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Detection Probabilities for Weak Regional Seismic Events

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Abstract. The direct estimation method for threshold magnitudes associated with teleseismic events is well-known and frequently used. A modification of the method is developed in order to make the approach applicable to events from regions with low seismicity. It is assumed that observations are supplied by a seismograph network located within the region. As an example, threshold magnitudes are given for the Uppsala station, Sweden. Reliability of the suggested approach is examined by means of a rockburst series.

Key words: Detection probability – Earthquakes in Sweden – Uppsala seismograph station – Weak regional Seismic events – Operational threshold magnitude.

1. Introduction

Capability of a network to detect weak regional events is essential for earthquake prediction and risk analysis. Besides engineering aspects, detectability estimates are of great importance for many seismological studies since they provide a measure of operational efficiency of the station and/or of the network *wrt* the given type of events.

Investigations considering teleseismic epicentral distances were carried out by Kelly and Lacoss (1969), Ringdal (1975), von Seggern and Blandford (1976), Pirhonen et al. (1976), Ringdal et al. (1977), among others. In the present work, we are concerned with the problem of evaluating the detection capability for weak regional events by means of a national seismograph station network. By regional events we mean events recorded merely by regional seismic networks. In general, we follow the direct estimation approach which compares the detection performance of a station in question with the detection performance of an independent reference system like e.g. ISC or NEIS. When considering events of low magnitudes, an immediate application of the direct method is not possible due to the lack of a reference system, because regional events are recorded by a very limited number of stations. Another problem emerges from the fact that the size of regional

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earthquakes is determined by means of a regional magnitude scale. Consequently, the detectability given in terms of magnitude thresholds is also dependent upon the procedures applied for the magnitude determination.

The station detectability of teleseismic events is expressed as a function of the reference system magnitude (hereafter, reference magnitude) and is related to a certain limited seismic region. However, in cases like, e.g., Swedish earthquakes any division of the region in a number of subregions is difficult. Moreover, for regions with low seismicity, additional grouping of events might jeopardize the statistical representation. Therefore, rather than relating the detectability of a station to a certain subregion we shall relate it to the epicentral distance.

Below, we present a modified direct estimation method adapted to Swedish regional conditions. As a quantitative example, threshold magnitudes are given for the seismograph station Uppsala (UPP). Results are examined by means of detectability estimates of rockbursts in Grängesberg (central Sweden). Although the observations treated in this work are limited to a specific region and to a specific network, the applicability of the method is rather general.

2. Modified Direct Estimation Method

Event detectability, of a particular seismograph station for a given seismic region, is expressed by means of the incremental detection probability as a function of magnitude. Ringdal (1975) and von Seggern and Blandford (1976) discuss the detection probability in greater detail. Here, we follow the approach of Ringdal.

2.1. Statistical Model

The statistical model used is based upon the following assumptions: Firstly, the reference magnitude, M_R, and the station magnitude, M_A, are distributed about the same true unknown magnitude, M, so that $M_R \sim N(M+C_R, \sigma_R^2)$, $M_A \sim N(M+C_A, \sigma_A^2)$. C and σ denote the magnitude bias and the magnitude variance, respectively. The threshold magnitude, M_t, is normally distributed with mean value μ and with variance σ^2, i.e., $M_t \sim N(\mu, \sigma^2)$. Secondly, M_A and M_R as well as M_A and M_t are mutually independent. Lastly, the number of detected earthquakes, N_c, exceeding a magnitude M_R, is expressible through the magnitude-frequency relationship

$$\log_e[N_c(M_R)] = \alpha - \beta M_R. \tag{1}$$

By accepting the above assumptions, the estimated probability of detection is (Ringdal, 1975)

$$P(M_R) = \Phi\left[\frac{M_R - b}{s}\right] \tag{2}$$

where Φ is the Gaussian cumulative distribution function and

$$b = \mu + C_R - C_A + \beta\sigma_R^2$$
$$s^2 = \sigma^2 + \sigma_A^2 + \sigma_R^2. \tag{3}$$

The observed detection curve, i.e., the percentage of events detected by the station, for different magnitude levels, is fitted to the $P(M_R)$-function in Eq. (2). The curve-fitting is carried out, e.g., by using the probit analysis technique (IBM Application Program, 1970) which yields the maximum likelihood estimate of b and s.

2.2. Operational and True Threshold Magnitudes

Following Eqs. (2) and (3) we may refer to three different threshold magnitudes:

(a) True threshold magnitudes, μ, which are more of theoretical importance but may also be used when comparing the detectability of various stations even from different networks. However, since our data are limited to regional observations, determination of C_R, and therefore also of μ in Eq. (3), is not possible (for details see Shapira et al., 1978).

(b) Biased or operational threshold magnitudes, $b \pm s$. These are directly obtained by fitting the observations to the function $P(M_R)$ in Eq. (2).

(c) Corrected operational threshold magnitudes, $\hat{b} \pm \hat{s}$, defined as

$$\begin{aligned} \hat{b} &= b + C_A = \mu + C_R + \beta\sigma_R^2 \\ \hat{s}^2 &= s^2 - \sigma_A^2 = \sigma^2 + \sigma_R^2. \end{aligned} \tag{4}$$

In this case we assume that $C_R + \beta\sigma_R^2$ remains constant, irrespective of the investigated station. Hence, $\hat{b} \pm \hat{s}$ can be used when comparing the detectability of stations within the network.

2.3. Reference Magnitude

For weak regional events we have to accept the regional network or part of it as the reference system. All magnitudes discussed throughout this work refer to regional magnitudes, M, defined for the region in question. Accordingly, we here define the reference magnitude of an event as

$$M_R = \frac{1}{K} \sum_{i=1}^{K} M_i \qquad \text{for } i \neq A \tag{5}$$

where K is the number of available station magnitudes (investigated station A excluded) for the event and M_i is the magnitude of the event measured at the i-th station. The exclusion of M_A in Eq. (5) is due to the required independence between M_R and M_A.

2.4. Relocation of Earthquakes

Our intention is to study the detection probability of the station A as a function of M_R for a chosen epicentral distance. To introduce the distance dependence, we transform all available earthquakes to a common epicentral distance, say Δ_k. Let us consider an earthquake of a known magnitude, M_R, at distance Δ from station A. We use the magnitude definition

$$M = f(D, T) + g(\Delta) \tag{6}$$

where D is the maximum ground amplitude, T is the corresponding period and $g(\Delta)$ is the calibrating function. A fictitious earthquake at distance Δ_k which causes a ground motion at A, with the same maximum amplitude, D, and period, T, will have, as follows from Eq. (6), the reference magnitude

$$M_R(\Delta_k) = M_R + g(\Delta_k) - g(\Delta). \tag{7}$$

Employment of Eq. (7) enables to determine magnitudes for fictitious events located at any chosen epicentral distance.

Throughout this work the decision 'detected' or 'not detected' is based upon the conserved parameters D and T. Thus, when the actual earthquake is detected by station A, it is assumed that all relocated earthquakes associated with this actual event are also detected. On the other hand, if the actual event is not detected, it means that none of the corresponding fictitious earthquakes are detected by A.

3. Detection Capability of UPP – A Quantitative Example

To demonstrate the applicability of the modified direct estimation method we determine the capability of UPP to detect Swedish events. UPP, the sole manned station within the Swedish seismograph station network (SSSN), is used for quick preliminary event locations and magnitude determinations. Thus, the knowledge of corresponding threshold magnitudes is of great practical importance.

3.1. Observational Material

The data used comprises 72 Swedish earthquakes detected during 1963–1972 (Kulhánek and Wahlström, 1977). By careful selection, man-made and possibly triggered seismic events (e.g., rockbursts) were excluded. All measurements, phase identifications and event discriminations have been carried out by one and the same seismologist (R.W.). In this respect, the data employed are homogeneous. It should be noted that a great majority of the 72 earthquakes were located merely by the SSSN. Only occasionally, closely located Finnish or Norwegian stations also contributed with readings. Epicentral and seismograph station (SSSN) locations are shown in Fig. 1.

3.2. Magnitude Bias and Variance

A regional magnitude scale for the Scandinavian area has been developed in Båth et al. (1976). The station-magnitude formula reads

$$M = \log_{10}(100\,\mathrm{D}) + F(\Delta, T) \tag{8}$$

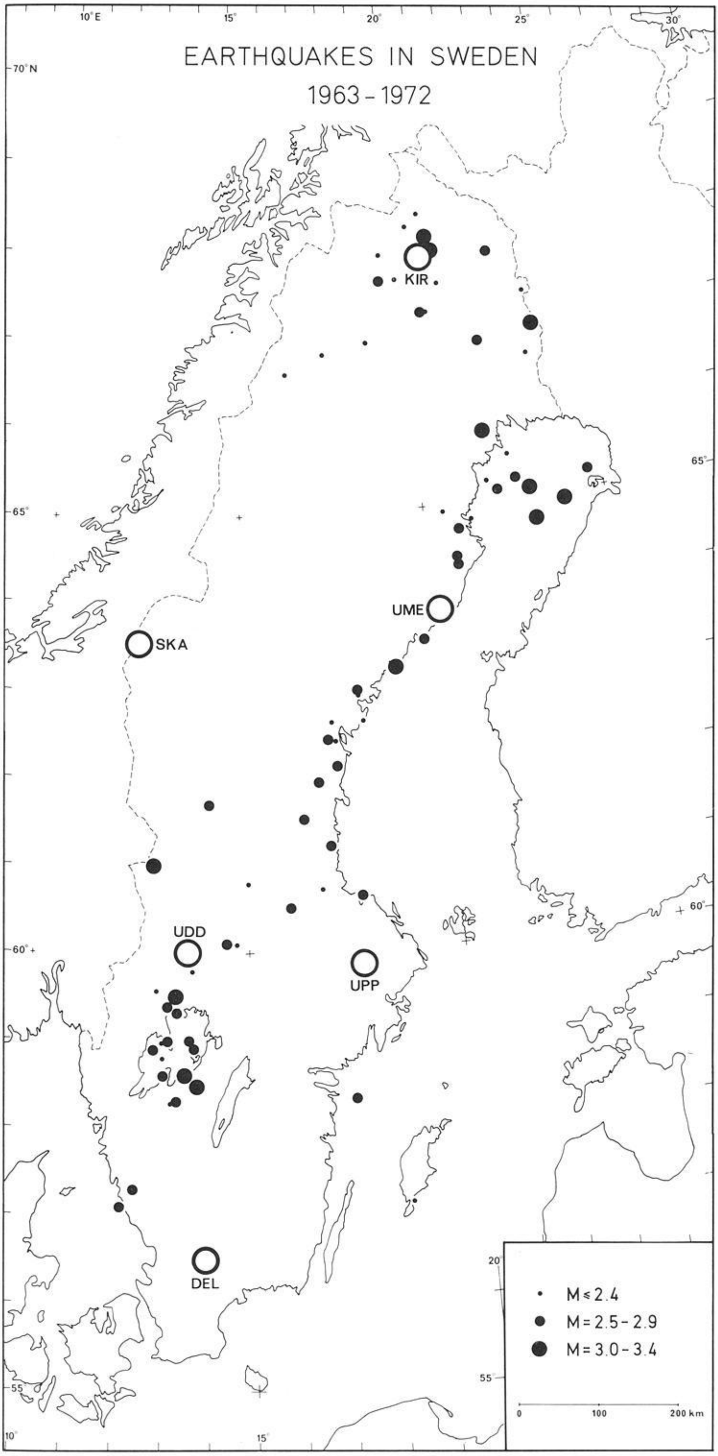

Fig. 1. Map showing the location of seismograph stations (*open circles*) and epicentres (*solid circles*) used in this paper

where D is the maximum ground amplitude of Sg in microns, measured on the short-period vertical-component seismogram. $F(\Delta, T)$ is the calibrating function available in tabular form (Båth et al., 1976) for $50\,\text{km} \leqq \Delta \leqq 1{,}500\,\text{km}$ and $0.3\,\text{s} \leqq T \leqq 1.4\,\text{s}$. As follows from Eq. (8), the calibrating function includes also the T parameter. Thus, Eq. (7) will be written as

$$M_R(\Delta_k) = M_R + F(\Delta_k, T) - F(\Delta, T). \tag{9}$$

Reference magnitudes are determined by applying Eq. (5), excluding UPP magnitude determinations. The station-magnitude bias, C_A, is determined by averaging the difference $M_A - M_R$ over all events with magnitudes reported by four or more stations. Then, $C_A = -0.06$ for $A = \text{UPP}$. The standard deviation of M_A *wrt* M_R is influenced by the number of stations, K, which contribute to estimations of M_R. Therefore, we define the variance as

$$s_R^2(K) = \frac{1}{N_K(K-1)} \sum_{j=1}^{N_K} \sum_{i=1}^{K} (M_{ij} - M_{Rj})^2 \tag{10}$$

where N_K is the number of events with magnitudes reported from K stations (UPP excluded), i is the station and j the event index, respectively. Numerical $s_R(K)$-values, for $K = 2, 3, 4, 5$ show that the relation $s_R^2(1) = K\, s_R^2(K)$ holds. The consistent $s_R(1)$-values are assumed to be good estimates of station magnitude variances, *wrt* the defined M_R, for any station within the SSSN. From our data we obtained $s_R(1) = 0.35$.

For those cases when the earthquake was not detected by UPP, the period, T, in Eq. (9) cannot be measured and has to be estimated. The present data do not confirm the expected pronounced correlation between the period and the magnitude and/or between the period and distance. The observations show that $T = (0.51 \pm 0.08)$ s. The mean value of 0.5 s is therefore used as the period for events which were not detected by UPP. The error (two standard deviations) in period measurements of about ± 0.15 s introduces inaccuracies into the $M_R(\Delta_k)$ determination which increase with distance. Using the $F(\Delta, T)$-table of Båth et al. (1976) and assuming $T = (0.50 \pm 0.15)$ s, we obtain the following $\sigma_A(\Delta)$ values

Δ	50–200 km	250–500 km	550–1000 km
$\sigma_A(\Delta)$	0.15	0.25	0.30.

We shall provide for the period effect by introducing a distance-dependent variance $\sigma_A^2 = (0.35)^2 + \sigma_A^2(\Delta)$, where here $\sigma_{\text{UPP}} = \sigma_A$.

3.3. Analysis and Results

Threshold magnitudes b_{50}, b_{90} and $\hat{b}_{50}$, $\hat{b}_{90}$ (indices denote probability level) are determined by the following procedure:

(a) All earthquakes are relocated to a common epicentral distance, Δ_k, from UPP. Magnitudes, $M_R(\Delta_k)$, of fictitious, i.e., relocated earthquakes are calculated according to Eq. (9).

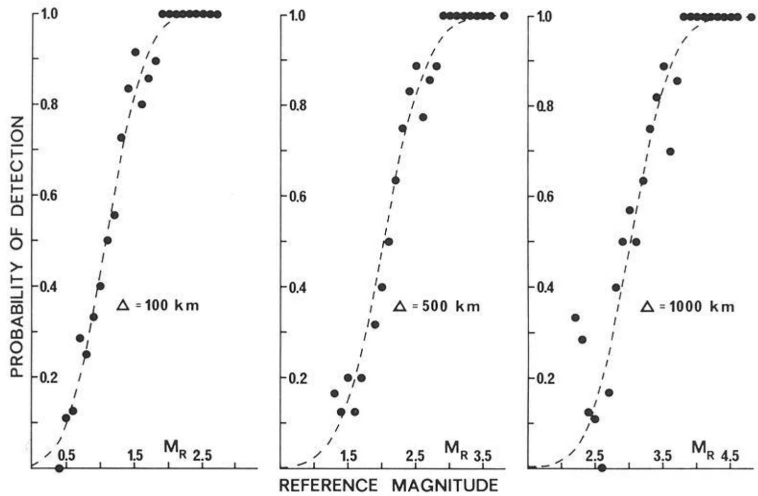

Fig. 2. Matched detection curves, $P(M_R)$, (*dashed lines*) and observed detection probabilities (*solid circles*) for UPP and epicentral distances of 100, 500, and 1000 km

(b) The observed detection probability for a chosen M_R is determined as the number of the detected events normalized by the total number of events. Magnitude intervals of 0.1 unit are used.

(c) Observed probabilities are matched with the probability distribution curve given by Eq. (2) to determine the operational threshold magnitude b_{50} and its corresponding variance s^2. From these values we calculate b_{90}, $\hat{b}_{50}$, and $\hat{b}_{90}$.

(d) We put $T = 0.5$ s in cases when the earthquake was not detected, whereas for detected events the period is measured from the seismograms and, in general, deviates from its mean value of 0.5 s. If we could assume $T = 0.5\,\text{s} = \text{const.}$, then the threshold estimates would be parallel to $F(\Delta, T = 0.5\,\text{s})$. Since this is not the case, we repeat the calculations for epicentral distances from 100 to 1000 km with steps of 100 km. Good correlation for the threshold magnitudes, m, is found empirically for the form

$$m = a_0 + a_1 \log_e \Delta + a_2 \Delta. \tag{11}$$

Table 1. Estimates of threshold magnitudes for UPP

Δ	b_{50}	b_{90}	$\hat{b}_{50}$	$\hat{b}_{90}$
100	1.1	1.7	1.0	1.3
200	1.5	2.1	1.4	1.7
300	1.7	2.4	1.7	1.9
400	1.9	2.6	1.9	2.2
500	2.0	2.7	2.0	2.3
600	2.2	2.9	2.2	2.5
700	2.5	3.2	2.4	2.8
800	2.6	3.3	2.6	2.9
900	2.8	3.5	2.8	3.1
1,000	3.0	3.6	3.0	3.2

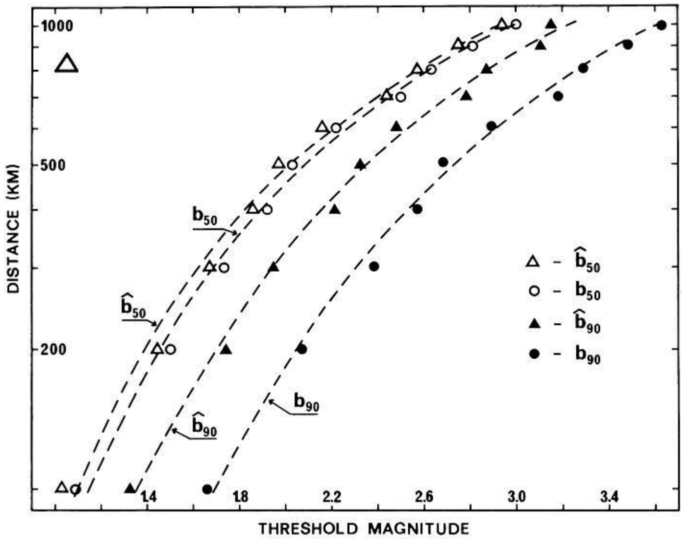

Fig. 3. Comparison of optimum fits (*dashed lines*) due to Eq. (11) and results obtained by the probit technique (*circles/triangles*), 50 % and 90 % levels as well as operational, b, and corrected operational, $\hat{b}$, threshold magnitudes are considered

As an example, Fig. 2 presents detectability curves for three chosen distances. Table 1 summarizes the estimates of the four threshold magnitudes for UPP. Coefficients a_0, a_1, a_2 are estimated in terms of the least-squares fits. Results are depicted in Fig. 3 for b_{50}, b_{90}, $\hat{b}_{50}$, and $\hat{b}_{90}$.

4. Examination of Results From a Sequence of Rockbursts

The mining area of Grängesberg (60.1°N, 15.0°E) in central Sweden, located approximately 150 km from UPP, is one of few seismic areas which may be used as an independent test of the estimated threshold magnitudes. During the period August 1974–October 1977 about 470 rockbursts, from the Grängesberg area, were recorded by at least one station from the SSSN (Båth and Wahlström, 1976 and Båth, 1977). Employing the available information on the rockburst series, we carried out the analysis as described by Ringdal (1975) or in other words, we applied the direct estimation method for UPP. Resulting values for UPP and $\Delta = 150$ km are: $b_{50} = 1.4$, $b_{90} = 1.8$, $\hat{b}_{50} = 1.4$, and $\hat{b}_{90} = 1.6$. These results compare favourably with estimates obtained from Eq. (11), differing by only 0.1 magnitude unit for all four threshold magnitudes (Shapira et al., 1978).

5. Discussion

The modification of the direct method enables one to investigate the detectability of weak events by means of seismograph stations located within the seismic region

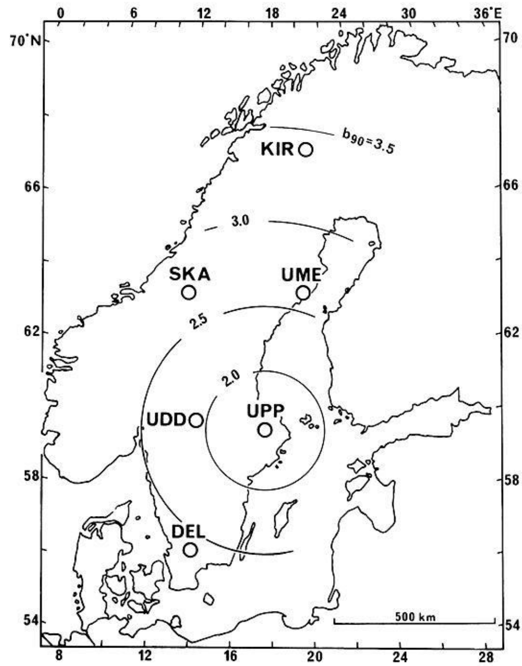

Fig. 4. Loci of constant operational threshold magnitudes, b_{90}, with respect to UPP

studied. As an example, loci of constant threshold magnitudes, b_{90}, *wrt* UPP are shown in Fig. 4. Certain practical considerations and limitations of the method proposed are discussed below.

Source Parameters. Generally speaking, Swedish earthquakes differ from each other by virtue of their different unknown source mechanisms, focal depths, etc. The relocation of events, which is the basic principle of the present approach, does not consider the possible influence of the source parameters. Varying source characteristics increase the standard deviations of estimated parameters. In spite of the undesired source influences, the described approach provides realistic estimates of threshold magnitudes as confirmed by the rockburst sequence.

True Magnitudes. Magnitudes considered in the present work are determined by means of a regional network consisting of six stations. In such a case, it becomes rather difficult to define the true magnitude of an event. Regarding Swedish recording conditions, the best estimate of the true (unknown) magnitude is provided by combined measurements from all the six stations.

The estimates of threshold magnitudes are obviously dependent upon the utilized magnitude scale. Therefore, direct comparison of threshold magnitude estimates resulting from different magnitude scales is not possible.

Frequency-Magnitude Relationship. An assumption of the random distribution of earthquakes with time has been made by present authors. Nevertheless, it is not impossible that the occurrence of Swedish earthquakes shows certain temporal variations. Caution has to be taken also when considering the frequency of the

Grängesberg rockbursts due to the possible time-depended triggering (Båth and Wahlström, 1976; Båth, 1977). However, the temporal effects are minimized by extending the period of observations. Thus, the potential, e.g., seasonal variations are considered to be smoothed out.

The parameter β in Eq. (1) is influenced by the relocation process. Nevertheless, Shapira et al. (1978) show that, for the present observational material, variations of β due to the event relocation are negligible.

Performance of UPP. The detection capability ascribed above to UPP is not necessarily the best one which may be achieved practically. Note, that the thresholds have been associated with short-period vertical-component Sg readings. It is true that, in general, Sg is the largest phase observed in seismograms of Swedish events recorded by the SSSN. However, horizontal amplitudes of Sg usually exceed the vertical amplitudes (Båth et al., 1976). Hence, inclusion of horizontal-component observations would very likely decrease, i.e., improve, the present threshold magnitudes.

6. Conclusions

Our main conclusions may be formulated as follows: (a) The applicability of the well-known direct estimation method has been extended to weak events recorded merely by regional seismic networks. (b) Threshold magnitudes are given numerically for the Uppsala station. (c) Reliability of the suggested approach has been confirmed by means of a rockburst series.

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