NIEDERSÄCHSISCHE STAATS- UND
UNIVERSITÄTSBIBLIOTHEK GÖTTINGEN

Werk

Jahr: 1979
Kollektion: fid.geo
Signatur: 8 Z NAT 2148:46
Digitalisiert: Niedersächsische Staats- und Universitätsbibliothek Göttingen
Werk Id: PPN1015067948_0046
PURL: http://resolver.sub.uni-goettingen.de/purl?PPN1015067948_0046

LOG Id: LOG_0019
LOG Titel: Accuracy of ray theoretical seismograms
LOG Typ: article

Übergeordnetes Werk

Werk Id: PPN1015067948
PURL: http://resolver.sub.uni-goettingen.de/purl?PPN1015067948
OPAC: http://opac.sub.uni-goettingen.de/DB=1/PPN?PPN=1015067948

Terms and Conditions

The Goettingen State and University Library provides access to digitized documents strictly for noncommercial educational, research and private purposes and makes no warranty with regard to their use for other purposes. Some of our collections are protected by copyright. Publication and/or broadcast in any form (including electronic) requires prior written permission from the Goettingen State- and University Library.
Each copy of any part of this document must contain there Terms and Conditions. With the usage of the library's online system to access or download a digitized document you accept the Terms and Conditions.
Reproductions of material on the web site may not be made for or donated to other repositories, nor may be further reproduced without written permission from the Goettingen State- and University Library.

For reproduction requests and permissions, please contact us. If citing materials, please give proper attribution of the source.

Contact

Niedersächsische Staats- und Universitätsbibliothek Göttingen
Georg-August-Universität Göttingen
Platz der Göttinger Sieben 1
37073 Göttingen
Germany
Email: gdz@sub.uni-goettingen.de

J. Geophys. 46, 135–149, 1979

Journal of Geophysics

Accuracy of Ray Theoretical Seismograms*

V. Červený

Institute of Geophysics, Charles University, Ke Karlovu 3, 121 16 Praha 2, ČSSR

Abstract. Theoretical seismograms computed by the ray method with certain modifications in the critical region are compared with theoretical seismograms computed by more exact methods (the reflectivity method). It is shown that the ray method with some simple modifications gives qualitatively satisfactory results for a broad class of models of media, even in certain singular regions (such as critical regions, the neighbourhood of caustics, the half-shadow, etc). Poor results are obtained for some non-ray waves. It would be possible, however, to use some modifications of the ray method (such as the ray method with a complex eikonal) even for certain non-ray waves and to include these waves into ray theoretical seismograms.

Key words: Ray theoretical seismograms – Accuracy of ray method – Modifications of ray method.

I. Introduction

At the present time, many different methods can be used to construct theoretical seismograms. Unfortunately, there is no method generally suitable for all situations. For various problems, various epicentral distances, and various models of media, certain methods are suitable and other cannot be used at all. Great differences exist also in the accuracy and in the computer time requirements.

The ray method may be applied to construct theoretical seismograms even for rather complicated laterally inhomogeneous media with curved interfaces, for which other methods can be hardly used.

The basic problem in the application of ray methods to the construction of ray theoretical seismograms lies in its insufficient accuracy in certain situations. There are three main reasons for this limited accuracy:

1. The ray method is not applicable in certain singular regions, such as

---

* Presented at the Workshop Meeting on Seismic Waves in Laterally Inhomogeneous Media, Liblice, ČSSR, February 27 – March 3, 1978

0340-062X/79/0046/0135/$03.00

the critical region, the neighbourhood of caustics, the transition zone between the illuminated region and the shadow zone, etc.

2. The standard ray method cannot be used to describe the properties of certain types of non-ray waves, such as various inhomogeneous waves, channel waves, tunnel waves, etc. (Of course, certain of these waves can be studied by the ray method with a complex eikonal.)

3. The number of waves which arrive at the receiver within a finite time window is often rather high. It is necessary to exclude a priori certain classes of waves (even regular zero-order ray waves) to make computations possible. Of course, this may lead to inaccuracies.

The object of this paper is to study the accuracy of ray theoretical seismograms. It would be difficult (practically impossible) to appreciate the accuracy of ray theoretical seismograms by the ray method itself. The simpler way is to compare the ray theoretical seismograms with the seismograms computed by another, more accurate method. We must, of course, perform computations only for simpler types of media, to which the more accurate methods are applicable. From this point of view, the most natural thing to do is to start with vertically inhomogeneous media, for which a number of various more exact methods can be now used to compute thoretical seismograms.

In this paper we shall compare the ray theoretical seismograms with theoretical seismograms computed by the reflectivity method, see Fuchs (1968), Fuchs and Müller (1971). We shall be mainly interested in the accuracy of ray theoretical seismograms in singular regions, see point (1) above. The accuracy of ray theoretical seismograms depends greatly on the epicentral distance. As we are interested here mainly in the interpretation of explosion seismology data, we shall study the range of epicentral distances from about 40 to 300 km for standard Earth's crust structures. The most important waves in this region are the refracted waves (also called diving waves) and the supercritical reflections from various first-order discontinuities. The frequently occurring singular regions at these epicentral distances are those listed sub (1) above. At shorter epicentral distances (~ 40 km) we have mainly subcritical reflections and the accuracy of ray theoretical seismograms is usually satisfactory even for more complicated structures. For large epicentral distances (~ 500 km), we expect lower accuracy in the ray description of waves propagating within the Earth's crust due to various reasons (such as the strong interference character of the wave field). The accuracy of ray theoretical seismograms in this region would require special investigation.

The problem of the accuracy of ray theoretical seismograms is far from being solved by the examples presented in this paper. The conclusions presented here are of course only of a very limited character. It would be necessary to perform a lot of other computations to be able to make some conclusions of more general character.

2. Construction of Ray Theoretical Seismograms

In the following we shall describe very shortly the main principles of the construction of ray theoretical seismograms. Details can be found in Červený et al. (1977), Hron and Kanasewich (1971), etc.

In the ray theory the wave field is decomposed into elementary waves, corresponding to individual rays. The seismograms of elementary waves, called elementary seismograms, are computed independently one after another. The resulting theoretical seismograms obtained as a superposition of elementary seismograms are called ray theoretical seismograms.

In the case of the medium being composed of homogeneous plane parallel layers, the elementary seismograms can be computed exactly (Cagniard-deHoop, Smirnov-Sobolev). It is, however, also possible to apply the standard ray theory, or some of its modifications.

For the computation of elementary seismograms, it is necessary to know the travel-time of the corresponding wave, its complex amplitude and the source-time function. The numerical methods of computation of these values in the case of vertically inhomogeneous media are well known. To compute elementary seismograms, it is also necessary to determine the Hilbert transform of the source-time function. For some important classes of functions, simple approximate formulae for the Hilbert transform were derived in Červený (1976). These functions can simulate with a good accuracy many real wavelets observed in explosion seismology studies. The approximate formulae expedite the computation of theoretical seismograms considerably.

To describe the type of the elementary wave, it is necessary to introduce certain numerical codes specifying the elementary waves. The main difficulty of the computation of ray theoretical seismograms does not lie in the computation of elementary seismograms, but in the algorithms for the generation of numerical codes and in the selection of waves. In case of a medium consisting of a finite number of plane-parallel homogeneous layers, the problem of the automatic generation of numerical codes becomes relatively simple, mainly when we do not consider converted phases. In this case, the elementary waves can be grouped into families of kinematically analogous waves. This grouping makes the computation substantially faster. Unfortunately, even the number of groups which arrive at the receiver within a finite time window is often rather high. This applies mainly to models with a large number of thin layers to large epicentral distances and to the long time window. In these cases, it is necessary to make certain a priori assumptions and to exclude from computations the waves that are expected to influence the theoretical seismograms only slightly.

To improve the accuracy of elementary ray theoretical seismograms, we can use certain modifications of the ray theory. In the case of a medium of parallel homogeneous layers, this applies mainly to the critical region modification. The critical region is of great importance in applications since the amplitude-distance curve of the corresponding reflected wave reaches its maximum there. It is not complicated to use critical region modifications when ray theoretical seismograms are computed. The computation is then substantially faster than the computation based on exact methods, and more exact than the computations based on the standard ray method. This modification was first used in the construction of ray theoretical seismograms in 1974, see Červený (1978). See also details in Červený et al. (1978).

It should also be noted that the ray theoretical seismograms can be easily supplemented by certain non-zero waves such as the head waves. In the ray theoretical seismograms presented in this paper the head waves are included automatically.

3. Discussion of the Accuracy of Ray Theoretical Seismograms

In this section we shall give some examples of theoretical seismograms computed by the ray method (or by some of its modifcations) and by the reflectivity method. To appreciate the accuracy of ray theoretical seismograms we shall choose certain simple vertically inhomogeneous structures. In the reflectivity method the smooth distribution of velocity and density with depth is simulated by a thin-layered medium with horizontal plane interfaces. Therefore, we shall consider only such types of media.

In all the computations (with the exception of the upper system in Fig. 11), the source-time function is given by the formula

$$f(t)=\exp\left(-4\pi^2 f_M^2 t^2/\gamma^2\right)\cos\left(2\pi f_M t+\nu\right), \tag{1}$$

with three free parameters: f_M, γ and ν. The quantity f_M corresponds approximately to the prevailing frequency of the source-time function. In all the examples we consider an explosive point source of P-waves with the symmetrical characteristics situated near the Earth's surface. Only ideal registration is considered, and a possible distortion of theoretical seismograms by the recording equipment is not taken into account. The receiver is also situated near to the Earth's surface, and the vertical component of the displacement is presented. Only P-waves are taken into account(no S-waves and converted waves). For simplicity, only primary reflections (with corresponding head waves) are considered, no multiple reflections. Some scaling of amplitudes with respect to the epicentral distance is applied in most cases (proportional to the epicentral distance). The scaling is, of course, the same for the ray theoretical seismograms and for theoretical seismograms computed by the reflectivity method.

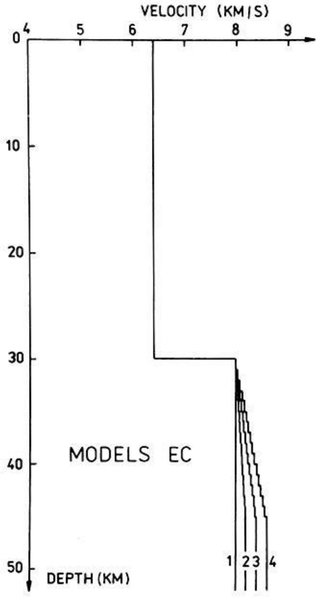

Fig. 1. P-wave velocity distribution for vertically inhomogeneous models EC1–EC4

3.1. Critical Region

First we shall consider simple models of a homogeneous Earth's crust with the Mohorovičić discontinuity situated at the depth of 30 km, see Fig. 1. In the uppermost mantle the velocity is either constant (see model EC1), or increases linearly with depth (models EC2, EC3, EC4). In this section we shall deal with model EC1 with the homogeneous upper mantle.

Figures 2 and 3 give theoretical seismograms computed by three different methods. The source time function in Figs. 2 and 3 is given by formula (1), with $\gamma=4$, $\nu=0$, it differs only in the predominant frequency, viz., $f_M=4$ Hz in Fig. 2 and $f_M=2$ Hz in Fig. 3. In both these cases, the uppermost figure (A) is computed by the reflectivity method, the middle figure (B) by the ray method with the critical region modification, and the lower figure (C) by the standard ray method.

In the standard ray method, the amplitude-distance curve of the reflected wave reaches its maximum right at the critical point. In our case, the critical

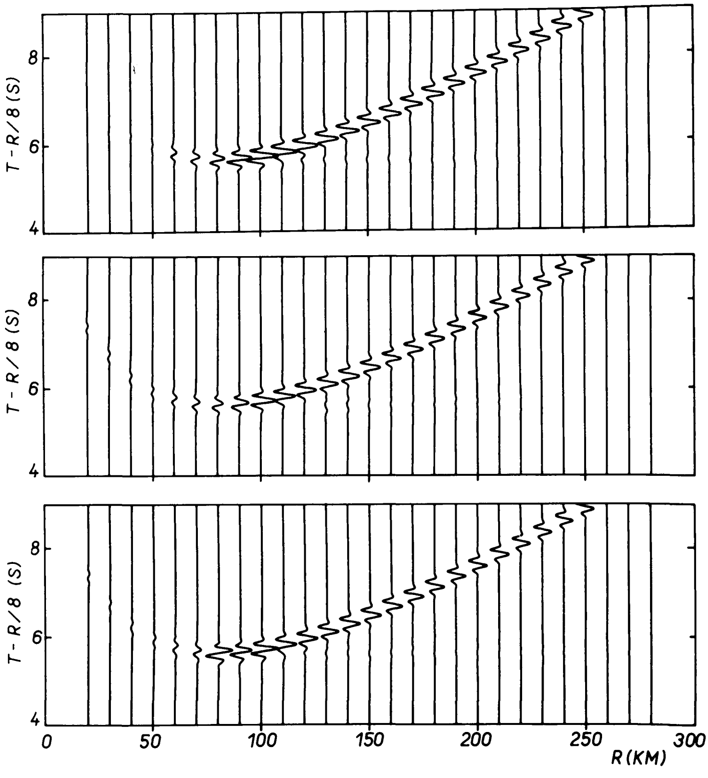

Fig. 2. A comparison of theoretical seismograms for the model EC1 shown in Fig. 1, computed by three different methods: *Upper* figure (A): the reflectivity method. *Middle* figure (B): the ray method, with a critical region modification. *Lower* figure (C): the standard ray method. The explosive point source and receiver are located close to the Earth's surface. The source-time function is given by (1), with $\gamma=4$, $\upsilon=0$ and with the predominant frequency $f_M=4$ Hz

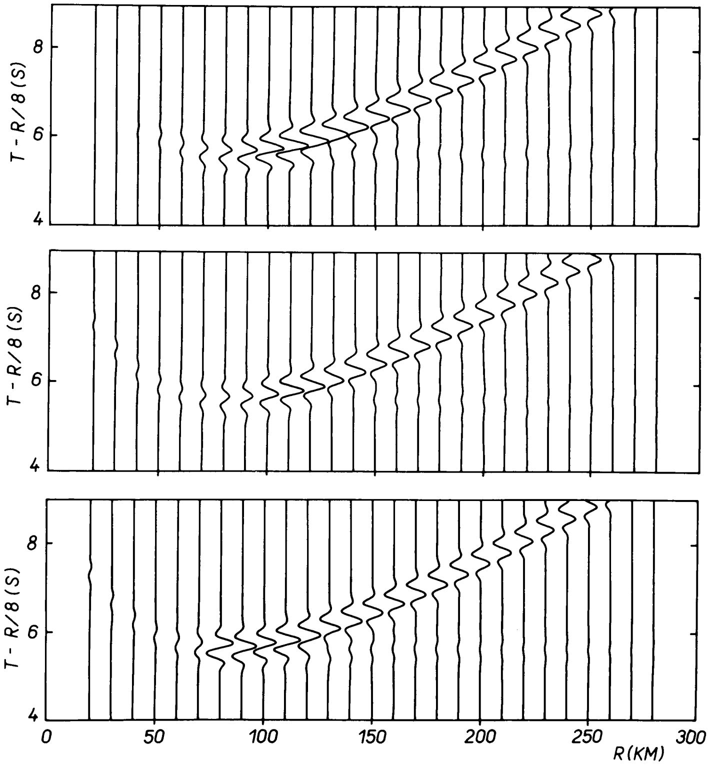

Fig. 3. The same as in Fig. 2, for the predominant frequency $f_M = 2$ Hz

point is situated at the epicentral distance of 80 km. The maximum amplitudes at the critical point can be clearly seen in both sets of theoretical seismograms computed by the standard ray method. In reality, the maximum of amplitude curves of reflected waves is shifted beyond the critical point, and the shift is frequency dependent. Both these effects can be clearly seen in theoretical seismograms computed by the reflectivity method and in theoretical seismograms computed by the ray method with the modification in the critical region (see B and C). In the case $f_M = 2$ Hz, the maximum amplitudes are shifted approximately to 110 km, and in the case of $f_M = 4$ Hz approximately to 100 km.

Although the results obtained by the two methods (see A and B) differ in details, the overall agreement of most important pecularities of wave fields is satisfactory. This agreement was obtained due to the applied modification of the ray method in the critical region.

(Let us mention one difference between the two systems of seismograms which is connected only with the numerical computation effects. Weak amplitudes of the wave reflected from the Moho at small epicentral distances obtained by the reflectivity method are caused by the velocity filtration applied in the reflectivity method.)

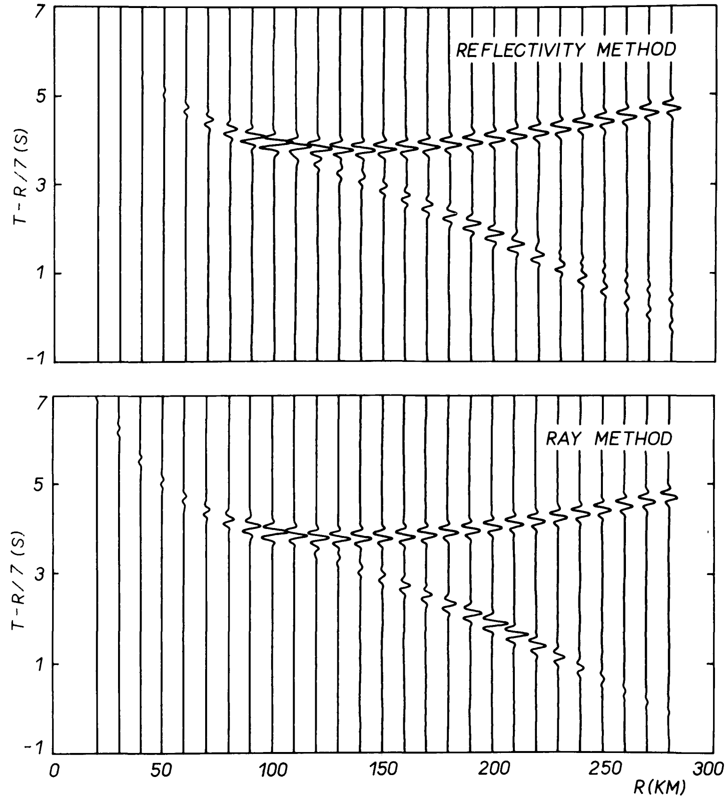

Fig. 4. A comparison of theoretical seismograms for the model EC4 shown in Fig. 1, computed by two different methods: *Upper* figure: reflectivity method. *Lower* figure: the ray method with a critical region modification for reflections from the Moho. The explosive source and the receiver are located close to the Earth's surface. The source-time function is given by (1) with $\gamma=4$ and $\nu=0$ and with the predominant frequency $f_M=4$ Hz

3.2. Interference Head Waves

Now we shall present theoretical seismograms computed for models EC2, EC3 and EC4, see Fig. 1. The continuous increase of velocities in the uppermost mantle is simulated by the sequence of thin homogeneous layers. The corresponding systems of theoretical seismograms are presented in Figs. 4–6 for the prevailing frequency $f_M=4$ Hz. The critical region modification is used for reflected waves from the Moho (in the computation of ray theoretical seismograms).

Two dominant waves are evident in these figures: the wave reflected from the Moho and the wave refracted in the uppermost mantle. This wave is also called the interference head wave, see Červený and Ravindra (1971). Let us note that the refracted wave is formed by a superposition of waves reflected from individual fictitious interfaces in the uppermost mantle.

The properties of the wave reflected from the Moho remain the same as in Figs. 2 and 3, see the previous ection. Here we shall be interested in the

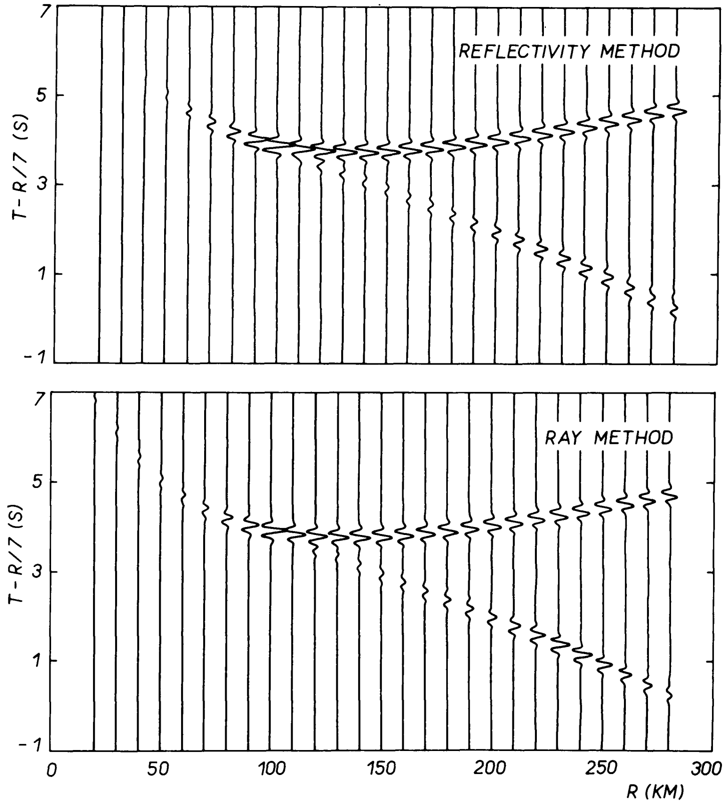

Fig. 5. The same as in Fig. 4, for the model EC3

properties of the refracted wave in the uppermost mantle. For this wave, the standard ray method is not applicable, see Červený and Ravindra (1971). Certain interesting pecularities of these waves, described earlier in the above shown reference, may be easily verified in Figs. 4–6: the amplitudes of refracted waves beyond the critical point first decrease with the increasing epicentral distance, then they increase, and at a certain epicentral distance the amplitudes reach their maximum values. The position of this maximum depends on the velocity gradient below the Moho. The larger the gradient, the smaller is the epicentral distance at which the maximum is situated.

Again, we can see that the theoretical seismograms computed by the two methods differ in details, but the most important pecularities (such as the position of maxima and minima of amplitude-distance curves) are satisfactory for many practical purposes. As a rule, the ray method gives larger amplitudes than the reflectivity method.

(Let us mention another difference which is connected with certain differences in computational procedures. The wave, which separates from the refracted wave at large epicentral distances in the reflectivity computations for the model EC4 corresponds to the multiple reflection from the bottom side of the Mohorov-

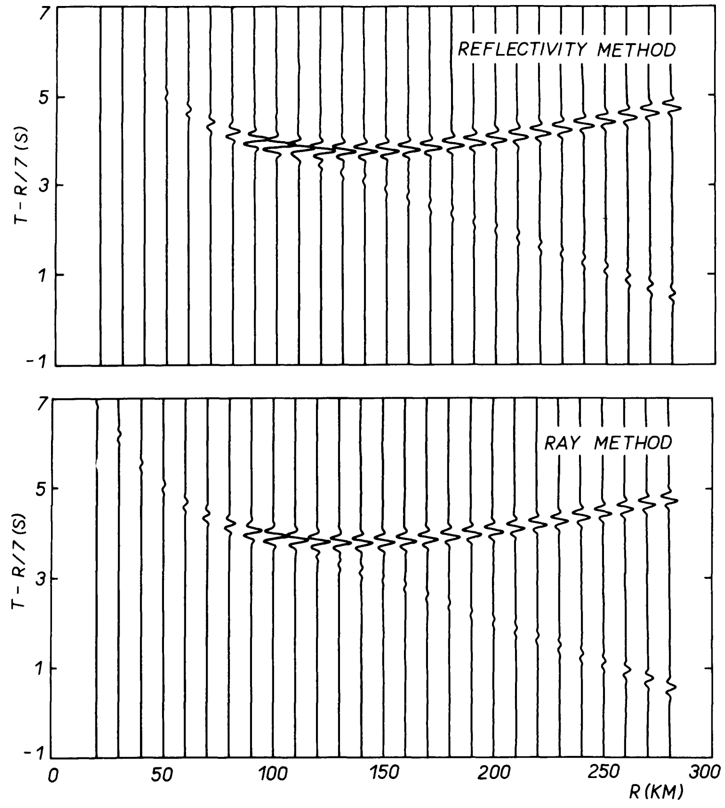

Fig. 6. The same as in Fig. 4, for the model EC2

ičić discontinuity. In ray theoretical seismograms, this wave is missing since only primary reflections were considered. It would be easy, though, to obtain this wave even in ray theoretical seismograms by choosing different input data controlling the generation algorithms. This possibility is included in the programs used for computations.)

3.3. Transition Region Between the Shadow and the Illuminated Zone

We shall now consider the models ECG shown in Fig. 7. The models are similar to those discussed above (see Fig. 1), but assume a positive velocity gradient within the Earth's crust. For the models ECG shown In Fig. 7, we obtain a strong refracted wave propagating within the Earth's crust, which did not exist in the case of models EC. The travel time curve of this refracted wave has a common tangent point with the travel-time curve of the wave reflected from the Moho. This tangent point is situated at about 195 km. At larger epicentral distances, a geometrical shadow zone for both waves is formed. We are not interested here in the deep shadow, as the amplitudes are very

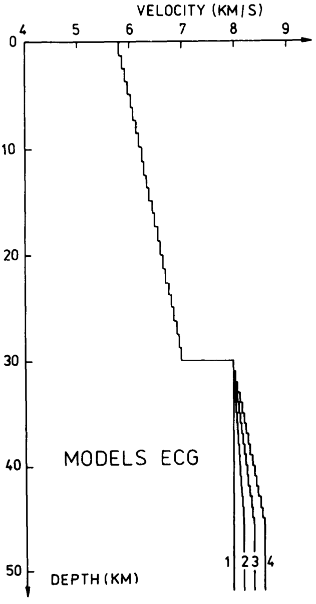

Fig. 7. P-wave velocity distribution for vertically inhomogeneous models ECG1–ECG4

small there, but in the wave field in the half-shadow (~ 195 km). It is well known that the standard ray theory fails there.

Now we shall present theoretical seismograms for the model ECG1, see Fig. 8 for $f_M = 4$ Hz and Fig. 9 for $f_M = 2$ Hz. Again, the upper figure is computed by the reflectivity method, the lower by the ray method, with the modification in the critical region for the wave reflected from the Moho.

We are interested mainly in the properties of the interference wave formed by the superposition of the refracted wave and the wave reflected from the Moho, in the vicinity of the gometrical boundary of the shadow zone (~ 195 km). The behaviour of this interference wave is very interesting. The amplitudes of the interference wave first strongly increase, form a maximum at about 170 km, and they continuously decrease. The level of amplitudes of the interference wave in the region of their maximum (~ 170 km) is really rather high, of the same order or even higher than the amplitudes in the critical region. Similarly as in previous cases, the agreement between the theoretical seismograms computed by the two methods is satisfactory. The agreement is in a way surprizing, no modification of the ray method was used to compute the interference wave in the transition region. The agreement is mostly caused by the simulation of the smooth velocity-depth distribution by thin layers. The agreement could not be obtained for a smooth velocity-depth distribution. Similar results were obtained for the models ECG2–ECG4. We shall not present them here for they would be, to some extent, only a combination of theoretical seismograms for the model ECG1 with those for models EC2, EC3, and EC4. As an example, we shall present here only the ray theoretical

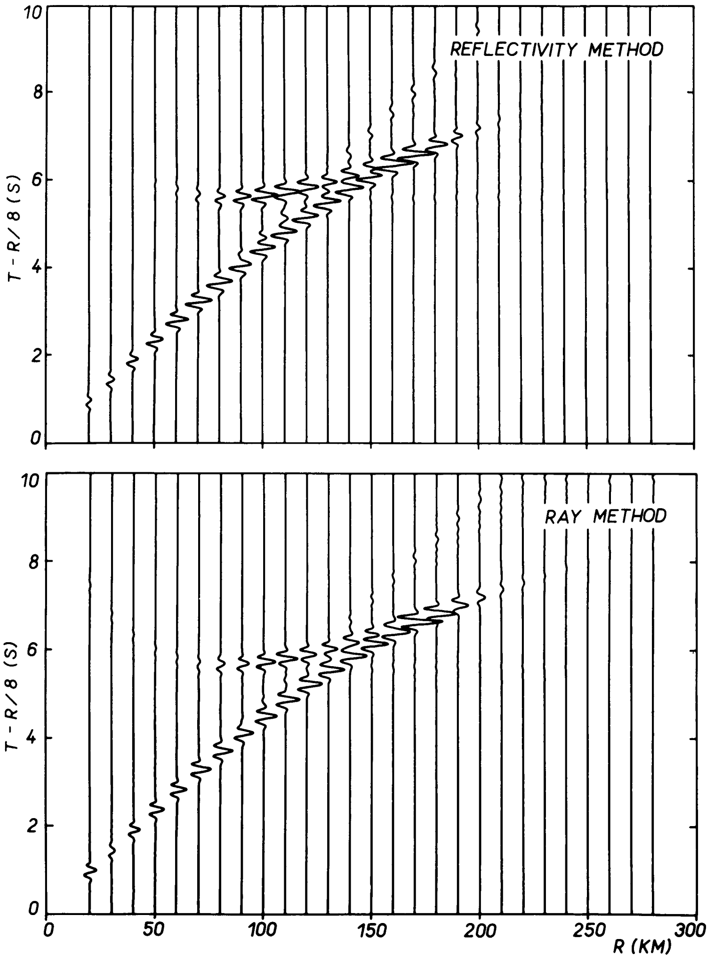

Fig. 8. A comparison of theoretical seismograms for the model ECG1 shown in Fig. 7, computed by two different methods: *Upper* figure: the reflectivity method. *Lower* figure: the ray method with a critical region modification for reflections from the Moho. The explosive point source and receiver are situated close to the Earth's surface. The source-time function is given by (1), with $\gamma = 4$ and with $\nu = 0$. The predominant frequency f_M equals 4 Hz

seismograms for the model ECG4 (with the critical region modification for reflections from the Moho), computed for the predominant frequencies $f_M = 4$ Hz and $f_M = 2$ Hz, see Fig. 10.

3.4. *Other Singular Regions*

A number of other ray theoretical seismograms were computed and compared with the reflectivity method. This applies to various layered structures composed

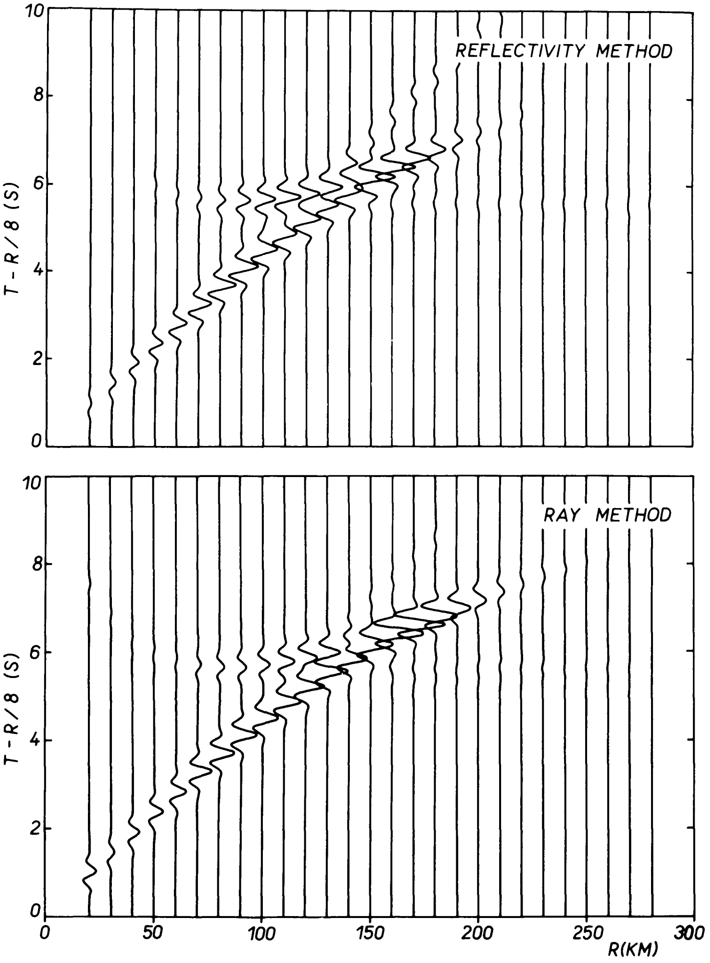

Fig. 9. The same as in Fig. 8, for the predominant frequency $f_M = 2$ Hz

of thick layers, to more realistic structures, etc. The agreement of the results was usually satisfactory. Great attention was also paid to the neighbourhood of a caustic. Some results of these computations can be found in Červený et al. (1977), Červený (1978), Červený and Pšenčík (1977). Similarly as in the case of the half-shadow the aggreement is satisfactory.

3.5. Non-Ray Waves

As shown above, the ray theory supplemented by some simple modifications may yield satisfactory results, even in singular regions.

As was mentioned in the introduction, the standard ray method does not include certain types of non-ray waves, such as various inhomogeneous waves,

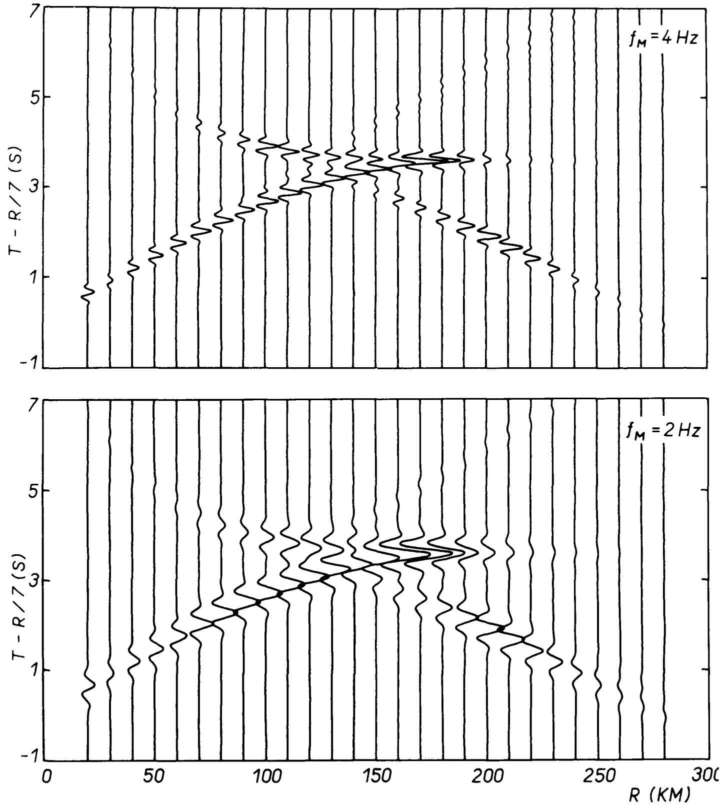

Fig. 10. Ray theoretical seismograms for the model ECG4. The critical region modification is applied to reflections from the Moho. The explosive point source and the receiver are situated close to the Earth's surface. The source-time function is given by (1), with $\gamma = 4$, $\nu = 0$. The predominant frequency $f_M = 4$ Hz in the upper figure, $f_M = 2$ Hz in the *lower* figure

channel waves, tunnel waves, diffracted waves. Some types of diffracted waves can be obtained by a proper simulation of the smooth velocity distribution by a thin-layered medium, see the waves penetrating to the shadow zone in Figs. 8 and 9. For other types of waves, such as the tunnel waves, the comparison of ray theoretical seismograms with the theoretical seismograms computed by the ray method has not given satisfactory results.

We shall present one example, see Fig. 11. Fuchs and Schulz (1976) studied the effect of the thin-high-velocity layer on the wave field in an attempt to explain the combination of low-frequency Moho reflections and high-frequency P_n arrivals which were observed in several cases. The theoretical seismograms for a structure given in the left upper corner of Fig. 11, computed by the reflectivity method, are shown in the upper part of the figure. They clearly demonstrate the tunnelling of waves through a thin high-velocity lamina. The first arrival is the reflection from the high-velocity layer, the second arrival comes from the Moho. As soon as the incidence of the wave from the source

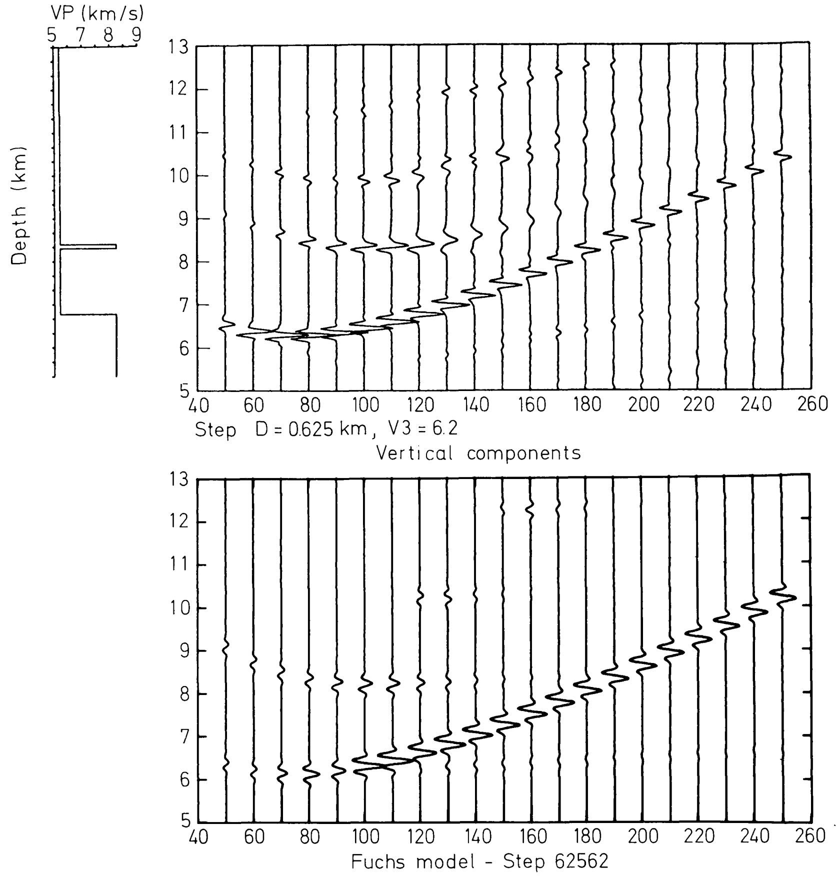

Fig. 11. A comparison of theoretical seismograms for the model of medium containing a thin high-velocity lamina, see upper left corner. *Upper* figure: theoretical seismograms computed by the reflectivity method. *Lower* figure: ray theoretical seismograms, with the critical region modification for reflections from lamina and from the Moho. The explosive point source and the receiver are located close to the Earth's surface, the source-time function in both sections are slightly different

on the lamina is supercritical, only the low frequencies propagate (tunnel) through the lamina, and the reflection from the next interface contains only low frequencies. It must be emphasized that the tunnel wave cannot be described by the standard ray theory. The ray theoretical seismograms computed for the same model are shown in the lower part of Fig. 11. The figure clearly demonstrates that the standard ray method does not properly describe the tunnel effects.

It would be possible, however, to use some modifications of the ray method (such as the ray method with a complex eikonal, see Červený et al., 1977) even for certain on-ray waves. These modifications have not yet been included in our programs for ray theoretical seismograms, but they have been checked in some test computations (e.g., for tunnel waves).

4. Conclusions

The comparisons of the ray theoretical seismograms with the reflectivity theoretical seismograms have shown that the ray method with some simple modifications gives qualitatively satisfactory results for a broad class of models of media, even in some singular regions. This applies, e.g. to the critical region, to the neighbourhood of a caustic, the half-shadow, etc. Satisfactory results have been also obtained for interference head waves, converted waves, etc. On the other hand, poor results have been obtained for some non-ray waves and inhomogeneous waves. We may expect that certain modications (the ray method with a complex eikonal, etc.) will be used in the near future to remove many of the remaining limitations and improve effectively the accuracy of ray theoretical seismograms.

These comparisons show that the ray theoretical seismograms can give valuable results even for laterally inhomogenous media with curved interfaces, especially when we use certain modifications to improve their accuracy in singular regions and to describe properly certain non-ray waves.

Acknowledgements. The author wishes to thank to Profesor K. Fuchs for the invitation to work for 3 months at the Institute of Geophysics, University of Karlsruhe, and to Deutscher Akademischer Ausstauschdienst for the financial support of this stay. All the computations presented in this paper were performed during this stay. The author is also greatly indebted to his colleagues from the Charles University (Prague) and from the University of Karlsruhe for helpful suggestions and valuable discussions.

References

Červený, V.: Approximate expressions for the Hilbert transform of a certain class of functions and their application to the theory of seismic waves. Stud. Geophys. Geod. **20**, 125–132, 1976

Červený, V.: Ray theoretical seismograms. In: Geophysikalische Interpretationsmethoden. Sammelwerk des Symposiums der Arbeitsgruppe 1.7 KAPG, Bratislava, Mai 1974, L. Kubáčková, ed.: pp. 49–64. Bratislava: Veda 1978

Červený, V., Fuchs, K., Müller, G., Zahradník, J.: Theoretical seismograms for inhomogeneous elastic media. Voprosy dinamicheskoy teorii rasprostraneniya seismicheskikch voln. Vol. 20, G.I. Petrashen, ed. Leningrad: Nauka 1978

Červený, V., Molotkov, I.A., Pšenčík, I.: Ray method in seismology. Praha: Univerzita Karlova 1977

Červený, V., Pšenčik. I.: Ray theoretical seismograms for laterally varying layered structures. In: Proceedings XV. General Assembly European Seismological Commission, Kraków 1976, Part I. Publ. Inst. Géophys. Pol. Acad. Sci A-4 (115), R. Teisseyre, ed.: pp. 173–185. Warszawa-Lódź: PWN 1977

Červený, V., Ravindra, R.: The theory of seismic head waves. Toronto and Buffalo: University Toronto Press 1971

Fuchs, K.: The reflection of spherical waves from transition zones with arbitrary depth-dependent elastic moduli and density. J. Phys. Earth **16** (Special Issue), 27–41, 1968

Fuchs, K., Müller, G.: Computation of synthetic seismograms with the reflectivity method and comparison with observations. Geophys. J.R. Astron. Soc. **23**, 417–433, 1971

Fuchs, K., Schulz, K.: Tunneling of low-frequency waves through the subcrustal lithosphere. J. Geophys. **42**, 175–190, 1976

Hron, F., Kanasewich, E.R.: Synthetic seimograms for deep seismic sounding studies using asymptotic ray theory. Bull. Seismol. Soc. Am. **61**, 1169–1200, 1971

Received August 1, 1978; Accepted July 2, 1979