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Ray Theoretical Seismograms for Laterally Inhomogeneous Structures*

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Abstract. Applications of the ray method to the construction of theoretical seismograms for laterally varying layered structures are discussed. Numerical examples are presented. It is shown that the refracted waves are very sensitive to the curvature of interfaces. Certain modifications and improvements of ray theoretical seismograms to increase their accuracy are suggested.

Key words: Ray theoretical seismograms – Laterally inhomogeneous structures – Modifications of ray method.

1. Introduction

Various methods can be used to compute theoretical seismograms for vertically (or radially) inhomogeneous media. It is not possible to give here a full survey of these methods, it can be found in Červený et al. (1978). We shall mention here only some of them. The most popular are probably the reflectivity method (Fuchs, 1968; Fuchs and Müller, 1971), the exact ray method (also called the generalized ray method) and its various modifications (Cagniard, 1962; de Hoop, 1960; Müller, 1968; 1969; Helmberger, 1968; Chapman, 1976; Gilbert and Helmberger, 1972; Wiggins, 1974; etc.), and methods based on modal summation (Buland, 1977). Recently some new important methods have been developed, such as the method of partial separation of variables in the combination with finite differences (Alekseev and Mikhailenko, 1976; 1977; 1978).

Most of these methods, however, can be hardly used to compute theoretical seismograms for general laterally inhomogeneous media with curved interfaces. It is the regions with strong lateral inhomogeneities, however, that play a very important role in the present geodynamic studies. Ray theoretical seismograms can give valuable results even for these types of media, especially when we

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use some modifications to increase their accuracy in singular regions and in some other situations.

Let us note that under 'ray theoretical seismograms' we understand here the theoretical seismograms computed by the standard ray method or by some of its modifications.

The application of ray methods to the construction of theoretical seismograms is most natural in the case of small epicentral distances (reflection methods in seismic prospecting, etc.). In this case, the construction of theoretical seismograms does not cause great difficulties and the accuracy of computations is usually quite satisfactory, even in the case of rather complicated structures. At larger epicentral distances, where we deal mainly with refracted waves and supercritical reflections (e.g., 50–300 km for the models of the Earth's crust), the situation is more complicated in several respects. However, it is shown in Červený (1979) that even at these epicentral distances the ray theoretical seismograms can give satisfactory results from an interpretational point of view. In this paper, we shall shortly describe the procedures to construct theoretical seismograms for such epicentral distances. We shall also present some numerical examples. For very large epicentral distances (say, 500 km or more for the models of the Earth's crust), the rays of waves propagating within the Earth's crust are mostly horizontal, and the waves are formed by a superposition of a large number of elementary waves. The accuracy of ray theoretical seismograms in this region is expected to be lower. These large epicentral distances are not considered here, they would need special investigation.

Many other details on various aspects of the construction of ray theoretical seismograms for laterally inhomogeneous media can be found in Červený et al. (1977), Červený and Pšenčík (1978), Smith (1977), Jedrzejowska-Zwinczak (1978).

2. Construction of Ray Theoretical Seismograms for Laterally Varying Layered Structures

The basic principles of the construction of ray theoretical seismograms are described in Červený et al. (1977), and in Červený (1979), for vertically inhomogeneous media. In principle, the same procedures can be used in the case of laterally inhomogeneous media. In some details, however, these procedures differ.

The first important difference consists in the computation of travel-times and complex amplitudes. At the present time, however, the methods of computation of travel times and complex amplitudes are well known, even for complicated laterally inhomogeneous media with curved interfaces, see Červený et al. (1977). The first and the most important step in these computations is the computation of rays. Rays in such media are described by a system of ordinary differential equation of the first order. The system can be solved by standard numerical procedures, such as the Runge-Kutta method. Some complications in these computations are connected with the fact that the computation of the ray trajectory is not a Cauchy initial value problem, but a two-point boundary value problem (since we are looking for the ray connecting the source with the receiver). The solution of a two-point boundary value problem is usually easier at smaller epicentral distances; with increasing epicentral distance it becomes more complicated and cumbersome. Moreover, the travel-time curves of certain important waves may have many branches at larger epicentral distances, and we must determine all of them. Methods of a two-point ray tracing are discussed elsewhere, see Julian and Gubbins (1977).

As soon as the ray connecting the source and the receiver is determined, the evaluation of amplitudes is easier. Some problems are connected with the computation of geometrical spreading. Several methods for computing geometrical spreading are described in Červený et al. (1977).

The second difference consists in the impossibility of grouping the elementary waves in laterally inhomogeneous media into families of kinematically analogous waves, as in the case of vertically inhomogeneous media. It is necessary to compute elementary waves independently, one after another. This makes the computations more cumbersome and lengthy. The number of elementary waves arriving at the receiver within a time window of a given length increases considerably with the increasing epicentral distance.

The third difference consists in the algorithms for the generation of numerical codes of individual elementary waves. In comparison with the same algorithms for vertically inhomogeneous media, the generation algorithms are more complicated. The problem of the generation of numerical codes of elementary waves is closely connected with the problem of parameterization of the medium, and with the system of numerical coding of elementary waves. It is not simple to find an automatic or semi-automatic generation system for quite general laterally inhomogeneous media with block structures. It is often more suitable to read the numerical codes of elementary waves used to construct the theoretical seismogram for a specific model of medium in cards, as input data. The preparation of these input data is rather cumbersome but the experience of the interpreter may be also very important. However, when we use proper parameterization of the medium and proper numerical codes of waves, it is not difficult to develop fully or partially automatic generation algorithms. For example, it is very convenient to use such a parameterization of a medium in which all the interfaces are continuous from the left border to the right border of the medium under investigation. Then the interfaces and the corresponding individual layers can be numbered from the top to the bottom. Some interfaces might in part coincide with other interfaces or they might be in part only fictitious (with the same velocities and densities on both sides). Then we can use quite similar numerical coding of elementary waves as in vertically inhomogeneous media, and very similar generation algorithms. Some elementary waves arriving at the receiver will be, of course, of zero amplitudes. The generation algorithms, however, become much more simple then in the general case.

Similar parametrization of the medium, coding of elementary waves and generation algorithms are used in our programs. They permit fully automatic generation of all primary P, S, PS, SP, PP, and SS reflected and refracted waves for an arbitrary surface or burried source and a receiver. The generation of other multiply reflected/refracted waves we wish to consider can be read in as input data.

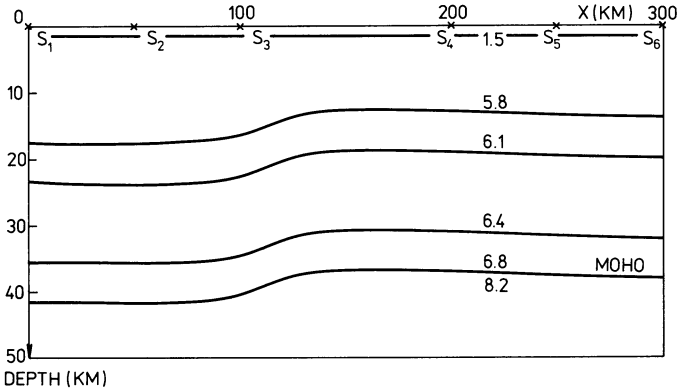


Fig. 1. Model of a laterally varying layered Earth's crust, used for a computation of theoretical seismograms. Thick lines denote the geometrical shape of interfaces, the numbers the velocity in individual layers. The points $S1$–$S6$ show the receiver positions

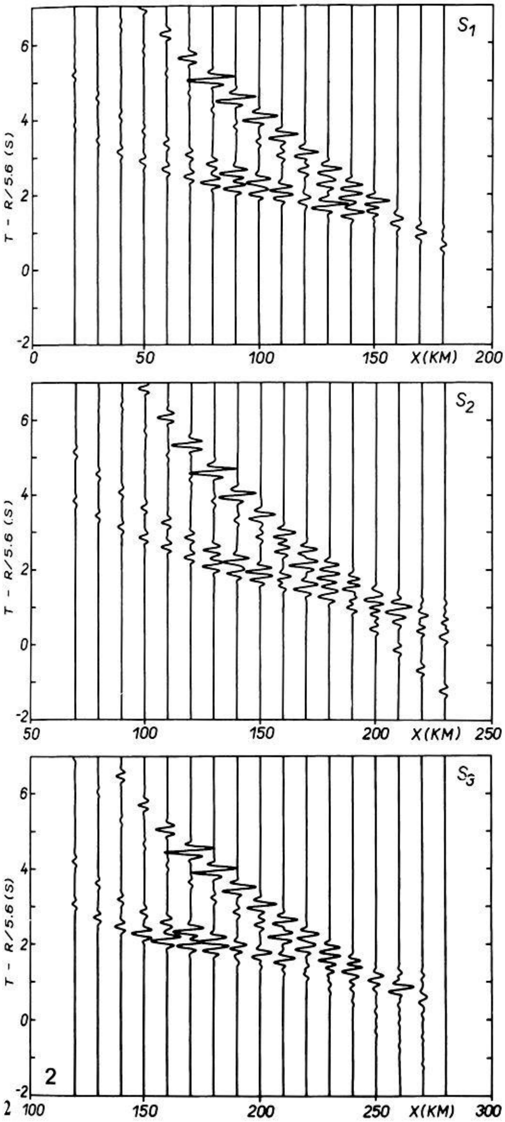
3. Examples

We shall now give examples of ray theoretical seismograms for laterally varying layered structures. Assume that the medium is composed of homogeneous layers separated by curved interfaces. (Let us note that a smooth laterally inhomogeneous medium can be simulated by a system of thin homogeneous layers with curved interfaces.) The interfaces are specified by a set of points and approximated by splines. A program allowing for lateral variations of velocity within individual layers is under preparation. We consider an explosive point source of P waves, with the symmetrical directional characteristics, situated near to the Earth's surface. The source-time function is given by the formula

$$f(t)=\exp(-4\pi^2 t^2 f_M^2/\gamma^2)\cos(2\pi f_M t+v),$$

with $f_M = 4\,\text{Hz}$, $\gamma = 4$, $v = 0$. Only ideal registration is considered, and a possible distortion by the recording equipment is not taken into account. The receiver is also situated near to the Earth's surface, and the vertical displacement component is presented. For simplicity, only primary reflected and refracted P waves are considered, no multiply reflected/refracted P waves, converted waves, S waves, etc. (Let us note that these waves can be optionally taken into account in the used programs.) No scaling of amplitudes with respect to the epicentral distance is applied.

The basic characteristics of the model of the Earth's crust, used for computations, see Fig. 1, is the slope of interfaces at distances $x = 100$–130 km. At other distances, the interfaces are roughly horizontal. In Fig. 2, three systems of theoretical seismograms corresponding to three various positions of source are shown (see $S1$, $S2$, and $S3$ in Fig. 1). It can be clearly seen from Fig. 2 that the theoretical seismograms for the model of medium shown in Fig. 1 depend considerably on the position of the source, especially at large epicentral distances. Mainly the refracted waves are very sensitive to the slope of interfaces. For example, in the case of the source $S2$ ($x = 50$ km), the refracted waves are



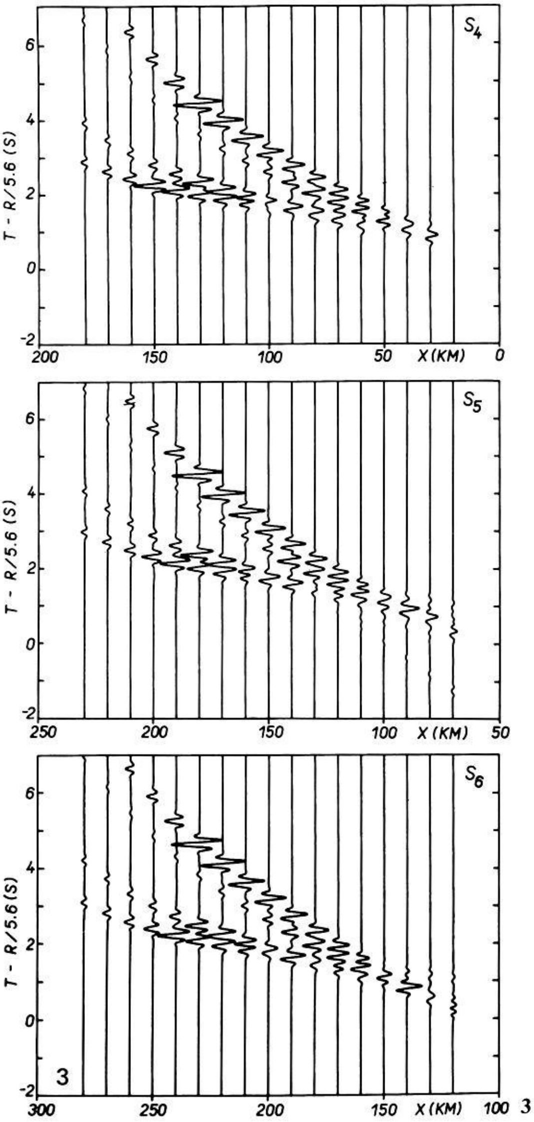


Fig. 2. Ray theoretical seismograms for the model of laterally varying Earth's crust shown in Fig. 1, for sources *S*1 (*top figure*), *S*2 (*middle figure*), *S*3 (*lower figure*). The epicentral distance is denoted by *R*, the distance along the profile by *x*. Reduction velocity equals 5.6 km/s

Fig. 3. Ray theoretical seismograms for the model of laterally varying Earth's crust shown in Fig. 1, for sources *S*4 (*upper figure*), *S*5 (*middle figure*) and *S*6 (*lower figure*). The epicentral distance is denoted by *R*, the distance along the profile by *x*. Reduction velocity equals 5.6 km/s

very strong. In fact, at the epicentral distance $R \sim 200$ km ($x \sim 250$ km), they are of the same order of magnitude or even higher than the primary reflections. (It should be noted that at these large epicentral distances the refracted wave propagating below the Moho comes at the receiver at the first arrivals.) In the case of the source $S3$ ($x = 100$ km), the refracted waves are rather weaker, as the curvatures of the interfaces are smaller in the relevant regions. In the case of the source at $S1$ ($x = 0$ km), the refracted waves vanish altogether; the reason can be simply understood from the geometry of interfaces.

The slope of interfaces in the region $x \sim 100$–130 km has also some influence on the amplitudes of the waves reflected from the corresponding interfaces at certain epicentral distances. This increase of amplitudes is connected in some cases with the loop on the travel-time curve, in other cases simply with the strong curvature of the travel-time curve.

Figure 3 shows three systems of theoretical seismograms computed along reversed profiles, corresponding to the sources situated at $S4$ ($x = 200$ km), $S5$ ($x = 250$ km) and $S6$ ($x = 300$ km). As in the preceding case, the refracted waves are not observed in the case of sources $S4$ and $S6$. They are, however, clearly visible in the case of the source $S5$. They again indicate the curvature of the interface. In this case, however, the refracted waves are weaker than in the case of the source at $S2$.

4. Possible Modifications of Ray Theoretical Seismograms

The main disadvantage of ray theoretical seismograms consists in their limited accuracy in some singular regions. Many of these limitations can be removed by means of various modifications of the ray method. An application of these modifications to the program for constructing ray theoretical seismograms needs primarily further development of the ray theory and of its various modifications, as well as progress in the numerical realization of these new theoretical approaches. The same can be said about a possibility of taking into account some of the non-ray and inhomogeneous waves, diffracted waves, etc. Much progress has been reached recently in the approximate computation of diffracted waves connected with wedges at interfaces (block structures, etc.), see Klem-Musatov et al. (1975). It would not be difficult to supplement the above described programs by the routines for an approximate computation of diffracted waves. Moreover, recent development in the theory of diffraction at curved interfaces suggests certain advantages of simulating smooth interfaces by a piece-wise linear approximation. This would remove some problems connected with caustics, triplications of travel-time curves, etc., of reflected waves from curved interfaces (Klem-Musatov, personal communication, 1977).

Some difficulties in constructing ray theoretical seismograms which are connected with a great number of elementary and with some non-ray waves (such as tunnel waves) can be simply removed by a combination of ray and matrix methods. This approach is effective mainly in studying the wave field in a medium composed of thick layers separated by thin transition layers. The mentioned modification was used successfully in seismic prospecting (Ratnikova, 1973). Up to this time, it has been used for vertically inhomogeneous media only. Nevertheless, it can be applied to laterally inhomogeneous media too.

Another promising approach is the computation of the wave field along the rays by more precise methods, see Kirpichnikova (1971). To perform these computations, it might be very convenient to use finite difference procedures. Generally, the combination of the ray method with the method of finite differences can be useful in various situations.

The ray method can be also generalized for some essentially different types of media, e.g., for anisotropic media, prestressed media, absorbing media, etc. Numerical computations of ray theoretical seismograms for such media should not cause too great difficulties, even in the case of horizontal inhomogeneities.

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