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On the Coastal Effect on Geoelectrical Soundings

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Abstract. To estimate the influence of the ocean on a geoelectrical sounding carried out on land near the coast, a model is chosen consisting of a perfectly conducting, infinitely thin, half-infinite sheet lying on the surface of a homogeneous earth. Model curves of the apparent resistivity are given for measurements made parallel or perpendicular to the edge of the sheet. From these model curves it is possible to derive apparent resistivity curves for other configurations of the current and potential electrodes.

As an example, we have considered a geoelectrical sounding made with large electrode separation in Southern Africa. For a homogeneous half-space, the maximum deviation of the apparent resistivity from the resistivity of the half-space is of the order of about 15% for the applied maximum electrode spacing of 1,250 km due to the effect of the Indian Ocean. Therefore it may be concluded that the measured sharp minimum of the apparent resistivity curve (ρ_a) is not produced by the ocean. If the measured ρ_a-curve is accordingly corrected for the coastal effect it yields nearly the same conductivity model as before.

Key words: Apparent resistivity – Effect of ocean – Conducting half-infinite sheet.

1. Introduction

Blohm et al. (1977) have reported on results of geoelectrical depth soundings carried out in Southern Africa from 1973 to 1975. The measurements were made using a Schlumberger configuration with electrode spacings up to 1,250 km. The array was nearly parallel to the Indian Ocean at an average distance of about 350 km from it. Therefore, the question may be asked to what extent the data could have been affected by the highly conductive salt water of the ocean. (Sea water has a resistivity of about 0.2 Ohm · m as compared with up to 100,000 Ohm · m of the South-African rocks at shallow depth.) The mean depth of the

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ocean (about 3 km) was very small compared with the distances of the electrodes from the coast as well as from each other. Accordingly, an infinitely thin, half-infinite sheet with vanishing resistivity at the boundary of a half-space may serve as a model of the ocean in order to estimate the possible effect on the sounding curve.

Such a simple model may also be used to estimate the effect of the sea water on soundings carried out immediately near the coast. Additionally, this model may be helpful for the interpretation of soundings made near the edge of shallow highly conductive deposits of large lateral extent.

Grant and West (1965) have quoted a paper by Sommerfeld (1897), in which the potential of a point source is derived for the case of an infinitely conducting half-plane embedded in a uniform space. Sommerfeld's formula was later corrected by Carslaw (1899). By means of this formula, corresponding Schlumberger sounding apparent resistivities may be derived from the gradients of the potential, especially from gradients parallel or perpendicular to the edge of a semi-infinite sheet lying at the earth's surface.

2. Theory

Within a uniform space with resistivity ρ_1, Sommerfeld (1897) assumes a half-infinite thin sheet with no resistivity which coincides with the half-plane $z=0$, $x \geqq 0$ in Cartesian coordinates. A point source A with current I is located at $A(x', y', z')$; the point P where the potential is measured is assumed to have the coordinates $P(x, y, z)$. If cylindrical coordinates (r, φ, y) are introduced so that the axis of the cylinder coincides with the edge of the sheet which is described by $\varphi=0$, then

$$\begin{aligned} x &= r \cdot \cos\varphi \\ y &= y \\ z &= r \cdot \sin\varphi. \end{aligned} \tag{1}$$

Now, the potential V may be written as

$$V=\frac{I\rho_1}{2\pi^2} \cdot \left[\frac{1}{R} \cdot \arctan\left(\frac{\sigma+\tau}{\sigma-\tau}\right)^{1/2} - \frac{1}{R'} \arctan\left(\frac{\sigma+\tau'}{\sigma-\tau'}\right)^{1/2}\right]. \tag{2}$$

This formula is derived by conformal mapping of the half-infinite plane onto an infinite plane applying additionally the method of images in a space of two revolutions. The following definitions are used:

$$R=[r^2+r'^2-2rr'\cos(\varphi-\varphi')+(y-y')^2]^{1/2} \tag{3a}$$

$$\tau=\cos\frac{\varphi-\varphi'}{2} \tag{3b}$$

$$\sigma=\left[\frac{(r+r')^2+(y-y')^2}{4rr'}\right]^{1/2}. \tag{3c}$$

The quantities R' and τ' are obtained from (3a) and (3b), respectively, by inserting $-\varphi'$ in place of φ'.

In our case the half-plane is assumed at the earth's surface $z=0$, for $x \geqq 0$ ($\varphi=0$), and the source A and the point of measurement P are also located at the earth's surface with $x<0$, i.e.

$$\varphi=\varphi'=\pi. \tag{4}$$

If a half-space rather than uniform space is considered, one has only to multiply the potential by a factor of two. Since $r=-x$, $r'=-x'$, $\tau=1$, $\tau'=-1$, $R=R'$, using definitions (3c) and the addition theorem for the arctan function, Eq. (2) takes the simple form

$$V=\frac{I\rho_1}{\pi^2}\cdot\frac{1}{R}\cdot\arctan u, \tag{5}$$

where

$$u=2(xx')^{1/2}/R \tag{5a}$$

and

$$R=[(x-x')^2+(y-y')^2]^{1/2}. \tag{5b}$$

If the point of measurement P approaches the source, one obtains

$$V\to V_0=\frac{I\rho_1}{2\pi}\cdot\frac{1}{R} \qquad (P\to A), \tag{6}$$

which is the potential of a point source at the boundary of a half-space without any disturbing bodies. If P approaches the edge of the sheet, we get

$$V\to 0 \qquad (x\to 0). \tag{7}$$

To obtain the gradient of the potential V at the point P in any direction s defined by the angle ψ with respect to the x-axis, we have to calculate the derivatives with respect to x and y:

$$\frac{\partial V}{\partial s}=\frac{\partial V}{\partial x}\cdot\cos\psi+\frac{\partial V}{\partial y}\cdot\sin\psi. \tag{8}$$

The apparent resistivity $\rho_a^{(s)}$ may be defined for a measurement of the gradient in the s-direction by

$$\rho_a^{(s)}=\rho_1\cdot\frac{\partial V/\partial s}{\partial V_0/\partial s}. \tag{8a}$$

Especially for a measurement perpendicular or parallel to the edge of the sheet, one obtains:

$$\frac{\rho_a^{(x)}}{\rho_1}=\frac{\partial V/\partial x}{\partial V_0/\partial x}=\frac{2}{\pi}\left[\arctan u+\frac{u^2(x-x')-2x'}{(x-x')\,u(1+u^2)}\right] \tag{9}$$

(perpendicular to the edge),

$$\frac{\rho_a^{(y)}}{\rho_1}=\frac{\partial V/\partial y}{\partial V_0/\partial y}=\frac{2}{\pi}\left[\arctan u+\frac{u}{1+u}\right] \tag{10}$$

(parallel to the edge).

3. Results

3.1. Model Curves of the Apparent Resistivity

From expressions (9) and (10) it is possible to calculate model curves of the apparent resistivity for a single electrode A (measurement of the gradient) by setting $y'=0$ and expressing all geometric quantities in terms of the distance D from the source A to the edge of the conducting sheet. The distance between the electrode A and the measuring point (center of MN in Fig. 1) is designated by d; in practice the gradient of the potential is obtained from the difference of the potentials between the potential electrodes M and N when placed close together.

Depending upon the position of the potential electrodes M and N, as shown in Fig. 1 (inset), three types of apparent resistivity curves will result. For example, when M and N are situated between electrode A and edge of the sheet ($d=x-x'$), we obtain the apparent resistivity curve (a) shown in Fig. 1. Similarly, when A lies between MN and the sheet, curve (b) results, and when MN is moved parallel to the edge, curve (c) will be obtained.

The gradient $\partial V/\partial s$ of the potential – and therefore the apparent resistivity according to (8a) – in the case of two electrodes A (with current I) and B (with current $-I$) can be obtained by superposition of the gradients for the case of a single current electrode A. Especially for a Schlumberger configuration (with potential electrodes M and N in the middle between A and B) with the electrode separation $L=\overline{AB}$ being perpendicular to the edge of the sheet and with the distance l from the midpoint of the electrode configuration to the edge of the conducting sheet, the following formula may be used:

$$\rho_a=\left(\frac{L/2}{l-L/2}\right)^2\cdot\rho_a^{(x+)}\left[\frac{L/2}{l-L/2}\right]+\left(\frac{L/2}{l+L/2}\right)^2\rho_a^{(x-)}\left[\frac{L/2}{l+L/2}\right], \tag{11}$$

where the arguments of $\rho_a^{(x+)}$ and $\rho_a^{(x-)}$ denote the abscissa of the curves (a) and (b) in Fig. 1. In case of a Schlumberger configuration parallel to the edge of the sheet (with two electrodes A and B) we obtain the same apparent resistivity curve as shown in curve (c), Fig. 1, for a single electrode A, with $d=L/2$ and D $=$distance of A and B from the edge. In our example, this case is of special interest. It may be concluded from curve (c) that the deviation of $\rho_a^{(y)}$ from ρ_1 is less than 3% for $d/D<0.85$. As d/D approaches infinity ($u\to 0$), the following expression for $\rho_a^{(y)}$ with a maximum error of 3% may be used:

$$\rho_a^{(y)}\approx\rho_1\cdot\frac{8}{\pi(d/D)}\qquad(d/D>9.4). \tag{12}$$

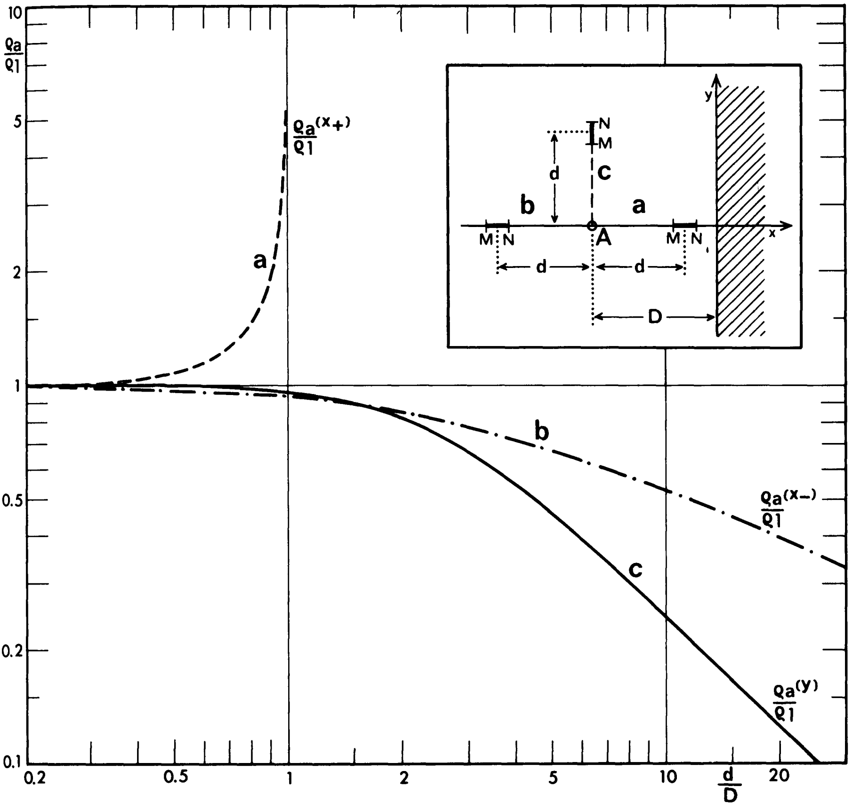


Fig. 1. Apparent resistivity (ρ_a) model curves for a single current electrode A and an infinitely thin perfectly conducting half-infinite sheet (hatched) situated on top of a homogeneous halfspace of resistivity ρ_1. Three different cases are shown for different positions (cf. inset) of the closely spaced potential electrodes M and N

Thus for large values of d/D, the apparent resistivity curve plotted on a logarithmic scale is a straight line with a slope of $-45°$.

3.2. Comparison With a Lateral Discontinuity of Resistivity

For measurements near a lateral change of resistivity, the question arises to what extent a finite thickness of a layer of high conductivity at the boundary of a half-space for $x>0$ may influence a sounding curve within the region $x<0$. For example, is it possible to obtain within a given accuracy of the measurements the model curve (c) for a sheet-like body with a finite or an infinite thickness? The extreme case of an infinite thickness of the layer with resistivity ρ_2 may be treated in the following simple manner, which allows a useful comparison with the model consisting of an infinitely thin sheet as treated in Sect. (3.1).

The model consists of a quarter-space $z>0$, $x<0$ with resistivity ρ_1 and a quarter-space $z>0$, $x>0$ with resistivity ρ_2. The potential V at the earth's surface in case of a point source $A(x', y', 0)$ in medium 1 as given in Van Nostrand and Cook (1966) is:

$$V=\frac{I\rho_1}{2\pi}\{[(x-x')^2+(y-y')^2]^{-1/2}+k[(x+x')^2+(y-y')^2]^{-1/2}\} \quad (13)$$

where $k=(\rho_2-\rho_1)/(\rho_2+\rho_1)$ is the reflection coefficient. The apparent resistivities, which correspond to curves (a)–(c) in Fig. 1 are now

$$\frac{\rho_a^{(x\pm)}}{\rho_1}=1\mp k\left(\frac{d/D}{2\mp d/D}\right)^2, \quad (14a, b)$$

$$\frac{\rho_a^{(y)}}{\rho_1}=1+k\left(\frac{d/D}{\sqrt{4+(d/D)^2}}\right)^3. \quad (14c)$$

By comparing formulas (10) and (14c), we find that for $d/D<3$, the apparent resistivity values from (10) are nearly equal (with 3% difference) to the apparent resistivity values of (14c) for the case when $\rho_2/\rho_1=0.3$. On the other hand, for $\rho_2=0$ $(k=-1)$ we get from (14c):

$$\rho_a^{(y)}\to\rho_1\frac{6}{(d/D)^2} \qquad (d/D\to\infty), \quad (15)$$

e.g. the corresponding curve has a slope of $-63.4°$ if plotted on a logarithmic scale. Furthermore, for a gradient measurement perpendicular to the boundary of this contact model, we have for $\rho_2=0$ about the same $\rho_a^{(x+)}$-values for $d/D<0.85$ as before. For larger values $(d/D\to 1)$, in the case of a conducting sheet, the apparent resistivity values rise to infinity, whereas in the case of the contact model, these values reach $2\cdot\rho_1$. Thus a distinction between both models is possible only for measurements made immediately near the edge of a layer of high conductivity.

3.3. Results and Conclusions for Measurements Made in Southern Africa

For further interpretation of measurements made in Southern Africa (Blohm et al., 1977) it should be helpful to simulate the distribution of the equipotential lines at the earth's surface as disturbed by the ocean. Therefore, two current electrodes A and B with current I and $-I$ respectively, are placed parallel to the edge of the sheet at a distance D and the equipotential lines calculated from Eq. (5) are shown in Fig. 2. $D=350$ km and $\overline{AB}=1{,}250$ km have been chosen according to the case of the maximum electrode distance applied (Blohm et al., 1977). As may be seen in Fig. 2, the potential lines are disturbed considerably only immediately near the coast line.

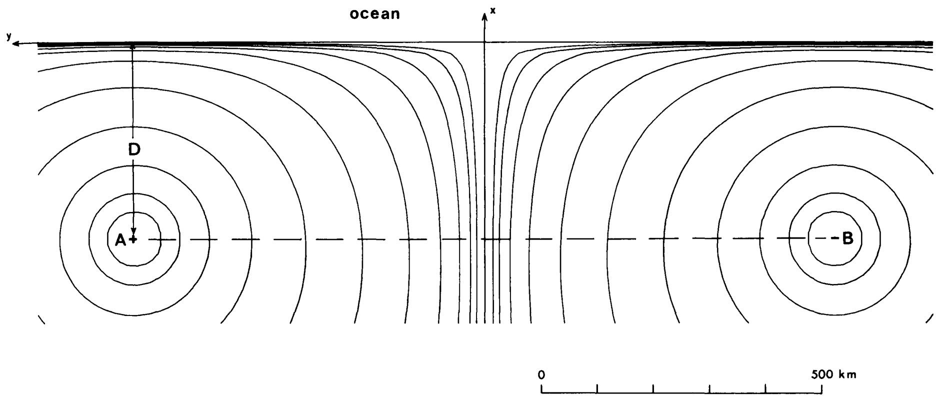


Fig. 2. Equipotential lines on the earth's surface $z=0$ for two electrodes A and B near an infinitely thin and infinitely conducting sheet ($x>0$) over a homogeneous half-space. The geometrical configuration corresponds to $\overline{AB}=1{,}250$ km and distance $D=350$ km [approximate model of an electrical sounding made in Southern Africa near the Indian Ocean (Blohm et al. 1977)]

For the mentioned maximum electrode spacing we have a ratio $d/D \approx 1.8$, and from curve (c) in Fig. 1 follows $\rho_a^{(y)}/\rho_1 \approx 0.85$. This means that the measured apparent resistivity is about 15% smaller due to the influence of the ocean if homogeneous space below the surface is assumed. The sharp minimum of the sounding curve was measured at half an electrode spacing of about 100 km (see Fig. 3), that means at a ratio $d/D \approx 0.3$. As curve (c) in Fig. 1 shows, nearly no influence of the ocean is to be seen at this ratio ($\rho_a \approx \rho_1$). Therefore, the apparently indicated well conducting layer at a depth of about 30 km cannot be due to the influence of the ocean.

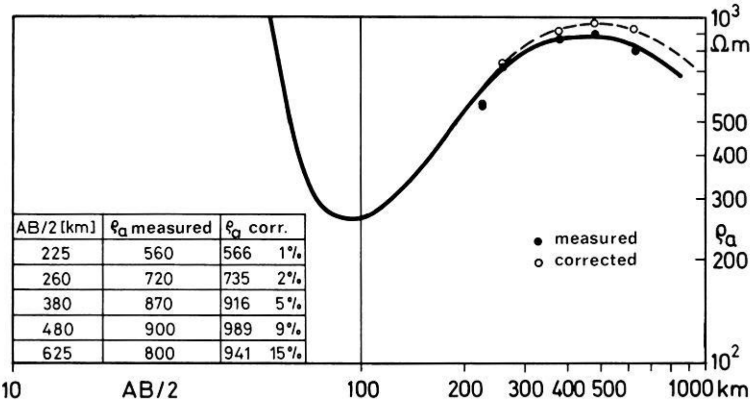
Even for larger electrode spacings the effect of the ocean on the sounding curve remains so small that it should be permitted to correct the ρ_a-values in a first approximation according to the model of a perfectly conducting, infinitely thin sheet lying on the surface $x>0$ of a homogeneous earth. A better correction would be obtained by taking a layered model below the ocean similar to that on land, but in this case an analytic solution does not seem to be possible. For the largest five electrode spacings from curve (c) in Fig. 1 the corresponding ratio $F=\rho_a/\rho_1$ is taken and the measured ρ_a-values are multiplied by $1/F$. The measured and corrected ρ_a-values of the geoelectrical sounding (Blohm et al., 1977) are shown in Fig. 3 and the results of the interpretation assuming a horizontally layered model are given below:

Best Model for Measured ρ_a-Values

Resistivity		100,000		5,000		50		7,800		1 Ωm
Depth	0		2.5		30.5		35.5		150 km	

Best Model for Corrected ρ_a-Values (for the influence of ocean)

Resistivity		100,000		5,000		50		9,250		1 Ωm
Depth	0		2.5		30.5		35.5		170 km	



AB/2 [km]	ρa measured	ρa corr.
225	560	566 1%
260	720	735 2%
380	870	916 5%
480	900	989 9%
625	800	941 15%

Fig. 3. Part (large electrode spacings *AB* only) of the geoelectrical sounding curve from measurements in Southern Africa (see Blohm et al., 1977). Due to the influence of the Indian Ocean the last five ρ_a-values have been corrected according to the thin sheet model as explained in the text

It may be seen from a comparison of these results that the resistivity and the depth of the next to last layer increase from 7,800 Ωm to 9,250 Ωm and from 150 km to 170 km, respectively.

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