\bar{\gamma}_l(\kappa)-n$ curve yields a reliable estimate of the radial travel time for PKIKP waves (from oscillations of PKIKP type, such as radial oscillations) or for S waves between the surface and the core-mantle boundary (from oscillations of SH or SV type, such as torsional free oscillations). Since we are dealing here mainly with free oscillations equivalent to PKIKP waves, for brevity we simply use ${}_n\bar{\gamma}_l$ for ${}_n\bar{\gamma}_l$ (PKIKP) and ${}_n\sigma_l$ for ${}_n\sigma_l$ (PKIKP) except where otherwise stated. It should be noted that the parameter ${}_n\bar{\gamma}_l$ differs slightly from the parameter ${}_n\gamma_l$ defined by Anderssen and Cleary (1974), which also yields a valid estimate of radial travel times.

In order to examine the effect of Earth structure on ${}_n\bar{\gamma}_l$, its values are normally plotted as a function of n. This paper is concerned mainly with the effects on ${}_n\bar{\gamma}_l$ of discontinuities and transition layers. These effects occur as oscillations in the ${}_n\bar{\gamma}_l$ curves, which are described in terms of *amplitude* (in units of

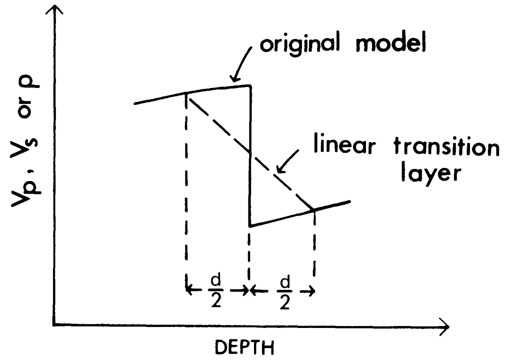

Fig. 2. Diagram showing how a discontinuity in a parameter is replaced by a linear transition layer

seconds) and *frequency* (in cycles per unit n) or *period* (in units of n, i.e. dimensionless).

2.2. Magnitudes of Discontinuities and Transition Layers

We define a *transition layer* as a spherically symmetric region of limited thickness in the Earth's interior, in which the density and the compressional and shear wave velocities are monotone functions of depth. It is clear that a discontinuity is the limiting case where a transition layer has zero thickness. In order to be consistent with normal terminology, however, we will restrict the term 'transition layer' to regions of finite thickness.

The radial oscillations are equivalent to vertically incident PKIKP waves. If the results for torsional eigenfrequencies are applicable to radial oscillations, the amplitude of the oscillation in the ${}_n\bar{\gamma}_o$ curve caused by a discontinuity is related to the reflection coefficient for these waves at the discontinuity, which in turn is related to the *magnitude* of the discontinuity which is defined as the *absolute value of the reflection coefficient for vertically incident P waves*, i.e.,

$$|\rho^+\alpha^+ - \rho^-\alpha^-|/(\rho^+\alpha^+ + \rho^-\alpha^-) \tag{1}$$

where (ρ^+, α^+) and (ρ^-, α^-) are pairs of density and P wave velocity on opposite sides of the discontinuity. This can be generalized to the case of a transition layer. Since the reflection coefficient for these waves at a transition layer is closely related to the contrast in the acoustic impedance (product of density and velocity) at the two ends of the transition layer (cf. Wolf, 1937), we define the magnitude of a transition layer to be (1), where (ρ^+, α^+) and (ρ^-, α^-) denote the pairs of density and P wave velocity values at the two ends of the transition layer.

Let $R(\omega, d)$ denote the absolute value of the reflection coefficient for waves of angular frequency ω normally incident at a transition layer with thickness d. Then $R(\omega, d)$ approaches (1) as ω approaches zero, and, generally speaking, $R(\omega, d)$ decreases as ω increases. The rate of decrease is closely associated with d: the larger d, the more rapid the decrease in $R(\omega, d)$ (cf. Wolf, 1937). This implies that, for waves with wavelengths much greater than *d*, the transition layer can be treated as a discontinuity, while the reflection of waves with wavelengths much smaller than *d* at the transition layer is negligible.

3. Effects of Core Boundaries

The effects of the I/O and C/M boundaries will be studied by systematic replacement of either or both of the I/O and C/M discontinuities by linear transition layers as shown in Fig. 2.

3.1. Earth Models Used

Models 1066A (Fig. 3A; Gilbert and Dziewonski, 1975) and HOMO (with homogeneous mantle, outer core and inner core; Fig. 3B), and a set of models derived from them by systematic replacement of either or both the I/O and C/M discontinuities by linear transition layers, are used in this section.

Model 1066A is continuous except at the crust-mantle, I/O and C/M boundaries, where the discontinuities in density and elastic wave velocities are large. It has been confirmed by numerical experiments that, compared to the effects of the core boundaries, the effect of the crust-mantle discontinuity on the radial oscillations is negligible for 1066A. It is therefore not necessary to take its existence into account. For convenience the Earth models derived from 1066A are referred to as ACxIy, where ACxIy contains a linear transition layer ACx of thickness x at the C/M boundary and a linear transition layer AIy of thickness y at the I/O boundary. Model AC0I0 corresponds to 1066A. Some other examples of ACxIy are shown in Fig. 3A. Although it is known from the observations of short period reflections such as PcP and PKIKP that the changes in elastic properties at both core boundaries are quite abrupt, for the purposes of this investigation models with transition layers of several hundred km at the core boundaries have been included.

The simple model HOMO was constructed from model PEM (Dziewonski et al., 1975) by taking the average values of density, P and S wave velocities for the mantle (including the crust), the outer core and the inner core, in the sense that total mass and the P and S wave radial travel times are preserved. The models derived from HOMO are referred to as HCxIy, where HCxIy, like ACxIy, is a model with a C/M transition layer of thickness x and an I/O transition layer of thickness y.

3.2. Results

Our results show that the ${}_n\bar{\gamma}_0$ curves for models 1066A and HOMO contain, for high overtone numbers (say, $n > 20$), two distinct oscillations with periods of 5.5 and 2.4, which are produced by the I/O and C/M discontinuities respectively. If either of the discontinuities is replaced by a linear transition layer, the amplitude

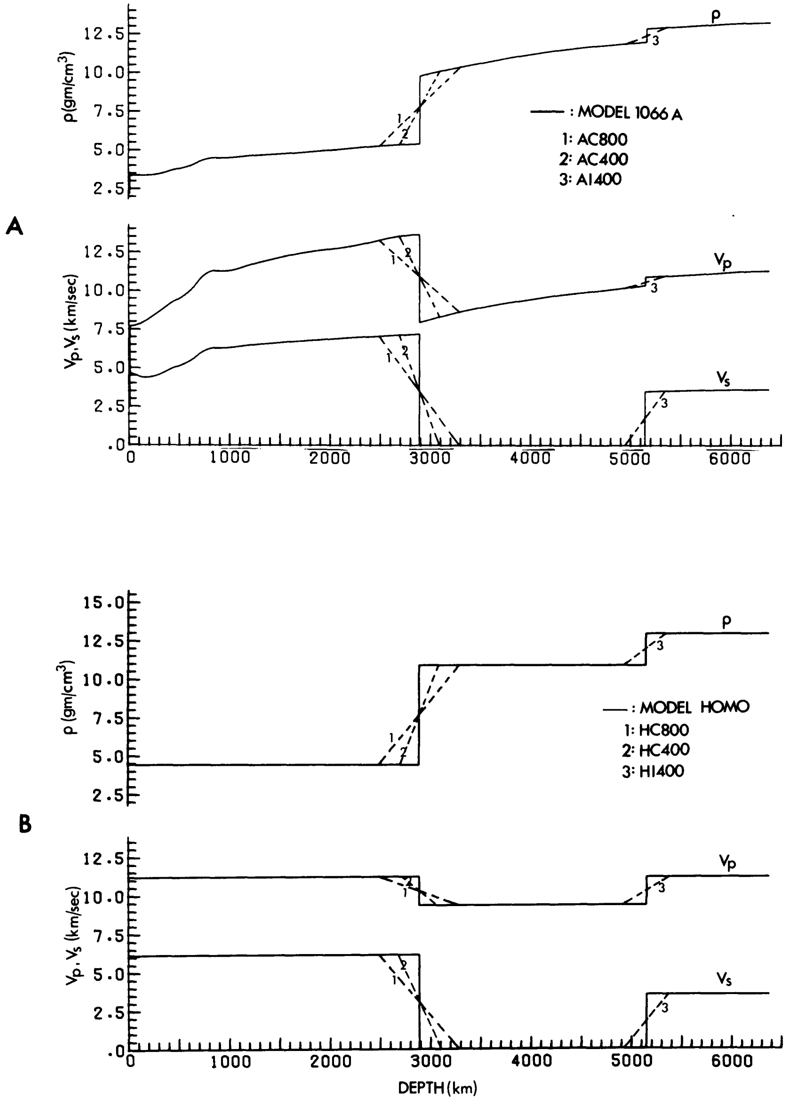

Fig. 3A and B. Models 1066A (**A**) and HOMO (**B**) and some other models ACxIy and HCxIy derived from them. See text for descriptions of ACxIy and HCxIy

of the corresponding oscillation decreases as the overtone number n increases. The rate of the decrease is closely related to the thickness of the transition layer: the thicker the transition layer, the more rapid the amplitude decrease. This is related to the above-mentioned property that, for a fixed value of d, $R(\omega, d)$ decreases as ω increases. We now discuss separately the effects at the core boundaries.

3.2.1 Effect of a Discontinuity or Transition Layer at the I/O Boundary. Figures 4 and 5 show the $_n\bar{\gamma}_o$ curves constructed from ACxIy and HCxIy, respectively, with

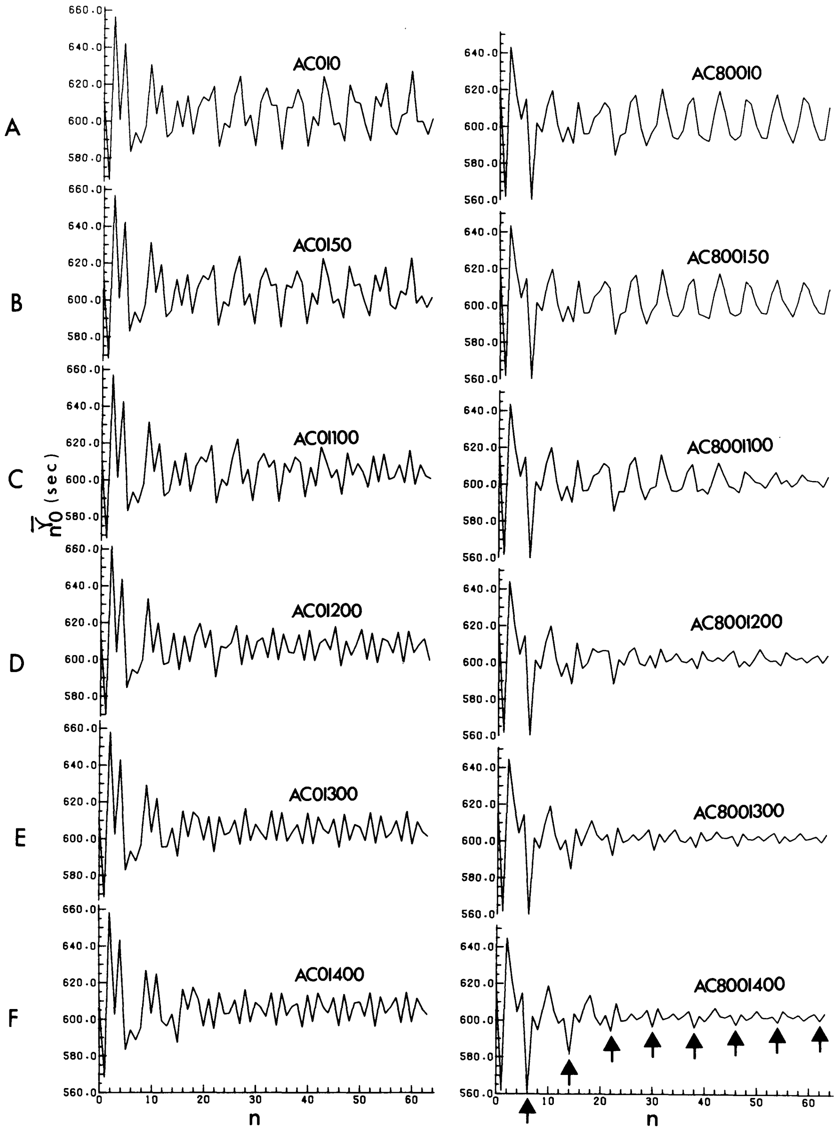

Fig. 4A–F. $_n\bar{\gamma}_0 - n$ curves constructed from ACxly with varying y and fixed x values of 0 km *(left column)* and 800 km *(right column)*

varying thickness y. The $_n\bar{\gamma}_0$ curves in the right-hand column of each figure contain, for $n \geqq 20$, an oscillation with period 5.5. Except for small n, these curves have a negligibly small C/M boundary effect, because the C/M discontinuity has been replaced by an 800 km transition layer. They are much simpler than those in the left-hand columns, where the discontinuity has been retained.

In both Figs. 4 and 5, when $y \neq 0$ the amplitude of the oscillation with period 5.5 in the $_n\bar{\gamma}_0$ curve decreases with increasing n, and the rate of the decrease is closely associated with the value of y. In addition, there is a close agreement in behaviour, as y increases, between the corresponding curves in Figs. 4 and 5. For

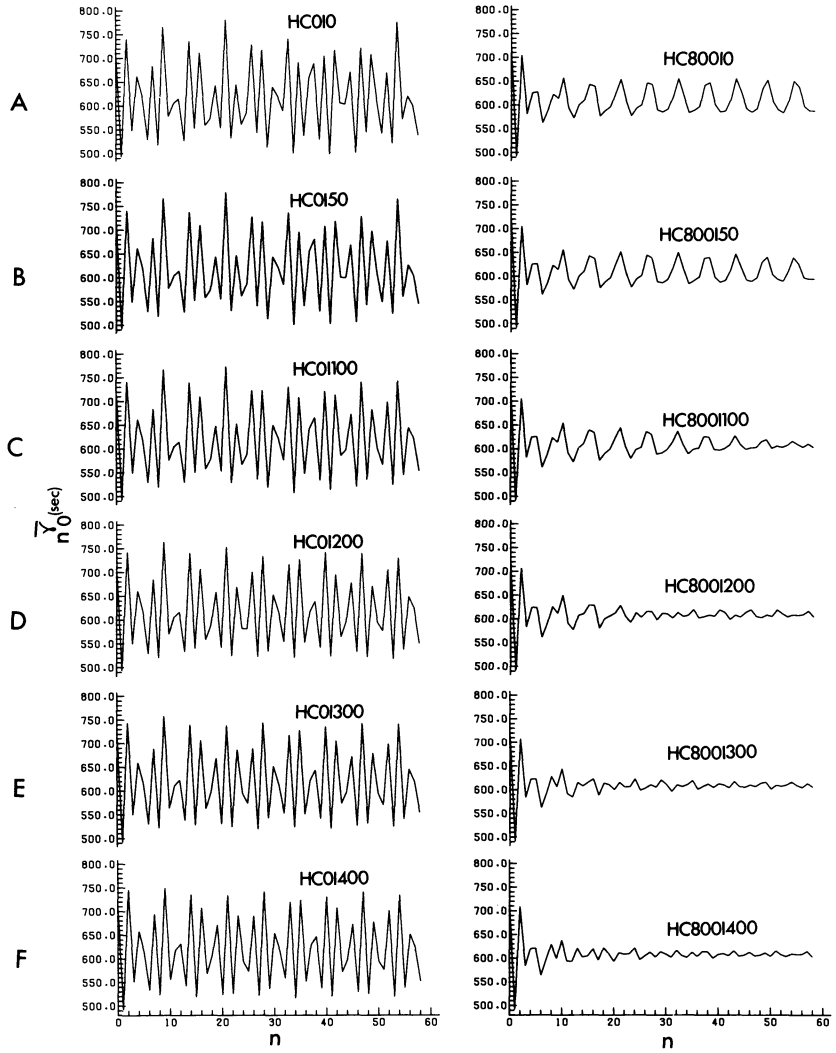

Fig. 5A–F. $_n\bar{\gamma}_o - n$ curves constructed from HCx1y with varying y and fixed x values of 9 km *(left column)* and 800 km *(right column)*

$y \geqq 200$ km, the decrease in the amplitude of the oscillation is restricted to the range $0 \leqq n \leqq 20$, except for a slowly-decreasing ripple (with a period of about 8) in Fig. 4, which is produced by the transition zone in the upper mantle of 1066A.

It is therefore clear that the oscillation in Figs. 4A and 5A having a period of 5.5 arises from the I/O discontinuity. It follows that the oscillation having a period of 2.4 in the left-hand columns of Figs. 4 and 5 is produced by the C/M discontinuity, since in both 1066A and HOMO the only two dominant structural features which can cause large amplitude oscillations in the $_n\bar{\gamma}_o$ curves are the I/O and C/M discontinuities.

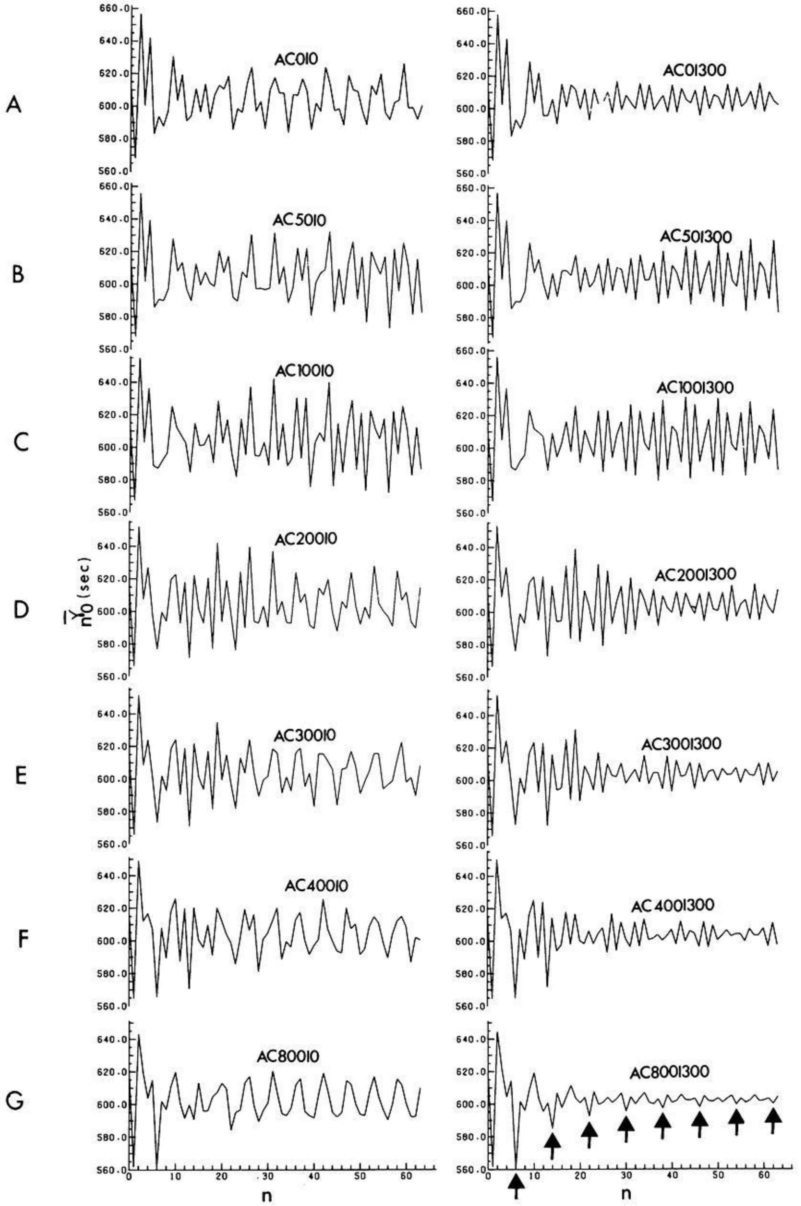

Fig. 6A–G. $_n\bar{\gamma}_o - n$ curves constructed from ACxIy with varying x and fixed y values of 0 km *(left column)* and 300 km *(right column)*

3.2.2. Effect of a Discontinuity or Transition Layer at the C/M Boundary. Figures 6 and 7 show $_n\bar{\gamma}_o$ curves contructed from models ACxIy and HCxIy, respectively, with varying x values. The effect of the I/O boundary on the $_n\bar{\gamma}_o$ curves in the right columns of Figs. 6 and 7 is greatly reduced when n is large, since it is replaced by a 300 km thick transition layer. For $n \geqq 20$, these $_n\bar{\gamma}_o$ curves contain

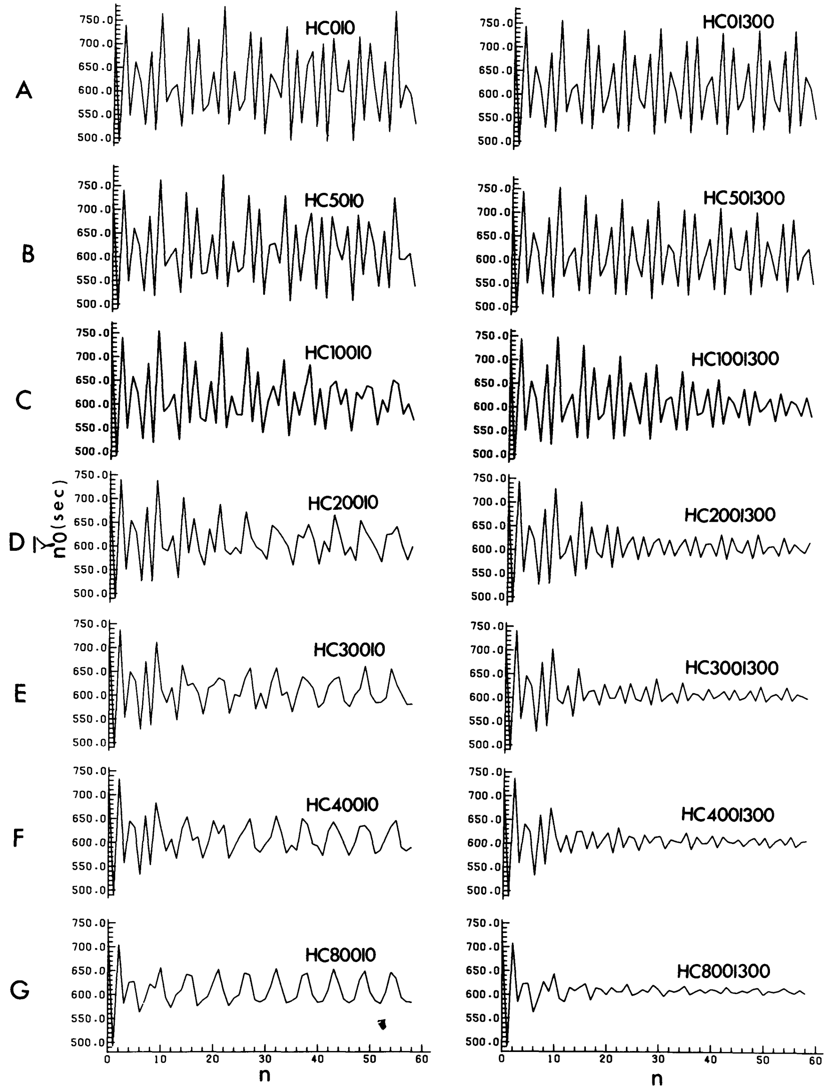

Fig. 7A–G. $_n\bar{\gamma}_o - n$ curves constructed from HCxIy with varying x and fixed y values of 0 km *(left column)* and 300 km *(right column)*

an oscillation with period 2.4. As the C/M transition layer is extended to 800 km, the amplitude of this oscillation is significantly reduced. It can therefore be identified as the effect of the C/M transition layer.

As x increases, the change in the form of the $_n\bar{\gamma}_o$ curves is more complex for ACxIy than for HCxIy. The character of the change common to both 1066A and HOMO is: when $x = 0$ (i.e., the C/M boundary is a discontinuity), the oscillation with period 2.4 is persistent (cf. Figs. 6A and 7A), whereas when $x \neq 0$ (i.e., the C/M boundary is a transition layer), the amplitude of this oscillation

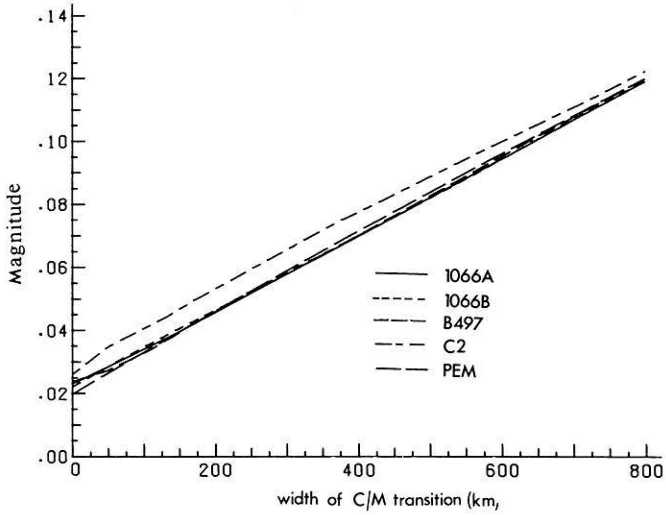

Fig. 8. The dependence of the magnitude of the C/M transition layer on the thickness of the layer for different Earth models

decreases with increasing n (cf. Figs. 6D through 6G and 7B through 7G). The rate of decrease is closely related to the vaue of x.

In addition, the character of the change for ACxIy has the following two properties: (i) The amplitude of the oscillation with period 2.4 increases with increasing x (cf. the right-hand curves of Fig. 6B through D for the range $10 \leqq n \leqq 30$). This increase corresponds to an increase in the magnitude of the C/M transition layer when x is increased. Such an increase in magnitude will occur for most existing Earth models. The dependence of this magnitude on x is shown in Fig. 8 for models 1066A, 1066B (Gilbert and Dziewonski, 1975), B497 (Dziewonski and Gilbert, 1973, Appendix Al), PEM (Dziewonski et al., 1975) and C2 (Anderson and Hart, 1976). (ii) The amplitude of the oscillation with period 2.4 increases with increasing n for certain values of x; in particular, for $n > 10$ when $x = 50$ km, and for $10 < n < 45$ when $x = 100$ km. Its connection with any special characteristic of the C/M boundary is unknown.

Because of (i) and (ii), the decrease (with n) of the amplitude of the oscillation with period 2.4 is much slower for ACxIy than for HCxIy.

4. Effect of the Crust and Upper Mantle Structure

From Sect. 3 it follows that a discontinuity or a transition layer will produce in the ${}_n\bar{\gamma}_o$ curve an oscillation with a particular frequency determined by its position. The oscillation produced by a discontinuity is persistent, whereas the amplitude of the oscillation produced by a transition layer decreases with n at a rate closely related to the thickness of the transition layer. In this section the effect of crust and upper mantle structure on the ${}_n\bar{\gamma}_o$ curve will be discussed based on this conclusion.

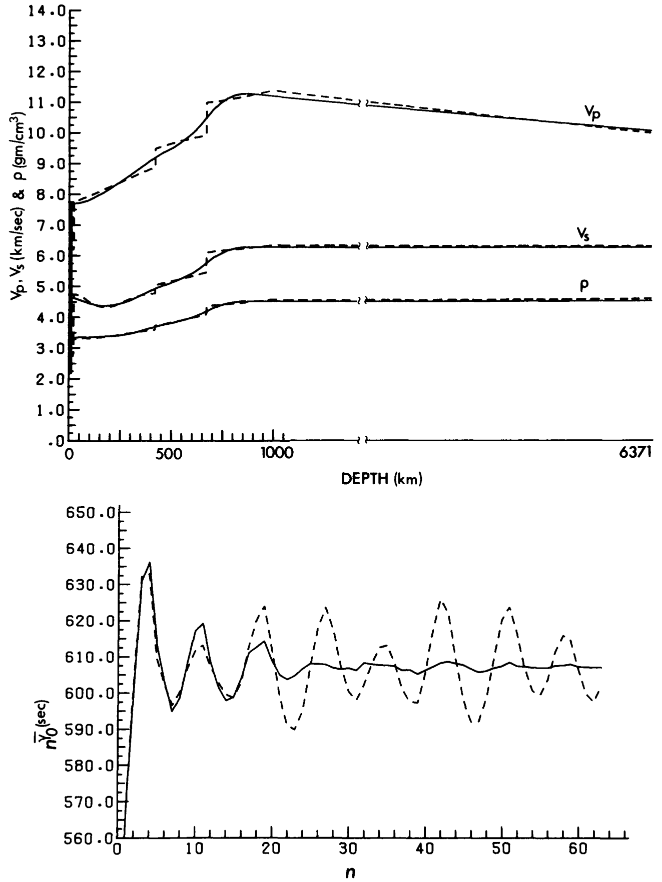

Fig. 9. Comparison of the effect of the continuous upper mantle of 1066A *(solid line)* and that of the discontinuous one of 1066B *(dashed line)* on the ${}_n\bar{\gamma}_o - n$ curves. The models used are derived in a way such that the radial P wave travel times from the core-mantle discontinuity to the Earth's center are kept the same as 1066A and 1066B respectively

The upper mantle (extending from the base of the crust to a depth of about 1000 km) is the region where the density and P and S wave velocities are most complicated and where the existing Earth models differ most. These Earth models can be divided into two classes: those containing a discontinuous upper mantle, which generates a persistent oscillation in the ${}_n\bar{\gamma}_o$ curve, and those containing a continuous upper mantle, which generates an oscillation with decreasing amplitude. To illustrate, the effect of the continuous upper mantle of 1066A and that of the discontinuous upper mantle of 1066B are compared in Fig. 9. The ${}_n\bar{\gamma}_o$ curves for models 1066A and 1066B (constructed from the same set of geophysical data) are also compared in Fig. 10.

The oscillation corresponding to the upper mantle structure has a period of 8 (approximately). Except for discontinuities, the effect of upper mantle structure on the ${}_n\bar{\gamma}_o$ curve is mainly restricted to $n \leqq 20$ (Fig. 9). Therefore, for $n > 20$, the

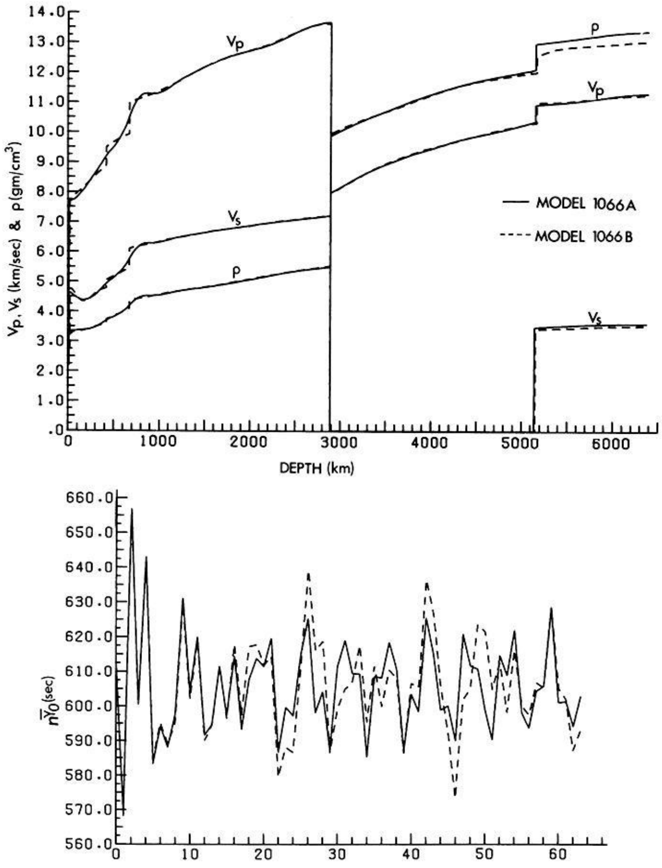

Fig. 10. Comparison of ${}_n\bar{\gamma}_o - n$ curves for models 1066A *(solid line)* and 1066B *(dashed line)*

oscillation only corresponds to the upper mantle discontinuities and rapid transition layers. For example, the slowly-decreasing ripples marked by arrows in Figs. 4 and 6 arise from the transition region in the upper mantle. Since the discontinuities and rapid transition layers generate oscillations with similar periods, their individual properties cannot be separated. Thus the effects of the 421 km and 671 km discontinuities in model 1066B combine to form an oscillation with period 8 (Figs. 9 and 10).

For torsional oscillations, it has been shown in Wang (1978, Ch. 6) that the amplitude of the solotone effect corresponding to a discontinuity in the upper mantle or crust is proportional to $k_1 \sin(t_1 \pi/\alpha)$ while the period of the solotone effect is α/t_1, where k_1 is the reflection coefficient at the discontinuity for normally incident SH waves, and t_1 and α are the shear wave radial travel times from the Earth's surface to the discontinuity and the core-mantle boundary respectively. Numerical experimentation has indicated that this result can be extended to radial oscillations. Thus the oscillation generated by a discontinuity or transition layer in the upper mantle or crust will have a period z_0/z_1 and an amplitude closely associated with the product of the magnitude of the discontinuity or transition layer and $\sin(z_1\pi/z_0)$, where $z_1$and z_0 are the Pwave radial travel times from the Earth's surface to the discontinuity or transition layer and the Earth's center respectively. Therefore, as a discontinuity or a transition layer in the upper mantle is moved toward the surface of a model Earth, the oscillation in the ${}_n\bar{\gamma}_o$ curve corresponding to it becomes smaller in amplitude and longer in period.

Accordingly, an oscillation corresponding to a discontinuity or transition layer in the crust will have very long period and a very small amplitude, since z_1/z_0 will be very small. In particular, for a discontinuity at a depth less than 40km and with a magnitude less than 0.5, the corresponding oscillation will have a peak-to-peak amplitude less than 10s.

It follows from the above that although the dominant effects in ${}_n\bar{\gamma}_o$ curves from radial oscillations of PKIKP type are produced by the C/M and I/O discontinuities (as indicated by Anderssen et al., 1975), it may be possible to separate out upper mantle effects on the basis of period. In particular, it can be seen in Fig. 9 that there is a significant difference in ${}_n\bar{\gamma}_o$ between continuous and discontinuous models at about $n=10$, which is within the presently observable range. As shown in Fig. 10, this difference has been compensated for in Models 1066A and 1066B. The result nevertheless shows that the effect of the upper mantle is in principle separable from the data at comparatively small values of n.

5. Comparison Between Observed and Synthetic Data

Figure 11B shows the differences between ${}_n\bar{\gamma}_o$ values for model 1066A (Curve 1) and for observed free oscillation data (designated by '×'). It can be seen that 1066A fits the observed ${}_n\bar{\gamma}_o$ values well for $n \leqq 2$ and that, in general, when n increases, the fit becomes worse. This trend indicates that the differences in ${}_n\bar{\gamma}_o$ values could be reduced by changing the parameter values concerning discontinuities and/or rapid transitions in the model.

Shown in Fig. 11A (Curves 2 and 3) are two perturbed models of 1066A which fit, except for ${}_5\bar{\gamma}_o$, the observed ${}_n\bar{\gamma}_o$ values better than 1066A. Figure 11 confirms that the radial normal mode data for small overtone numbers cannot distinguish an Earth model with discontinuous core boundaries from an Earth model with continuous ones. Although the perturbed models are not realistic, the results show that the differences between ${}_n\bar{\gamma}_o$ values derived from observed data and eigenfrequencies of 1066A are sensitive to the changes in parameter values defining the core boundaries, and indicate that if sufficiently refined normal mode data, including high overtone numbers, can be obtained, the Earth model can be improved significantly.

Figure 11B also shows the error ranges (designated by triangles) of the observed ${}_n\bar{\gamma}_o$ values, based on the observation errors listed by Gilbert and Dziewonski (1975). It is clear from these that, in addition to data of higher overtone number, a refinement of the present data is required for satisfactory constraint on the Earth's fine structure.

If lateral variations in the radius of the C/M boundary exist, as suggested, for example, by Hide and Horai (1968), then because of the averaging properties of

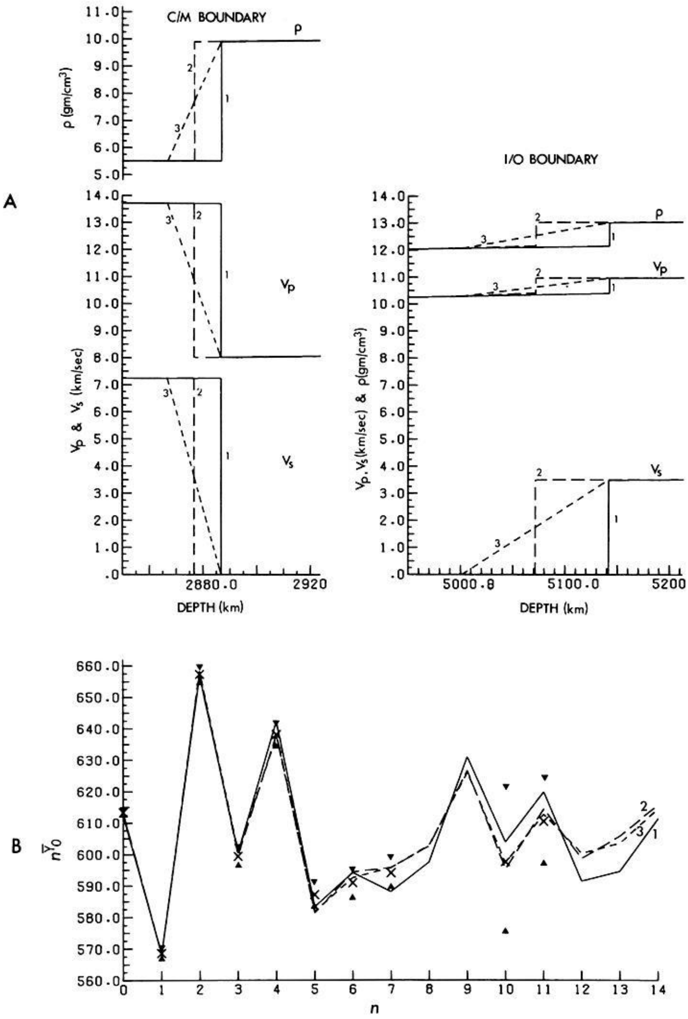

Fig. 11 A and B. Error ranges *(designated by triangles)* and comparison of observed ${}_n\bar{\gamma}_o$ values *(designated by '×')* and those constructed from 1066A *(curve 1)* and two perturbed models of 1066A *(curves 2 and 3)*. One perturbed model is derived from 1066A by shifting the C/M and I/O discontinuities toward the Earth's surface by 10 and 70 km respectively, and the other by replacing the discontinuities by 20 and 140 km thick transition layers in the way shown

free oscillations, the effect of these variations would be similar to that of a transition layer at the boundary. It follows from the above results that such variations would not be detectable by the present radial oscillation data.

6. Conclusions

The above results may be summarized as follows:

(1) A ${}_n\bar{\gamma}_o$ curve comprises a baseline component whose value is determined by the P wave radial travel time from the surface to the centre of the Earth, and

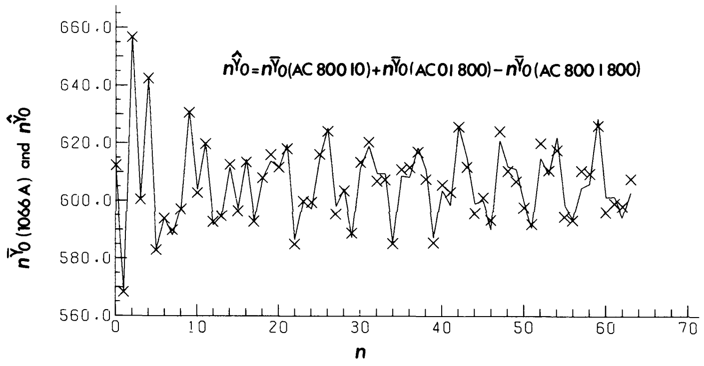

Fig. 12. Comparision of $_n\bar{\gamma}_o$(1066A) *(solid line)* and $[_n\bar{\gamma}_o(AC800IO) + {}_n\bar{\gamma}_o(AC0I800) - {}_n\bar{\gamma}_o(AC800I800)]$ *(designated by '×')* values

an oscillatory component which reflects the structure in the Earth's interior, including gradual slopes, transition layers and discontinuities.

(2) The effect of a discontinuity on the $_n\bar{\gamma}_o$ curve is seen as a persistent oscillation, while the effect of a transition layer is seen as an oscillation whose amplitude decreases with n at a rate closely associated with the thickness of the layer. The amplitude of the oscillation is closely related to the magnitude of the discontinuity or transition layer, while the period is related to its position. A consequence of this is that the effects of gradual slopes on the $_n\bar{\gamma}_o$ curve are limited to small overtone numbers (say $n \leqq 20$) only.

(3) It is known from the observation of short period waves that the I/O and C/M boundaries are discontinuities or very rapid transition layers of large magnitudes, therefore their effects on the Earth's radial oscillations are persistent or almost persistent oscillations in the $_n\bar{\gamma}_o$ curve having periods of about 5.5 and 2.4 respectively. Except for very rapid transition layers or discontinuities, the effect of upper mantle structure is mostly restricted to $n \leqq 20$. The oscillation corresponding to the structure of the upper mantle has a period of about 8. The effect of crustal structure on the radial oscillations is seen as an oscillation of very long period and very small amplitude (less than 10s from peak-to-peak).

(4) As a consequence of the above, the oscillations of very small n (say, $n < 5$) in the $_n\bar{\gamma}_o$ curve are a composite effect of all the Earth's interior structures (gradual slopes, transition layers and discontinuities), i.e., they reflect the overall spherically symmetric structure of the Earth. When n increases, the oscillations tend to reflect local structures such as very rapid transitions and discontinuities, i.e., their effects are proportionally greater. When $n > 20$, the oscillations are mostly the effects of these very rapid transition layers and discontinuities.

(5) The exact manner in which the effects of all the structural features of the Earth on the radial free oscillations are composed still requires a detailed study. By neglecting the minor effects and the interference among the effects of major structural features, however, we can represent the overall composite effect of the Earth's structure as the sum of all the individual oscillations in the $_n\bar{\gamma}_o$ curve. Figure 12 shows the comparison of $_n\bar{\gamma}_o$ (1066A) (solid line) and $[_n\bar{\gamma}_o$(AC800IO)

$+{}_n\bar{\gamma}_o$(AC0I800) $-{}_n\bar{\gamma}_o$(AC800I800)] (designated by '×') structures, where ${}_n\bar{\gamma}_o(\kappa)$ is the ${}_n\bar{\gamma}_o$ value for model κ.

(6) A comparison between observed and synthetic ${}_n\bar{\gamma}_o$ values confirms that the radial normal mode data for small overtone number cannot distinguish a continuous Earth model from a discontinuous one. Because of the large error ranges of the observed ${}_n\bar{\gamma}_o$ values and the nature of free oscillations of low overtone number, information about the fine structure of the Earth and lateral variations in the radius of the C/M boundary cannot be detected by the present radial oscillation data.

Acknowledgements. We wish to thank Professor Freeman Gilbert of the Scripps Institution of Oceanography, University of California at San Diego for making his computer program MODE available to us.

References

Anderson, D.L., Hart, R.S.: An Earth model based on free oscillations and body waves. J. Geophys. Res. **81**, 1461–1475, 1976

Anderssen, R.S., Cleary, J.R.: Asymptotic structure in torsional free oscillations of the Earth-I. Overtone structure. Geophys. J. Roy. Astron. Soc. **39**, 241–268, 1974

Anderssen, R.S., Cleary, J.R., Dziewonski, A.M.: Asymptotic structure in eigenfrequencies of spheroidal normal models of the Earth. Geophys. J. Roy. Astron. Soc. **43,** 1001–1006, 1975

Chang, A.C., Cleary, J.R.: Precursors to PKKP. Bull. Seism. Soc. Am. **68**, 1059–1079, 1978

Dziewonski, A.M., Gilbert, F.: Observations of normal modes from 84 recordings of the Alaskan earthquake of 1964 March 28, Part II. Further remarks based on new spheroidal overtone data. Geophys. J. Roy. Astron. Soc. **35**, 401–437, 1973

Dziewonski, A.M., Hales, A.L., Lapwood, E.R.: Parametrically simple Earth models consistent with geophysical data. Phys. Earth. Planet. Inter. **10**, 12–48, 1975

Gilbert, F.: Some asymptotic properties of the normal modes of the Earth. Geophys. J. Roy. Astron. Soc. **43**, 1007–1011, 1975

Gilbert, F., Dziewonski, A.M.: An application of normal mode theory to the retrieval of structural parameters and source mechanisms from seismic spectra. Philos. Trans. Roy. Soc. Lond. [A] **278**, 187–269, 1975

Haddon, R.A.W.: Corrugations on the mantle-core boundary or transition layers between inner and outer cores? Trans. Am. Geophys. Union **53**, 600 (abstract), 1972

Hales, A.L., Roberts, J.L.: Shear velocities in the lower mantle and the radius of the core. Bull. Seism. Soc. Am. **60**, 1427–1436, 1970

Hide, R., Horai, K.: On the topography of the core-mantle interface. Phys. Earth Planet. Inter. **1**, 305–308, 1968

McNabb, A., Anderssen, R.S., Lapwood, E.R.: Asymptotic behavior of the eigenvalues of a Sturm-Liouville system with discontinuous coefficients. J. Math. Anal. Appl. **54**, 741–751, 1976

Wang, C.: Ray-mode duality for SH waves and the effect of Earth structure on free oscillation eigenfrequencies. Ph.D. thesis. Australian National University, 1978

Wolf, A.: The reflection of elastic waves from transition layers of variable velocity. Geophysics **2**, 357–363, 1937

Received November 23, 1978; Accepted February 6, 1979