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Radio Pulse Dispersion in the Ionosphere*

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Abstract. A computer simulation of the propagation of a radio pulse shows that the best estimate of the virtual heights can be obtained by the phase change with frequency (principle of stationary phase). This is especially important for those frequencies where the penetrated part of the ionosphere is highly dispersive indicated by a rapid change of the virtual heights with frequency.

Key words: Ionosphere – Virtual height pulse dispersion.

Introduction

The travel time of a radio pulse reflected from the ionosphere can be measured in two different ways. For the classical approach a certain characteristic of the pulse shape is selected and the time difference between the appearance of this characteristic in the transmitted pulse and the reflected echo is measured. The other approach is based on the principle of stationary phase and the travel time can be derived from the phase change over a small frequency increment. Using the Dynasonde Wright (1977) compared the two methods. Systematic differences were found between the two measurements in those parts of the ionogram where the virtual height varies rapidly with frequency. In the following we show some further examples of those discrepancies and also the results of a computer simulation leading to the conclusion that the stationary phase principle gives a much better estimate of the virtual height than any characteristic of the pulse shape.

Dynasonde Data

In one standard mode of operation the Dynasonde transmits four pulses with a radio frequency f and the echoes are received in sequence at four different

* Dedicated to Professor Dr. K. Rawer on the occation of his 65th birthday

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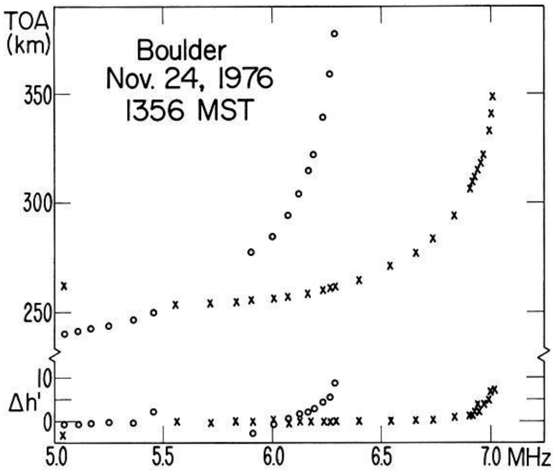


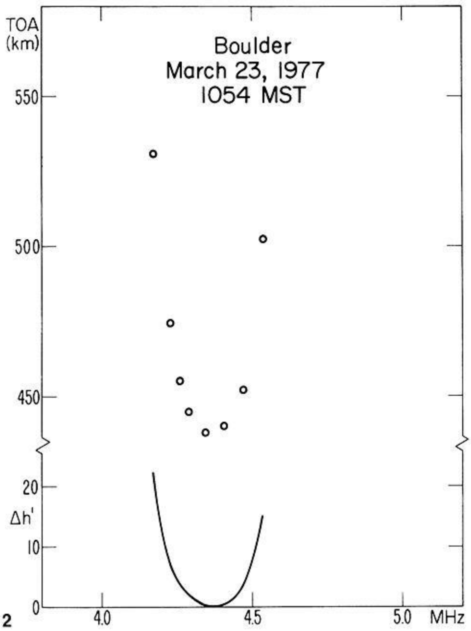
Fig. 1. Virtual heights and discrepancies between TOA (derived from the leading edge of an echo) and DPH (derived from the phase change with frequency), *Boulder, November 24, 1976* (*circles:* ordinary wave; *crosses:* extraordinary wave)

receiving antenna, which are linearly polarized and located at the corners of a square. This sequence is then repeated at a slightly higher frequency $f+\Delta f$, where Δf is always 8 kHz. For each echo the time of arrival TOA is counted to the instant of maximum second derivative of the leading edge of the pulse and data are also obtained on echo amplitude and phase. The latter two quantities are measured at a later time, approximately in the center of the echo (Wright and Pitteway, 1977). The phase resolution is 4° C and correspondingly a phase change of one unit over 8 kHz gives a resolution of 0.208 km for the virtual height (DPH) derived from the stationary phase principle. The TOA virtual height has practically the same resolution of 0.200 km, but is actually the average of eight values measured at the four antenna for the two frequencies. The TOA values are corrected for amplitude dependent delays in the system.

Four phase differences, one for each antenna, are obtained over the 8 kHz frequency interval. Figure 1 shows the high frequency portion of an ionogram where ordinary and extraordinary traces approach their critical frequencies. In the upper part the TOA values are plotted and in the lower portion the difference

$$\Delta h' = \text{DPH-TOA}.$$

We see that in the flat portions of each trace this difference is very small and increases rapidly when the slope of the trace becomes very large. Figure 2 shows that this effect is not restricted to steep positive slopes, it also appears when the virtual heights are rapidly decreasing with frequency. In this ionogram, a well pronounced $F1$ region maximum was present and only the $F2$ region portion of the ordinary component is shown. In both cases the differences between TOA and DPH amount to 10 km or more. Those differences can be significant in the calculation of electron density profiles, especially in the extrapolation to obtain the parameters of the maximum of a layer.



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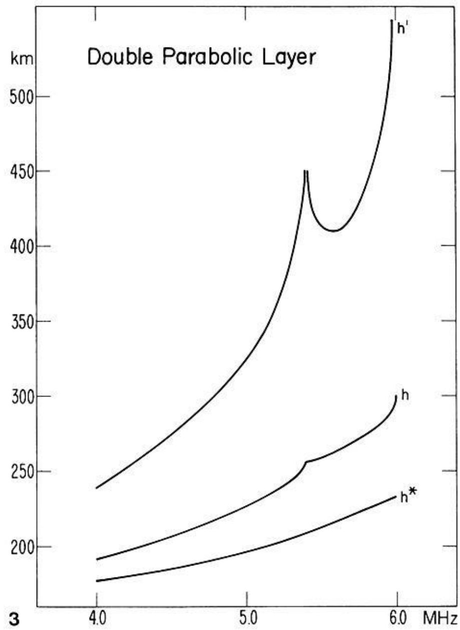

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Fig. 2. Virtual heights and discrepancies between TOA and DPH (ordinary wave), *Boulder, March 23, 1977*

Fig. 3. Virtual heights (h'), real heights (h) and phase height (h^*) of a *double parabolic layer*

Computer Simulation

The problem of the propagation of electromagnetic waves through a dispersive medium was first treated by Sommerfeld (1914) and Brillouin (1914). Several authors (e.g., Rydbeck, 1942; Budden, 1961; Wait, 1965) studied specific problems of the pulse propagation in the ionosphere using in general the following procedure: A rectangular radio pulse is decomposed by the Fourier transform into its spectral components. The propagation of the pulse through the ionosphere adds a frequency dependent phase term for the argument of the individual waveletts. The phase term is then expanded into a Taylor series and higher order terms of this series are neglected. The integral describing the echo returned from the ionosphere can then be expressed by well known functions. Those studies concentrated mainly on the propagation velocity and the deformation of the pulse shape, while the effect on the phase of the echo was neglected.

In order to resolve the experimental discrepancy between the virtual heights derived from the echo shape and those derived from the change of phase with frequency, the propagation of a radio pulse in the ionosphere was simulated numerically. The results were amplitude and phase as a function of time. In our calculations the radio pulse has a cosine shape and is given by

$$f(t)=\begin{cases}0 & -\infty \leqq t \leqq -T \\ \frac{1}{2}\left(1+\cos \pi \frac{t}{T}\right) \cos 2\pi f_0 t & -T \leqq t \leqq T \\ 0 & T \leqq t \leqq \infty\end{cases} \tag{1}$$

where T is the nominal pulse width (measured at half maximum amplitude level) and f_0 is the carrier frequency. A cosine pulse appears to be more realistic than a rectangular pulse since the transmitter network including antenna will always smooth the edges of an originally rectangular pulse. It also has the advantage of a faster decrease of the amplitude density of the spectral components with increasing distance of the frequency from the carrier frequency and we avoid the Gibb's phenomena which causes an artificial additional distortion of the echo shape as apparent in the papers by Rydbeck (1942) and Budden (1961).

The amplitude density of the pulse as obtained by the Fourier integral is then

$$a(f)=\frac{1}{4\pi}\left(\frac{\sin 2\pi(f-f_0)T}{(f-f_0)(1-4(f-f_0)^2 T^2)}+\frac{\sin 2\pi(f+f_0)T}{(f+f_0)(1-4(f+f_0)^2 T^2)}\right). \tag{2}$$

The echo reflected from the ionosphere is then given by

$$g(t)=\int_{-\infty}^{+\infty} a(f)\exp[i(2\pi f t-\phi(f))]\,df \tag{3}$$

where the phase $\phi(f)$ is proportional to the phase height $h^*(f)$

$$\phi=2\pi p=\frac{4\pi}{C} f h^*=\frac{4\pi}{C} f \int_0^{h_r} \mu\,dz \tag{4}$$

(μ refractive index, h_r reflection height, p phase in cycles).
Since μ depends only of f^2 we see that

$$\phi(-f)=-\phi(f) \tag{5}$$

and according to (2) that

$$a(-f)=a(f).$$

Therefore we can write instead of (3)

$$\begin{aligned} g(t) &= 2\int_0^{\infty} a(f)\cos 2\pi[f t-p(f)]\,df \\ &= 2R\left(\int_0^{\infty} a(f)\exp[i\,2\pi(f t-p(f))]\,df\right). \end{aligned} \tag{6}$$

Assuming that p can be expanded into a Taylor series we write

$$p = p(f_0) + \frac{dp}{df}(f - f_0) + \delta p \tag{7}$$

where δp includes all the higher order terms. Since by definition

$$\frac{dp}{df} = \tau$$

where τ is the total travel time of the pulse we can write instead of (6)

$$g(t) = 2R\left(\exp[i2\pi(f_0\tau - p(f_0))]\int_0^\infty a(f)\exp[i2\pi(f(t-\tau) - \delta p)]\,df\right). \tag{8}$$

If δp can be neglected the integral in (8) is then the inverse Fourier transform and gives the original pulse at the later time $t = \tau$. The factor in front of the integral in (8) is simply a phase shift of the carrier frequency corresponding to the difference between virtual height and phase height.

Instead of a series expansion we use the exact expression of p for a model electron density profile neglecting the earth's magnetic field. The model consists of two half parabolas with different critical frequencies and is shown in Fig. 3 together with the virtual and phase heights (Paul, 1967). For the numerical evaluation of (6) we used the following formulation

$$g(t) = 2R\left(\exp(2\pi i(f_0 t - p_0))\int_0^\infty a(f)\exp[2\pi i(\Delta f t - \Delta p)]\,df\right) \tag{9}$$

with $\Delta f = f - f_0$ and $\Delta p = p(f) - p(f_0)$.

The integral in (9) gives both the amplitude of the echo and its phase deviation from the nominal value p_0 as a function of time while the high frequency oscillation of the carrier frequency is contained in the factor in front of the integral.

The integral in (9) was evaluated numerically for several radio frequencies f_0. Some of the results are shown in Fig. 4. In the upper part of this figure a portion of the virtual height curve of Fig. 3 is shown, in the lower part the echo shape for three frequencies (indicated in the virtual height curve by the vertical arrows). The amplitudes of the echos are shown on a relative virtual height scale with zero in the nominal center of the pulse. The vertical dashed lines indicate the half-amplitude width of the undistorted pulse. The pulse form at the minimum of the virtual height trace ($f = 5.55$ MHz) is practically undistorted. At the other two frequencies, at $f = 5.44$ where the virtual height is rapidly decreasing with frequency and at $f = 5.95$ where the virtual height is rapidly increasing the echo shapes are flattened out, the amplitude is lower and the pulse width wider. A rough estimate of the product of the square of the maximum amplitude times the pulse width at half amplitude level shows that this quantity is equal for all three pulses shown, which means that the energy is conserved as expected. For each of the frequencies shown in Fig. 4 the

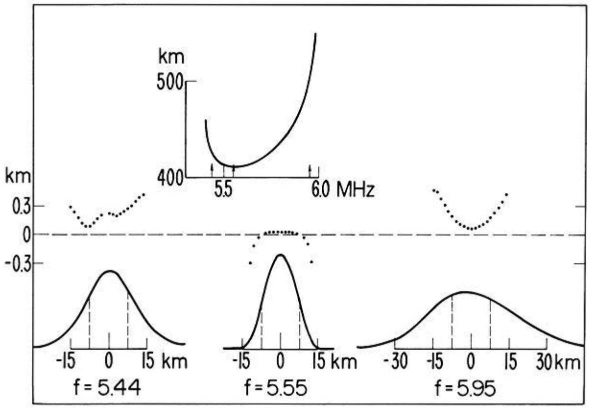

Fig. 4. Dispersion effect on phase change with frequency (*middle, dotted curves*) and on amplitude (*bottom*) for three frequencies reflected from the double parabolic model. The high frequency portion of the virtual heights of the model is shown in the *upper part* of the figure where the three frequencies are indicated by *arrows*

calculation was repeated for a second frequency 8 kHz higher simulating the Dynasonde mode of operation. At a given frequency f_0 the phase deviation from the nominal value $\phi(f_0)$ as defined in (4) can be significant. However, the change of this deviation with frequency is rather small, and therefore causes only small errors in the virtual heights, if those are derived from the phase change with frequency. Those errors are shown in the middle of Fig. 4 by the dotted curves, if the phase change over a 8 kHz interval is used. The horizontal scale is the same as for the echo shapes in the lower part of this figure. In the three cases shown the error of the virtual height (DPH) caused by the dispersion effect is less than 200 m.

More details are presented in Table 1, where the dispersion errors are listed for different methods of measuring the virtual height of the echo. The first line shows the average error over the nominal pulse width around the center of the echo obtained by the phase change over the 8 kHz frequency interval (see center line of Fig. 4). The following three lines give the errors, if a characteristic of the echo shape is used, first the location of the maximum, then the location of the steepest slope and in the last line the location of the point where the amplitude reaches half the value of its maximum. Those values were obtained by numerical interpolation and/or differentiation.

The phase change with frequency causes by far the smallest error. Next best results are obtained, if the time of maximum of the amplitude is used, but at least the error for $f = 5.95$ is not negligible anymore. Even larger errors occur if the steepest slope or the half amplitude level are used especially close to the critical frequency ($f = 6.0$ MHz). The discrepancies in the last line are comparable with the experimental errors found from the Dynasonde data (Figs. 1 and 2).

The results of the computer simulation indicate that the best estimates of the virtual heights are obtained if the principle of the stationary phase is applied. Any characteristic of the leading edge of the echo may lead to very large errors

Table 1. Virtual height error (calculated–theoretical height)

Frequency	5.44 MHz	5.55 MHz	5.95 MHz
Phase	0.196 km	0.022 km	0.118 km
Max. amplitude	−0.629 km	0.135 km	− 1.253 km
Steepest slope	−0.223 km	0.181 km	−10.982 km
Half amplitude	−3.089 km	0.039 km	−12.326 km

near the critical frequency. For profile calculations (e.g., Howe and McKinnis, 1967) this will in turn cause erroneous values for peak parameters.

The use of the maximum of the amplitude as a characteristic would reduce those errors significantly, but may not be practical.

It also should be pointed out that according to the computer simulation the amplitude decreases rapidly with increasing dispersion, an effect which has to be corrected for in deviative absorption measurements.

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